Indications for new scalar resonances at the LHC and a possible interpretation

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Over the last few years, the CMS and ATLAS Collaborations at the Large Hadron Collider (LHC) have reported excesses that could hint at several new scalar resonances. Although none of them have touched the discovery level, at least two of them, at about 95 and 650 GeV, have been indicated by more than one experiment and have reached statistical significance worthy of a serious investigation. Conservatively using only the numbers given by the experimental collaborations, we find combined global significances around 3σ and 4σ, respectively, for the 95 and 650 GeV putative resonances. There are some more, like the one at 320 GeV, which have also been hinted at. We show that the data on only the 650 GeV resonance, assuming they stand the test of time, predict the existence of a doubly charged scalar and make the more common extensions of the scalar sector like those by gauge singlet scalars, the two-Higgs doublet models, or the Georgi-Machacek model highly disfavored. We provide the readers with a minimalistic model that may possibly explain all the indications. Such a model can also accommodate the hints of a singly charged scalar at about 375 GeV and a doubly charged scalar at about 450 GeV, as found by both the major LHC collaborations, the combined global significance for each of them being above 2.5σ. We show that even the scant data, with large error bars, have the potential to strongly constrain our model containing four scalar multiplets, which makes the model easily testable and falsifiable. Our analysis comes with the obvious caveat that the allowed parameter space that we find depends on the available data on all the new resonances and may change in future. One may also note that this is an exploratory exercise that illustrates the difficulties when it comes to fitting several resonances simultaneously, even for next-to-minimal extensions of the Standard Model.

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I. INTRODUCTION

Twelve years after the discovery of the Higgs boson [1,2], some more interesting hints about new resonances have been found by both the ATLAS and the CMS Collaborations [3–16] at the Large Hadron Collider (LHC). It is way too early to say whether these hints will stand the test of time, but they surely deserve a close scrutiny. There have already been several such attempts in the context of specific models, e.g., in Refs. [17–22], although hardly any analysis considers all the indications simultaneously.

In this paper, we will deal with all the reasonable hints or indications of such scalar resonances. By reasonable, we mean only those resonances for which the combined global significance, as opposed to local significance, which does not take into account the look-elsewhere effect (LEE), is at the level of 2.5σ or higher. In most of the cases, these channels come from independent final states, and therefore the combined significance is simply obtained through a product of probabilities. Special emphasis will be given to those resonances that have been indicated by both the major

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TABLE I. The existing indications for some scalar resonances, apart from h_{125}.

New scalar	Process studied	Local significance	Global significance	Combined significance	References
h_{95}	$\to \gamma\gamma$	2.9σ	1.3σ		[6,7,18]
	$\to \tau^+\tau^-$	2.6–3.1σ	2.3–2.7σ	2.4–2.75σ	[8]
	$Z^* \to Zh_{95} \to Zb\bar{b}$	2.3σ	Not quoted	3.1–3.4σ	[24]
H_{650}	VBF, $\to W^+W^-$	3.8σ	$(2.6 \pm 0.2)\sigma$		[12]
	$\to ZZ$	2.4σ	0.9σ	$(4.08^{+0.12}_{-0.11})\sigma$	[10,11]
	$\to h_{95}h_{125}$	3.8σ	2.8σ		[13]
	$\to A_{400}Z \to \ell^+\ell^- t\bar{t}$	2.85σ	2.35σ		[16]
A_{400}	$\to t\bar{t}$	3.5σ	1.9σ	3.17σ	[14]
	$\to ZH_{320} \to Zh_{125}h_{125}$	3.8σ	2.8σ		[15]

LHC collaborations or maybe by even earlier experiments. A preliminary overview of the scenario, including the signals and their significances, may be found in Ref. [23]; however, in the present paper, we have been more conservative about the significance estimates. A comprehensive list of the hints is given in Sec. II and Table I of this paper. We will assume all resonances to be of spin-0 nature. They have been found to have nonzero coupling with the Standard Model (SM) gauge bosons, so the mass eigenstates can hardly be dominated by gauge singlets. We will show that they cannot come from simple multi-Higgs doublet extensions either. In fact, the apparent indications tend to rule out, or at least disfavor, a number of minimal extensions of the scalar sector of the SM.

There are three major constraints on any such scalar extension. The first one is the existence of custodial symmetry (CS), parametrized by the ρ parameter. In the SM, $\rho = 1$ at the tree level, but the Yukawa interaction in the SM breaks the CS. Once the radiative corrections are incorporated, the ρ parameter shifts from unity, the dominant contribution coming from the top quark loop and being proportional to m_t^2. To absorb this correction, one defines the parameter ρ_0, so that in the SM, ρ_0 is unity by definition. The experimental value is close to unity [25],

$$\rho_0 = 1.00031 \pm 0.00019, \tag{1}$$

so CS must be close to an exact symmetry of nature. Models with SU(2) singlet and doublet scalar multiplets respect CS; it is also possible to include larger multiplets of scalars in a way that CS is respected. The most famous example of this genre is the Georgi-Machacek (GM) model [26], where a complex triplet and a real triplet of scalars engender $\rho = 1$ at the tree level.[1] Scalar extensions that violate CS must have very small vacuum expectation values (VEVs) of the CS-breaking fields; thus, they are unlikely to be singly produced by gauge boson fusion, or decay to the gauge bosons with any significant branching ratio (BR), as the (neutral) scalar-gauge-gauge coupling is always proportional to the VEV of that scalar.

The GM model is interesting in the sense that this is the minimal model that preserves the CS and introduces both singly and doubly charged scalars in the spectrum. One may even construct a more generalized version of the GM model [27] where the CS is respected in the gauge sector only but not in the scalar sector; i.e., the components of the custodial scalar multiplets can potentially be nondegenerate. This opens up more decay channels for the scalars. A comprehensive analysis of this model has been done recently [28–30]. We stress this point because this will be important in our subsequent discussions.

The second major constraint is the nature of the scalar resonance at 125 GeV, which we call h_{125}. From the measurement of its couplings to the fermions and the gauge bosons, we know that its properties must be very close to that of the SM Higgs boson $h_{\rm SM}$ [31,32]. Denoting the generic multiplicative modifier to the coupling of any particle-antiparticle pair with the SM Higgs boson by κ, we have [31]

$$\kappa_W = 1.02 \pm 0.08, \quad \kappa_Z = 1.04 \pm 0.07,$$
$$\kappa_t = 1.01^{+0.11}_{-0.10}, \quad \kappa_b = 0.99^{+0.17}_{-0.16}. \tag{2}$$

At this point, one may note that, while $h_{125} \to \gamma\gamma$ enforces κ_t and κ_W to have the same sign, the sign of κ_b is yet to be determined. ATLAS and CMS Collaborations [33,34] showed, from the analysis of Wh_{125} production in the vector boson fusion (VBF) channel, that $\kappa_W = -\kappa_Z = 1$ is ruled out with significance greater than 8σ. Such conclusions, however, strongly rely on the underlying assumption that the processes under investigation are mediated by the SM particles only. It was shown in Ref. [35] that such a possibility can still be entertained if there is new physics below at about 600 GeV. However, to be conservative, we will consider all κ's for h_{125} to be positive and consistent with Eq. (2).

[1]The CS is respected in the GM model even at the one-loop level if one considers only the SU(2) part of the weak interaction. Once $\mathrm{U}(1)_Y$ is introduced and gauged, the tadpole diagrams break the CS.

The third major constraint is the preservation of unitarity in gauge boson scattering processes, like $W_L W_L \to W_L W_L, Z_L Z_L$, where L denotes the longitudinally polarized state. There are important sum rules that must be valid for any extensions with elementary scalars for the underlying gauge theory to remain renormalizable and for the cross section to fall at high energies [36]. For example, if the $h_{125} W^+ W^-$ coupling is found to be exactly what is expected in the SM, no other neutral scalar can couple to W^+W^- unless there is at least one doubly charged scalar in the theory. As this will play an important role in analyzing the signal and developing our model, we discuss this in a separate subsection.

Two of the strongest hints, as discussed in Ref. [23], are the resonances roughly peaked at 95 and 650 GeV, which we will call h_{95} and H_{650}, respectively.[2] While h_{95} has received ample attention in the literature, H_{650} has been relatively overlooked, a strange fact considering that H_{650} has, as we will see, the strongest combined global significance, above 4σ. The former, h_{95}, is a narrow resonance while the latter, H_{650}, is a broad one. CMS observed h_{95} decaying to $\gamma\gamma$ and $\tau^+\tau^-$ while the Large Electron Positron (LEP) found a hint of h_{95}, subsequently decaying to $b\bar{b}$, through $e^+e^- \to Z^* \to Z h_{95}$. What we mean by observation is actually an excess in the respective channels, which we discuss later. There have been attempts to interpret this resonance as a stand-alone gauge singlet scalar [19] or a gauge singlet along with a second Higgs doublet [18].

Both ATLAS and CMS found an excess at about 650 GeV [10–13,16], the new resonance decaying mostly to W^+W^- and also to ZZ. It also decays to $h_{95}h_{125}$. What is important is the fact that, in almost every channel, the local significance for H_{650} is more than 3σ, as can be seen from Table I. According to CMS, VBF is the preferred channel for H_{650} production, but whether it can also be produced through the gluon-gluon fusion (ggF) channel is still uncertain. Thus, whether H_{650} has a fermionic coupling is an open issue. We will see that its interpretation poses severe problems within the popular extensions of the scalar sector of the SM and will discuss a minimalistic model where this resonance may be accommodated.

There are more hints, most prominent among them are one at 400 GeV and another at about 320 GeV. While we are still very far from determining the CP properties of these bumps, the pattern indicates that h_{95}, H_{650}, and H_{320}, along with h_{125}, are CP even, while A_{400} is CP odd, assuming that all of them are CP eigenstates. Obviously, they need not be, and that opens up more interesting possibilities. For completeness, we should also mention the scalar A_{151}, tentatively CP odd, as surmised by the authors of Ref. [20] and further explored in Refs. [37,38], through an analysis of the CMS and ATLAS data. There are also two mild hints of charged scalars, a singly charged scalar H^+ at about 375 GeV and a doubly charged scalar H^{++} at about 450 GeV [39–41]. They will be discussed in more detail later.

We will show that the coupling of H_{650} to W^+W^- violates the unitarity sum rules [36] if H_{650} is a part of an extended scalar sector consisting of only SU(2) singlets and doublets. A similar conclusion has been reached by the authors of Ref. [42]. In other words, existence of at least one doubly charged scalar is needed. Complex triplets include doubly charged scalar fields, but in general, they do not respect the custodial symmetry; the GM model and its extensions being possible exceptions. We will, however, show that even the canonical GM model [26] falls short for this; it can only partially explain the scalar spectrum. Thus, one needs a further extension to accommodate all the resonances. We will propose such a model here, without going into any details of the corresponding scalar potential, by augmenting the generalized GM model proposed in Ref. [27] with another SU(2) doublet. We call this model the two-Higgs doublet extended Georgi-Machacek (2HDeGM) model. We emphasise that this is a minimal model, but highly predictive and restrictive and hence easily falsifiable.

The paper is organized as follows. In Sec. II, we enlist all the interesting resonances that we talk about and show why they cannot possibly be explained in typical extensions like the multi-Higgs doublet models or the canonical GM model. In Sec. III, we propose 2HDeGM, an extension of the GM model with an extra scalar SU(2) doublet, and show that this may satisfactorily explain all the existing hints without being in conflict with any existing data. In particular, we show that, even with whatever little data we have and without any knowledge of the full scalar potential, we can infer a lot and severely constrain the parameter space of the model and also make useful and testable predictions. We would like to stress again that 2HDeGM is a minimal type of model to accommodate all the hints and their production and decay patterns as far as possible, but we will not venture into a study of the exact scalar potential and hence the spectrum, necessitating involved stability and unitarity constraints, similar to [29,43], as this is not even required to establish the central theme of the paper. In short, we never claim that, even if all the signals persist, 2HDeGM has to be the new SM, as there may be numerous other alternatives, and if some of these signals go away, we may even do with a more minimal model. Section IV discusses some of the ramifications of this model, like searches for singly and doubly charged scalars, as well as its further tests at the LHC and future leptonic colliders. Section V summarizes and concludes the paper. Some details have been relegated to the Appendixes.

[2]We will generically use h for scalars whose masses are equal to or smaller than 125 GeV and H for the heavier ones.

II. THE INDICATIONS AND CONSEQUENCES

We will assume no *CP* violation in the scalar sector, so that all the new resonances are *CP* eigenstates. The *CP*-even states, as may be surmised from the decay pattern, will be denoted either by h or H, the *CP*-odd resonances by A, and the charged scalars by $H^{\pm}$ and $H^{\pm\pm}$, with subscripts denoting their (tentative) masses, i.e., the approximate position of the peak of the resonance, in GeV. One may note that the heavier resonances, like H_{650}, are quite broad and the mass value is the approximate position of the peak, which is prone to a significant uncertainty with such preliminary data. In this notation, as already mentioned, the SM-like Higgs boson will be called h_{125}.

A resonance may be observed in different channels, with p values p_1, p_2, etc., where p denotes the probability of observing the event assuming the null hypothesis. The p values are easily obtained from the σ values denoting the significances, as quoted by the individual experiments. The combined p value is obtained following the method outlined in Appendix C. We will use the numbers quoted by the experiments only and refrain from using any data for which such numbers are not explicitly quoted, and then combine them following the prescription mentioned in Appendix C. Also, we will be dealing with only global significances, not the local ones, so that the LEE is taken care of.

A summary of the signals is shown in Table I. We would like to warn our readers again that the resonance peak positions are not at all precise, and for broad resonances, we club all the signals together that lie within ∼10% of the tentative peak. We also do not consider the case of degenerate or close-lying resonances just for simplicity, although this is a distinct possibility for broad resonances. We have made a conservative estimate of the global significance of each resonance and have included only those where this is above 3σ. The case of h_{95} and H_{650} merits further discussion, for which we refer the reader to Secs. II A and II B, respectively.

A. h_{95}

The CMS Collaboration [7], from the first year of the Run II data, found the hint of a resonance at about 95 GeV, decaying into two photons. A similar signal with slightly less significance was also observed in the Run I data [6]. Combined, this gives a local significance of 2.9σ [18,44] and a modest global excess of 1.3σ. An analysis for light resonances decaying into two photons has also been released by CMS [45].

This resonance has also been observed to decay to $\tau^+\tau^-$ with a local (global) significance of $2.6(2.3)\sigma$ [8,9]. One may note that these numbers are obtained with the assumption that the parent scalar resonance peaks at 95 GeV; CMS actually found it at 100 GeV, with a local (global) significance of $3.1(2.7)\sigma$ [9].[3] Thus, we have the following from LHC:

$$\mu_{\gamma\gamma} = \frac{\sigma(pp \to h_{95} \to \gamma\gamma)}{\sigma(pp \to \phi \to \gamma\gamma)} = 0.33^{+0.19}_{-0.12}, \tag{3}$$

$$\mu_{\tau^+\tau^-} = \frac{\sigma(pp \to h_{95} \to \tau^+\tau^-)}{\sigma(pp \to \phi \to \tau^+\tau^-)} = 1.2 \pm 0.5, \tag{4}$$

where ϕ is a fictitious 95 GeV Higgs boson, i.e., a spin-0 object of mass 95 GeV whose couplings to the gauge bosons and fermions are exactly like the SM Higgs boson. The local significances are all less than 3σ; a combined global significance reaches $2.75(2.4)\sigma$ if one uses the higher (lower) significance value from Refs. [8,9], as can be seen in Table I.

Earlier, LEP also found a similar excess close by [24], at about 98 GeV, with a local significance of 2.3σ, where we need to take into account the possibility of coarser mass resolution so that these two hints may come from one and the same particle,

$$\mu_{b\bar{b}} = \frac{\sigma(e^+e^- \to Zh_{95} \to Zb\bar{b})}{\sigma(e^+e^- \to Z\phi \to Zb\bar{b})} = 0.117 \pm 0.057. \tag{5}$$

Thus, if the branching fraction of $h_{95} \to b\bar{b}$ is the same as that of a SM-like 95 GeV scalar, the ZZh_{95} coupling is approximately 0.33 ± 0.09 times the $ZZ\phi$ coupling.[4] This shows that, unless the ZZh_{95} coupling is much smaller than the $ZZ\phi$ one, (i) h_{95} must have a significant SU(2) doublet admixture, and (ii) a single SU(2) scalar doublet giving mass to all the fermions is not enough to explain the decay pattern, as it would have led to similar values for $\mu_{\tau^+\tau^-}$ and $\mu_{b\bar{b}}$. The LEP indication in $b\bar{b}$ is, however, prone to controversy, as stressed in [47].

B. H_{650}

It was first pointed out in Ref. [17] that an analysis of the then total LHC data of 113.5 fb^{-1} gives the signal of a broad resonance, of width $\mathcal{O}(100)$ GeV, that peaks

[3]The authors of Ref. [46] showed that the cross section for $pp \to t\bar{t}h_{95}$, $h_{95} \to \tau^+\tau^-$ may be even larger than the corresponding cross section with h_{125}. However, one may note that the background coming from $t\bar{t}Z$, $Z \to \tau^+\tau^-$ is large, and this decay channel for a scalar whose mass is so close to M_Z may still remain hidden at the LHC (e.g., a Z veto may severely suppress the signal). Of course, if the backgrounds can be controlled, this should be a very promising channel, maybe in upcoming leptonic colliders.

[4]The significance quoted in the LEP paper is a local one, no global significance has been quoted. If we combine the local significance of LEP with the global numbers of CMS, the combined significance just goes above 3σ. This is in contrast with Ref. [18] that claimed a combined significance of more than 4σ, which can be obtained by combining local significances only and neglecting the LEE.

between 600 and 700 GeV, with total number of signal events given by

$$N_{\rm sig} = 34.21^{+12.78}_{-6.03}. \qquad (6)$$

We will rather work with the more recent updates published by the LHC collaborations themselves. As the properties of H_{650} constitute an important part of this paper, we will briefly summarize the experimental results.

The resonance H_{650} has been indicated in various channels. The most important results came from CMS [12], who found the cross section of $pp \to H_{650} \to W^+W^-$ to be 160 fb, assuming the production to be purely through the VBF process. This has a local (global) significance of $3.8(2.6)\sigma$. The significance becomes insignificant if the production, at least a large part of it, goes through ggF.

The ATLAS Collaboration [10,11] looked for any possible resonance in the ZZ decay channel in their 139 fb^{-1} of data, ultimately leading to the four-lepton as well as $\ell\nu\ell\nu$ signals. They found a mild excess of local (global) significance $2.4(0.9)\sigma$ [10], peaking at 620 GeV. Considering the large width of this resonance, we may assume that this is the identical resonance that we call H_{650}. An important point to note is that, for ATLAS too, the significance, however mild, shows only if the production is assumed to be entirely through VBF. This, of course, does not mean that ggF production is absent; it is just that such events are swamped under the background because of their topology. Also, the broadness of the line shape means that there is always the possibility of the signal being a superposition of more than one closely spaced resonance. While this is an interesting alternative, we will stick to the simpler assumption of only a single resonance in the present paper. One may, however, note that the null result of CMS in the four-lepton channel is consistent with a small enough ZZH_{650} coupling and/or a broad resonance [48].

CMS [13] also found a $3.8(2.8)\sigma$ local (global) excess at 650 GeV for the decay channel $H_{650} \to h_{95}h_{125}$. The exact analysis of this channel depends on the scalar potential.

Finally, let us mention the ATLAS result [16], which looked for the channel $A \to HZ \to t\bar{t}\ell^+\ell^-$, where H and A are generic neutral CP-even and CP-odd scalars. There is an excess of local (global) significance $2.85(2.35)\sigma$ for $(m_A, m_H) = (650, 450)$ GeV. However, one may note that, apart from being opposite CP neutral scalars, the CP assignment of the parent and the daughter is quite arbitrary; it could very well be $(m_H, m_A) = (650, 450)$ GeV. This is what we will assume in this paper.

We convert all the significances to corresponding p values and then combine them following the method outlined in Appendix C. Taking everything together, the global significance comes out to be just above 4σ. This, therefore, is the strongest evidence of a new resonance, and we will therefore mostly focus upon this, as well as h_{95}, while the other resonances play a secondary role.

We examine now in some detail how to extract the $H_{650}W^+W^-$ coupling from the CMS data of the production cross section $\sigma(pp \to H_{650} \to W^+W^-)$. The excess reported by CMS [12] for $\sigma(pp \to W^+W^- \to 2\ell 2\nu)$ is found to be consistent with a heavy SM-like scalar, where the data and the SM-like Higgs boson expectation intersect around a mass of 650 GeV and a $\sigma(pp \to H_{650} \to W^+W^-)$ cross section around 160 fb, under the assumption that the latter proceeds only through VBF. For a SM-like Higgs boson of mass 650 GeV, the theoretical VBF production cross section $\sigma_{\rm VBF}^{\rm (SM)}$ is around 272 fb at $\sqrt{s} = 13$ TeV [49]. Thus, assuming no ggF contribution and neglecting consistently the H_{650} partial width to fermions, this leads to the estimate $\sigma_{pp\to H_{650}\to W^+W^-} \simeq \sigma_{\rm VBF}^{\rm (SM)} \times {\rm BR}^{*{\rm (SM)}}_{H_{650}\to W^+W^-} \simeq 184$ fb, where the asterisk indicates the neglect of fermion contributions to the total H_{650} width, and we used the SM-like updated widths [49],[5]

$$\Gamma_{H_{650}\to WW} \simeq 81 \text{ GeV}, \qquad \Gamma_{H_{650}\to ZZ} \simeq 39 \text{ GeV}. \qquad (7)$$

This theoretical cross section, being in excess of the 160 fb reported by CMS under the same VBF dominance assumption, can be viewed as reflecting the effect of the experimental selection of the VBF-like category in the CMS analysis, which we estimate to be around 13%. To access the $H_{650}W^+W^-$ coupling, one needs to separate the *WW*-fusion (WBF) from the *ZZ*-fusion (ZBF) contributions to the VBF cross section. This is not straightforward, if meaningful at all, due to interference terms among s, t, and u channels in $qq \to qqH$ [50]. However, the latter are expected to be small for a SM-like Higgs boson, especially when VBF cuts selecting hard forward or backward jets in $pp \to Hjj$ are applied to reduce ggF contributions [51,52]. Assuming this separation is obtained from a theoretical simulation, one can then, in principle, write a correlation between the (reduced) couplings $H_{650}W^+W^-$ and $H_{650}ZZ$ by requiring $\sigma_{\rm VBF} \times {\rm BR}^*{}_{H_{650}\to W^+W^-} \simeq 160$ fb. However, this will be consistent only if the separation between the ZBF and WBF contributions to the VBF is performed within the CMS categories selection procedure, which is not available to us. Fortunately, one can proceed differently, avoiding the use of the absolute magnitude of the cross section given by CMS. This is obtained by using the fact, noted above, that at 650 GeV the data cross section coincides with the SM-like expectation. One can thus simply write

$$\sigma_{\rm VBF} \times {\rm BR}^*_{H_{650}\to W^+W^-} = c\,\sigma_{\rm VBF}^{\rm (SM)} \times {\rm BR}^{*{\rm (SM)}}_{H_{650}\to W^+W^-}. \qquad (8)$$

[5] We used here a narrow-width approximation that is not fully justified, since $\Gamma/M \sim 18\%$. However, this allows for a consistent comparison with [12] where the same approximation has been used to obtain the red line of their Fig. 4.

The left-hand side depends on the reduced couplings $\kappa_W^{H_{650}}$ and $\kappa_Z^{H_{650}}$ as well as on the SM-like separate contributions to the VBF cross section with appropriate cuts, $\sigma_{\rm WBF}^{\rm SM}$ and $\sigma_{\rm ZBF}^{\rm SM}$, while the right-hand side is now expressed formally in terms of the latter cross sections rather than taking it as a number. In this way, no explicit assumption is needed about category selection criteria or VBF cuts, except that it should be the same on both sides of Eq. (8). The use of narrow-width approximation in Eq. (8) is safe since the ensuing error is expected to be roughly the same on both sides of the equation, see also footnote 5. We have, however, introduced a multiplicative factor c to account for possible order-one corrections. The ensuing correlation between $\kappa_W^{H_{650}}$ and $\kappa_Z^{H_{650}}$ takes the form

$$|\kappa_Z^{H_{650}}|^2 = \frac{\left(|\kappa_W^{H_{650}}|^2 - c(1+r)\mathrm{BR}^{*(\mathrm{SM})}_{H_{650}\to W^+W^-}\right)|\kappa_W^{H_{650}}|^2 - c(1+r)\mathrm{BR}^{*(\mathrm{SM})}_{H_{650}\to t\bar{t}}|\kappa_t^{H_{650}}|^2}{c(1+r)\mathrm{BR}^{*(\mathrm{SM})}_{H_{650}\to ZZ} - r|\kappa_W^{H_{650}}|^2}, \qquad r = \frac{\sigma_{\rm ZBF}^{\rm SM}}{\sigma_{\rm WBF}^{\rm SM}}, \tag{9}$$

where we have allowed for later use a nonvanishing reduced coupling to the top quark $\kappa_t^{H_{650}}$, but the starred BR still indicates that it is evaluated without including the corresponding width in the H_{650} *total* width. The ratio r is not expected to depend on the applied categorization method to separate ggF from VBF cross sections. We estimate it to be around $r \simeq 0.37$.[6] From the structure of Eq. (9), one sees that $|\kappa_W^{H_{650}}|$ has to remain in a finite range in order to be consistent with the positivity of $|\kappa_Z^{H_{650}}|^2$. Plugging in the actual values of the starred BRs and r for a 650 GeV SM-like Higgs boson, the allowed range for $|\kappa_W^{H_{650}}|$ is found to be very narrow, the minimum being around $0.96\sqrt{c}$ and the maximum well below $1.1\sqrt{c}$ in order for $|\kappa_Z^{H_{650}}|$ to remain perturbative, say $\lesssim \mathcal{O}(1)$. We infer the following bounds:

$$\begin{aligned} &0.96\sqrt{c}\,gM_W + \mathcal{O}(|\kappa_t^{H_{650}}|^2) \\ &\quad \lesssim |g_{WWH_{650}}| \lesssim (0.05 + 0.95\sqrt{c})gM_W + \mathcal{O}(|\kappa_t^{H_{650}}|^2), \end{aligned} \tag{10}$$

with c of order one. The lower bound estimate coincides with neglecting the VBF production through ZZ fusion, as well as the coupling to fermions. While we can guess that ZZ fusion is less effective than WW fusion from the ATLAS data, it is not completely negligible. Taking it into account will increase further the coupling $g_{WWH_{650}}$ above the lower bound given in Eq. (10). Moreover, allowing for a nonvanishing coupling to fermions, which has to remain small to be consistent with the CMS indications, would slightly increase the lower and upper bounds themselves. One may also note that the allowed range can be scaled up or down by roughly a factor $\sqrt{c}$. Finally, let us mention that any other H_{650} decay channel beyond the SM-like ones can be accounted for by shifting $\mathrm{BR}^{*(\mathrm{SM})}_{H_{650}\to t\bar{t}}|\kappa_t^{H_{650}}|^2$ in Eq. (9) by the corresponding branching fraction,

$$\mathrm{BR}^{*(\mathrm{SM})}_{H_{650}\to t\bar{t}}|\kappa_t^{H_{650}}|^2 \to \mathrm{BR}^{*(\mathrm{SM})}_{H_{650}\to t\bar{t}}|\kappa_t^{H_{650}}|^2 + \mathrm{BR}^{*}_{H_{650}\to h_1h_2}. \tag{11}$$

This would always decrease the value of $|\kappa_Z^{H_{650}}|$ as long as the denominator in Eq. (9) remains positive. We will come back to this point in Sec. III D when discussing the possible decays of H_{650} to the lighter scalars of the model.

C. A_{400} and H_{320}

The best evidence of A_{400} comes from the CMS analysis (see Fig. 4 of Ref. [14]) where the invariant mass reconstruction of the final $t\bar{t}$ state was used. The analysis is extremely challenging, given the large QCD background that interferes with the signal. CMS has performed a spin-parity analysis, which apparently selects a CP-odd candidate at about 400 GeV, with $\Gamma/M \sim 4\%$. This has a local (global) significance of $3.5(1.9)\sigma$.

While this is a modest evidence, ATLAS has also observed $A_{400} \to Zh_{125}h_{125}$ with about 3.8σ local and 2.8σ global significance [15]. The final state Higgs bosons apparently come from another scalar resonance, H_{320}. Admittedly, this is slightly below our 3σ global criterion, but this channel does not suffer from combinatorial background, since it originates from A_{400}. Assuming CP conservation in the scalar sector, this makes H_{320} CP even and hence A_{400} CP odd.

We may also note that the daughter scalar appearing in the final decay channel of H_{650}, namely, $H_{650} \to AZ$, may indeed be A_{400}, albeit with a slightly less significance. However, it still remains a good possibility that the two resonances at 400 and 450 GeV are one and the same.

[6]This estimate has been obtained by generating the process $pp \to H_{650} + 2\mathrm{jets}$ at $\sqrt{s} = 13$ TeV with MadGraph [53,54] version MG5_aMC_v3.5.4, with no loop-induced ggF contribution, applying VBF cuts and comparing the cross section when the Z-boson is excluded in the process, to that when W^+, W^- are excluded. As a consistency cross-check of the reduction of interference terms, the sum of the latter two is found to agree very well with the VBF cross section when the VBF cuts are applied.

D. h_{151} or A_{151}

A combined analysis of both CMS and ATLAS data was performed by the authors of Ref. [20], where they found a local (global) significance of $4.3(3.9)\sigma$ for a neutral scalar at around 151 GeV, with the particle decaying to $\gamma\gamma$ or $Z\gamma$. This claim has been disputed in Ref. [55], with a recalculated significance of $4.1(3.5)\sigma$. In spite of its potential interest, we will not discuss this resonance any further in this paper, since its hint is not explicitly mentioned in any official publication, including public notes, from either ATLAS or CMS. One may note that its CP properties are still uncertain, so it can be either h_{151} (CP even) or A_{151} (CP odd). However, if it exists, it can easily be accommodated in our model, namely, 2HDeGM.

E. On sum rules

If the underlying theory is renormalizable, the scattering unitarity bounds give strong constraints on the couplings, in the form of sum rules. Such sum rules exist in the literature for a long time; we refer the reader to, e.g., Ref. [36]. We will be interested in the sum rules involving gauge bosons and scalars only.

Suppose we parametrize the coupling between two gauge bosons and one scalar in the following way:

$$V_{a\alpha}V_{b\beta}\phi_i \quad : \quad i g_{abi} g_{\alpha\beta}, \tag{12}$$

where α and β are the Lorentz indices. This leads to the following sum rules for $W^+W^- \to W^+W^-$ and $W^+W^- \to ZZ$ scattering processes [36], assuming the custodial symmetry to be valid[7] so that $\cos^2\theta_W = M_W^2/M_Z^2$,

$$\begin{aligned} WW \to WW &: g^2 M_W^2 = \sum_k g^2_{W^+W^-\phi_k^0} - \sum_\ell g^2_{W^+W^+\phi_\ell^{--}}, \\ WW \to ZZ &: g^2 M_Z^2 = \sum_k g_{W^+W^-\phi_k^0} g_{ZZ\phi_k^0} - \sum_\ell g^2_{W^+Z\phi_\ell^-}, \end{aligned} \tag{13}$$

where the summations are over all scalar states. If CP is not conserved in the scalar sector, the sum rules also depend on the CP phases of the scalars.

One may draw the following conclusions:

(i) If the couplings of h_{125} to W^+W^- and ZZ are exactly like the SM (which we call the alignment limit), then no other neutral scalar can couple to W^+W^- or ZZ unless doubly charged scalars exist. This is true in the exact alignment limit, but even if there is a small deviation, the coupling of other neutral scalars to W^+W^- or ZZ would be tiny.

(ii) If doubly charged scalars exist, there must be some charged Higgs boson that couples to W^+Z. The reverse is also true [36].

(iii) The first equation of (13) can be rephrased as

$$\left[g_{WWh}^{(\mathrm{SM})}\right]^2 = \sum_i g^2_{W^+W^-\phi_i^0} - \sum_k |g_{W^-W^-\phi_k{}^{++}}|^2, \tag{14}$$

where $g_{WWh}^{(\mathrm{SM})}$ is the SM WWh coupling. In writing Eq. (14) one assumes CP conservation and all the neutral scalars ϕ_i^0 to be CP-even eigenstates, and for simplicity, $\rho = 1$ at the tree level.[8]

Once we have a renormalizable model to start with, the sum rules are automatically satisfied. However, a more challenging and interesting way is to determine the couplings experimentally and see how far the sum rules are satisfied; this may lead to the discovery of new scalars.

F. Inadequacy of Higgs doublets and the canonical Georgi-Machacek model

So far, all the new resonances that we have talked about are neutral scalars. There has been no definite hint of any charged scalar, but there are strong constraints, both direct and indirect. One may ask why the spectrum—with the usual *caveat emptor* that some, most, or all of them may vanish in near future—cannot be accommodated in a multi-Higgs doublet model or even in a more exotic extension like the canonical GM model.

To answer this, let us just focus on H_{650}, the exact mass value being of little relevance right now. If we take the data at face value and assume that it will stand the test of time, the analysis of Sec. II B leads to the following:

(i) The scalar sector of the SM has to be extended. The 125 GeV resonance h_{125} behaves almost exactly like the Standard Model Higgs boson h_{SM}. Thus, to a first approximation, we may assume that the extended scalar sector is at the "alignment limit"; i.e., the gauge and fermionic couplings of h_{125} are almost exactly that of h_{SM}. There may be small deviations, that are generally parametrized by the κ parameters. We will keep $|\kappa - 1| < 0.1$, consistent with Eq. (2), a conservative estimate for all couplings of h_{125}, which means a maximum deviation of 10% from the corresponding SM couplings.

(ii) Equation (10) shows that $|g_{WWH_{650}}|$ is confined to a narrow range with large values. The implication of such a large $|g_{WWH_{650}}|$ is a clear violation of the sum rule, Eq. (14), as the sum on i includes now at least

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[7]If custodial symmetry is not valid, the left-hand side of the second equation of (13) is replaced by $g^2 M_Z^4 \cos^2\theta_W/M_W^2$.

[8]The sum rule, which essentially means that unitarity in $2 \to 2$ scattering must not be violated, comes from longitudinal $W^+W^+ \to W^+W^+$ scattering. The neutral scalars lead to s-channel processes, while a doubly charged Higgs boson leads to a t-channel process. This new contribution softens the sum rule constraint.

h_{125} and H_{650} and the SM-like WWh_{125} coupling almost saturates the sum rule, unless there are doubly charged scalars in the spectrum. In order to fulfil the sum rule within a coherent extension of the SM, let us examine the various possibilities. Considering only $\mathrm{SU}(2)_\mathrm{L}$ singlets would obviously not work. On the one hand, a doubly charged state would then not couple to W^+W^+ and thus will not help in restoring the sum rule; on the other hand, H_{650} would also then dominantly be a gauge singlet and its coupling to W^+W^- would contradict the CMS indications. Adding just an $\mathrm{SU}(2)_L$ doublet to the SM doublet would not work either. Indeed this would require both the components to be electrically charged to account for a doubly charged state. This second doublet should then not develop a VEV, to avoid spontaneous breaking the electric charge conservation. However, even then, one would be left with 5 physical degrees of freedom, four of which are taken by the charged states and no room is left for a second neutral state. Adding a third doublet with a neutral and a charged component, there are now nine physical states, of which only two would be CP-even neutral scalars, thus discarding the inclusion of h_{95} in the spectrum. Going further with more singlets or doublets, although a logical possibility, would be at the expense of minimality. Thus, avoiding the unusual construct of a doublet with two charged components, one may say that multi-Higgs doublet models fail to explain the spectrum and its characteristics. Thus, there must be SU(2) representations higher than doublet.

(iii) The minimal way of including a doubly charged scalar respecting the gauge symmetries of the model would be by adding an $\mathrm{SU}(2)_\mathrm{L}$ complex triplet. A major problem with scalar multiplets higher than the doublet is that they generally do not respect the custodial symmetry, which keeps $\rho = 1$ at the tree level. For instance, adding only one complex triplet like in the type-II seesaw model and enforcing ρ to remain consistent with unity, Eq. (1), induces a totally different phenomenology from the one expected for H_{650} or its (doubly) charged partners, suppressing single productions at the LHC (see, e.g., Ref. [56] for a very recent appraisal.). Thus, one complex triplet would not work in our case. One elegant way out that ensures $\rho = 1$ without further constraints at the tree level, was suggested by Georgi and Machacek [26]. We refer the reader to Ref. [27] for a detailed discussion on CS for the GM models. Thus, the canonical GM model could be a good starting point. One may even look at the extended GM proposed in Ref. [27]. The GM model has one doublet, one real triplet, and one complex triplet of scalars under the weak SU(2). After spontaneous symmetry breaking, the custodial symmetry still remains intact, and the physical scalars can be grouped in custodial multiplets, for which there are two singlets, one triplet, and one fiveplet.

(iv) Does the canonical GM model have enough scalars to fit the bill? Assuming that H_{650} is a CP-even neutral scalar, there are only three such objects in the GM model. Can we ascribe them to h_{95}, h_{125}, and H_{650}? The answer is, unfortunately, no. The reason is twofold. The first and major objection is that, in the GM model, the $H_{650}ZZ$ coupling is $2/\cos^2\theta_W \approx 2.6$ times as strong as the $H_{650}WW$ coupling [57], if H_{650} is a custodial singlet or member of the custodial fiveplet, respectively. In both cases, this is clearly contradictory to the hints. A milder objection is that, in the GM model, the custodial fiveplet consists entirely of SU(2) triplets and hence the neutral member does not couple to fermions. Thus, there would be no ggF production of H_{650}.[9]

III. THE TWO-HIGGS DOUBLET EXTENDED GEORGI-MACHACEK MODEL

We would like to propose a new model here, which we call 2HDeGM for short. The extension from the canonical GM model occurs in two steps. First, we add one more $\mathrm{SU}(2)_\mathrm{L}$ scalar doublet with the same weak hypercharge as the first doublet. Second, we impose CS only on the gauge part of the Lagrangian and not on the scalar potential, in the same vein as Ref. [27]. The latter condition removes the mass degeneracy among the scalar states in any nonsinglet custodial multiplet. Thus, 2HDeGM consists of two SU(2) doublets Φ_1 and Φ_2 (with VEVs v_1 and v_2, respectively), a real triplet Ξ, and a complex triplet X (both with VEV u), and with the $\mathrm{SU}(2)_L$ and $\mathrm{U}(1)_Y$ quantum numbers (T, Y) as

$$\Phi_1: \left(\frac{1}{2}, 1\right), \quad \Phi_2: \left(\frac{1}{2}, 1\right), \quad \Xi: (1, 0), \quad X: (1, 2),$$

with the electric charge $Q = T_3 + Y/2$.

The CP-even neutral members of these multiplets are denoted by ϕ_1^0, ϕ_2^0, ξ^0, and χ^0, respectively. This is a minimal extension of the canonical GM model and, by construction, preserves CS. We do not show the full scalar potential, but we will assume CS to be respected in the gauge sector only, keeping $\rho = 1$ at the tree level. It need not be respected by the scalar potential, as is explained in Ref. [27]. For the subsequent analysis, we will assume that CS is not respected in the scalar potential.

There are at least four two-Higgs doublet models (2HDMs) that forbid tree-level flavor-changing neutral current through some discrete symmetries on the scalar sector. They are typically called type I, type II, type X (or

[9] $pp \to H_{650} \to WW$ through ggF has not yet been detected, which is probably due to the nature of the background.

lepton specific, sometimes called type III), and type Y (or flipped). We will follow the notation of Ref. [58], so that the couplings of the SU(2) scalar doublets Φ_1 and Φ_2 with fermions look as follows:

(1) Type I: Φ_2 couples with all quarks and charged leptons, Φ_1 has no fermionic coupling.
(2) Type II: Φ_2 couples with weak isospin projection $I_3 = +\frac{1}{2}$ fermions and Φ_1 with $I_3 = -\frac{1}{2}$ fermions.
(3) Type X: Φ_2 couples with quarks and Φ_1 couples with leptons. Thus, type I and type X have identical roles in quark flavor physics.
(4) Type Y: Couplings with the quarks are identical to that of type II, but Φ_2 couples with charged leptons.

Thus, as far as quark flavor physics is concerned, type I and type X behave identically, and type II and type Y are also indistinguishable.

There are 17 scalar fields to start with, and 3 of them are eaten up by the gauge bosons. The construction obviously respects CS, and the custodial multiplets have to be

$$\mathbf{5} \oplus \mathbf{3} \oplus \mathbf{3} \oplus \mathbf{3} \oplus \mathbf{1} \oplus \mathbf{1} \oplus \mathbf{1}. \tag{15}$$

That this is the unique decomposition into irreducible multiplets can be easily understood just by counting the number of charged fields. Note that these need not be the mass eigenstates [27].

One of the $\mathbf{3}$'s is the Goldstone triplet. In the final spectrum, we have four CP-even and two CP-odd neutral scalars. We may identify the CP-even neutrals with h_{95}, h_{125}, H_{320}, and H_{650}, respectively. This is what we will assume in the rest of the paper and proceed accordingly. Similarly, we may identify the CP-odd neutrals with A_{400} and something else, yet to be detected. For example, this can be A_{151}.

There are three independent VEVs, and so three custodial singlets. The neutral members of the higher multiplets, $\mathbf{5}$ or $\mathbf{3}$, cannot develop a VEV, as that will break the CS. One of the CP-even neutral scalars must go into $\mathbf{5}$; let us call that H_5. It should be a linear combination of $\phi_1^0 \sin\beta - \phi_2^0 \cos\beta$ (where $\tan\beta = v_2/v_1$) and $\chi^0 - \sqrt{2}\xi^0$. However, vertices like $H_5 W^+ W^-$ or $H_5 ZZ$ can exist [57]. One may note that H_5 need not be a mass eigenstate, except $H_5^{\pm\pm}$.

h_{95} decays to $\tau^+\tau^-$ and $b\bar{b}$, so it must contain some doublet fields.[10] The 125 GeV resonance h_{125} must be very close to the SM Higgs boson $h_{\rm SM}$, i.e., in the alignment limit. H_{650} may or may not have any doublet components; whether it is produced in ggF is still not clear.

The SU(2) triplet fields do not couple to fermions (except, possibly, neutrinos, which are outside our scope), but the doublet fields do. To prevent tree-level flavor-changing neutral current, we consider two of the most popular choices: type I, where the doublet Φ_1 has no fermionic coupling, and the doublet Φ_2 gives mass to all the fermions, and type II, where Φ_1 (Φ_2) gives mass to isospin projection $-\frac{1}{2}$ ($+\frac{1}{2}$) fermions. Other choices are also possible.

Let us also recall that, on top of the unitarity sum rules considered in Sec. II E, one can also impose unitarity requirements on scattering processes involving only the scalar states. The latter lead to constraints involving combinations of the couplings in the scalar potential. These can translate into upper bounds on the physical scalar masses, as shown, for example, in Ref. [59] for the GM model when a Z_2 symmetry is imposed. However, relaxing the latter discrete symmetry, one can identify decoupling limits where the mass upper bounds are totally relaxed [57]. In the present study, we do not need to specify the scalar potential for the extended Higgs sector. We will assume it to be the most generic compatible with the gauge symmetries and renormalizability, so that one expects to find decoupling limits despite the unitarity constraints and thus no upper bounds on the scalar masses. Note, however, that even imposing some extra global symmetries, the bounds of [59] need not hold any longer in our case; there are more $|{\rm in}\rangle$ and $|{\rm out}\rangle$ states, and, unlike the GM model, the scalar masses are no longer expressible in terms of the common triplet mass m_3 and common fiveplet mass m_5. The CMS bound on the mass of H^{++}, coming from $H^{++} \to W^+ W^+$, need not be valid in the presence of competing channels opening up, like the decay to one charged scalar and W^+, or two singly charged scalars. This opens up the possibility of a comparatively large triplet VEV u than what is usually found in the context of the GM model.

A. Convention

For any generic multiplet Φ, the neutral component $\Phi_{Q=0}$ gets a VEV,

$$\Phi_{Q=0} = \begin{cases} v + \frac{1}{\sqrt{2}}(\Phi^0 + i\Phi'^0) & \text{if the multiplet } \Phi \text{ is complex,} \\ v + \Phi^0 & \text{if the multiplet } \Phi \text{ is real.} \end{cases} \tag{16}$$

In this convention, $v = 174$ GeV in the SM, with $M_W^2 = \frac{1}{2} g^2 v^2$. The vacuum shifts for the CP-even neutral scalars of 2HDeGM are as follows:

$$\phi_1^0 \to \phi_1^0 + \sqrt{2} v_1, \quad \phi_2^0 \to \phi_2^0 + \sqrt{2} v_2,$$
$$\chi^0 \to \chi^0 + \sqrt{2} u, \quad \xi^0 \to \xi^0 + u,$$

and

$$M_W^2 = \frac{1}{2} g^2 (v_1^2 + v_2^2 + 4u^2), \tag{17}$$

[10] There are three bases we will be talking about: the gauge basis, with two doublets and two triplets, the custodial basis, whose irreducible representations are $\mathbf{5}, \mathbf{3}, \mathbf{3}, \mathbf{3}, \mathbf{1}, \mathbf{1}, \mathbf{1}$, and the mass basis.

where now

$$v = \sqrt{v_1^2 + v_2^2 + 4u^2}. \tag{18}$$

We also define

$$u = \frac{v}{2}\sin\theta_H, \quad v_1 = v\cos\theta_H\cos\beta, \quad v_2 = v\cos\theta_H\sin\beta, \tag{19}$$

so that

$$\tan\theta_H = \frac{2u}{\sqrt{v_1^2+v_2^2}}, \qquad \tan\beta = \frac{v_2}{v_1}. \tag{20}$$

While we take all the VEVs to be real, their relative signs are unconstrained, i.e., β and θ_H need not be fixed in the first quadrant. The custodial singlets of 2HDeGM are given by

$$S_1 = \phi_1^0, \quad S_2 = \phi_2^0, \quad S_3 = \frac{1}{\sqrt{3}}(\sqrt{2}\chi^0 + \xi^0), \tag{21}$$

and the neutral component of the fiveplet, orthogonal to all of them, must be

$$F_0 = \frac{1}{\sqrt{3}}(\chi^0 - \sqrt{2}\xi^0). \tag{22}$$

The gauge-gauge-scalar terms in the Lagrangian, relevant for the sum rules, are [27]

$$\mathcal{L}_{\rm cubic} = \frac{g^2 v_1}{2\sqrt{2}} S_1 \boldsymbol{W}\cdot\boldsymbol{W} + \frac{g^2 v_2}{2\sqrt{2}} S_2 \boldsymbol{W}\cdot\boldsymbol{W} + \frac{2g^2 u}{\sqrt{3}} S_3 \boldsymbol{W}\cdot\boldsymbol{W} + g^2 u[\boldsymbol{W}\otimes\boldsymbol{W}]\cdot\boldsymbol{F}, \tag{23}$$

where

$$\boldsymbol{W}\cdot\boldsymbol{W} = 2W^+W^- + (W_3)^2,$$
$$\boldsymbol{W}\otimes\boldsymbol{W}|_0 = \frac{1}{\sqrt{6}}[-2W^+W^- + 2(W_3)^2]. \tag{24}$$

This immediately gives the couplings of the CP-even neutral scalars to W^+W^-. To obtain the coupling with ZZ, one just has to replace $g^2W_3^2$ with $(g^2/\cos^2\theta_W)Z^2$. $\boldsymbol{F}$ is the fiveplet custodial multiplet, which is dotted with the fiveplet combination $\boldsymbol{W}\otimes\boldsymbol{W}$. It is easy to check that the $W_3W_3\xi^0$ terms cancel out, as they have to be.

It is important to note here that $\mathcal{L}_{\rm cubic}$, by construction, respects the CS. As a consequence, if the potential is assumed to be that of the canonical GM that also respects the CS, the physical H_{650} mass eigenstate would be either one of the custodial singlets or part of the custodial fiveplet. Its couplings to WW and ZZ would then be uniquely fixed by $\mathcal{L}_{\rm cubic}$, bringing us back to the inconsistency related to these couplings noted in the last item of Sec. II F. This justifies departing from the canonical GM potential in the 2HDeGM model as previously stated.

Note that any combination of custodial singlets is a custodial singlet. For example, instead of S_1 and S_2, we can define

$$S_h = \phi_1^0\cos\beta + \phi_2^0\sin\beta, \qquad S_H = -\phi_1^0\sin\beta + \phi_2^0\cos\beta \tag{25}$$

as two custodial singlets. Note that $\langle S_H\rangle = 0$ and $\langle S_h\rangle = \sqrt{v_1^2+v_2^2}$.

So, the terms that we get from $\mathcal{L}_{\rm cubic}$ are

$$\mathcal{L}_{\rm cubic} = g^2\Bigg[\frac{v_1}{2\sqrt{2}}\phi_1^0 W_3^2 + \frac{v_1}{\sqrt{2}}\phi_1^0 W^+W^- + \frac{v_2}{2\sqrt{2}}\phi_2^0 W_3^2 + \frac{v_2}{\sqrt{2}}\phi_2^0 W^+W^- + \sqrt{2}u\chi^0W^+W^- + \sqrt{2}u\chi^0W_3^2 + 2u\xi^0W^+W^-\Bigg]. \tag{26}$$

Now, let us define the matrix $\mathcal{X}$ as the one that rotates the gauge basis to the mass basis,

$$\begin{pmatrix} h_{95} \\ h_{125} \\ H_{320} \\ H_{650}\end{pmatrix} = \mathcal{X}_{4\times4}\begin{pmatrix}\phi_1^0 \\ \phi_2^0 \\ \chi^0 \\ \xi^0\end{pmatrix}. \tag{27}$$

$\mathcal{X}$ is orthogonal, so $H = \mathcal{X}\Phi$ means $\Phi = \mathcal{X}^\top H$, or $\phi_i = x_{ji}H_j$, where all x_{ij}'s are real valued satisfying,

$$\sum_j x_{ij}x_{kj} = \delta_{ik}. \tag{28}$$

For example,

$$h_{125} = x_{21}\phi_1^0 + x_{22}\phi_2^0 + x_{23}\chi^0 + x_{24}\xi^0. \tag{29}$$

From Eq. (26), one gets

$$\begin{aligned}\mathcal{L}_{W^+W^-h} &= g^2W^+W^-\Bigg[\left\{\frac{v_1}{\sqrt{2}}x_{11} + \frac{v_2}{\sqrt{2}}x_{12} + \sqrt{2}ux_{13} + 2ux_{14}\right\}h_{95} \\ &+ \left\{\frac{v_1}{\sqrt{2}}x_{21} + \frac{v_2}{\sqrt{2}}x_{22} + \sqrt{2}ux_{23} + 2ux_{24}\right\}h_{125} \\ &+ \left\{\frac{v_1}{\sqrt{2}}x_{31} + \frac{v_2}{\sqrt{2}}x_{32} + \sqrt{2}ux_{33} + 2ux_{34}\right\}H_{320} \\ &+ \left\{\frac{v_1}{\sqrt{2}}x_{41} + \frac{v_2}{\sqrt{2}}x_{42} + \sqrt{2}ux_{43} + 2ux_{44}\right\}H_{650}\Bigg],\end{aligned} \tag{30}$$

where the obvious Lorentz indices have been suppressed.

Similarly,

$$\mathcal{L}_{ZZh} = \frac{g^2}{\cos^2\theta_W} ZZ \left[\left\{ \frac{v_1}{2\sqrt{2}} x_{11} + \frac{v_2}{2\sqrt{2}} x_{12} + \sqrt{2} u x_{13} \right\} h_{95} + \left\{ \frac{v_1}{2\sqrt{2}} x_{21} + \frac{v_2}{2\sqrt{2}} x_{22} + \sqrt{2} u x_{23} \right\} h_{125} + \left\{ \frac{v_1}{2\sqrt{2}} x_{31} + \frac{v_2}{2\sqrt{2}} x_{32} + \sqrt{2} u x_{33} \right\} H_{320} + \left\{ \frac{v_1}{2\sqrt{2}} x_{41} + \frac{v_2}{2\sqrt{2}} x_{42} + \sqrt{2} u x_{43} \right\} H_{650} \right]. \quad (31)$$

Equations (30) and (31) are consistent with the sum rules given in Ref. [36]. For example, if all other x_{ij}'s are zero except $x_{22} = 1$, the sum on the rhs of Eq. (4.1) of [36] reduces to just one term, $g^4 v_2^2/2 = g^2 M_W^2$. If one keeps all the terms and also brings in the doubly charged Higgs boson, that will give some more nontrivial constraint on x_{ij}'s. This is what we get from Eqs. (30) and (31):

(i) H_{650} need not be a pure fiveplet. Even if it is, the x_{44} term creates a difference between $\sigma(pp \to H_{650} \to WW)$ and $\sigma(pp \to H_{650} \to ZZ)$. If it is a pure fiveplet (this happens if the full scalar potential is also invariant under CS), then

$$\sqrt{v_1^2 + v_2^2}(x_{41}\cos\beta + x_{42}\sin\beta) + u(x_{43} + \sqrt{2}x_{44}) = 0. \quad (32)$$

(ii) $h_{95} \to \tau^+\tau^-$ and $h_{95} \to b\bar{b}$ depend not only on v_1 and v_2 but also on x_{11} and x_{12}, and definitely on what type of 2HDM we have in mind.

(iii) Equation (13) gives the tightest constraint on this model. First, take the limit $v_1, v_2 \gg u$, so that we can neglect x_{i3} and x_{i4} terms. This always makes $\sum_k g^2_{WW\phi_k} > g^2 M_W^2$. So we need to bring in the doubly charged Higgs boson, but the $W^+W^+\chi^{--}$ coupling is just $g^2 u$, and if u is too small, it can never satisfy the unitarity sum rule Eq. (13).

B. Input strategy

Note that Eq. (27) is in line with the implicit assumption of absence of explicit CP violation in the potential, so that the neutral scalar mass matrix splits into two separate sectors of CP-even and CP-odd eigenstates. Requiring also real VEVs v_1, v_2, and u, we avoid discussing new sources of CP violation and the possibly related constraints. To determine the elements of this orthogonal matrix, we need six inputs: three to determine the first row, two for the second, and one for the third, and the rest get determined from the orthogonality conditions. Unfortunately, it is not straightforward to obtain these six parameters, and this is a more speculative job than just showing the violation of the sum rule. This is how we choose the six parameters:

(i) Three of them come from the coupling of h_{125} with $t\bar{t}$, W^+W^-, and ZZ, consistent with Ref. [31].

(ii) Two of them are obtained from the couplings of h_{95} to $t\bar{t}$ and W^+W^-. The decay $h_{95} \to \gamma\gamma$ depends on both these couplings, both in production and in decay. We assume the ratio of ggF to VBF production for a 95 GeV SM Higgs boson-like scalar to be 90:10, which is scaled by the corresponding coupling modifiers or the κ factors. A similar thing happens for the decay too. This gives us allowed regions in the $\kappa_t - \kappa_W$ plane for h_{95}. If we assume the underlying 2HDM to be of type I, $\kappa_t = \kappa_\tau$, and we get another constraint on κ_t from $h_{95} \to \tau^+\tau^-$. Of course, we need to check that the numbers are consistent with the LEP result on $e^+e^- \to Zh_{95} \to Zb\bar{b}$. How this is done will be discussed in Sec. III D.

(iii) Finally, the last number is obtained from the coupling of H_{650} to W^+W^-. We do not consider the $H_{650}ZZ$ coupling, as it is small and plagued with a large uncertainty.

C. Possible solutions and implications

Based on Eqs. (30) and (31) and further assumptions on the Yukawa sector, we explore in this section various theoretical configurations in order to delineate the v_1, v_2, u, and x_{ij}'s parameter-space regions that are consistent with the experimental indications described in Sec. II. Obviously, one should start with the best known state, namely, h_{125}, to put constraints on the three VEVs and on its doublet and triplet content given by $x_{2i}, (i = 1, \ldots 4)$, cf. Eq. (29). As we will see, this is quite constraining by itself due to the precise experimental knowledge of the h_{125} couplings to gauge bosons and fermions. As shown in Appendix A 1, if the reduced couplings of h_{125} to a pair of W's and to a pair of Z's, denoted, respectively, $\kappa_W^{h_{125}}$ and $\kappa_Z^{h_{125}}$, are equal to each other, then, either $u = 0$, or x_{23} is correlated to x_{24} through Eq. (A7). This holds true even if these reduced couplings differ from one, i.e., even if they are not exactly SM-like. The experimentally determined central values of the signal strengths or the fitted coupling modifiers [31,32] indeed suggest that we are in such a configuration. This very constrained configuration can be somewhat relaxed within 1 or 2 standard deviations of the measured quantities. In this case, combining Eqs. (A4) and (A5) leads to

$$u(x_{23} - \sqrt{2}x_{24}) = \frac{v}{2}\delta, \quad (33)$$

where $\delta = \kappa_Z^{h_{125}} - \kappa_W^{h_{125}}$. It is, however, instructive to treat first the case $\delta = 0$ assuming Eq. (A7). Also the results strongly depend on what one assumes about the Yukawa sector.

1. Type-II Yukawa

If the couplings of the Φ_1 and Φ_2 doublets to fermions are taken to be those of the type-II 2HDM, then the reduced couplings of h_{125} to the down- and up-type fermions read, respectively,

$$\kappa_d^{h_{125}} = \frac{v}{v_1} x_{21}, \qquad \kappa_u^{h_{125}} = \frac{v}{v_2} x_{22}, \tag{34}$$

and more generally,

$$\kappa_d^{\mathcal{H}_a} = \frac{v}{v_1} x_{a1}, \quad \kappa_u^{\mathcal{H}_a} = \frac{v}{v_2} x_{a2}, \quad a = 1,2,3,4, \tag{35}$$

where we define $(\mathcal{H}_a)_{a=1,2,3,4} = (h_{95}, h_{125}, H_{320}, H_{650})$. In particular, there is no extra freedom from the Yukawa couplings here; the reduced couplings $\kappa_d^{\mathcal{H}_a}$ ($\kappa_u^{\mathcal{H}_a}$) being obtained by equating directly the SM expressions of the down-type (up-type) fermion masses to those of the type II. The experimentally well-determined values of $\kappa_t^{h_{125}}$, $\kappa_b^{h_{125}}$, $\kappa_\tau^{h_{125}}$, and $\kappa_\mu^{h_{125}}$ will thus lead to further constraints on the parameters of the scalar/gauge boson sector of our model.

Since the experimental values of the $\kappa^{h_{125}}$'s are very close to 1, let us illustrate first the case $\kappa_t^{h_{125}} = \kappa_b^{h_{125}} = 1$. Upon use of Eqs. (34), (A6), and (A7) and in the normality condition $\sum_j x_{2j}^2 = 1$, one finds readily a simple relation between u and x_{24}, namely, $4u^2 = 3x_{24}^2 v^2$. Injecting this relation as well as Eq. (34) in Eq. (A5) determines u uniquely (up to a global sign),

$$u^2 = \frac{3}{4} \frac{1 - \kappa_Z^{h_{125}}}{3 - \mathrm{sgn}(u) 2\sqrt{6}} v^2. \tag{36}$$

This relation illustrates a tension in the type-II case for obtaining a sufficiently large value of u necessary for the consistency of our approach, as explained in Sec. II E. For instance, taking $\kappa_Z^{h_{125}} \simeq 1.04\,(0.985)$ and $v = 174$ GeV, one finds $u \simeq 22$ GeV ($\simeq -6.6$ GeV); these values are too small. Indeed, we anticipate here the needed value of $|u| \gtrsim 65$ GeV to ensure real-valued reduced couplings of h_{95}, H_{320}, and H_{650} to $t\bar{t}$, W^+W^-, and ZZ, the determination of which will be discussed in the following sections. This calls for a relaxation of the two simplifying assumptions made so far, namely, taking $\delta \neq 0$ in Eq. (33) and/or $\kappa_t^{h_{125}} \neq \kappa_b^{h_{125}} \neq 1$ within the allowed experimental error bars. In this case, the analysis becomes much more involved and is actually included in a general strategy applicable to any of the Yukawa types. The reader is referred to Appendix A 1 for a description of its main ingredients.

Suffice it to say here that a numerical scan, varying the reduced couplings within the experimental ranges given in Eq. (2), still shows that the maximal $|u|$ does not exceed $\simeq 50$ GeV. However, the value can be raised further if κ_b is negative. While right now the sign of κ_b is a free parameter, there is a possibility that it may be determined unambiguously in future [60]. For our subsequent discussions, we will be rather conservative and treat both $\kappa_b^{h_{125}}$ and $\kappa_\tau^{h_{125}}$ as positive numbers. As pointed out above, such small values of $|u|$ fail to ensure real-valued couplings in the CP-even scalar sector.

2. Type-X and type-Y Yukawa couplings

The quarks in type-X 2HDM get their masses either from Φ_1 or from Φ_2, exactly like type-I 2HDM, which is discussed below in detail. However, in type X the charged leptons get their masses from the other scalar, like type II. The reduced couplings $\kappa_\tau^{h_{125}}$ and $\kappa_\mu^{h_{125}}$ being close to unity, the rationale remains the same as above and the triplet VEV u has a similar upper bound to what is found for type II. As for type-Y 2HDM, the quarks get their masses exactly like in type II, so the triplet VEV u has obviously the same upper bounds as what is found for type II.

3. Type-I Yukawa couplings

If the couplings of the doublets to fermions are taken to be those of the type-I 2HDM, i.e., taking all the charged fermions to couple only to Φ_2, then the reduced couplings of the down- and up-type fermions to the CP-even scalars read

$$\kappa_d^{\mathcal{H}_a} = \kappa_u^{\mathcal{H}_a} = \frac{v}{v_2} x_{a2}, \qquad a \in \{1,2,3,4\}. \tag{37}$$

Specifying to h_{125},

$$\kappa_d^{h_{125}} = \kappa_u^{h_{125}} = \frac{v}{v_2} x_{22} \tag{38}$$

and considering, just as before, $\kappa_t^{h_{125}} = \kappa_b^{h_{125}} = 1$, and using Eqs. (38), (A6), and (A7) along with $\sum_j x_{2j}^2 = 1$, we find

$$v_1^2 + 4u^2 = (x_{21}^2 + 3x_{24}^2) v^2, \tag{39}$$

which indicates that the constraints are less stringent than those in the type-II case, as x_{21} remains free. On top of this, u can be raised even further if we allow a reasonable relaxation of $\delta = 0$ and $\kappa_t^{h_{125}} = \kappa_b^{h_{125}} = 1$, following the strategy described in Appendix A 1. In this case, $|u|$ can exceed $\simeq 65$ GeV allowing for solutions with real-valued couplings. We will thus stick to type I in the rest of the paper.

4. Determining the full sector

Starting from the input strategy described in Sec. III B and taking into account the features discussed at the beginning of the present section and in Secs. III C 1–III C 3 that favor type I over the three other types, we devise a well-defined procedure to determine the content of all four physical

TABLE II. Inputs and constraints, assuming type-I Yukawa.

h_{125}: central values, cf. Eq. (2)	$\kappa_t^{h_{125}} = \kappa_b^{h_{125}} = 0.99, \kappa_Z^{h_{125}} = 1.04, \kappa_W^{h_{125}} = 1.02$
h_{95}:	Scan on $\kappa_t^{h_{95}} = \kappa_\tau^{h_{95}}$ and $\kappa_W^{h_{95}}$ consistent with Eqs. (3) and (4)
H_{650}: cf. Eq. (10)	$\kappa_W^{H_{650}} = 0.89, 0.91$ with $c = 0.85$ or 0.78 $\kappa_W^{H_{650}} = 0.97, 1.0$ with $c = 1$.
H_{320}:	$\lvert\kappa_W^{H_{320}}\rvert, \lvert\kappa_Z^{H_{320}}\rvert \lesssim 0.45$

states h_{95}, h_{125}, H_{320}, and H_{650} as well as their couplings to W^+W^-, ZZ, and $f\bar{f}$. We list hereafter its main steps, relegating more technical details to Appendixes A 1, A 2, and B[11]:

(1) We choose six inputs as discussed in Sec. III B: (a) $\kappa_W^{h_{125}}, \kappa_Z^{h_{125}}, \kappa_b^{h_{125}}$ (b) $\kappa_W^{h_{95}}, \kappa_t^{h_{95}}$, and (c) $\kappa_W^{H_{650}}$. A consistent choice of input numbers is described in the next subsection.

(2) The input in (a) allows us to determine uniquely v_1, v_2, and the x_{2i}'s and to choose u arbitrarily, albeit in a given domain of consistency; see Appendix A 1 for details.

(3) The knowledge of the four-vector with components x_{2i} constrains the four-vector x_{1i} to live on a unit sphere in the hyperplane orthogonal to x_{2i}. Combined with the input in (b) and the first lines of Eqs. (30), (31), and (38), the x_{1i}'s are fixed up to a possible sign ambiguity; see Appendix A 2 for details.

(4) The knowledge of the four-vectors x_{1i} and x_{2i} constrains the four-vector x_{4i} to lie on a unit circle in the plane orthogonal to x_{1i} and x_{2i}. Combined with the input in (c) and the fourth line of Eq. (30), this fixes the x_{4i}'s up to a discrete multiplicity; see Appendix A 2 for details.

(5) From x_{1i}, x_{2i}, and x_{4i} the four-vector x_{3i} is uniquely fixed up to a global sign, and all the remaining reduced couplings $\kappa_Z^{h_{95}}, \kappa_Z^{H_{650}}, \kappa_t^{H_{650}}, \kappa_W^{H_{320}}, \kappa_Z^{H_{320}}$, and $\kappa_t^{H_{320}}$ are predicted.

(6) Consistency with the various experimental indications can now be checked, in particular, that of $\kappa_W^{H_{650}}$, $\kappa_Z^{H_{650}}, \kappa_t^{H_{650}}$ with the correlation given by Eq. (9).

D. Results

In the two following subsections, rather than presenting the results with the theoretical and experimental constraints all compiled in one step, we find it more instructive to somewhat break them up in order to understand their relative importance and effects. This is justified, in particular, since the present data from the LHC will change with future data taking.

1. The general scan

We show in this subsection the result of scans, taking as input the six reduced couplings $\kappa_W^{h_{125}}, \kappa_Z^{h_{125}}, \kappa_b^{h_{125}}$, $\kappa_W^{h_{95}}, \kappa_t^{h_{95}}$, and $\kappa_W^{H_{650}}$ as indicated in Table II.

(i) For h_{125} we take the experimental central values for the couplings to $b\bar{b}$, W^+W^-, and ZZ values as given in Eq. (2).

(ii) For h_{95}, we scan on $\kappa_t^{h_{95}}$ and $\kappa_W^{h_{95}}$ values that are consistent with the experimental indications, Eqs. (3) and (4), allowing 1 or 2 standard deviations from their central values; recall that here $\kappa_\tau^{h_{95}} = \kappa_b^{h_{95}} = \kappa_t^{h_{95}}$ since we are in the type-I-2HDM-Yukawa scenario. Note that only the gauge boson and fermion virtual contributions to the $\gamma\gamma$ channel are included. We refer the reader to Appendix B for details; let us just mention here that charged and doubly charged states can also contribute, however, in a model-dependent way controlled by the couplings in the potential, a sector that we do not study in detail in the present paper. Consideration of the charged scalars, however, will not change the typical pattern presented in the figures and tables below.

Note that, given its controversial status [47], the LEP indication, Eq. (5), for h_{95} is *not* systematically taken into account, but only shown as a further possible constraint referred to as LEPh95.

(iii) For $\kappa_W^{H_{650}}$, we choose four representative values within the allowed narrow range given by Eq. (10) for three different values of c.

(iv) Since the indication for H_{320} involves a substantial $H_{320}h_{125}h_{125}$ coupling (see the last line of Table I), the couplings of H_{320} to ZZ and WW should remain sufficiently small to account for the nonobservation of decays to gauge bosons [15]. While this is difficult to quantify precisely, the last line of Table II should not be seen as an input motivated by quantitative experimental indications, but rather as a qualitative guide for the scan presented in this section. We will, however, be brought to relax these bounds on $|\kappa_W^{H_{320}}|, |\kappa_Z^{H_{320}}|$ in the subsequent section.

[11]We rely on the *Mathematica* package [61] for the symbolic calculations and numerical scans, as well as for the production of the figures.

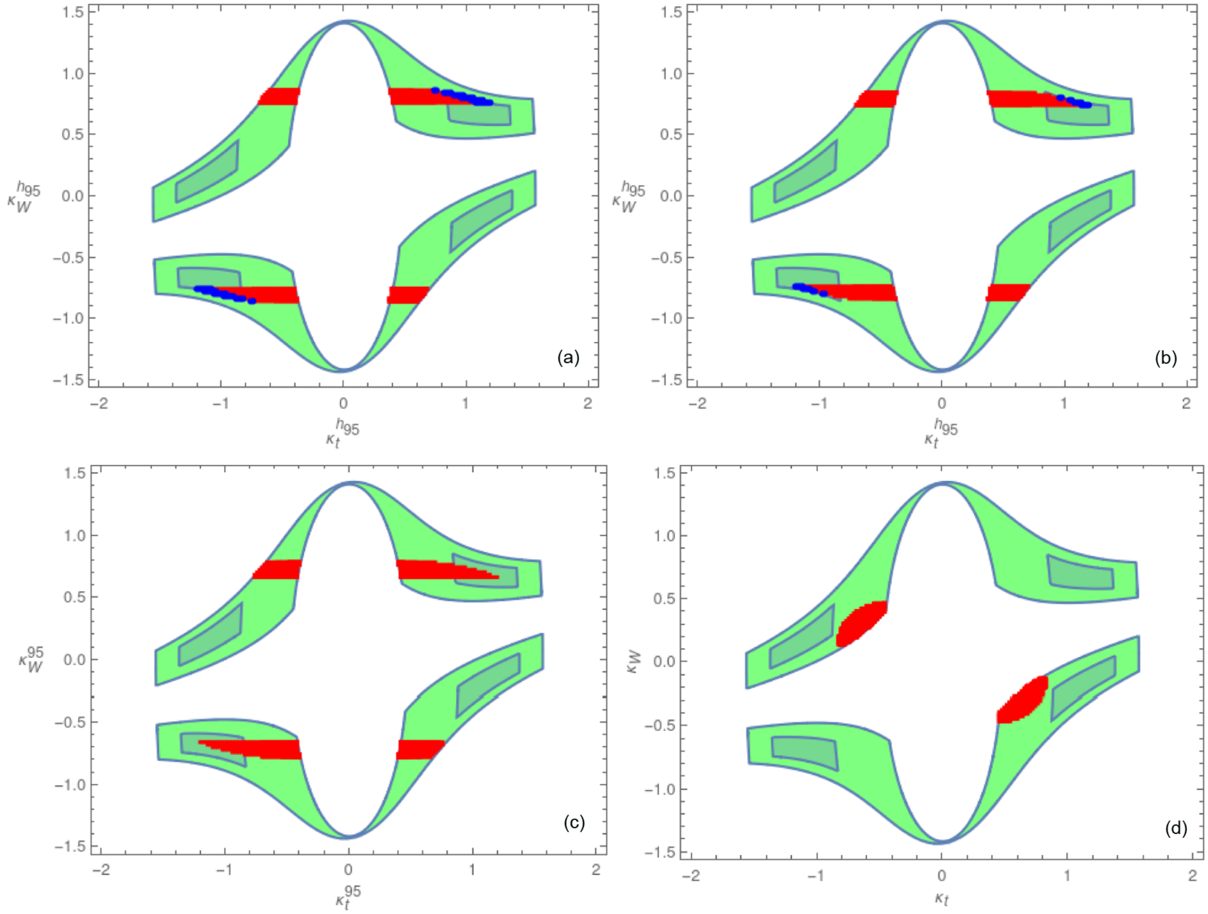

FIG. 1. $\kappa_t^{h_{95}}$ versus $\kappa_W^{h_{95}}$. The light (dark) green regions correspond to the $2(1)\sigma$ constraints, Eqs. (3) and (4). Fixing $\kappa_t^{h_{125}} = \kappa_b^{h_{125}} = 0.99, \kappa_Z^{h_{125}} = 1.04, \kappa_W^{h_{125}} = 1.02$, the red regions correspond to requiring no complex-valued couplings or mixings and varying the H_{320} reduced couplings to W and Z in the range $-0.45 \le \kappa_W^{H_{320}}, \kappa_Z^{H_{320}} \le +0.45$; the blue regions correspond to overlaying the constraint given by Eq. (5) taken at the 2σ level: (a) $\kappa_W^{H_{650}} = 0.89$, $u \simeq 78$ GeV, $v_1 \simeq 16$ GeV, $v_2 \simeq 76$ GeV; (b) $\kappa_W^{H_{650}} = 0.91$, $u \simeq 78$ GeV, $v_1 \simeq 16$ GeV, $v_2 \simeq 76$ GeV; (c) $\kappa_W^{H_{650}} = 0.97$, $u \simeq 78$ GeV, $v_1 \simeq 16$ GeV, $v_2 \simeq 76$ GeV; (d) $\kappa_W^{H_{650}} = 1$, $u \simeq 69$ GeV, $v_1 \simeq 14$ GeV, $v_2 \simeq 104$ GeV.

Figure 1 shows the result of the allowed regions for $\kappa_t^{h_{95}} (= \kappa_b^{h_{95}} = \kappa_\tau^{h_{95}})$ and $\kappa_W^{h_{95}}$, consistent with Eqs. (3) and (4), at the 1σ level (dark green) and 2σ level (light green). Obviously the green regions do not depend on the other inputs and are identical in the four panels of the figure. On top of that, the fixed $\kappa^{h_{125}}$'s and the real-valued nature of $\mathcal{X}$ and couplings constrain the allowed ranges of the VEVs v_1, v_2, and u for each of the four chosen values of $\kappa_W^{H_{650}}$. Among the latter, the first three require u at least as large as $\sim$78 GeV, with $v_1 \simeq 16$, $v_2 \simeq 76$ GeV, corresponding to Figs. 1(a)–1(c), while a somewhat smaller value $u \simeq 69$ GeV is obtained with $v_1 \simeq 14, v_2 \simeq 104$ GeV for $\kappa_W^{H_{650}} = 1$, as in Fig. 1(d). Even then, large fractions of the green regions are found not to be compatible with real-valued couplings of the remaining sectors. The red regions are the allowed ones where reality of $\mathcal{X}$ as well as the requirements of the last line of Table II are fulfilled. Note also that only the smaller values of $\kappa_W^{H_{650}}$ allow for regions consistent with LEPh95, the blue regions in Figs. 1(a) and 1(b), while slightly larger $\kappa_W^{H_{650}}$ would not be consistent with the LEP indications. Thus, ignoring the latter conservatively, the red regions are the ones allowed by all theoretical constraints and experimental data.

Moving around in the allowed red regions, the construction of the $\mathcal{X}$ entries as described in Sec. III C 4 leads to the discrete branches for the reduced couplings of H_{650} shown in Figs. 2 and 3. In particular, for a given fixed value of $\kappa_W^{H_{650}}$, there exists a one-branch solution for $\kappa_t^{H_{650}}$ but a two-branch solution for $\kappa_Z^{H_{650}}$. One can now bring in the correlation among $\kappa_Z^{H_{650}}, \kappa_W^{H_{650}}$, and $\kappa_t^{H_{650}}$, induced by the CMS indications, Eq. (9). The ensuing $\kappa_Z^{H_{650}}$ as a function of $\kappa_t^{H_{650}}$ for each fixed value of $\kappa_W^{H_{650}}$ and illustrative choices of the normalization factor c is depicted by the red dashed

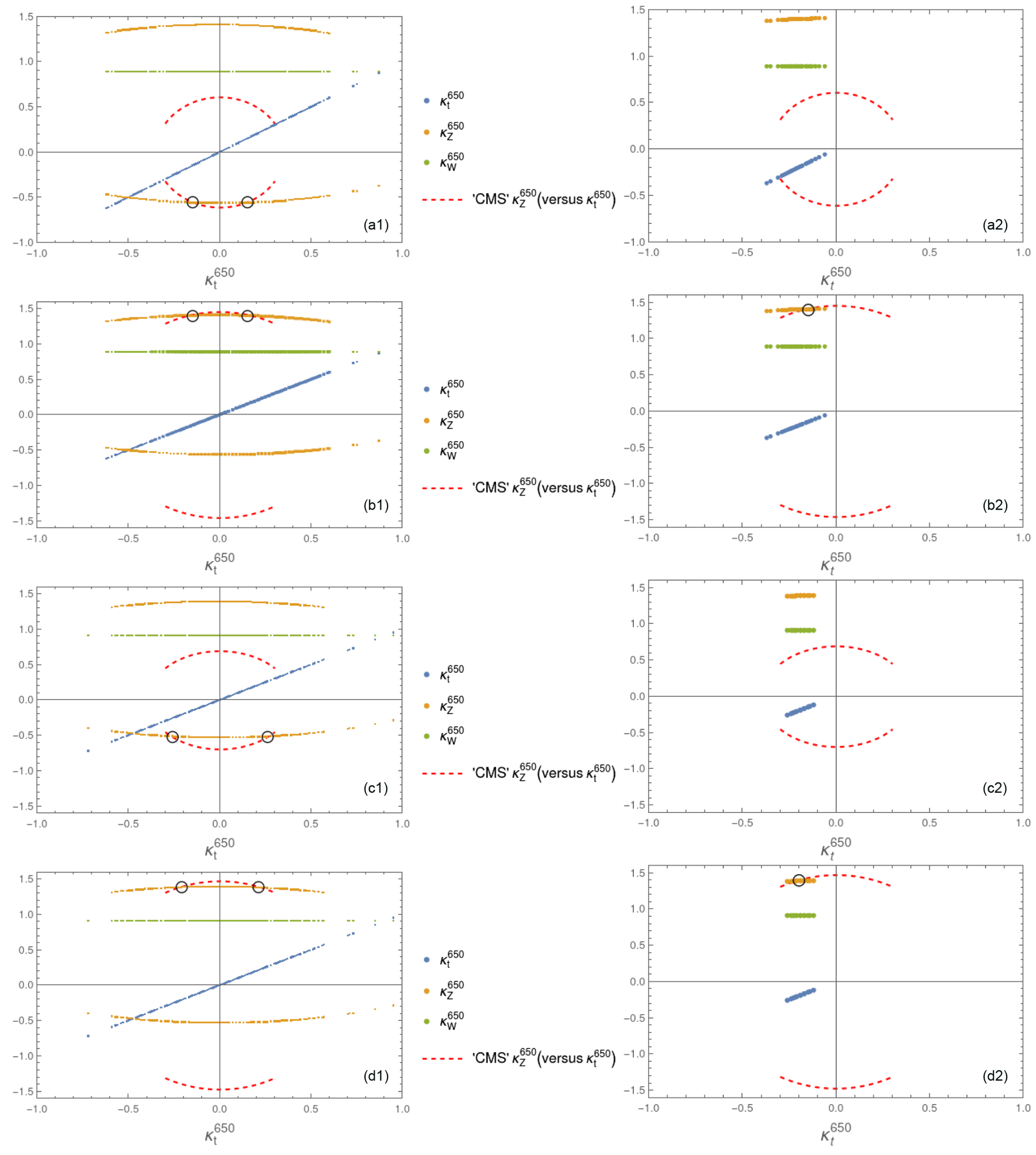

FIG. 2. Parameters and constraints taken from Fig. 1, showing all $\kappa^{H_{650}}$ against $\kappa_t^{H_{650}}$. (a1)–(b2) Correspond to Fig. 1(a): $\kappa_W^{H_{650}} = 0.89$, $u \simeq 78$ GeV, $v_1 \simeq 16$ GeV, $v_2 \simeq 76$ GeV, with (a1),(a2) $c = 0.82$; (b1),(b2) $c = 0.75$. (c1)–(d2) Correspond to Fig. 1(b): $\kappa_W^{H_{650}} = 0.91$, $u \simeq 78$ GeV, $v_1 \simeq 16$ GeV, $v_2 \simeq 76$ GeV, with (c1),(c2) $c = 0.85$; (d1),(d2) $c = 0.78$. (a1),(b1),(c1),(d1) All constraints are imposed except for LEPh95, Eq. (5). (a2),(b2),(c2),(d2) Correspond to overlaying the constraint given by LEPh95, Eq. (5), taken at the 2σ level. The black circles highlight the location of points that are also consistent with the CMS indication for H_{650} [12]. The corresponding full solutions are given in Table III for the cases of (c1) and (d2).

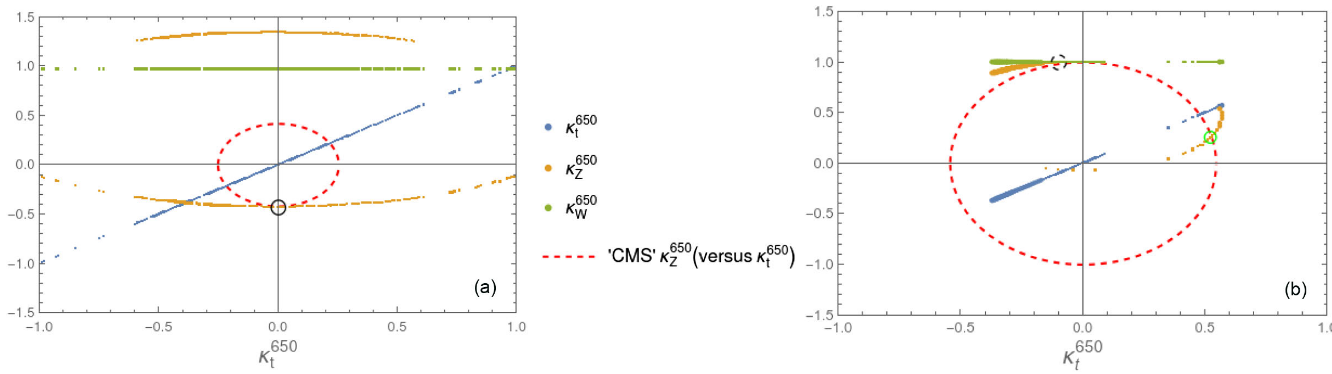

FIG. 3. Similar to Fig. 2: parameters and constraints taken from Fig. 1. (a) Corresponds to Fig. 1(c): $\kappa_W^{H_{650}} = 0.97$, $u \simeq 78$ GeV, $v_1 \simeq 16$ GeV, $v_2 \simeq 76$ GeV. (b) Corresponds to Fig. 1(d): $\kappa_W^{H_{650}} = 1$, $u \simeq 69$ GeV, $v_1 \simeq 14$ GeV, $v_2 \simeq 104$ GeV. For both cases, $c = 1$. The black circles highlight the location of points that are also consistent with the CMS indication for H_{650} [12]. None of these solutions satisfy LEPh95.

lines in the figures. The final solutions are obtained as the overlap between this correlation and the lines of $\kappa_Z^{H_{650}}$. Examples are indicated by the black circles in Figs. 2 and 3.

TABLE III. Fractions of the scalar fields of the gauge basis as contained in the scalar fields of the mass basis, as well as the reduced couplings to fermions and gauge bosons, for the case $\kappa_W^{H_{650}} = 0.91$, $u \simeq 78$ GeV, $v_1 \simeq 16$ GeV, $v_2 \simeq 76$ GeV. (a) corresponds to the solution shown in Fig. 2(d2) that satisfies all constraints including LEPh95, Eq. (5). (b) and (c) correspond to the two solutions shown in Fig. 2(c1) that satisfy all constraints but LEPh95, Eq. (5).

(a)

$c = 0.78$	ϕ_1^0	ϕ_2^0	χ^0	ξ^0	κ_t	κ_Z	κ_W
h_{95}	0.35	−0.45	0.36	−0.74	−1.04	0.47	−0.78
h_{125}	0.73	0.43	0.44	0.29	0.99	1.04	1.02
H_{320}	0.26	−0.77	−0.06	0.57	−1.78	−0.42	0.36
H_{650}	−0.52	−0.09	0.82	0.21	−0.21	1.39	0.91

$u \simeq 78$ GeV, $v_1 \simeq 16$ GeV, $v_2 \simeq 76$ GeV

(b)

$c = 0.85$	ϕ_1^0	ϕ_2^0	χ^0	ξ^0	κ_t	κ_Z	κ_W
h_{95}	−0.62	0.3	0.72	0.03	0.68	1.37	0.76
h_{125}	0.73	0.43	0.44	0.29	0.99	1.04	1.02
H_{320}	0.12	−0.84	0.44	0.29	−1.94	0.43	0.4
H_{650}	−0.25	0.12	−0.3	0.91	0.26	−0.52	0.91

(c)

$c = 0.85$	ϕ_1^0	ϕ_2^0	χ^0	ξ^0	κ_t	κ_Z	κ_W
h_{95}	0.41	0.26	−0.86	−0.13	0.6	−1.39	−0.78
h_{125}	0.73	0.43	0.44	0.29	0.99	1.04	1.02
H_{320}	−0.52	0.86	0.01	0.02	1.96	0.34	0.36
H_{650}	−0.16	−0.11	−0.25	0.95	−0.26	−0.51	0.91

In Fig. 2, corresponding to Figs. 1(a) and 1(b), we compare the cases when LEPh95 is taken into account (the right panels) to the cases where this indication is ignored (the left panels). A common feature is that LEPh95 either eliminates the solutions altogether that are otherwise consistent with all the other indications, see Fig. 2(a1) versus 2(a2) and Fig. 2(c1) versus 2(c2), or requires large values of $\kappa_Z^{H_{650}}$ that can cause tension with the hints for a 650 GeV object decaying to ZZ [10,11], Figs. 2(b1), 2(b2), 2(d1), and 2(d2). Thus, the LEPh95 indications appear to be only marginally consistent, when all the other indications are considered together.

A given solution fixes the couplings and field content of the four scalar states. Table III shows the outcome for the full CP-even sector corresponding to the three solutions for the case $\kappa_W^{H_{650}} = 0.91$. Figure 3 and Table IV show features similar to those discussed above, for the cases $\kappa_W^{H_{650}} = 0.97$ and $\kappa_W^{H_{650}} = 1.0$; these are, however, inconsistent with LEPh95. It is noteworthy that, for all the solutions, the predictions for $\kappa_t^{H_{650}}$ come out naturally small to very small, in line with the CMS indications of suppressed ggF contributions [12].

The solutions are, however, quite sensitive to the precise value of $\kappa_W^{H_{650}}$. Going from 0.97 to 1 drastically changes the solutions for $\kappa_Z^{H_{650}}$, but $\kappa_t^{H_{650}}$ is found to be consistently small. On the other hand, $\kappa_t^{H_{320}}$ is predicted to be significantly large. This could be in tension with the experimental indications [15] of H_{320} decaying mainly to a pair of SM-like Higgs bosons. This would require large triple-scalar couplings to cope with the subdominance of the tt^*, WW, and ZZ decay channels.

We have also checked the consistency of our benchmark points with HiggsTools [62] and, in particular, its sublibrary HiggsBounds-5 [63]. To begin with, let us first drop the triple-scalar couplings and also forget the LEPh95 constraints. We find that three of the five benchmark points in Tables III

TABLE IV. Fractions of the scalar fields of the gauge basis as contained in the scalar fields of the mass basis, as well as the reduced couplings to fermions and gauge bosons, satisfying all constraints except for LEPh95, Eq. (5): (a) corresponds to the solution $\kappa_W^{H_{650}} = 0.97$ shown in Fig. 3(a); (b) corresponds to the solution $\kappa_W^{H_{650}} = 1$ shown in Fig. 3(b).

(a)

$c=1$	ϕ_1^0	ϕ_2^0	χ^0	ξ^0	κ_t	κ_Z	κ_W
h_{95}	0.42	0.24	−0.87	−0.1	0.56	−1.41	−0.76
h_{125}	0.73	0.43	0.44	0.29	0.99	1.04	1.02
H_{320}	−0.48	0.87	0.03	−0.12	1.99	0.38	0.21
H_{650}	−0.24	0	−0.23	0.94	0	−0.43	0.97

$u \simeq 78$ GeV, $v_1 \simeq 16$ GeV, $v_2 \simeq 76$ GeV

(b)

$c=1$	ϕ_1^0	ϕ_2^0	χ^0	ξ^0	κ_t	κ_Z	κ_W
h_{95}	0.24	−0.37	−0.37	0.82	−0.62	−0.8	0.42
h_{125}	0.65	0.59	0.4	0.26	0.99	1.04	1.02
H_{320}	−0.42	0.71	−0.52	0.21	1.19	−0.43	0.22
H_{650}	−0.59	−0.01	0.66	0.47	−0.02	1	1

$u \simeq 69$ GeV, $v_1 \simeq 14$ GeV, $v_2 \simeq 104$ GeV

and IV, namely, Tables III(b), III(c), and IV(a), remain allowed, while Tables III(a) and IV(b) are ruled out by ATLAS [64,65] as well as CMS [48], stemming from the large value of $|\kappa_Z^{H_{650}}|$, which results in an unacceptably large four-lepton rate.

Now let us switch on the LEP constraints from negative searches for light scalars. This immediately rules out the three surviving benchmark points Tables III(b), III(c), and IV(a). This is due to the large $|\kappa_Z^{h_{95}}|$ values that exceed the experimental upper bound of $\simeq 0.48$ for an h_{95} state with SM-like decays [24,66]. This is irrespective of whether one believes in the h_{95} signal indications from LEP.

These five benchmark points illustrate a tension between the LEP and LHC constraints due to a seesaw effect between $\kappa_Z^{H_{650}}$ and $\kappa_Z^{h_{95}}$ resulting from the orthogonality of the rotation matrix $\mathcal{X}$ in Eq. (27). However, one may note that such conclusions need not be true if the triple-scalar couplings, involving one H_{650} state, are large enough to lead to sizable decays to two lighter scalars. This would result in a smaller value of $|\kappa_Z^{H_{650}}|$, as discussed after Eq. (9), and thus in a smaller BR$(H_{650} \to ZZ)$. A detailed discussion of this issue follows now.

2. Focusing on viable solutions

We just saw that, without the triple-scalar couplings, it is hard to find solutions consistent with both LEP and LHC. This is true not only for the benchmark points shown, but over the entire parameter space, the reason being that both $h_{95}ZZ$ and $H_{650}ZZ$ couplings are very tightly constrained. In this subsection, we will see what happens when a sizable triple-scalar coupling is introduced. This is a two-step process. In the first step, we introduce triple-scalar couplings leading to discalar decays of H_{650}.

The triple-scalar couplings can appear through both trilinear and quartic terms in the potential. Let us first start from the dimension-four operator $\lambda(\Xi^\dagger\Xi + X^\dagger X)(\Phi_1^\dagger\Phi_1 + \Phi_2^\dagger\Phi_2)$, where λ is a dimensionless coupling. Let us consider, more specifically, the decay $H_{320} \to h_{125}h_{125}$, originating, e.g., from the term $\lambda X^\dagger X \Phi_2^\dagger \Phi_2$. Taking only the CP-even neutral component, shifting the vacuum and rotating to the mass basis, the contribution to the trilinear coupling $H_{320}h_{125}h_{125}$ comes out to be

$$H_{320} \to h_{125}h_{125}: \quad 2\sqrt{2}\lambda[v_2(x_{23}^2x_{32} + 2x_{22}x_{23}x_{33}) + u(x_{22}^2x_{33} + 2x_{22}x_{23}x_{32})]. \tag{40}$$

It is easy to see that all such terms must be of the form λ times some VEV. As all the VEVs that we consider are less than 100 GeV, and the quartic couplings are expected to be perturbative, we may surmise that the coefficients for the triple-scalar operators coming from the quartic terms in the scalar potential are typically less than 100 GeV.

On the other hand, one may have gauge-invariant trilinear terms of the form

$$\frac{1}{\sqrt{2}}[M_\Xi \Phi_i^\dagger \boldsymbol{\tau}\Phi_j \cdot \Xi + M_X \Phi_i^\top \sigma_2 \boldsymbol{\tau}\Phi_j \cdot \tilde{X} + M_{X\Xi} X^\dagger \boldsymbol{t} X \cdot \Xi], \quad i,j \in \{1,2\}, \tag{41}$$

where the couplings M_Ξ, M_X (taken here, for simplicity, the same for all three scalar doublet combinations) and $M_{X\Xi}$ have mass dimension one, and σ_2 is the second Pauli matrix.[12] If M_X, M_Ξ, and $M_{X\Xi}$ are at least of the order of 100 GeV or more, we may neglect, as a first approximation, the contributions coming from the quartic terms. For example, the M_X term in Eq. (41) gives rise to a factor of

$$\frac{M_X}{\sqrt{2}}(x_{22}^2x_{33} + 2x_{22}x_{23}x_{32}) \tag{42}$$

for the same decay, namely, $H_{320} \to h_{125}h_{125}$, and this does not involve any VEVs. We will thus illustrate examples of consistent benchmark points in terms of the free couplings M_Ξ, M_X, and $M_{X\Xi}$.

[12] We refer the reader to [27] for the definitions of the $\boldsymbol{\tau}$ and $\boldsymbol{t}$ SU(2)-representation matrices in the spherical basis relevant for the vector representation of the triplets. The $\frac{1}{\sqrt{2}}$ normalization factor allows direct matching with the couplings of the same operators in the matrix representation of the triplets [43].

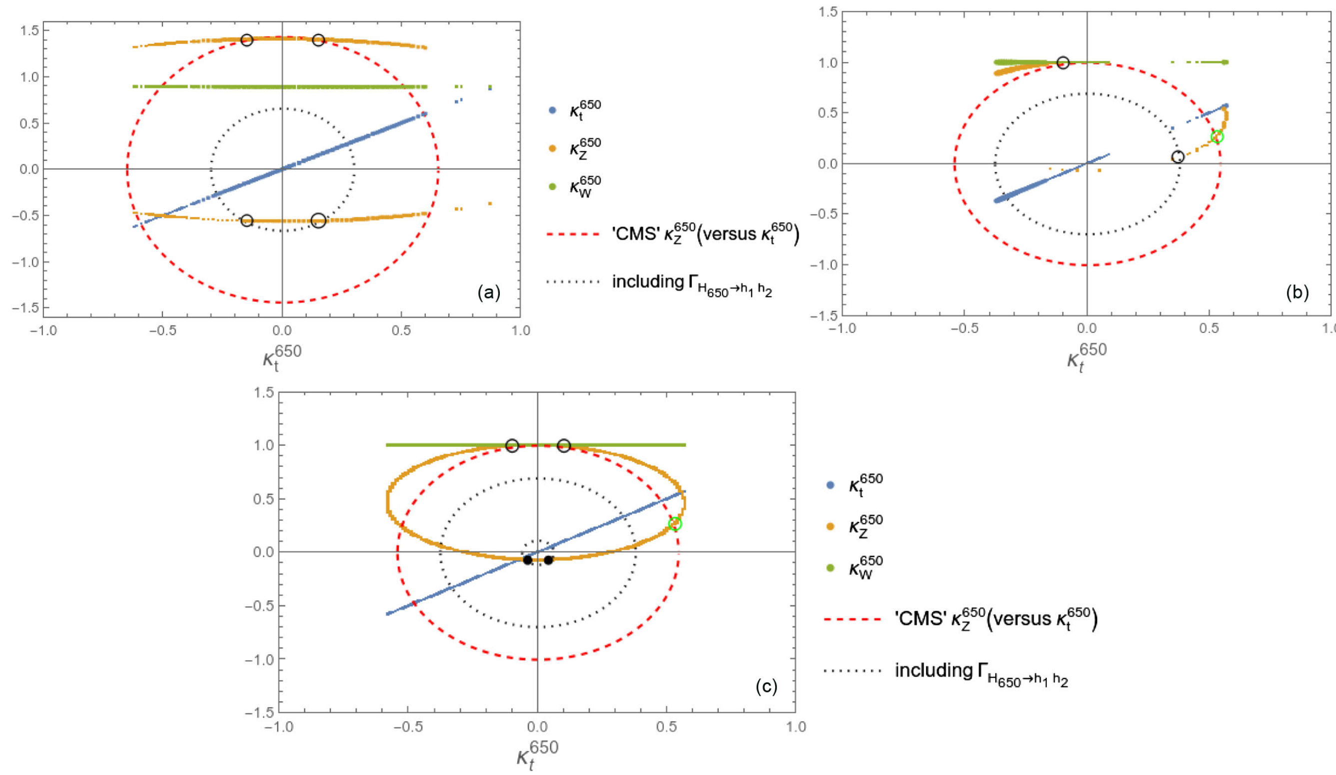

FIG. 4. (a),(b) The effect of $H_{650} \to h_1 h_2$ decay: (a) Taking $\Gamma_{H_{650}\to h_1 h_2} \simeq 7.7$ GeV illustrates the modification to the solutions given in Fig. 2(b1), where $\kappa_W^{H_{650}} = 0.89$, $u \simeq 78$ GeV, $v_1 \simeq 16$ GeV, $v_2 \simeq 76$, and $c = 0.75$. (b) Taking $\Gamma_{H_{650}\to h_1 h_2} \simeq 3.5$ GeV illustrates the modification to the solution given in Fig. 3(b), where $\kappa_W^{H_{650}} = 1.0$, $u \simeq 69$ GeV, $v_1 \simeq 14$ GeV, $v_2 \simeq 104$ GeV, and $c = 1$. (c) Illustrates the combined effect of relaxing the constraint $|\kappa_W^{H_{320}}|, |\kappa_Z^{H_{320}}| \lesssim 0.45$, cf. Table II, with the remaining input as in (b), and $H_{650} \to h_1 h_2$ decays: $\Gamma_{H_{650}\to h_1 h_2} \simeq 5$ GeV (respectively, $\simeq$3.5 GeV) corresponds to the smaller (respectively, larger) black dotted ellipse. The two black blobs indicate the solutions given in Table V. See also the main text for further discussions.

Before considering explicitly the possible sources of H_{650} decays to lighter scalars in the 2HDeGM, we stress first that the inclusion of such decay channels, although necessary, would not be sufficient to delineate viable portions of the parameter space. We show, in Figs. 4(a) and 4(b), the effect of the two-body decays $H_{650} \to h_1 h_2$, where $h_{1,2}$ can be any of the three lighter scalar states. The red dashed ellipses now shrink to the black dotted ones as a result of the substitution in Eq. (9) according to Eq. (11). The new solutions correspond to the intersections of the black dotted ellipses with the orange lines, yielding indeed smaller values for $|\kappa_Z^{H_{650}}|$ than the ones initially obtained in Figs. 2(b1) and 3(b), and with $\mathrm{BR}^*_{H_{650}\to h_1 h_2} \simeq 6\%$ and 3%, respectively, that remains sufficiently small to be consistent with the nonobservation of these channels. However, this does not ameliorate the seesaw effect noted in the previous subsection. For instance, the solution with $\kappa_Z^{H_{650}} \simeq -0.55$ in Fig. 4(a) and the solution with $\kappa_Z^{H_{650}} \simeq 0.07$ in Fig. 4(b) imply $|\kappa_Z^{h_{95}}| \simeq 1.4$ and 1.2, respectively, values that are way above the LEP upper bounds.

We now go one step further. It turns out that, to reduce the seesaw effects, one needs to relax the bounds shown in the last line of Table II. As stressed previously, these bounds on $|\kappa_Z^{H_{320}}|$ and $|\kappa_W^{H_{320}}|$ are merely qualitative working assumptions. By relaxing them, one allows the seesaw effect to be shared among the κ's of the three states H_{650}, H_{320}, and h_{95} and to populate regions where $|\kappa_Z^{h_{95}}|$ and $|\kappa_Z^{H_{650}}|$ take small values simultaneously, thus satisfying the experimental limits. This is illustrated in Fig. 4(c), which corresponds to the same input as for Fig. 4(b) but with $|\kappa_Z^{H_{320}}|$ and $|\kappa_W^{H_{320}}|$ allowed to vary from 0 to 1.5. The orange line is now a complete ellipse and viable solutions can be obtained for moderately small widths for $H_{650} \to h_1 h_2$. As an example, assuming a width of $\simeq$5 GeV that corresponds to $\mathrm{BR}^*_{H_{650}\to h_1 h_2} \lesssim 6\%$, and restricting to $|\kappa_t^{H_{650}}|, |\kappa_t^{H_{320}}| \lesssim 0.1$, one finds the two benchmark points, albeit very close in the parameter space (because the space is so tightly constrained), given in Table V for which the h_{95} and H_{650} states satisfy all the corresponding constraints discussed so far in the paper (on top of those for the SM-like h_{125} state).

The second step is not yet complete. It still remains to be seen whether H_{320} complies with the corresponding experimental indications, in particular, since now $|\kappa_Z^{H_{320}}|$ and $|\kappa_W^{H_{320}}|$ are allowed to have large values. Taking the numbers

TABLE V. New benchmark points assuming $\mathrm{BR}^*_{H_{650}\to h_1 h_2} \lesssim 6\%$, requiring $|\kappa_t^{H_{650}}|, |\kappa_t^{H_{320}}| \lesssim 0.1$ and relaxing $|\kappa_W^{H_{320}}|, |\kappa_Z^{H_{320}}| \lesssim 0.45$, for which the h_{95} and H_{650} constraints are satisfied. See main text for further discussions.

(a)

$c = 1$	ϕ_1^0	ϕ_2^0	χ^0	ξ^0	κ_t	κ_Z	κ_W
h_{95}	−0.48	0.8	−0.27	−0.22	1.34	0.01	−0.02
h_{125}	0.65	0.59	0.4	0.26	0.99	1.04	1.02
H_{320}	−0.46	−0.02	0.88	−0.15	−0.03	1.35	0.48
H_{650}	−0.37	0.02	−0.03	0.93	0.03	−0.07	1.0

(b)

$c = 1$	ϕ_1^0	ϕ_2^0	χ^0	ξ^0	κ_t	κ_Z	κ_W
h_{95}	−0.47	0.8	−0.31	−0.17	1.34	−0.06	0.0
h_{125}	0.65	0.59	0.4	0.26	0.99	1.04	1.02
H_{320}	−0.48	0.02	0.86	−0.16	0.03	1.35	0.48
H_{650}	−0.35	−0.02	−0.02	0.94	−0.03	−0.07	1.0

$u \simeq 69$ GeV, $v_1 \simeq 14$ GeV, $v_2 \simeq 104$ GeV

in Table V as a benchmark and using HiggsBounds-5 [63], we find that the total decay width of H_{320} should be increased by 250% to satisfy the experimental limits on VBF-produced objects decaying to a pair of Z bosons, [67], while decays to W's and fermions remain within the experimental limits. This means a decay branching fraction into final states, other than gauge boson and fermion pairs, of about 70%, which is in accordance with the experimental indications of decays to pairs of h_{125}, see Table I. One will, however, have to cope with exclusion limits applicable to both H_{650} and H_{320}, from searches for heavy scalars decaying to a SM-like Higgs boson and a lighter scalar [68,69].

The trilinear couplings among the physical states, $H_{650}h_a h_b$ and $H_{320}h_a h_b$, where $a, b = 95$ and/or 125, are uniquely fixed in terms of the x_{ij}'s and the three mass parameters. Moreover, the production cross sections for H_{650} and H_{320}, as well as their decays to fermions and gauge bosons, are fixed by the knowledge of the corresponding reduced couplings κ to fermions and gauge bosons. We have computed the branching ratio $\mathrm{BR}[H_{650} \to \sum_{a,b} h_a h_b]$ of the H_{650} total decay to lighter scalars, requiring it to be $\lesssim 6\%$ that corresponds to the solution given in Fig. 4(c). We also computed the production cross sections times branching ratios $\sigma(pp \to H_{650} \to h_{95}(b\bar{b})h_{125}(\tau^+\tau^-))$ and $\sigma(pp \to H_{320} \to h_{95}(b\bar{b})h_{125}(\tau^+\tau^-))$, akin to the limits from Fig. 5 of Ref. [68], and $\sigma(pp \to H_{650} \to h_{95}(b\bar{b})h_{125}(\gamma\gamma))$ and $\sigma(pp \to H_{320} \to h_{95}(b\bar{b})h_{125}(\gamma\gamma))$, akin to the limits from Fig. 3 of Ref. [69]. Specifying to the benchmark points of Table V, we find that all the experimental constraints discussed above can be satisfied, the one for $\sigma(pp \to H_{320} \to h_{95}(b\bar{b})h_{125}(\gamma\gamma))$ being the most stringent. Taking a conservative upper bound of $\simeq 0.8$ fb for the latter [69], we find

$$
\begin{aligned}
&115 \lesssim M_X \lesssim 150\,\mathrm{GeV}, \quad |M_\Xi| \lesssim 150\,\mathrm{GeV},\\
&400 \lesssim M_{X\Xi} \lesssim 500\,\mathrm{GeV},
\end{aligned} \tag{43}
$$

a range compatible with the physical masses of the four scalars. Relaxing the $\sigma(pp \to H_{320} \to h_{95}(b\bar{b})h_{125}(\gamma\gamma))$ bound to, say, 1.5 fb opens up several disjoint domains in the $(M_X, M_\Xi, M_{X\Xi})$ space, the one compatible with the four physical masses being enlarged to

$$
\begin{aligned}
&200 \lesssim M_X \lesssim 430\,\mathrm{GeV}, \quad 150 \lesssim M_\Xi \lesssim 300\,\mathrm{GeV},\\
&200 \lesssim M_{X\Xi} \lesssim 560\,\mathrm{GeV}.
\end{aligned} \tag{44}
$$

Finally, we give in Table VI the production cross section, which is dominated by the VBF process, and the main decay widths of H_{650}, resulting from the benchmark points of Table V that are consistent with all the constraints.

In summary, the discussion in this subsection has shown why the allowed space is so tightly constrained. Based on this discussion, one can now see pictorially from Figs. 2 and 3(a) why none of these configurations can give viable solutions even when including decays to scalars and/or relaxing the last constraint of Table II. These configurations correspond to the three sets of values of u, v_1, v_2, and $\kappa_W^{H_{650}}$ corresponding to Figs. 1(a)–1(c). The rationale for discussing these benchmark points is to show explicitly how the parameter space gets constrained by data. We are thus left with the set of values $u \simeq 69$, $v_1 \simeq 14$, $v_2 \simeq 104$ GeV and $\kappa_W^{H_{650}} = 1$ corresponding to Fig. 1(d) that lead to the benchmark points of Table V, thus determining allowed regions in the vicinity of these points. However, one should not conclude that the only consistent solutions are restricted to these regions. For instance, allowing now $\kappa_t^{H_{650}}$ and $\kappa_t^{H_{320}}$

TABLE VI. The VBF production cross section (see also footnote 6) and main decay widths of H_{650}, for the benchmark point of Table V(a) corresponding to the solution given in Fig. 4(c). The illustrated value of the total decay width to lighter scalars, $\Gamma_{H_{650}\to\sum_{a,b} h_a h_b}$, corresponds to $M_X \simeq 148$, $M_\Xi \simeq -60$, and $M_{X\Xi} \simeq 500$ GeV, consistent with the conservative bounds given by Eq. (43).

Benchmark	$\sigma^{\mathrm{VBF}}_{pp\to H_{650}jj}$ (fb)	$\Gamma_{H_{650}\to W^+W^-}$ (GeV)	$\Gamma_{H_{650}\to ZZ}$ (GeV)	$\Gamma_{H_{650}\to t\bar{t}}$ (GeV)	$\Gamma_{H_{650}\to\sum_{a,b} h_a h_b}$ (GeV)
Table V(a)	124	81.1	0.19	0.02	5

to lie in the range $0.1 \lesssim |\kappa_t^{H_{650}}|, |\kappa_t^{H_{320}}| \lesssim 0.2$, one finds several other benchmark points with the latter $|\kappa_t|$'s ranging from 0.14 to 0.19 and acceptable regions around these points, each of them leading to acceptable ranges for M_X, M_Ξ, and $M_{X\Xi}$, similar to the ones discussed above. It remains true, though, that the compilation of all available constraints reduces drastically the possibility of fitting simultaneously the four scalar resonances. In particular, very stringent constraints originate from requiring small $\kappa_Z^{H_{650}}$ and $\kappa_Z^{h_{95}}$, due, respectively, to the present limits on four-lepton events from heavy resonance decays and to limits on Higgsstrahlung production of light scalars decaying to fermions.

E. Indirect constraints

For the indirect constraints, like those coming from flavor physics, or muon $(g-2)$, one needs the fermionic couplings of the scalars. Only the SU(2) doublets couple with fermions (if we neglect the lepton-number violating coupling of the triplets). Thus, one expects the flavor constraints to be typically close to those coming from the different two-Higgs doublet models.

Muon $(g-2)$ shows a tension of more than 5σ with the SM expectation [25],

$$\Delta a_\mu = a_\mu^{\mathrm{exp}} - a_\mu^{\mathrm{SM}} = [24.9 \pm 2.2(\mathrm{expt}) \pm 4.3(\mathrm{theo})] \times 10^{-10}. \tag{45}$$

The dominant contribution from extra neutral scalars is the one-loop magnetic moment amplitude, mediated by CP-even or CP-odd scalars. A similar contribution comes from the charged Higgs mediated diagram too. The coupling is proportional to $\tan\beta$ for type-II and type-X models and $\cot\beta$ for type I and type Y. Thus, we expect the tension Δa_μ to go down for type-II and type-X models and remain the same for type-I and type-Y models, if we work in the large $\tan\beta$ limit. One must remember that, in 2HDeGM, components of Φ_1 and Φ_2 are distributed among all the physical scalars, some of them being quite heavy, so we do not expect as large a contribution as in 2HDMs.

The most important constraint from quark flavor physics is on the mass of the charged Higgs boson m_{H^+} that comes from $b \to s\gamma$. The charged Higgs boson contribution tends to increase the difference between the theoretical prediction from SM and the experimental number and so gives a very tight constraint on the H^+ mass when such contributions are significant. For 2HDM type II and type Y, Ref. [70] finds an absolute limit of $m_{H^+} > 583$ GeV at 95% confidence limit, which is valid for all $\tan\beta$. Bounds from type I and type X are much weaker; for $\tan\beta = 1$, one finds $m_{H^+} > 445$ GeV at 95% confidence limit, but the lower limit drops sharply with increasing $\tan\beta$. Thus, for these two types of 2HDM, there is essentially no constraint from flavor physics in the large $\tan\beta$ limit. A similar conclusion holds for $B_s \to \mu^+\mu^-$.

Other indirect constraints come from Higgs boson signal strengths and oblique parameters. Authors of Ref. [71] performed a global fit of 2HDM by combining all theoretical and experimental bounds and presented the allowed parameter space. The latest Run II signal strength data from LHC have been imposed and the results show that the deviation from the alignment limit $\beta - \alpha = \pi/2$ cannot be larger than 0.26, 0.055, 0.069, and 0.056 for 2HDM types I, II, X, and Y, respectively, at 2σ confidence level. The fit includes electroweak precision observables too.

One may wonder whether the constraints on the parameter space of the canonical GM model [26], like those on the triplet VEV u, are still valid. For example, Ref. [72] finds u to be about 40 GeV or less, mostly from $b \to s\gamma$. From a global fit, the authors of Ref. [73] draw a similar conclusion. We have consistently worked with a larger value of u. One may note that not only does our model have more fields than canonical GM, but there is also no degeneracy among the member of the custodial multiplets. The latter fact opens up new decay channels, like that of the doubly charged Higgs boson, and so the constraints coming from the LHC search on $H^{++} \to W^+W^+$ need not be valid any longer. The former fact may also allow for possible cancellation among different radiative corrections.

IV. FURTHER RAMIFICATIONS

A. Charged scalars

2HDeGM is a minimalistic model to explain all the indications, and it can be pruned down further if some of the indications go away. If they all persist, one would like to ask where the charged scalars and the CP-odd scalars are.

We note that there are some mild hints. For example, the CMS Collaboration [39] shows a mild excess for $H^+ \to W^+Z \to 3\ell + \nu$ at $m_{H^+} \sim 375$ GeV and another mild excess for $H^{++} \to W^+W^+ \to 2\ell + e\nu$ at $m_{H^{++}} \sim 450$ GeV. They do not quote any numbers. Surprisingly, ATLAS also found a mild signal of H^+ at 375 GeV [40], where the production is through VBF, with a local (global) significance of $2.8(1.6)\sigma$. They also found [41] a $3.2(2.5)\sigma$ local (global) significance signal for H^{++} at 450 GeV, decaying to W^+W^+.

There are indeed very strong bounds on the mass of H^{++} of the canonical GM model from the CMS Collaboration [39]. From the nonobservation of $H^{++} \to W^+W^+$, a lower bound of 2 TeV $\lesssim m_{H^{++}}$ has been inferred. Similarly, the nonobservation of $H^+ \to W^+Z$ gives a lower bound 1.1 TeV $\lesssim m_{H^+}$. They have been obtained for $\sin\theta_H = 1$, while the VBF cross section and hence the total number of events depend on $\sin^2\theta_H$. Even without taking that into account, we have seen in the previous section that such bounds are not watertight. For example, in 2HDeGM where the scalar custodial multiplets are not degenerate, H^{++} can decay to H^+W^+, where H^+ is some singly charged scalar of the model. It can also decay to two singly charged scalars. While the former branching ratio can be estimated from the knowledge of the charged Higgs boson mass matrix (i.e., the components of the custodial fiveplet singly charged scalar in different mass eigenstates), the latter needs a knowledge of the scalar potential and hence cannot even be estimated at this stage. What can definitely be said is that the CMS bounds should not hold any longer and the mass of H^{++} can indeed be less than 1 TeV.

A similar conclusion holds for all the singly charged scalars. If they are above the $t\bar{b}$ threshold and have a significant SU(2) doublet component, they should be searched in the $H^+ \to t\bar{b}$ channel; otherwise, one has to look for $H^+ \to c\bar{b}, \tau^+\nu_\tau$. If the charged scalars are sufficiently apart, e.g., one close to 375 GeV and the other, say, at about 130 GeV, there can be a cascade like $H_1^+ \to H_2^+ + Z$.

B. *CP*-odd neutral scalars

One possible CP-odd scalar is A_{400}. For the CP-odd scalars, there is a different mixing matrix between the mass and the gauge eigenstates, one of whose rows is determined from the fact that, among the three CP-odd scalars in the theory, one has to be the neutral Goldstone boson. To determine the matrix, it is important to get the rate for $A_{400} \to t\bar{t}$. One may note that the production must proceed through ggF or associated production with $t\bar{t}$.

2HDeGM can accommodate two CP-odd neutral scalars, and hopefully the second slot will be taken up by A_{151} when it is confirmed by the experimentalists. Since these are broad resonances, there is also the possibility that two or more resonances are closely spaced and cannot be resolved. There is also the possibility that the scalar potential is not CP invariant, and hence the mass eigenstates are no longer CP eigenstates. These issues may all be resolved in the next e^+e^- or muon collider.

V. SUMMARY AND CONCLUSION

Over the last few years, there have been strong hints of more resonances from the LHC experiments, at around 95 and 650 GeV, and possibly at 320 and 400 GeV, even though none of them are officially confirmed as yet. Assuming that these are all scalar resonances, we try to accommodate them, together with the SM-like 125 GeV resonance, in a minimalist model. As far as we know, this is the first attempt to include all such hints. To be conservative, we take into account only the significances announced by the experimental collaborations. We thus find the combined global significance for the 95 GeV resonance to be around 3σ, while that for the 650 GeV resonance has reached the 4σ level. Another resonance at around 400 GeV, which is in all probability a CP-odd neutral scalar, has also breached the 3σ mark, and the significance of the one at 320 GeV is just below 3σ.

We first show that the coupling strengths of these new scalars to the weak gauge bosons predict the existence of doubly charged scalars, if the underlying theory is renormalizable and the unitarity bounds respected. This, in turn, leads us to discard the simplest scalar extensions like those with $\mathrm{SU}(2)_L$ singlets and doublets and help us to arrive at the minimal model, with two $\mathrm{SU}(2)_L$ doublets, one real triplet, and one complex triplet. The triplet structure is similar to the Georgi-Machacek model, with identical VEVs for the triplet fields to ensure $\rho = 1$ at the tree level, except that the custodial symmetry should not be respected in the scalar sector and hence the custodial multiplets are no longer mass degenerate. We call this the 2HDeGM model. This model accommodates not only all the neutral scalars, both CP even and CP odd, but also singly and doubly charged scalars for which there are some mild hints from both ATLAS and CMS Collaborations.

While the model may seem to contain a lot of fields and free parameters, and the amount of data may seem scant, we note with pleasant surprise that even the existing data, taken at face value, severely constrain the allowed parameter space, without even going into a detailed discussion of the scalar potential and resulting constraints. This, in turn, indicates very narrow ranges for the couplings of the 650 GeV scalar H_{650} to the electroweak gauge bosons. This also indicates a large discalar decay width for H_{320}, which is still allowed from the CMS and ATLAS analyses, but more data and/or a more sensitive analysis may easily rule this model out.

We assume CP to be conserved in the scalar sector and all mass eigenstates to be CP eigenstates also. Within the narrow ranges of the couplings, rotating the four CP-even states to their mass eigenstates entails theoretical consistency constraints that cut down the possibilities to very few solutions. Let us summarize the major characteristics of the solutions:

(i) Over most of the parameter space, defined in terms of the masses, or the parameters of the scalar potential, there exists irreconcilable tension among the various observables, coming mostly from the LHC. Two of the most important constraints are the paucity of events in the four-lepton channels coming from the decay of H_{650} and the constraints on a light Higgs boson at 95 GeV produced through Higgs-strahlung and decaying into light fermions that put strong upper bounds on $\kappa_Z^{H_{650}}$ and $\kappa_Z^{h_{95}}$. While $\kappa_Z^{H_{650}}$ has to be small, the couplings of H_{650} with W^+W^- can be large (and have to be large to be consistent with the CMS indications).

(ii) The orthogonality of the $\mathcal{X}$ matrix makes the implementation of the above condition difficult. We find that we must allow for a large discalar decay width for H_{320}, in all probability to $h_{95}h_{125}$.

(iii) There are many ways to refine the analysis, but almost all of them require a thorough knowledge of the scalar potential, which we do not have. Even then, it is interesting to note that we can put nontrivial constraints on the allowed values of the trilinear scalar couplings. The decay $h_{95} \to \gamma\gamma$ can also be modified by singly and doubly charged scalars in the loop, although such effects are expected to be small.

(iv) We also find that, under conservative assumptions about the signs of the SM-like Yukawa couplings, type-I Yukawa structure is favored over the three other conventional types.

Given the large number of searches for new resonances by ATLAS and CMS, few excesses above 2σ are bound to occur, eventually disappearing with more statistics. Nonetheless, in the absence of robust theoretical guide to where physics beyond the SM, if any, might show up, the interpretations of excesses remains a precious tool that might shed light on the right way to go. Our study illustrates possibilities and issues when trying to interpret simultaneously several excesses within a given theoretical model. It also sets the stage for further investigations depending on future data. For example, (i) $H_{650} \to W^+W^-$ and/or ZZ and also $t\bar{t}$ if backgrounds can be controlled, (ii) $H_{320} \to W^+W^-, ZZ$, and (iii) $h_{95} \to \tau^+\tau^-$ in associated production with the top, should be prioritized, while one should also look for a heavy scalar decaying to two light scalars or a light scalar and a weak gauge boson. We would also like to mention that this is an exploratory exercise that illustrates the difficulties when one tries to fit several resonances simultaneously, even for next-to-minimal extensions of the SM. A further extension of the model that we propose should be much less constrained, but that is outside the scope of this paper.

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DATA AVAILABILITY

The data that support the findings of this article are not publicly available upon publication because it is not technically feasible and/or the cost of preparing, depositing, and hosting the data would be prohibitive within the terms of this research project. The data are available from the authors upon reasonable request.

APPENDIX A: DETERMINING v_1, v_2, u AND THE x_{ij}'s

1. Information from h_{125}

The reduced couplings $\kappa_W^{h_{125}}$ and $\kappa_Z^{h_{125}}$ are defined as follows:

$$g_{h_{125}WW} = g^2 \frac{v}{\sqrt{2}} \kappa_W^{h_{125}}, \qquad g_{h_{125}ZZ} = g^2 \frac{v}{2\sqrt{2}\cos^2\theta_W} \kappa_Z^{h_{125}}, \tag{A1}$$

where $g_{h_{125}WW}$ and $g_{h_{125}ZZ}$ denote the true couplings. The SM couplings are retrieved in the limit $\kappa_W^{h_{125}} = \kappa_Z^{h_{125}} = 1$ and taking $\cos^2\theta_W = M_W^2/M_Z^2$ at the tree level.

Let us assume $\kappa_W^{h_{125}}$ and $\kappa_Z^{h_{125}}$ are extracted from experiment (ignoring experimental errors at this stage) and try to determine v_1, v_2, and u in general, that is, without necessarily adopting the "alignment limit" $h_{125} \approx h_{\rm SM}$. Of course this limit will be a special case of the general solutions. One reason for trying the general solution is to fully control the value of u which is essential for the discussion of the constraints on the doubly charged Higgs boson. Furthermore, once the solutions are obtained, experimental errors can be easily propagated.

Equations (30) and (31) give the theoretical expressions of the scalar to W^+W^- and ZZ couplings, with, for the latter, an extra symmetry factor of 2 to account for the identity of the two Z bosons. After canceling some common factors, the relevant expressions for the reduced couplings read

$$v\kappa_W^{\mathcal{H}_a} = v_1 x_{a1} + v_2 x_{a2} + 2u(x_{a3} + \sqrt{2}x_{a4}), \tag{A2}$$

$$v\kappa_Z^{\mathcal{H}_a} = v_1 x_{a1} + v_2 x_{a2} + 4u x_{a3}, \tag{A3}$$

where $a = 1, \ldots, 4$. Let us consider, in particular, the reduced couplings involving h_{125},

$$v\kappa_W^{h_{125}} = v_1 x_{21} + v_2 x_{22} + 2u(x_{23} + \sqrt{2}x_{24}), \tag{A4}$$

$$v\kappa_Z^{h_{125}} = v_1 x_{21} + v_2 x_{22} + 4u x_{23}, \tag{A5}$$

$$v^2 = v_1^2 + v_2^2 + 4u^2, \tag{A6}$$

where we also added the constraint on the VEVs to reproduce the weak scale (in our normalization, $v = 246/\sqrt{2} \simeq 174$ GeV). Note that, since the linear combination of v_1 and v_2 is identical in Eqs. (A4) and (A5), strict equality of the two reduced couplings $\kappa_W^{h_{125}}$ and $\kappa_Z^{h_{125}}$ implies either $u = 0$ or $x_{23} - \sqrt{2}x_{24} = 0$.

Barring the $u = 0$ solution, the two equations become degenerate when

$$x_{23} - \sqrt{2}x_{24} = 0. \tag{A7}$$

Even when adding Eq. (A6), one has now some freedom in the values of the VEVs v_1, v_2, u even for fixed x_{2i}'s.

To proceed as already anticipated in Sec. III C, we relax the above relation assuming the two reduced couplings not equal,

$$\delta = \kappa_Z^{h_{125}} - \kappa_W^{h_{125}} \neq 0, \tag{A8}$$

which leads to the consistency relation implied by Eqs. (A4) and (A5), namely, Eq. (33). Equations (A4) and (A5) degenerate then into one single equation,

$$\kappa_W^{h_{125}} = \delta + \frac{1}{v}(v_1 x_{21} + v_2 x_{22} + 4\sqrt{2}u x_{24}). \tag{A9}$$

Moreover, in order to treat the various Yukawa-type couplings within the same setup, we take κ_d and κ_u defined by Eq. (34) as input. Choosing $|\kappa_d| \simeq 1$ *and* $|\kappa_u| \simeq 1$ within the experimental error bars obviously means we are considering type II (or type Y and possibly type X). On the other hand, taking $|\kappa_u| \simeq 1$ within the experimental error bars for all quarks and charged leptons, and $|\kappa_d|$ outside the error bars, means implicitly that we consider type I (and exclude type X). Thus, taking κ_d defined by Eq. (34) as a free input allowed to be very different from 1 is interpreted as a parametrization of how much one deviates from type II. Moreover, taking κ_d and κ_u as input has the merit of transforming Eq. (A9) into a linear equation in v_1^2, v_2^2 which, coupled with Eq. (A6), give straightforwardly v_1^2 and v_2^2 in terms of $\kappa_W^{h_{125}}, \delta, u, x_{24}, \kappa_d$, and κ_u.

Since we stick to real-valued VEVs, the conditions $v_1^2 > 0$ and $v_2^2 > 0$ imply constraints that can be recast in terms of allowed intervals on x_{24} that depend on the remaining input parameters $\kappa_W^{h_{125}}, \delta, \kappa_d, \kappa_u$, as well as u, the latter being so far a free parameter. In addition, assuming all x_{2j} real valued, the normality condition $\sum_j x_{2j}^2 = 1$ should be required, cf. Eq. (28). Using Eqs. (33) and (34) and the expressions of v_1^2 and v_2^2 in this condition leads to a twofold solution for x_{24} again in terms of $\kappa_W^{h_{125}}, \delta, \kappa_d, \kappa_u$, and u. The real valuedness of x_{24} is then found to imply a lower bound on $|u|$.

All the above constraints are controlled analytically. Finally, putting together any of the twofold solutions for x_{24} and its allowed domain that secures $v_1^2 > 0$ and $v_2^2 > 0$ leads to upper bounds on $|u|$. The value of u can then be arbitrarily chosen within the determined upper and lower bounds on $|u|$ and only the solution fulfilling $|x_{24}| < 1$ is consistently kept. The VEVs v_1 and v_2 are then uniquely fixed (up to global signs), as well as x_{21}, x_{22}, and x_{23} through Eqs. (33) and (34).

In summary, providing the data for the couplings of the SM-like state h_{125} to W, Z and fermions and a consistent choice of the triplet VEV, fix entirely the doublets VEVs and the doublet/triplet content of h_{125}. The next step is to use plausible data for the other scalars, extracted from the experimental indications, in order to fully determine the neutral CP-even sector's couplings. We describe the procedure in the following section.

2. Information from h_{95} and H_{650}

Since v_1, v_2, u and the x_{2i}'s are now determined, one still needs to reconstruct the rest of the orthogonal matrix $\mathcal{X}$, with all its entries real valued. The set of constraints Eq. (28) boils down to reconstructing an orthonormal basis in four dimensions starting from the known vector (x_{2i}). To obtain, say, (x_{1i}), one generates randomly three four-dimensional vectors (hopefully) forming with (x_{2i}) a linearly independent set, then construct, e.g., via the Gram-Schmidt method, an orthonormal basis for the three-dimensional space orthogonal to (x_{2i}). The vector (x_{1i}) lives on the unit sphere centered at the origin in this space and thus depends on two angles. These angles are then fixed from the data of two reduced couplings, e.g., $\kappa_W^{h_{95}}$ and $\kappa_t^{h_{95}}$, upon use of

$$\kappa_W^{h_{95}} = \frac{1}{v}(v_1 x_{11} + v_2 x_{12} + 2u(x_{13} + \sqrt{2}x_{14}))$$
$$\text{and} \quad \kappa_t^{h_{95}} = \frac{v}{v_2} x_{12}. \tag{A10}$$

Knowing (x_{1i}) and (x_{2i}), the same Gram-Schmidt procedure is repeated to determine, say, the unit vector (x_{4i}) which is parametrized by one angle in the two-dimensional space orthogonal to (x_{1i}) and (x_{2i}). This angle is then determined from the data of, for example, $\kappa_W^{H_{650}}$ through the relation

$$\kappa_W^{H_{650}} = \frac{1}{v}(v_1 x_{41} + v_2 x_{42} + 2u(x_{43} + \sqrt{2}x_{44})). \tag{A11}$$

Finally, the remaining vector (x_{3i}) is determined (up to a global sign), by applying Gram-Schmidt on $(x_{1i}), (x_{2i}), (x_{4i})$ and a randomly chosen four-dimensional vector.

APPENDIX B: GETTING THE PARAMETER SPACE FOR h_{95}

Let us briefly show how we got the allowed regions in the $\kappa_t - \kappa_W$ plane for h_{95} from the decays $h_{95} \to \gamma\gamma$ and $h_{95} \to \tau^+\tau^-$.

Suppose the interaction Lagrangian is

$$\mathcal{L}_{\text{int}} = -\frac{g m_t \kappa_t}{2 m_W} \bar{t} t h + g m_W \kappa_W W_\mu^+ W^{\mu-} h - \frac{g m_H^2 \kappa_H}{m_W} H^+ H^- h, \tag{B1}$$

where $h = h_{95}$, but it can be any other neutral scalar, even the SM Higgs boson. $\mathcal{L}_{\rm int}$ also includes the contribution of some possible charged scalars in the theory.

The decay width is given by [75]

$$\Gamma(h \to \gamma\gamma) = \frac{\alpha^2 g^2}{1024\pi^3} \frac{m_h^3}{m_W^2} \left| \sum\nolimits_i N_i e_i^2 \kappa_i F_i \right|^2, \quad \text{(B2)}$$

where i can be 0, $\frac{1}{2}$, or 1, depending on the spin of the particle in the loop, and $\kappa_0 = \kappa_H$, $\kappa_{1/2} = \kappa_t$, $\kappa_1 = \kappa_W$. They modify the SM couplings. $N_i = 1$ for W and charged scalars and 3 for top; e_i is the electric charge of the particle in the loop (1 for W and H^+, $\frac{2}{3}$ for top), and $\alpha = 1/137$.

With the shorthand

$$\tau = \frac{4m_i^2}{m_h^2}, \qquad f(\tau) = [\sin^{-1}(1/\sqrt{\tau})]^2, \quad \text{(B3)}$$

one has

$$\begin{aligned} F_1 &= 2 + 3\tau + 3\tau(2-\tau)f(\tau), \\ F_{1/2} &= -2\tau[1 + (1-\tau)f(\tau)], \\ F_0 &= \tau[1 - \tau f(\tau)]. \end{aligned} \quad \text{(B4)}$$

Note that these expressions hold only if $2m_i > m_h$ so that there is no absorptive part.

Suppose $h = h_{95}$, and we consider a type-I 2HDM plus scalar triplets. h_{95} can go only to $\tau^+\tau^-$ and $b\bar{b}$ at tree level; these are the only significant two-body channels that are open. So

$$\mathrm{Br}(h_{95} \to \tau^+\tau^-) \approx \mathrm{Br}(\phi \to \tau^+\tau^-), \quad \text{(B5)}$$

where ϕ is a 95 GeV scalar with exactly SM-like couplings. Now, $\sigma(pp \to h_{95} \to \tau^+\tau^-) = \sigma(pp \to h_{95}) \times \mathrm{Br}(h_{95} \to \tau^+\tau^-)$. If h_{95} is produced by ggF alone, $\mu_{\tau\tau} = \kappa_t^2$; if it is produced by VBF alone, $\mu_{\tau\tau} = \kappa_W^2$. A better approximation, with a 90:10 ratio for ggF and VBF modes of production, seems to be

$$\mu_{\tau\tau} = 0.9\kappa_t^2 + 0.1\kappa_W^2. \quad \text{(B6)}$$

Without any charged Higgs boson, $\mu_{\gamma\gamma}$ is given by

$$\begin{aligned} \mu_{\gamma\gamma} &= (0.9\kappa_t^2 + 0.1\kappa_W^2) \frac{\mathrm{Br}(h_{95} \to \gamma\gamma)}{\mathrm{Br}(\phi \to \gamma\gamma)} \\ &= (0.9\kappa_t^2 + 0.1\kappa_W^2) \frac{|\kappa_W F_1 + \frac{4}{3}\kappa_t F_{1/2}|^2}{|F_1 + \frac{4}{3}F_{1/2}|^2}. \end{aligned} \quad \text{(B7)}$$

Putting in all the values, this gives

$$0.0295(0.9\kappa_t^2 + 0.1\kappa_W^2)(-1.81\kappa_t + 7.62\kappa_W)^2 = 0.33^{+0.19}_{-0.12}, \quad \text{(B8)}$$

while $\mu_{\tau\tau}$ gives $\sqrt{0.9\kappa_t^2 + 0.1\kappa_W^2} = 1.2 \pm 0.5$.

APPENDIX C: CALCULATION OF SIGNIFICANCES

The p value for any signal is related to the probability that the null hypothesis is acceptable; the smaller the p value is, the more strongly the null hypothesis (i.e., no new physics) is rejected. This can also be converted to the significance quoted in terms of the standard deviation σ for a normalized distribution.

The experimentalists quote both the global and local significances, in terms of σ or p values. The local significance (LS) also includes the possibility of a random statistical fluctuation, called the look-elsewhere effect. To eliminate LEE, we use the global significance (GS) numbers, with $\sigma_{\rm GS} < \sigma_{\rm LS}$. The ratio of $\sigma_{\rm LS}:\sigma_{\rm GS}$ can be taken as the LEE factor.

Suppose a new resonance is observed in two different channels, or maybe in the same channel by two different experiments. To start with, one may take the observations to be independent (this need not be correct; there may be correlations). A conservative, and according to us the correct, way is to combine the GS numbers, not the LS ones, at least as long as the existence is confirmed. Once the existence is confirmed, there is no question of any LEE factor, and one may safely use the LS numbers for any further observations (e.g., for any signal coming from the 125 GeV Higgs boson resonance). The combined significance that one gets is naturally the most conservative one. A good example is the significance of the h_{95} signal as discussed in the text.

An important question is how to combine two different p values. Different methods exist with different statistical properties, see, e.g., Ref. [76]. We will use two methods, the well-known Fisher method of obtaining the test statistic χ^2 [77] and the method of Ref. [74]. We found that both methods give identical results.

While the p value is related with the probability of rejecting the null hypothesis, it is not directly interpretable as a probability. Thus, the combination of two p values, namely, p_1 and p_2, is not simply $p_1 p_2$. If this were true, the product of a large number of statistically insignificant p values could have resulted in a sufficiently small p value to discard the null hypothesis, which is obviously counterintuitive. We describe here briefly the method of [74].

Suppose one has n number of measurements with p values denoted by p_1, $p_2 \ldots p_n$, respectively, and let $p_* = p_1 p_2 \ldots p_n$, the product of all the p values. Assuming the null hypothesis, let us compute the probability $R_n(\rho)$ such that $p_* < \rho$. Then $R_n(p_*)$ is the combined p value. For $n = 1$, $R_1(\rho) = \rho$ by definition, and then one can use the recursion formula

$$R_n(\rho) = \rho + \int_\rho^1 R_{n-1}(\rho/p)dp, \tag{C1}$$

which leads to

$$p = p_1 p_2[1 - \ln(p_1 p_2)] \tag{C2}$$

for two measurements. The algorithm goes on *ad infinitum*.

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