Twistor approach to classical and quantum D0-brane

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We develop the (super)twistor approach to D0-brane, which is the massive type IIA superparticle in ten-dimensional spacetime. The basic variables are hexadecuplet of constrained $OSp(32|1)$ supertwistors similar but not identical to the ones which have been used for the description of 11D massless superparticle, also known as M0-brane. We show how the constrained supertwistor formulation is related to the spinor moving frame approach to D0-brane. We perform the quantization of the model by two different methods and discuss the relation of the results with the quantization of the spinor moving frame formulation of D0-branes. The quantum state spectrum describes the massive counterpart of the linearized type IIA supergravity.

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I. INTRODUCTION

It is well known that dynamical nonperturbative objects, p-branes and particularly Dirichlet p-branes or Dp-branes, being an integral part of string theory, are essential for its overall consistency and for establishing various strong/weak coupling dualities between different branches of string/M-theory. D0-branes or Dirichlet particles are the simplest and yet nontrivial examples of Dp-branes belonging to a non-perturbative spectrum of the type IIA string theory. Its standard (Brink-Schwarz type) action can be obtained by dimensional reduction of the standard action of M0-brane (or M waves), which is the massless $D = 11$ superparticle [1]. This fact is one of the manifestations of the duality between type IIA superstring string theory and other "corners" of M theory, which can be considered as a decompactification limit of the former in its strong coupling regime [2]. The quantum state spectrum of the D0-brane was shown to be the linearized massive type IIA super-multiplet, with 128 bosonic and 128 fermionic degrees of freedom, which can be obtained from the 11-dimensional supergravity as a nontrivial mode in the Kaluza-Klein dimensional reduction [3].

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The above mentioned quantum state spectrum of D0-brane was found and studied in [3] devoted to the quantization of this dynamical system in the so-called spinor moving frame formalism (see [4] for an earlier discussion of quantization of a D0-brane). This formalism was proposed in [5] and used there as the basis for generalized action principle and superembedding approach to this object. Notice that the action for multiple D0-branes (mD0 system) recently constructed and elaborated in [6,7] is presently known only in the frame of the spinor moving frame approach.

In this paper we develop a twistor approach to D0-brane in the classical and quantum domain. As it could have been expected from studies of similar systems in $D = 4$ (see for example [8–12]), the formulation in terms of twistorial variables considerably simplifies both the Hamiltonian analysis and the quantization procedure. Furthermore, the results of such quantization provided us with a one-particle counterpart of the on-shell superamplitudes [13,14] being an important tool for reaching an impressive progress in multiloop calculations in maximal $D = 4$, $\mathcal{N} = 4$ supersymmetric gauge theory and $D = 4$, $\mathcal{N} = 8$ supergravity [15,16].

The generalization of the twistor approach [8,9,17–19] to the case of massive $D = 4$ superparticle was elaborated in [20–22] under the name of two-twistor formalism (see, e.g., [23–30] for further related studies). The amplitudes with $D = 4$ massive particles in the frame of $D = 4$ spinor helicity formalism, which can be related to the two-twistor approach, was the subject of [31–33]. The first stages toward generalization of these results for the case of amplitudes and superamplitudes of the processes involving

D0-branes were discussed in [3,34]. We believe that the development of pure twistor description of D0-brane, which we present below, will be useful for further development of this line.

The paper is organized as follows.

In Sec. II, as a warm-up, we briefly review a well-known twistorlike model of massless $D = 4$ $\mathcal{N} = 1$ superparticle [9], emphasizing its reformulation in terms of Ferber supertwistors [8] and quantization in terms of supertwistor variables. We also discuss there the relation of Ferber-Schirafuji formulation with a spinor moving frame or a Lorentz harmonic approach to the $D = 4$ superparticle [35] because this allows for the ten-dimensional generalization, which we will use in Sec. III.

In Sec. III we develop the constrained supertwistor approach to D0-brane in the classical domain. Our starting point is the spinor moving frame approach for D0-brane [3,5] which we review in Sec. IV A. In Sec. III B we introduce the constrained helicity spinors, related to the spinor moving frame variables by $SO(16)$ transformations. These are used as a basic element of the constrained $OSp(32|1)$ supertwistor approach. The action of D0-brane in this approach is obtained in Sec. III C where the gauge symmetries of this action and its relation to the so-called tensorial superspace are also discussed. Hamiltonian mechanics of the constrained supertwistor formulation of D0-brane is developed in Sec. III D.

In Sec. IV we perform the quantization of D0-brane in its constrained supertwistor formulation. First, in Sec. IV A, we do this by the method that has been used in [36] for quantization of supertwistor formulation of the massless $D = 11$ superparticle (see [37] for a similar method in the frame of light-cone quantization). The result clearly indicates that the quantum state spectrum of the D0-brane is given by the massive counterpart of $D = 10$ type IIA supergravity supermultiplet, in agreement with the results of [3]. In Sec. IV B we perform an alternative quantization resulting in a multi-superfield description of quantum state vector of D0-brane. This allows us in Sec. IV C to reproduce the analytic on-shell superfield description of the quantum state vector of D0-brane obtained in [3].

The conclusion and discussion of possible applications of our results can be found in Sec. V.

Appendixes collect the description of our notation and some useful technicalities.

II. MASSLESS $D = 4$ $\mathcal{N} = 1$ SUPERPARTICLE. A REMINDER

Let us briefly recall the main features of classical and quantum description of the massless $D = 4$ $\mathcal{N} = 1$ superparticle model in terms of twistorial variables [8,9]. The suitable classical action to start with is given by [9]

$$S = \int_{\mathcal{W}^1} \Pi^{\alpha\dot{\alpha}} \lambda_\alpha \bar{\lambda}_{\dot{\alpha}} = \int d\tau\, \Pi_\tau^{\alpha\dot{\alpha}} \lambda_\alpha \bar{\lambda}_{\dot{\alpha}}, \qquad \alpha, \dot{\alpha} = 1, 2, \tag{1}$$

where $\Pi^{\alpha\dot{\alpha}}$ is the pull-back of the Volkov-Akulov 1-form

$$\Pi^{\alpha\dot{\alpha}} = dx^{\alpha\dot{\alpha}} + id\theta^\alpha \bar{\theta}^{\dot{\alpha}} - i\theta^\alpha d\bar{\theta}^{\dot{\alpha}} = d\tau \Pi_\tau^{\alpha\dot{\alpha}} \tag{2}$$

to the worldline $\mathcal{W}^1$ in the superspace enlarged by bosonic spinor variables λ^α and λ^α. This line is defined parametrically in terms of coordinate functions of the proper time τ,

$$\mathcal{Z}^{\mathcal{M}}(\tau) = (x^{\alpha\dot{\alpha}}(\tau), \theta^\alpha(\tau), \bar{\theta}^{\dot{\alpha}}(\tau), \lambda^\alpha(\tau), \bar{\lambda}^{\dot{\alpha}}(\tau)). \tag{3}$$

Constructing the Hamiltonian mechanics starting from the action (1), one obtains the primary constraints

$$\Phi_{\alpha\dot{\alpha}} = p_{\alpha\dot{\alpha}} - \lambda_\alpha \bar{\lambda}_{\dot{\alpha}} = 0, \tag{4}$$

$$p_\alpha = 0, \qquad p_{\dot{\alpha}} = 0, \qquad D_\alpha = -\pi_\alpha + ip_{\alpha\dot{\alpha}}\bar{\theta}^{\dot{\alpha}}, \\ D_{\dot{\alpha}} = -\pi_{\dot{\alpha}} + ip_{\alpha\dot{\alpha}}\theta^\alpha \tag{5}$$

where $p_{\alpha\dot{\alpha}}$, p_α, $\bar{p}_{\dot{\alpha}}$, π_α, and $\bar{\pi}_{\dot{\alpha}}$ are momenta, canonically conjugated to $x_{\alpha\dot{\alpha}}$, λ^α, $\bar{\lambda}^{\dot{\alpha}}$, θ^α, and $\bar{\theta}^{\dot{\alpha}}$. After introducing the Poisson brackets between canonical variables

$$[p_{\alpha\dot{\alpha}}, x^{\beta\dot{\beta}}]_{P.B.} = \delta_\alpha^\beta \delta_{\dot{\alpha}}^{\dot{\beta}}, \quad [p^\alpha, \lambda_\beta]_{P.B.} = \delta_\beta^\alpha, \\ [\bar{p}_{\dot{\alpha}}, \bar{\lambda}^{\dot{\beta}}]_{P.B.} = \delta_{\dot{\alpha}}^{\dot{\beta}}, \quad \{\pi_\alpha, \theta^\beta\}_{P.B.} = \delta_\alpha^\beta, \quad \{\bar{\pi}_{\dot{\alpha}}, \bar{\theta}^{\dot{\beta}}\}_{P.B.} = \delta_{\dot{\alpha}}^{\dot{\beta}}, \tag{6}$$

one can see that the part of the total set of (eight bosonic and four fermionic) constraints are of the second class (see, e.g., [11] for details), which makes the quantization procedure complicated. In particular, two bosonic and two fermionic constraints are of the first class, the rest of the constraints are of the second class. This makes the total number of the independent bosonic phase space variables[1] equal to 6 and the number of independent fermionic phase space variables equal to 2.

Despite its simplicity, the quantization of the model is nontrivial due to the presence of the second-class constraints. A possible approach that simplifies the problem is to change the variables to supertwistors

$$\mathcal{Z}_A = (\lambda_\alpha, \bar{\mu}^{\dot{\alpha}}, \chi), \qquad \bar{\mathcal{Z}}^A = (\mu^\alpha, \bar{\lambda}_{\dot{\alpha}}, \bar{\chi}), \tag{7}$$

the components of which are related to the original coordinate functions (3) by the supersymmetric generalization of the Penrose incidence relations [8]

[1]The counting goes as follows. The phase space contains $2 \times (4 + 4) = 16$ bosonic and $2 \times 4 = 8$ fermionic degrees of freedom. Two bosonic first class constraints eliminate 4, and six bosonic second class constraints eliminate 6 degrees of freedom. Similarly, two fermionic first class constraints eliminate 4 and two fermionic second class constraints eliminate 2 degrees of freedom, thus leaving us with six bosonic and two fermionic degrees of freedom.

$$\mu^\alpha = x^{\alpha\dot\alpha}\bar\lambda_{\dot\alpha} + i\theta^\alpha(\bar\theta^{\dot\beta}\bar\lambda_{\dot\beta}), \qquad \bar\mu^{\dot\alpha} = x^{\alpha\dot\alpha}\lambda_\alpha + i\bar\theta^{\dot\alpha}(\theta^\beta\lambda_\beta),$$
$$\chi = \theta\lambda, \qquad \bar\chi = \bar\theta^{\dot\alpha}\bar\lambda_{\dot\alpha}. \tag{8}$$

The action (1) can now be written in terms of the supertwistor variables,

$$S = -\int_{\mathcal{W}^1} \bar{\mathcal{Z}}^A d\mathcal{Z}_A, \tag{9}$$

without any reference to $x^{\alpha\dot\alpha}$ θ^α and $\bar\theta^{\dot\alpha}$, provided these variables obey the following constraint

$$\bar{\mathcal{Z}}^A\mathcal{Z}_A = \mu^\alpha\lambda_\alpha - \bar\mu^{\dot\alpha}\bar\lambda_{\dot\alpha} + 2i\bar\chi\chi = 0. \tag{10}$$

This is correct because the general solution of Eq. (10) is provided by Eq. (8).

In the Hamiltonian formalism the constraint (10) reflects the presence (generates) the $U(1)$ gauge symmetry of the system, which makes the number of independent phase space variables equal to 6 in agreement with our previous counting.

To perform quantization of the system defined by the free twistorial action (9) restricted by (10) is easy. Indeed, after reducing the phase space by accounting for explicitly solved bosonic second class constraints and passing to the standard Dirac brackets for the fermionic second class constraints, one finds that the only condition that the state vector has to satisfy, is the quantum version of the constraint (10). Replacing the canonical Dirac brackets

$$[\mu_\alpha, \lambda^\beta]_{D.B.} = \delta_\alpha^\beta, \qquad [\bar\mu_{\dot\alpha}, \bar\lambda^{\dot\beta}]_{D.B.} = \delta_{\dot\alpha}^{\dot\beta}, \qquad \{\chi, \bar\chi\}_{D.B.} = -\frac{i}{2} \tag{11}$$

with commutators and anticommutator according to (updated) Dirac rules, $[\ldots,\ldots\}_{D.B.} \to \frac{1}{i}[\ldots,\ldots\}$, and passing to the coordinate (λ-) representation for bosonic and holomorphic (χ-) representation for fermionic variables, one finds that the quantum version of (10) reads

$$\left(\lambda_\alpha\frac{\partial}{\partial\lambda_\alpha} - \bar\lambda_{\dot\alpha}\frac{\partial}{\partial\bar\lambda_{\dot\alpha}} + \chi\frac{\partial}{\partial\chi} - s\right)\Phi(\lambda,\bar\lambda,\chi) = 0. \tag{12}$$

The constant s in this equation is due to the ordering ambiguity. It is quantized and has the meaning of (super) helicity of the massless supermultiplet describing the quantum state spectrum of the superparticle (see, e.g., [11]).

In the simplest case of $s = 0$, the solution of Eq. (12) is a chiral on-shell superfield

$$\Phi(\lambda,\bar\lambda,\chi) = \phi(\lambda) + i\chi\bar\psi(\lambda), \tag{13}$$

the components of which describe the general solutions $\phi(p)|_{p^2=0}$, $\bar\psi_{\dot\alpha}(p) = \bar\lambda_{\dot\alpha}\bar\psi(\lambda)$ of the linearized equations of motion for scalar supermultiplet fields, $p^2\phi(p) = 0$ and $p_{\alpha\dot\alpha}\bar\psi^{\dot\alpha} = 0$. This solution is based on solving $p^2 = 0$ by the Cartan-Penrose representation $p_{\alpha\dot\alpha} = \lambda_\alpha\bar\lambda_{\dot\alpha}$.

The spinor moving frame description of $D = 4$ superparticle [35] is closely related to the above Ferber-Schirafuji supertwistor formalism described above, but allows for an easy generalization for higher dimensions and for supersymmetric extended objects, including $D = 4$ and $D = 10$ superstrings [38,39], $D = 11$ supermembrane [40,41], and other p-branes [42,43]. We briefly describe it here as its generalization for the case of $D = 10$ D0-brane [3,5], which we review in Sec. III A, will be used as a starting point to develop supertwistor approach to this object beginning in Sec. III B.

To make its appearance natural, let us consider a $D = 4$ massless particle in a special Lorentz frame in which its lightlike momentum reads $P_{(a)} = (1,0,0,1)\rho^{++}(\tau)$, where $\rho^{++}(\tau)$ is the energy of the particle. In an arbitrary frame the momentum is $p_\mu = U_\mu^{(a)}P_{(a)} \equiv u_\mu^{--}\rho^{++}$, where

$$U_\mu^a = \left(\frac{1}{2}(u_\mu^{++} + u_\mu^{--}), u_\mu^i, \frac{1}{2}(u_\mu^{++} - u_\mu^{--})\right)$$
$$\in SO(1,3), \quad i = 1,2 \tag{14}$$

$U_\mu^a(\tau)$ is a Lorentz group valued matrix constructed from two lightlike vectors u_μ^{++}, u_μ^{--} and spacelike vectors u_μ^i orthogonal to these and among themselves. This follows from (14), which is tantamount to saying that $U_\mu^a U^{\mu b} = \eta^{ab}$ i.e.,

$$u_\mu^{++}u^{\mu++} = 0, \qquad u_\mu^{--}u^{\mu--} = 0, \qquad u_\mu^{--}u^{\mu++} = 2,$$
$$u_\mu^i u^{\mu\pm\pm} = 0, \qquad u_\mu^i u^{\mu j} = -\delta^{ij}. \tag{15}$$

The above equations, but with $i = 1,\ldots,D-2$, are actually valid for any D. In $D = 4$ it is convenient to represent $D-2 = 2$ orthogonal moving frame vectors by complex conjugate $u_\mu^{+-} = \frac{1}{2}(u_\mu^1 + iu_\mu^2)$ and $u_\mu^{-+} = \frac{1}{2}(u_\mu^1 - iu_\mu^2)$.

The $D = 4$ spinor moving frame variables consist of a pair of complex bosonic spinors $v_\alpha^\pm$ normalized by

$$v^{\alpha-}v_\alpha^+ = 1, \qquad \bar v^{\dot\alpha-}\bar v_{\dot\alpha}^+ = 1, \tag{16}$$

which are "square root" from the above moving frame variables in the sense of

$$u_\mu^{--} = v^-\sigma_\mu\bar v^-, \qquad u_\mu^{++} = v^+\sigma_\mu\bar v^+,$$
$$u_\mu^{+-} = v^+\sigma_\mu\bar v^-, \qquad u_\mu^{-+} = v^-\sigma_\mu\bar v^+. \tag{17}$$

This is tantamount to saying that spinor moving frame matrix $v_\alpha^{(\beta)} = (v_\alpha^+, v_\alpha^-) \in SL(2,C)$ is a "square root" of the moving frame matrix (14).

Using (17) to write the action (1) in terms of the spinor moving frame variables,

$$S = \int_{\mathcal{W}^1} \rho^{++} u_\mu^{--} \Pi^\mu = \int d\tau\, \rho^{++} (v^- \sigma_\mu \bar{v}^-) \Pi_\tau^\mu \tag{18}$$

we see that it is related to the Ferber-Schirafuji action $\int d\tau (\lambda \sigma_\mu \bar{\lambda}) \Pi_\tau^\mu$ by simple redefinition of the bosonic spinor variables $\lambda_\alpha = \sqrt{\rho^{++}} v_\alpha^-$ and $\bar{\lambda}_{\dot{\alpha}} = \sqrt{\rho^{++}} \bar{v}_{\dot{\alpha}}^+$.

To resume, the action the massless $D = 4$ $\mathcal{N} = 1$ superparticle can be reformulated in terms of (super)twistor variables, which simplifies essentially the quantization. In the next Sec. III we will develop, on the basis of spinor moving frame formulation of D0-brane, its formulation in terms of constrained supertwistors. The relations of these latter with the spinor moving frame variable is not so straightforward as in the four-dimensional case and requires the use of $SO(16)$, $SU(8)$, and also of a more exotic symmetry realized by the Stueckelberg mechanism.

III. SUPERTWISTOR APPROACH TO D0-BRANE LAGRANGIAN AND HAMILTONIAN MECHANICS

A. Spinor moving frame formulation of D0-brane

We begin by reviewing the generalization of the above derivation of spinor moving frame formulation for a more complicated case of massive superparticle in ten dimensions [5], which is to say for Dirichlet superparticle or D0-brane [1]. This object lives in $D = 10$ type IIA superspace with coordinates

$$Z^M = (x^\mu, \theta^{\alpha 1}, \theta_\alpha^2) \quad \mu = 0, \ldots, 9, \quad \alpha = 1, \ldots, 16 \tag{19}$$

and is invariant (up to boundary terms) under the rigid supersymmetry transformations

$$\delta x^\mu = i\theta^1 \sigma^\mu \epsilon^1 + i\theta^2 \tilde{\sigma}^\mu \epsilon^2, \quad \delta\theta^{\alpha 1} = \epsilon^{\alpha 1}, \quad \delta\theta_\alpha^2 = \epsilon_\alpha^2. \tag{20}$$

Here $\theta^1 \sigma^\mu \epsilon^1 = \theta^{1\alpha} \sigma_{\alpha\beta}^\mu \epsilon^{1\beta}$ and $\theta^2 \tilde{\sigma}^\mu \epsilon^2 = \theta_\alpha^2 \tilde{\sigma}^{\mu\alpha\beta} \epsilon^{2\beta}$, where $\sigma_{\alpha\beta}^\mu = \sigma_{\beta\alpha}^\mu$ and $\tilde{\sigma}^{\mu\alpha\beta} = \tilde{\sigma}^{\mu\beta\alpha}$ are 10D generalizations of 4D relativistic Pauli matrices; they obey

$$\sigma^\mu \tilde{\sigma}^\nu + \sigma^\nu \tilde{\sigma}^\mu = \eta^{\mu\nu} \mathbb{I}_{16\times 16}, \quad \eta_{\mu\nu} = \mathrm{diag}(1, -1, \ldots, -1). \tag{21}$$

The first order form of the standard D0-brane action [1] reads

$$\int d\tau \left(p_\mu \Pi_\tau^\mu - \frac{1}{2} e (p^2 - m^2) \right) - im \int_{\mathcal{W}^1} (\mathrm{d}\theta^{\alpha 1} \theta_\alpha^2 - \theta^{\alpha 1} \mathrm{d}\theta_\alpha^2), \tag{22}$$

where $\Pi_\tau^\mu = \Pi^\mu / \mathrm{d}\tau$ and Π^μ is the type IIA counterpart of the Volkov-Akulov 1-form,

$$\Pi^\mu = \mathrm{d}x^\mu - i\mathrm{d}\theta^1 \sigma^\mu \theta^1 - i\mathrm{d}\theta^2 \tilde{\sigma}^\mu \theta^2 = d\tau \Pi_\tau^\mu. \tag{23}$$

The moving frame action for the D0-brane [5] can be written as

$$\begin{aligned} S^{\mathrm{D0}} &= \int_{\mathcal{W}^1} \mathcal{L}_1^{\mathrm{D0}} \equiv \int \mathrm{d}\tau L^{\mathrm{D0}} \\ &= m \int_{\mathcal{W}^1} \Pi^\mu u_\mu^0 - im \int_{\mathcal{W}^1} (\mathrm{d}\theta^{\alpha 1} \theta_\alpha^2 - \theta^{\alpha 1} \mathrm{d}\theta_\alpha^2), \\ u_\mu^0 u^{0\mu} &= 1, \end{aligned} \tag{24}$$

where the unit timelike vector u_μ^0 is considered as a part of the moving frame matrix

$$(u_\mu^0, u_\mu^I) \in \mathrm{SO}(1,9) \quad I = 1, \ldots, 9. \tag{25}$$

Equation (25) is tantamount to stating that the moving frame vectors obey

$$\begin{aligned} &u^{\mu 0} u_\mu^0 = 1, \qquad u^{\mu 0} u_\mu^I = 0, \qquad u^{\mu I} u_\mu^J = -\delta^{IJ}, \\ &\epsilon^{\mu_1 \ldots \mu_{10}} u_{\mu_1}^0 u_{\mu_2}^{i_1} \ldots u_{\mu_{10}}^{i_9} = \epsilon^{i_1 \ldots i_9}. \end{aligned} \tag{26}$$

The action (24) can be obtained from the standard D0-brane action (22) when solving the mass-shell condition for auxiliary momentum variable, $p^2 = m^2$, by $p_\mu = m u_\mu^0$.

The spinor moving frame action is given by (24) in which the moving frame vectors are considered to be composites of the spinor moving frame matrix

$$v_\alpha{}^q \in \mathrm{Spin}(1,9), \qquad \alpha = 1, \ldots, 16, \quad q = 1, \ldots, 16 \tag{27}$$

in the sense of the following constraints (see [5–7] for more details)

$$\begin{aligned} u_\mu^0 \sigma_{\alpha\beta}^\mu &= v_\alpha{}^q v_\beta{}^q, \\ u_\mu^I \sigma_{\alpha\beta}^\mu &= v_\alpha{}^q \gamma_{qp}^I v_\beta{}^p, \end{aligned} \tag{28}$$

$$v_\alpha^q \tilde{\sigma}_\mu^{\alpha\beta} v_\beta^p = u_\mu^0 \delta_{qp} + u_\mu^I \gamma_{qp}^I, \tag{29}$$

where γ_{qp}^I are 9d gamma matrices, $I = 1, \ldots, 9$ obeying

$$\gamma_{qp}^I = \gamma_{pq}^I, \qquad \gamma_{qq}^I = 0, \qquad \gamma^I \gamma^J + \gamma^J \gamma^I = 2\delta^{IJ} \mathbb{I}_{16\times 16}. \tag{30}$$

Taking the sigma-trace of the first equation in (28), or taking the trace of (29) (and taking into account that $\gamma_{qq}^I \equiv 0$), we find

$$u^{0\mu} = \frac{1}{16} v_\alpha{}^q \tilde{\sigma}^{\mu\alpha\beta} v_\beta{}^q. \tag{31}$$

Substituting this into the action (24), one can see that it provides a $D = 10$ massive generalization of the

Ferber-Schirafuji action (1) for massless $D=4$ superparticle model considered in Sec. II.

The integration of the Lagrangian form $\mathcal{L}_1^{D0}$ in the action (24) is over the worldline in an enlarged superspace (Lorentz harmonic superspace), which can be defined parametrically

$$\mathcal{W}^1 \in \Sigma^{(10+45|16+16)}:\ \mathcal{Z}^{\mathcal{M}} = \mathcal{Z}^{\mathcal{M}}(\tau) = (x^\mu(\tau), \theta^{\alpha 1}(\tau), \theta^2_\alpha(\tau), v_\alpha{}^q(\tau)) \tag{32}$$

with the use of coordinate functions $\mathcal{Z}^{\mathcal{M}}(\tau)$ of the proper time τ. Let us stress that the set of these includes the spinor moving frame matrix (27).

The derivatives of the spinor moving frame matrix are expressed in terms of SO(1,9) Cartan forms

$$\Omega^I = u_\mu^{(0)} du^{\mu I}, \qquad \Omega^{IJ} = u_\mu^I du^{\mu J} \tag{33}$$

by [6,7]

$$v_p{}^\alpha \mathrm{d} v_\alpha{}^q = -\frac{1}{4}\Omega^{IJ}\gamma^{IJ}_{pq} + \frac{1}{2}\Omega^I \gamma^I_{pq}. \tag{34}$$

We have written this equation using a $v_p{}^\alpha$ matrix, which is the inverse to $v_\alpha{}^p$,

$$v_p{}^\alpha v_\alpha{}^q = \delta^{pq}, \qquad v_\alpha{}^q v_q{}^\beta = \delta_\alpha{}^\beta \tag{35}$$

as this will be useful below. Notice that, as the charge conjugation matrix for the $SO(9)$ group is symmetric and can be chosen to coincide with δ_{qp}, the position of $Spin(9)$ indices is not important and we chose it for aesthetic and transparency reasons.

Of the Cartan forms (33), Ω^{IJ} transforms under the gauge $SO(9)$ symmetry of the action (24) as a connection while Ω^I transforms covariantly under this symmetry. These reflect the fact that of 45 degrees of freedom in a moving frame matrix obeying (27) [this is to say they are restricted by constraints (28) and (29)], 36 are pure gauge while nine are physical, the same as present in a timelike momentum of the massive $D=10$ particle.

The *covariant harmonic derivatives*, this is to say the vector fields (differential operators) that are dual to the Cartan forms (33), are

$$D^{IJ} = \frac{1}{2} v^p_\alpha \gamma^{IJ}_{pq}\frac{\partial}{\partial v^p_\alpha}, \qquad D^I = \frac{1}{2} v^p_\alpha \gamma^I_{pq}\frac{\partial}{\partial v^p_\alpha}. \tag{36}$$

These give zero when acting on the constraints (28) and (29).

In checking that (28) and (29) are in a kernel of the operators (36), one should consider the moving frame vectors to be composed from spinor moving frame variables. One can also write the operators that can act on expressions involving both moving frame and spinor moving frame variables as if the latter were independent. These are

$$D^{IJ} = \frac{1}{2} v^p_\alpha \gamma^{IJ}_{pq}\frac{\partial}{\partial v^p_\alpha} + u^I_\mu \frac{\partial}{\partial u^J_\mu} - u^J_\mu \frac{\partial}{\partial u^I_\mu}, \tag{37}$$

$$D^{I} = \frac{1}{2} v^p_\alpha \gamma^{I}_{pq}\frac{\partial}{\partial v^p_\alpha} + u^0_\mu \frac{\partial}{\partial u^I_\mu} + u^I_\mu \frac{\partial}{\partial u^0_\mu}. \tag{38}$$

B. From spinor moving frame to constrained helicity spinor: Prequel on SO(16) symmetry of twistor action

A rigorous development of the twistor approach to D0-brane begins by introducing a new set of constrained bosonic spinors $\lambda_\alpha{}^q$ related to the spinor moving frame matrix $v_\alpha{}^q$ by SO(16) rotation

$$\lambda_\alpha{}^p = v_\alpha{}^q S^{qp}, \qquad SS^T = I_{16\times 16} \tag{39}$$

$$\Rightarrow v_\alpha{}^q = S^{qp}\lambda_\alpha{}^p. \tag{40}$$

These new objects with 165 degrees of freedom (vs 45 degrees of freedom in $v_\alpha{}^q$ summing "physical" and pure gauge), which we will call helicity spinors, obey the constraints[2]

$$\begin{aligned} u^0_\mu \sigma^\mu_{\alpha\beta} &= \lambda_\alpha{}^q \lambda_\beta{}^q, \\ u^I_\mu \sigma^\mu_{\alpha\beta} &= \lambda_\alpha{}^q (S^T\gamma^I S)_{qp}\lambda_\beta{}^p, \end{aligned} \tag{41}$$

$$\lambda^q_\alpha \tilde{\sigma}^{\alpha\beta}_\mu \lambda^p_\beta = u^0_\mu \delta_{qp} + u^I_\mu (S^T\gamma^I S)_{qp}, \quad I = 1, \ldots, 9. \tag{42}$$

The matrices $(S^T\gamma^I S)_{qp}$ carrying indices of $SO(16)$ are not constant but are symmetric and obey the same Clifford algebra as the constant γ^I_{qp} do. Notice that the upper and lower vector indices of $SO(16)$ group are transformed in the same way so that we are free in choosing their position and use this to make equations lighter and their structure more transparent.

Notice that (41) as well as the contraction of (42) imply that u^0_μ is a composite of the constrained helicity spinors so that the spinor moving frame action for D0-brane from Eq. (24) can be equivalently written in the following Ferber-Schirafuji-like form

$$S^{\mathrm{D0}} = \int_{\mathcal{W}^1} \mathcal{L}_1^{\mathrm{D0}} = \frac{m}{16}\int_{\mathcal{W}^1} \lambda^q_\alpha \tilde{\sigma}^{\alpha\beta}_\mu \lambda^q_\beta \Pi^\mu - im\int_{\mathcal{W}^1} (\mathrm{d}\theta^{\alpha 1}\theta^2_\alpha - \theta^{\alpha 1}\mathrm{d}\theta^2_\alpha), \tag{43}$$

[2] Hence, the name of constrained helicity spinors.

where λ_α^q variables are constrained by (41) and (42). This action is clearly invariant under the $SO(16)$ gauge symmetry which acts on the Latin indices (p) of λ_α^p and on the matrix S^{pq}, involved in the constraints (41) and (42), by multiplication from the right. The $SO(9)$ gauge symmetry of the original form of the spinor moving frame action acts now on moving frame vectors u_μ^I, entering the constraints (41) and (42), and also on the matrix S^{pq} by multiplication from the left.

Thus our dynamical system, as described by the action (43) and constraints (41) and (42), is invariant under the $SO(9)\times SO(16)$ gauge symmetry. The $SO(16)$ valued $S^{pq}(\tau)$ is clearly the Stueckelberg field for $SO(16)$ gauge symmetry, which can be used to gauge $S^{pq}(\tau)$ to unity. Nevertheless, as we will see below, to maintain this field and to use constrained helicity spinors $\lambda_\alpha{}^q$ instead of the original spinor moving frame variables $v_\alpha{}^q$ is very convenient for developing and for future using the twistor approach to D0-brane. This allows us to avoid the use of covariant momenta and covariant derivative in the formulation of the twistor approach while maintaining the constraints (41) and (42) as strong equalities. To explain why this is possible, in Appendix B we introduce the covariant derivatives with respect to helicity spinors.

C. Constrained supertwistor approach to D0-brane: I. Action and SO(16) symmetry

As it was in the case of the massless $D=4$ superparticle, we would like to pass to a formulation in terms of twistor and supertwistor variables. In our $D=10$ case these supertwistor variables will be multiple and constrained.

To this end, we use (41) and the Leibnitz rule to present the action (43) in the following form

$$
\begin{aligned}
S^{\mathrm{D0}} &= m\int_{\mathcal{W}^1}(\mathrm{d}\tilde{x}^{\alpha\beta}\cdot\lambda_\alpha{}^p\lambda_\beta{}^p - i\mathrm{d}\theta^{\alpha1}\cdot\theta^{\beta1}\lambda_\alpha{}^p\lambda_\beta{}^p - i\mathrm{d}\theta^2_\alpha\theta^2_\beta\lambda_p{}^\alpha\lambda_p{}^\beta - i\mathrm{d}\theta^{\alpha1}v_\alpha{}^p\theta^2_p + i\theta^{p1}\mathrm{d}\theta^2_\alpha\lambda_p{}^\alpha)\\
&= m\int_{\mathcal{W}^1}(\mathrm{d}(\tilde{x}^{\alpha\beta}\lambda_\alpha{}^p)\lambda_\beta{}^p + (-\tilde{x}^{\alpha\beta}\lambda_\beta{}^q + i(\theta^{p1}\theta^{q1} + \theta^2_p\theta^2_q + 2\theta^{p1}\theta^2_q)\lambda_p{}^\alpha)\mathrm{d}v_\alpha{}^q - i\mathrm{d}(\theta^{p1}+\theta^2_p)\cdot(\theta^{p1}+\theta^2_p)). \qquad (44)
\end{aligned}
$$

Here $\lambda_q{}^\alpha$ is inverse to the helicity spinor matrix $\lambda_\alpha{}^q$,

$$
\lambda_\beta{}^p\lambda_p{}^\alpha = \delta_\beta{}^\alpha, \qquad \lambda_p{}^\alpha\lambda_\alpha{}^q = \delta_{pq}, \qquad (45)
$$

and we have also defined

$$
\tilde{x}^{\alpha\beta} = \frac{1}{16}x^\mu\tilde{\sigma}_\mu{}^{\alpha\beta}, \qquad \theta^{q1} = \theta^{\alpha1}\lambda_\alpha^q, \qquad \theta^2_q = \theta^2_\alpha\lambda_q^\alpha. \qquad (46)
$$

Further, let us notice that [as can be seen from Eq. (B1) in Appendix B] the helicity spinors satisfy $\lambda_p{}^\alpha\mathrm{d}\lambda_\alpha{}^p = 0$. Therefore, the action (44) can be written in the following twistorial form

$$
S^{\mathrm{D0}} \coloneqq \int_{\mathcal{W}^1}\mathcal{L}_1^{\mathrm{D0}} = m\int_{\mathcal{W}^1}(\mathrm{d}\mu^{\alpha p}\cdot\lambda_\alpha{}^p - \mu^{\alpha p}\cdot\mathrm{d}\lambda_\alpha{}^p - i\mathrm{d}\eta^p\cdot\eta^p) = m\int_{\mathcal{W}^1}\mathrm{d}Y^{Mp}\Omega_{MN}Y^{Np}, \qquad (47)
$$

as the difference between this functional and (44) vanishes, $-\frac{i}{16}\theta^{p1}\theta^2_p\lambda_q{}^\alpha\mathrm{d}\lambda_\alpha{}^q = 0$. In (44)

$$
\begin{aligned}
\mu^{\alpha p} &= \tilde{x}^{\alpha\beta}\lambda_\beta{}^p - \frac{i}{2}(\theta^{q1}\theta^{p1} + \theta^2_q\theta^2_p + 2\theta^{q1}\theta^2_p)\lambda_q{}^\alpha + \frac{i}{16}\theta^{q1}\theta^2_q\lambda_p{}^\alpha,\\
\eta^p &= \theta^{p1} + \theta^2_p \qquad (48)
\end{aligned}
$$

and

$$
\Omega_{MN} = \begin{pmatrix} 0 & \delta_\alpha{}^\beta & 0\\ -\delta^\alpha{}_\beta & 0 & 0\\ 0 & 0 & -i\end{pmatrix}, \qquad (49)
$$

is an $OSp(32|1)$ invariant matrix.

In the second part of Eq. (47) we have introduced hexadecuplet of constrained orthosymplectic supertwistors Y^{Mp}

$$Y^{Mp} = (\mu^{\alpha p}, \lambda_\alpha{}^p, \eta^p). \tag{50}$$

The symplectic symmetry $OSp(32|1) \times SO(16)$, which would be present in the action (48) with unconstrained supertwisors (50), is broken down to $Spin(1,9) \times SO(16)$ by the constraints (41) and (42) imposed on the helicity spinors λ_α^q, which is used as the second 16×16 bosonic block of the $(32+1) \times 16$ supermatrix Y^{Mp}.

To see what at least the GL(16) subgroup $Sp(32) \in OSp(32|1)$ is about, it is useful to note that gauge equivalent description of the system is given by the same action (47) but with $\mu^{\alpha p}$ of (48) replaced by

$$\mu^{\alpha p} = \tilde{X}^{\alpha\beta}\lambda_\beta{}^p - \frac{i}{2}(\theta^{q1}\theta^{p1} + \theta^2_q\theta^2_p + 2\theta^{q1}\theta^2_p)\lambda_q{}^\alpha + \frac{i}{16}\theta^{q1}\theta^2_q\lambda_p{}^\alpha, \qquad \eta^p = \theta^{p1} + \theta^2_p \tag{51}$$

where $\tilde{X}^{\alpha\beta} = \tilde{X}^{\beta\alpha}$ is an arbitrary symmetric spin-tensor. The enlarged superspace with such 136 bosonic coordinates and some number n of fermionic coordinates clearly has $GL(16)$ automorphism symmetry of its natural supersymmetry algebra.[3]

The $SO(1,9)$ invariant decomposition of an arbitrary $D = 10$ spin-tensor reads

$$\tilde{X}^{\alpha\beta} = \frac{1}{16}x^\mu(\tilde{\sigma}_\mu)^{\alpha\beta} + \frac{1}{2\cdot 16\cdot 5!}y^{\mu_1\ldots\mu_5}\tilde{\sigma}^{\alpha\beta}_{\mu_1\ldots\mu_5}. \tag{52}$$

The gauge equivalence of such a model in enlarged superspace with our original spinor moving frame formulation of D0-branes is suggested by the observation that the action (47) is *formally* invariant under

$$\delta\mu^{\alpha p} = b^{\mu_1\ldots\mu_5}(\tilde{\sigma}_{\mu_1\ldots\mu_5})^{\alpha\beta}\lambda_\beta{}^p \tag{53}$$

[due to the constraints on the helicity spinors, see Eqs. (41), (42), and (B1) in the Appendix B]. The gauge symmetry (53) allows one to gauge away 126 additional bosonic coordinates described by the $D = 10$ fifth rank self-dual antisymmetric tensor, $y^{\mu_1\cdots\mu_5} = 0$, thus reducing (52) to the original form (48) of our massive $D = 10$ generalization of the Ferber-Penrose incidence relation (8).

Actually, to make the above arguments on gauge symmetry (53) of the action (47) literally valid, we should first notice that Eqs. (51) with (52) provide the general solution of the constraint

$$\mathcal{J}^{pq} \coloneqq \mu^{\alpha[p}\lambda_\alpha{}^{q]} - \frac{i}{2}\eta^p\eta^q = 0. \tag{54}$$

Thus we can consider our dynamical system to be described by the action (47) with supertwistor variables (50) restricted, besides (41) and (42), by an additional constraint (54) only. Such action does possess the gauge symmetry (53) as well as $SO(16)$ symmetry. As we will see in a moment, the constraint (54) serves as generator of this latter symmetry in the Hamiltonian approach.

D. Constrained supertwistor approach to D0-brane: II. Hamiltonian mechanics

In order to perform the Hamiltonian analysis of the action (47), we shall follow the standard procedure, applicable for systems with bosonic and fermionic degrees of freedom [53]. In particular, we find that the calculation of canonical momentum for fermionic variable, $\pi^p_{(\eta)} \coloneqq \frac{\partial\mathcal{L}_1^{D0}}{\partial d\eta^p} = -im\eta^p$, gives us the constraint

$$d^p_{(\eta)} \coloneqq \pi^p_{(\eta)} + im\eta^p \approx 0 \tag{55}$$

stating that the real Grassmann variable η^p is the momentum for itself. To be explicit in this statement, we introduce the Poisson brackets

$$\{\pi^p_{(\eta)}, \eta^q\}_{P.B.} = \{\eta^q, \pi^p_{(\eta)}\}_{P.B.} = -\delta^{pq}, \qquad \{\eta^p, \eta^q\}_{P.B.} = 0, \qquad \{\pi^p_{(\eta)}, \pi^q_{(\eta)}\}_{P.B.} = 0, \tag{56}$$

and find that our fermionic constraints (55) obey the algebra

$$\{d^p_{(\eta)}, d^q_{(\eta)}\}_{P.B.} = -2im\delta^{pq} \tag{57}$$

and, hence, are of the second class in classification of Dirac [54].

To treat these constraints as strong equalities, i.e., equalities that can be used before calculating all the Poisson brackets [54], we introduce Dirac brackets, defined as

$$[\ldots,\ldots\}_{DB} = [\ldots,\ldots\}_{PB} - \frac{i}{2m}[\ldots, d^p_{(\eta)}\}_{PB}[d^p_{(\eta)},\ldots\}_{PB}. \tag{58}$$

With respect to these Dirac brackets our real fermionic coordinate is self-conjugate,

$$\{\eta^p, \eta^q\}_{D.B.} = -\frac{i}{2m}\delta^{pq}, \tag{59}$$

which is the exact expression of the statement that the fermionic coordinate function plays the role of its own momentum.

[3] Such tensorial superspaces were studied in relation with a description of massless higher spin theories [11,12,44–47], alternative to the unfolding approach of [48], as well as in relation with the so-called BPS (Bogomol'nyi-Prasad-Sommerfield) preons concept of [49], [50–52].

To analyze the bosonic sector of the dynamical system described by supertwistor action (47), it is better to perform first the integration by parts reducing the second term of the Lagrangian form to the first one,

$$\mathcal{L}_1^{\mathrm{D0}} \mapsto -m(2\mu^{\alpha p}\cdot \mathrm{d}\lambda_\alpha{}^p + i\mathrm{d}\eta^p\cdot\eta^p). \tag{60}$$

Then the canonical momentum conjugate to $\lambda_\alpha{}^p$ is equal to $-2m\mu^{\alpha p}$, while momentum conjugate to $\mu^{\alpha p}$ vanishes. These two relations are clearly the pair of the resolved second class constraints, which can be used as string relations reducing the phase space after we pass to Dirac brackets

$$[\mu^{\alpha p}, \lambda_\beta{}^q]_{D.B.} = \frac{1}{2m}\delta_\beta^\alpha\delta^{pq}. \tag{61}$$

The same results can be obtained by calculating the canonical momenta for bosonic variables directly from the Lagrangian form in (47) and then passing to Dirac brackets for the resulting second class constraints.

Of course, the helicity spinor matrix λ_α^p, which is now a component of hexadecuplet of $OSp(32|1)$ supertwisors (50), is strongly constrained by (41), (42), which makes the rigorous development of Hamiltonian analysis quite complicated. However, there exists a shortcut based on using the so-called covariant momenta for λ_α^p [classical counterparts of the covariant derivatives presented in Eqs. (B5) and (B6) of Appendix B] instead of canonical momenta. This allows one to treat the constraints (41) and (42) as string relations without explicit search for corresponding Dirac brackets (see [3] and references therein). But moreover, studying the quantization in the frame of supertwistor formulation (47), we actually do not need to specify in detail this procedure, as the only additional constraint we will need to use in the quantization, Eq. (54), contains the expression for one of such covariant momenta [the classical counterpart of $\mathcal{D}^{pq}$ in (87) and (B5)] as its bosonic part and, thus, is well defined when the constraints (41), (42) are treated as strong relations.

One can check that the constraints (54) form an algebra

$$[\mathcal{J}^{pq}, \mathcal{J}^{rs}]_{D.B.} = \frac{1}{2m}(\delta^{p[s}\mathcal{J}^{r]q} - \delta^{q[s}\mathcal{J}^{r]p}) \tag{62}$$

under the Dirac brackets (61) and (59). This means that these constraints are of the first class and generate 120 parametric $Spin(16)$ gauge symmetry.

IV. QUANTIZATION

A. Quantum D0-brane in the constrained supertwistor approach

The quantization of the supertwisor formulation of the D0-branes (Dirichlet superparticle), which we describe in this (sub)section, is quite similar to that for the quantization of the $D=11$ massless superparticle sketched in [36] (see [37] for quantization of the massless $D=11$ superparticle in the light-cone gauge, [55] for similar quantization in spinor moving frame formalism and [56] for quantization in terms of pure spinor variables). This could have been expected, taking into account the well-known correspondence between massless 11D and massive 10D fields (see, e.g., Sec. 5.3 of [57] and also Sec. 3 of [3]).

According to Dirac prescription, we should replace the basic Dirac brackets (61) and (59) by the commutator and anticommutator,

$$[\hat{\mu}^{\alpha p}, \lambda_\beta{}^q] = \frac{i}{2m}\delta_\beta^\alpha\delta^{pq}, \tag{63}$$

$$\{\hat{\eta}^p, \hat{\eta}^q\} = \frac{1}{2m}\delta^{pq}. \tag{64}$$

One immediately observes that fermionic operators $\hat{\eta}^p$ obey the 16-dimensional Clifford algebra. Hence they can be represented by SO(16)-invariant counterparts of Dirac matrices,

$$\hat{\eta}^q \mapsto \frac{1}{2\sqrt{m}}(\Gamma_q)_{\mathfrak{A}}{}^{\mathfrak{B}} \tag{65}$$

provided the quantum state vector of our dynamical system, Ξ carries the 256-valued index $\mathfrak{A}$ of the Dirac spinor of $SO(16)$,

$$\hat{\eta}^q\Xi_{\mathfrak{A}} = \frac{1}{2\sqrt{m}}(\Gamma_q)_{\mathfrak{A}}{}^{\mathfrak{B}}\Xi_{\mathfrak{B}}. \tag{66}$$

Then, using the coordinate representation for the bosonic commutation relation (63),

$$\hat{\mu}^{\alpha q} = \frac{i}{2m}\frac{\partial}{\partial\lambda_\alpha^q}, \qquad \hat{\lambda}_\alpha^q = \lambda_\alpha^q, \tag{67}$$

we find that the first class constraint (54) imposed on the state vector reads

$$\left(\delta_{\mathfrak{A}}{}^{\mathfrak{B}}\left(\lambda_\alpha{}^q\frac{\partial}{\partial\lambda_{\alpha p}} - \lambda_\alpha{}^p\frac{\partial}{\partial\lambda_{\alpha q}}\right) - \frac{1}{2}\Gamma^{qp}{}_{\mathfrak{A}}{}^{\mathfrak{B}}\right)\Xi_{\mathfrak{B}}(\lambda) = 0. \tag{68}$$

It implies that the state vector transforms as a Majorana spinor under $SO(16)$ symmetry acting also on the constrained helicity spinors. These carry degrees of freedom of the massive momentum $p_\mu = mu_\mu^0 = \frac{1}{16}\lambda^q\tilde{\sigma}_\mu\lambda^q$ as well as an information on the polarization of different massive particle states (see below).

Now let us introduce the representation of $SO(16)$ gamma matrices

$$(\Gamma_q)_{\mathfrak{A}}{}^{\mathfrak{B}} = \begin{pmatrix} 0 & (\sigma_q)_{\mathcal{A}\tilde{\mathcal{B}}} \\ (\tilde{\sigma}_q)^{\tilde{\mathcal{A}}\mathcal{B}} & 0 \end{pmatrix} \tag{69}$$

in terms of $SO(16)$ counterparts of Pauli matrices carrying indices of different Majorana-Weyl spinor representations of $SO(16)$ and obeying

$$(\sigma_q)_{\mathcal{A}\tilde{\mathcal{C}}}(\tilde{\sigma}_p)^{\tilde{\mathcal{C}}\mathcal{B}} + (\sigma_p)_{\mathcal{A}\tilde{\mathcal{C}}}(\tilde{\sigma}_q)^{\tilde{\mathcal{C}}\mathcal{B}} = \delta_{pq}\delta_{\mathcal{A}}{}^{\mathcal{B}},$$
$$(\tilde{\sigma}_p)^{\tilde{\mathcal{A}}\mathcal{C}}(\sigma_q)_{\mathcal{C}\tilde{\mathcal{B}}} + (\tilde{\sigma}_q)^{\tilde{\mathcal{A}}\mathcal{C}}(\sigma_p)_{\mathcal{C}\tilde{\mathcal{B}}} = \delta_{pq}\delta^{\tilde{\mathcal{A}}}{}_{\tilde{\mathcal{B}}}, \tag{70}$$

and also split the quantum state vector of our dynamical system, Ξ carries the 256-valued index $\mathfrak{A}$ of the Dirac spinor of $SO(16)$ on two Majorana-Weyl spinors of $SO(16)$

$$\Xi_{\mathfrak{A}} = \begin{pmatrix} \Phi_{\mathcal{A}} \\ \Psi^{\tilde{\mathcal{A}}} \end{pmatrix}. \tag{71}$$

The action of the fermionic operator $\hat{\eta}^q$ on these Weyl spinor components of the quantum state vector field are thus given by

$$2\sqrt{m}\hat{\eta}^q\Phi_{\mathcal{A}} = (\sigma_q)_{\mathcal{A}\tilde{\mathcal{B}}}\Psi^{\tilde{\mathcal{B}}}, \quad 2\sqrt{m}\hat{\eta}^q\Psi^{\tilde{\mathcal{A}}} = (\tilde{\sigma}_q)^{\tilde{\mathcal{A}}\mathcal{B}}\Phi_{\mathcal{B}}. \tag{72}$$

Equation (68) can be split into two equations each for just one of two Majorana-Weyl components of the quantum state vector field,

$$\left(\lambda_\alpha{}^q \frac{\partial}{\partial\lambda_{\alpha p}} - \lambda_\alpha{}^p \frac{\partial}{\partial\lambda_{\alpha q}}\right)\Phi_{\mathcal{A}}(\lambda) = \frac{1}{2}\sigma^{qp}{}_{\mathcal{A}}{}^{\mathcal{B}}\Phi_{\mathcal{B}}(\lambda), \tag{73}$$

$$\left(\lambda_\alpha{}^q \frac{\partial}{\partial\lambda_{\alpha p}} - \lambda_\alpha{}^p \frac{\partial}{\partial\lambda_{\alpha q}}\right)\Psi^{\tilde{\mathcal{A}}}(\lambda) = \frac{1}{2}\tilde{\sigma}^{qp\tilde{\mathcal{A}}}{}_{\tilde{\mathcal{B}}}\Psi^{\tilde{\mathcal{B}}}(\lambda). \tag{74}$$

The fermionic nature of $\hat{\eta}^q$ operator and Eq. (71) suggests one consider these two Majorana-Weyl spinor components of the state vector field to be of different statistics, e.g., bosonic $\Phi_{\mathcal{B}}$ and fermionic $\Psi^{\tilde{\mathcal{B}}}$. The suitable $SO(9)$ covariant representation of this latter can be given in terms of a gamma-traceless 9d vector-spinor, while the former can be split into symmetric traceless second rank tensor and antisymmetric third rank tensor

$$\Phi_{\mathcal{B}} = \begin{pmatrix} h_{IJ} = h_{JI}, \quad h_{II} = 0, \\ a_{IJK} = a_{[IJK]} \end{pmatrix},$$
$$\Psi^{\tilde{\mathcal{A}}} = \left(\sqrt{\frac{2}{m}}\tilde{\psi}^I_q, \tilde{\psi}^I_q(S^T\gamma^I S)_{qp} = 0\right). \tag{75}$$

The explicit $SO(9)$ invariant representation of $SO(16)$ Pauli matrices in this split notation can be found in [55]. With it, the realization of the $SO(16)$ Clifford algebra on the above set of fields is given by

$$\hat{\eta}^q h_{IJ} = \frac{1}{m}(S^T\gamma^{(I}S)_{qp}\psi_p^{J)},$$
$$\hat{\eta}^q a_{IJK} = \frac{3}{2m}(S^T\gamma^{[IJ}S)_{qp}\psi_p^{K]}, \tag{76}$$

$$\hat{\eta}^q\psi^I_p = \frac{1}{2}h_{IJ}(S^T\gamma^J S)_{qp} + \frac{1}{12}a_{JKL}(S^T\gamma^{IJKL}S - 6\delta^{I[J}S^T\gamma^{KL]}S)_{qp}, \tag{77}$$

(cf. [36,55,58] where quantization of massless $D = 11$ superparticle was considered).

The above set of fields with $SO(9)$ indices are related with the solutions of linearized equations of the massive counterpart of type IIA supergravity by

$$h_{\mu\nu}(p) = u^I_\mu u^J_\nu h_{IJ}(\lambda), \qquad a_{\mu\nu\rho}(p) = u^I_\mu u^J_\nu u^K_\rho a_{IJK}(\lambda),$$
$$\psi_{\mu\alpha}(p) = u^I_\mu \lambda_{\alpha q}\tilde{\psi}_{Iq}(\lambda). \tag{78}$$

Here, let us recall, the moving frame vectors u^I_μ are spacelike, orthogonal, normalized, and related to the helicity spinors $\lambda_{\alpha q}$ by the constraints (42) while the momentum is timelike and expressed by

$$p_\mu = m u^0_\mu = \frac{m}{16}\lambda_{\alpha q}\tilde{\sigma}^{\alpha\beta}_\mu\lambda_{\beta q}. \tag{79}$$

Fixing the $SO(16)$ gauge $S^{qp} = \delta^{qp}$, in which $\lambda^q_\alpha = v^q_\alpha$, Eq. (78) can be reduced to the ones obtained in [3], where it was shown that quantum state spectrum of D0-brane was given by the massive counterpart of type IIA supergravity and its relation with the fields of linearized $D = 11$ supergravity was considered.[4] This conclusion about quantum state spectrum of D0-brane is thus confirmed by supertwistor quantization of the dynamical system, which we have discussed above.

Our supertwistor approach to quantum D0-brane possesses invariance under the $SO(16)$ symmetry, which implies imposing the Eq. (73) on the bosonic fields and Eq. (74) on the fermionic fields. The rhss of these equations can be specified either by using the explicit representation for $SO(16)$ Pauli matrices or from antisymmetric $[rq]$ part of the action by $\hat{\eta}^r$ on Eqs. (76) and (77). We do not need the explicit form of these equations for our discussion below. Let us only note that equations for h_{IJ} (A_{IJK}) hidden in (73) involve a_{IJK} (h_{IJ}) while the equation for the fermionic field, (73) does not involve other fields.

In the next Section we will use another quantization scheme for our dynamical system. This will relate the results of supertwistor quantization as it is performed in this

[4] In [3] one can also find the explanation of why we prefer not to use the name "massive type IIA supergravity" for the massive counterpart of type IIA supergravity.

Section with the spinor moving frame description of quantum D0-brane given in [3].

B. Alternative quantization: I. Constrained on-shell superfields from the supertwistor approach

Let us come back to the fermionic second class constraints (55), which obey the algebra (57) on the Poisson brackets. In the coordinate representation the quantum counterpart of these constraint, $\hat{d}^q_{(\eta)}$, is proportional to

$$D_q = \frac{\partial}{\partial \eta^q} - m\eta^q \tag{80}$$

($\hat{d}^q_{(\eta)} = -iD_q$), which obeys

$$\{D_q, D_p\} = -2m\delta_{qp}. \tag{81}$$

Clearly we cannot impose these constraints on the state vector in the sense that state vector vanishes under their action. However, what we can is to *represent the state vector by a set of superfields* in such a way that the result of the action of D_q on one of such superfields is expressed through the other superfields in a self-consistent manner.

Such a way of quantization was briefly discussed in [59] in the context of $D = 11$ supergravity for which it resulted in the set of superfield equations first proposed in [60]. These are quite similar to the system of superfield equations for the massive counterpart of type IIA supergravity appearing in the quantum state spectrum of D0-brane [3], which we will describe below.

The suitable set of superfields to describe the quantum state spectrum of the D0-brane in the frame of our constrained supertwistor approach is suggested by the study in the previous Section [as the operator $\hat{\eta}^q$ there obeys the Clifford-like algebra similar to (81)]. It contains the bosonic symmetric traceless SO(9) tensor superfield $H_{IJ}(\lambda, \eta)$, antisymmetric third rank tensor superfield $A_{IJK}(\lambda, \eta)$, and fermionic superfield with $SO(9)$ vector and $SO(16)$ vector indices, $\Psi^I_q(\lambda, \eta)$ obeying $\Psi^I_q(S^T\gamma^I S)_{qp} = 0$,

$$\begin{aligned} &H_{IJ}(\lambda, \eta) = H_{JI}(\lambda, \eta), \qquad H_{II}(\lambda, \eta) = 0 \\ &A_{IJK}(\lambda, \eta) = A_{[IJK]}(\lambda, \eta), \\ &\Psi^I_q(\lambda, \eta), \qquad \Psi^I_q(S^T\gamma^I S)_{qp} = 0. \end{aligned} \tag{82}$$

The superfield equations representing the fermionic constraint action on the state vectors are

$$D_q H_{IJ} = 2i(S^T\gamma^{(I} S)_{qp}\Psi^{J)}_p, \tag{83}$$

$$D_q A_{IJK} = 3i(S^T\gamma^{[IJ} S)_{qp}\Psi^{K]}_p, \tag{84}$$

$$\begin{aligned} D_q\Psi^I_p = {}& imH_{IJ}(S^T\gamma^J S)_{qp} \\ &+ \frac{im}{3!}A_{JKL}(S^T\gamma^{IJKL}S - 6\delta^{I[J}S^T\gamma^{KL]}S)_{qp}. \end{aligned} \tag{85}$$

To deal with the constraints (54) in this approach we should first use the fermionic constraint (55) to write it as

$$\mathcal{J}^{pq} = \mu^{\alpha[p}\lambda_\alpha{}^{q]} + \frac{1}{2m}\eta^{[q}\pi^{p]}_{(\eta)}, \tag{86}$$

which is the true first class constraint in the presence of second class constraint (55).

The quantum version of $2mi\mathcal{J}^{pq} = -2mi\mathcal{J}^{qp}$ is given by differential operator

$$\mathfrak{D}^{qp} = \lambda_\alpha{}^{[q}\frac{\partial}{\partial\lambda_\alpha{}^{p]}} + \eta^{[q}\frac{\partial}{\partial\eta^{p]}}, \tag{87}$$

which generates $SO(16)$ transformations of $\lambda_\alpha{}^p$, η^p as well as of $S^{pq} = \lambda_\alpha{}^p v^\alpha_q$ and D_q,

$$\mathfrak{D}^{qp}\lambda_\alpha{}^r = \lambda_\alpha{}^{[q}\delta^{p]r}, \qquad \mathfrak{D}^{qp}\eta^r = \eta^{[q}\delta^{p]r}, \tag{88}$$

$$\mathfrak{D}^{qp}S^{rs} = S^{r[q}\delta^{p]s}, \tag{89}$$

$$[\mathfrak{D}^{qp}, D_r] = D_{[q}\delta_{p]r}. \tag{90}$$

Our multicomponent state vector superfield should be restricted by imposing in some manner the quantum constraint generating $SO(16)$ symmetry. The consistency with (83), (85), and (90) does not allow one to consider all the component superfields describing our state vector to be in kernel of the differential operator $\mathcal{J}^{pq} = -\frac{i}{2m}\mathfrak{D}^{qp}$. To be precise, this is possible for the bosonic superfield, but not for the fermionic one, so that we impose

$$\mathfrak{D}^{qp}H^{IJ} = 0, \qquad \mathfrak{D}^{qp}A_{IJK} = 0, \tag{91}$$

$$\mathfrak{D}^{qp}\Psi^I_r = \Psi^I_{[q}\delta_{p]r}. \tag{92}$$

Actually, this is logical since the fermionic field carries the $SO(16)$ vector index q while the bosonic superfields carry SO(9) vector indices only. The technical reason is the nontrivial action on the fermionic derivative D_p and on the S^{pq} matrix variable, (90) and (89). To resume, the constraints (91) and (92) imply the invariance of bosonic superfields and covariance of the fermionic superfield under $SO(16)$ symmetry, which also acts on bosonic and fermionic components of constrained supertwistor coordinates.

It can be shown that the field content of the set of superfields (82) is given by the set of their leading components,

$$h^{IJ} = H^{IJ}|_{\eta^q=0}, \quad a_{IJK} = A_{IJK}|_{\eta^q=0}, \quad \psi^I_q = \Psi^I_q|_{\eta^q=0}, \tag{93}$$

while the higher components of each of these superfields are expressed in terms of leading components of the other ones. Indeed, the next to leading component of fermionic superfield, $D_p\Psi^I_q|_{\eta=0}$, is expressed through the leading components of the bosonic superfields by leading component of Eq. (85), while the next to leading components of the bosonic superfields are expressed in terms of leading component of the fermionic superfield by using Eqs. (83) and (84). The independent component fields (93), in their turn, describe the solutions (78) of the linearized equations of massive analog of the type IIA supergravity multiplet [3].

C. Alternative quantization: II. Analytic on-shell superfields from the constrained supertwistor approach

In this Section we will show how the above constrained superfield description of the D0-brane quantum state vector can be related to the description by an analytic (chiral) superfield subject to a duality constraint obtained in [3].

To this end we first have to introduce, following [61,3], the complex null vector U^I, $I = 1, \ldots, 9$, normalized on 2 in its contraction with its conjugate $\bar{U}^I$, and seven real normalized vectors orthogonal to these and among themselves,

$$U_I U_I = 0 = \bar{U}_I \bar{U}_I, \qquad U_I \bar{U}_I = 2,$$
$$U_I U_I{}^{\check{J}} = 0 = \bar{U}_I U_I{}^{\check{J}}, \qquad U_I{}^{\check{J}} U_I{}^{\check{K}} = \delta^{\check{J}\check{K}}. \tag{94}$$

This set of vectors forms a basis in 9d Euclidean space and can be collected in the $SO(9)$ matrix

$$U_I^{(J)} = \left(U_I{}^{\check{J}}, \frac{1}{2}(U_I + \bar{U}_I), \frac{1}{2i}(U_I - \bar{U}_I)\right) \in SO(9). \tag{95}$$

The splitting in (95) is invariant under $SO(7) \times U(1)$ symmetry, which allows one to consider $U_I, \bar{U}_I$, and $U_I{}^{\check{J}}$ as constrained homogeneous coordinates of the coset $\frac{SO(9)}{SO(7)\times U(1)}$.

The “covariant harmonic derivatives” by which is meant such differential operators acting on the vectors $U_I, \bar{U}_I, U_I{}^{\check{J}}$ that preserve constraints (94), are

$$\mathbb{D}^{\check{I}} = \frac{1}{2} U_J \frac{\partial}{\partial U_J^{\check{I}}} - U_J^{\check{I}} \frac{\partial}{\partial \bar{U}_J}, \qquad \bar{\mathbb{D}}^{\check{I}} = \frac{1}{2} \overline{U_J} \frac{\partial}{\partial U_J^{\check{I}}} - U_J^{\check{I}} \frac{\partial}{\partial U_J}, \tag{96}$$

$$\overline{\mathbb{D}}^{\check{I}\check{J}} = \frac{1}{2} U_K^{\check{I}} \frac{\partial}{\partial U_K^{\check{J}}} - U_K^{\check{J}} \frac{\partial}{\partial U_K^{\check{I}}}, \qquad \mathbb{D}^{[0]} = U_J \frac{\partial}{\partial U_J} - \bar{U}_J \frac{\partial}{\partial \bar{U}_J}. \tag{97}$$

Of these, (96) are vector fields tangential to the coset $\frac{SO(9)}{SO(7)\times U(1)}$ while (97) are in $SO(7)$ and $U(1)$ directions. Their commutators provide us with a manifestly $SO(7) \times U(1)$ covariant representation of $so(9)$ algebra.

Next, we have to introduce the complex 16×8 complex matrix variables $\mathrm{w}_q^A = (\bar{\mathrm{w}}_{qA})^*$ providing the bridges between the 16 dimensional vector representation of $SO(16)$ and fundamental (antifundamental) representation of the $SU(8)$ group (cf. [3]). The complex rectangular matrix $\bar{\mathrm{w}}_{qA} = (\mathrm{w}_q^A)^*$ is related to the block $\bar{w}_{p\tilde{A}} = (w_q^{\tilde{A}})^*$ of a complex representation of a $Spin(9)$ valued matrix, $(\bar{w}_{p\tilde{A}}, w_p^{\tilde{A}}) \in Spin(9)$ with $SO(7)$ spinor index $\tilde{A} = 1, \ldots, 8$ and $SO(9)$ spinor index p, by,

(i) left multiplication by $SO(16)$ matrix S^{qp} [see (39) and (B7)], so that q is converted to the $SO(16)$ index p, and

(ii) action of the $SU(8)$ group in its antifundamental representation to obtain $\bar{\mathrm{w}}_{qA} = (\mathrm{w}_q^A)^*$.

The action of antifundamental representation of $SU(8)$ on the indices of $\delta_{\tilde{A}\tilde{B}}$, which plays the role of the SO(7) charge conjugation matrix, gives a nonconstant complex symmetric matrix $\mathcal{U}_{AB}$ while the action on that by fundamental representation gives the complex conjugate matrix $\bar{\mathcal{U}}^{AB}$. Clearly this complex matrix $\mathcal{U}_{AB}$ is symmetric and unitary, i.e., it obeys

$$\mathcal{U}_{AC} \bar{\mathcal{U}}^{CB} = \delta_A{}^B, \qquad \mathcal{U}_{AB} = \mathcal{U}_{BA} = (\bar{\mathcal{U}}^{AB})^*. \tag{98}$$

See Appendix C and [3] for more details on representations of $SU(8)$ and $SO(7)$.

When Spin(9) valued matrix $(\bar{w}_{q\tilde{A}}, w_q^{\tilde{A}})$ is the double covering of the SO(9) frame (95), the bridge variables $(\bar{\mathrm{w}}_{qA}, \mathrm{w}_q^A)$ are related with this SO(9) vector frame by Eqs. (C1) and (C2) presented in Appendix C. The true constraints that restrict these variables are

$$\bar{\mathrm{w}}_{qA} \bar{\mathrm{w}}_{qB} = 0 = \mathrm{w}_q^A \mathrm{w}_q^B, \qquad \bar{\mathrm{w}}_{qA} \mathrm{w}_q^B = \delta_A{}^B,$$
$$\bar{\mathrm{w}}_{qA} \mathrm{w}_p^A + \mathrm{w}_q^A \bar{\mathrm{w}}_{pA} = \delta_{qp}. \tag{99}$$

These imply that they provide a complex representation for the $SO(16)$ matrix.

Using these variables and the complex null vector from the SO(9) frame (95) we can construct from (82) complex bosonic and fermionic superfields inert under SO(9)

$$\Phi = H^{IJ} U_I U_J, \qquad \Psi^{A(U)} = \Psi_q^I U_I \mathrm{w}_q^A,$$
$$\bar{\Psi}_A^{(U)} = \Psi_q^I U_I \bar{\mathrm{w}}_{qA}, \quad \text{etc.}, \tag{100}$$

as well as mutually complex conjugate octuplets of the fermionic covariant derivatives

$$D^A = \mathrm{w}_q^A D_q, \qquad \bar{D}_A = \bar{\mathrm{w}}_{qA} D_q. \tag{101}$$

It is not difficult to find that $D_q\Phi = 4i\bar{w}_{qA}\mathcal{U}^{AB}\bar{\Psi}^{(U)}_B$. Contracting this with w^A_q we find that the superfield Φ is chiral,

$$\bar{D}_A\Phi = 0, \tag{102}$$

while contracting with w^A_q we find $D^A\Phi = 4i\mathcal{U}^{AB}\bar{\Psi}^{(U)}_B$, which implies the dependence of the fermionic superfield, $\bar{\Psi}^{(U)}_B = -\frac{i}{4}\bar{\mathcal{U}}_{BA}D^A\Phi$.

The superfield Φ also obeys

$$D^AD^BD^CD^D\Phi = \frac{1}{4!}\epsilon^{ABCDEFGH}\bar{D}_E\bar{D}_F\bar{D}_G\bar{D}_H(\Phi)^*. \tag{103}$$

To prove this is more involved; however the direct calculations are actually not necessary to be convinced that it holds (at least up to the sign). Indeed, it is sufficient to notice that:

(i) the set of superfields (82) obeying (83)–(85) describes the massive counterpart of the type IIA supergravity multiplet (see [3]);

(ii) as it was shown in [3], the same multiplet is described by the chiral superfield obeying (102) and (103); and

(iii) all the set of superfields (82) can be reproduced from the analytical superfield Φ in (100) with the use of the covariant harmonic derivative of the $SO(9)$ internal frame (96), the fermionic derivative, and Eqs. (83)–(85).

[Only the sign on (103) requires an explicit calculation to be fixed as it can be changed by replacing Φ with $i\Phi$].

The last item above probably needs some evidence. To get this, one can easily check that, for instance,

$$\begin{aligned}&\bar{\mathbb{D}}^{\check{I}}\Phi = -2U_JH^{JK}U^{\check{I}}_K,\\ &\bar{\mathbb{D}}^{\check{I}}\bar{\mathbb{D}}^{\check{J}}\Phi = 2U^{\check{I}}_KH^{KL}U^{\check{J}}_L - U^{\check{I}'}_KH^{KL}U^{\check{I}'}_L\delta^{\check{I}\check{J}} = \mathbb{D}^{\check{I}}\mathbb{D}^{\check{J}}\overline{\Phi},\end{aligned} \tag{104}$$

which implies

$$\begin{aligned}H^{IJ} = &\frac{1}{4}\bar{U}_I\bar{U}_J\Phi - \frac{1}{2}\bar{U}_{(I}U^{\check{K}}_{J)}\mathbb{D}^{\check{K}}\Phi\\ &+ U^{\check{K}}_IU^{\check{L}}_J\left(\frac{1}{4}\mathbb{D}^{\check{K}}\mathbb{D}^{\check{L}}\Phi - \frac{1}{36}\mathbb{D}^{\check{I}'}\mathbb{D}^{\check{I}'}\Phi\delta^{\check{K}\check{L}}\right)\\ &+ \frac{1}{36}\bar{U}_{(I}U_{J)}\mathbb{D}^{\check{I}'}\mathbb{D}^{\check{I}'}\Phi + \text{c.c.}\end{aligned} \tag{105}$$

The action of the covariant derivatives (96) on the $w^A_q, \bar{w}_{qA}$ variables is described in Eq. (119) of Appendix C. These can be reproduced using the $SO(16)$ rotations and $SU(8)$ transformations from their action on $Spin(9)$ variables, which can be found in [59].

V. CONCLUSIONS AND DISCUSSION

In this paper we have developed a constrained supertwistor approach to D0-brane in the classical and quantum domains. This provides a ten-dimensional generalization of the two-twistor approach to a $D = 4$ massive particle and superparticle from [21–23,25]. However, the structure and symmetries in the ten-dimensional case are much richer.

Although the basis for this supertwistor approach has been provided by spinor moving frame formulation of D0-brane from [3,5], the basic bosonic spinor variables of the supertwistor approach are not identified with spinor moving frame matrix $v_\alpha{}^q \in Spin(9)$ but with a helicity spinor matrix $\lambda_\alpha{}^p$, which is obtained from $v_\alpha{}^q$ by $SO(16)$ transformation, $\lambda^p_\alpha = v^q_\alpha S^{qp}$. This allows one to avoid the use of the so-called covariant momenta and covariant derivative formalisms (described in Appendix B) and to study the properties of the quantum state vector of D0-brane using the standard partial derivatives with respect to the bosonic spinors λ^p_α in a manner consistent with the constraints imposed on the latter.

The variables of the supertwistor approach to D0-braness are hexadecuplet of the constrained $OSp(32|1)$ supertwistors $Y^{Mp} = (\mu^{\alpha p}, \lambda_\alpha{}^p, \eta^p)$ subject to the constraint generating the $SO(16)$ symmetry in the Hamiltonian approach. The index $p = 1, \ldots, 16$ enumerating the supertwistors is transformed by vector representation of $SO(16)$. $OSp(32|1)$ symmetry is broken by the constraints imposed on the $\lambda_\alpha{}^p$ component of the supertwistor. However, at least the $GL(16)$ subgroup of this symmetry can be followed to the bosonic sector of the so called tensorial superspace, which appears in the general solution of the $SO(16)$ constraints on supertwistor variables, which provides the generalization of the Ferber-Penrose incidence relations [8].

We quantize the supertwistor formulation of D0-brane by two schemes. In the first the quantum state vector is represented by the 256 component Majorana spinor of $SO(16)$, the Majorana-Weyl "constituents" of which are taken to be with opposite statistics and [after gauge fixing of $SO(16)$ symmetry] can be represented by a fermionic gamma-traceless $SO(9)$ vector-spinor field, and by the set of the bosonic third rank antisymmetric and bosonic second rank symmetric traceless tensors of $SO(9)$.

The second of the quantization schemes describes the quantum state vector by a set of superfields subject to some equations with fermionic covariant derivatives. The leading components of these constrained superfields can be associated with the above described set of fields. These, in their turn, can be used to write the solutions of the linearized spacetime equations of the fields of the massive counterpart of type IIA supergravity multiplet, which thus describes the quantum state spectrum of D0-branes.

Using these constrained superfield approach to quantization, we have shown explicitly how the description of the

D0-brane quantum state vector by analytical on-shell superfield obtained in [3] can be derived from the supertwistor approach.

The fact that the on-shell superfields are one-particle counterparts of the on-shell superamplitudes, which in the $D=4$ case were the basis of an impressive progress in calculation of tree and loop processes in maximally supersymmetric Yang-Mills and supergravity theories, gives us the reason to hope that our results will be useful for development of the on-shell superamplitude approach in type IIA string theory, in its part involving D0-branes besides supergravity multiplets. The preliminary studies in [3] indicated a problem hampering the way toward an analytic (chiral) superamplitude formalism for such types of processes. A possible way out might pass through the development of a superamplitude counterpart of the constrained on-shell superfield formalism obtained in Sec. IV B or of an amplitude counterpart of the $SO(16)$ spinor description of the quantum state vector of D0-brane that we have obtained in Sec. IV A.

Another possible application of our approach can be related to the study of (super)twistor structures in curved $D=10$ spacetimes on the line of the $D=4$ two-twistor approach to studying Kerr-Newman black holes developed recently in [30].

We hope to address these problems in the near future.

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DATA AVAILABILITY

There are no publicly available research data or software supporting this manuscript. Requests for further information or data should be sent to the authors.

APPENDIX A: NOTATION AND CONVENTIONS

In $D=10$ we use the following sets of indices

(i) The Greek symbols from the middle of the alphabet correspond to vector indices: $\mu,\nu,\ldots=0,\ldots,9$.

(ii) The Greek symbols from the beginning of the alphabet $\alpha,\beta,\ldots=1,\ldots,16$ denote the $SO(1,9)$ spinor indices.

(iii) The small letters from the middle of Latin alphabet, $p,q,\ldots=1,\ldots,16$ denote $SO(16)$ vector indices of some variables, in particular $\lambda_\alpha{}^q$, and spinor $SO(9)$ $[Spin(9)]$ indices of other variables, like $v_\alpha{}^q$. We believe this will not produce any confusion, and certainly lighten the notation. The orthogonal matrix S^{qp} carries the first index, q, of Spin(9) and the second index, p, of $SO(16)$; $S^{qp}=S^{-1pq}$.

(iv) The capital Latin letters $I,J,K,\ldots=1,\ldots,9$ denote nine-vector indices, which are transformed by the $SO(9)$ group.

(v) The capital Latin letters with check symbol, $\check{I},\check{J},\check{K},\ldots=1,\ldots,7$ denote seven-vector indices, which are transformed by the $SO(7)$ group.

The generalized $D=10$ Pauli matrices $\sigma^\mu_{\alpha\beta}=\sigma^\mu_{\beta\alpha}$ and $\tilde{\sigma}^{\mu\alpha\beta}=\tilde{\sigma}^{\mu\beta\alpha}$ obey

$$\sigma^\mu\tilde{\sigma}^\nu+\sigma^\nu\tilde{\sigma}^\mu=\eta^{\mu\nu}\mathbb{I}_{16\times16},\quad \eta_{\mu\nu}=\mathrm{diag}(1,-1,\ldots,-1).\quad (\mathrm{A1})$$

The $d=9$ Dirac matrices $\gamma^I_{qp}=\gamma^I_{pq}$ obey the Clifford algebra

$$\gamma^I_{qp}=\gamma^I_{pq},\qquad \gamma^I\gamma^J+\gamma^J\gamma^I=2\delta^{IJ}\mathbb{I}_{16\times16}.\quad (\mathrm{A2})$$

The complex antisymmetric 8×8 matrices $\sigma^{\check{J}}_{AB}=-\sigma^{\check{J}}_{BA}$ and $\tilde{\sigma}^{\check{J}AB}=-\tilde{\sigma}^{\check{J}BA}=-(\sigma^{\check{J}}_{AB})^*$, carrying the indices of fundamental and antifundamental representations of $SU(8)$, are obtained from $SO(7)$ counterparts of Pauli matrices, imaginary antisymmetric $\sigma^{\check{J}}_{\tilde{A}\tilde{B}}=-\sigma^{\check{J}}_{\tilde{B}\tilde{A}}$, by $SU(8)$ transformations in the corresponding representation. They obey (C4) (see [3] for more details).

APPENDIX B: COVARIANT DERIVATIVES OF THE CONSTRAINED HELICITY SPINOR FORMALISM AND SOME COMMENTS

In this appendix we introduce the counterparts of the covariant harmonic derivatives (36) for the constrained helicity spinors λ^q_α obeying (41), (42). The first step in this direction is to find the expression for admissible derivative of the helicity spinor matrix, which is given by

$$\lambda_p{}^\alpha \mathrm{d}\lambda_\alpha{}^q=\mathcal{A}^{pq}+\frac{1}{2}\Omega^I(S^T\gamma^I S)_{pq},\qquad (\mathrm{B1})$$

where $\lambda_p{}^\alpha$ is the matrix inverse to $\lambda_\alpha{}^q$,

$$\lambda_p{}^\alpha=v^\alpha_q S^{qp},\qquad v^\alpha_q=S^{qp}\lambda_p{}^\alpha,\qquad (\mathrm{B2})$$

and

$$\mathcal{A}^{pq} = -\mathcal{A}^{qp} = S^{lp} DS^{lq} = \left(S^{-1}dS - \frac{1}{4}\Omega^{IJ}S^T\gamma^{IJ}S\right)_{pq} \tag{B3}$$

is the composite $SO(16)$ connection. In (B3) the symbol D denotes (as before) the $SO(8)$ covariant derivative, $DS = dS - \frac{1}{4}\Omega^{IJ}\gamma^{IJ}S$. Due to its presence, the $\mathcal{A}^{pq}$ [carrying two $SO(16)$ indices] is invariant under local $SO(8)$ symmetry acting on S^{qp} from the left. (Notice that $SO(16)$ acts on S^{qp} from the right.)

The covariant derivatives of the constrained spinor helicity formalism can be obtained by decomposing the differential in the space of constrained helicity spinors on the basis of $\mathcal{A}^{pq}$ and Ω^I Cartan forms,

$$d^{(\lambda)} \coloneqq d\lambda_\alpha{}^p \frac{\partial}{\partial\lambda_\alpha{}^p} = \frac{1}{2}\mathcal{A}^{pq}\mathcal{D}^{pq} + \Omega^I\mathcal{D}^I. \tag{B4}$$

They read

$$\mathcal{D}^{pq} = \lambda_\alpha{}^p \frac{\partial}{\partial\lambda_\alpha{}^q} - \lambda_\alpha{}^q \frac{\partial}{\partial\lambda_\alpha{}^p}, \tag{B5}$$

$$\mathcal{D}^I = \frac{1}{4}(S^T\gamma^I S)_{pq}\left(\lambda_\alpha{}^p \frac{\partial}{\partial\lambda_\alpha{}^q} + \lambda_\alpha{}^q \frac{\partial}{\partial\lambda_\alpha{}^p}\right), \tag{B6}$$

and, as it is not difficult to check, give zero when applied to the constraints (41) and (42).

Let us stress that in the description of our dynamical system, D0-brane, by the action with the Lagrangian form (43), the dynamical variables are helicity spinors $\lambda_\alpha{}^q$ constrained by (41) and (42), while the Stueckelberg field S^{pq} is a part of it appearing in these constraints. Explicitly it can be defined as

$$S^{qp} = v_q^\alpha \lambda_\alpha{}^p \tag{B7}$$

where v_q^α is the (inverse to the) spinor moving frame matrix related to the moving frame vectors u_μ^0 and u_μ^I, which enter (41) and (42), by constraints (28) and (29).

Gauging S^{pq} to unit 16×16 matrix δ^{pq} reduces the constraints (41) and (42) to the constraints (28) and (29) for the spinor moving frame matrix and the $SO(16)$ Cartan form (B3) to the spinor representation of the $SO(9)$ Cartan form Ω^{IJ}. The advantage of using the constrained helicity spinor variables is that the set of covariant momenta, the counterpart of covariant derivatives, will include the classical counterpart of the covariant derivative (B5) representing the $SO(16)$ symmetry generator. This will simplify essentially the formulation of the supertwistor approach and its possible future applications.

APPENDIX C: PROPERTIES OF THE $SO(16)$ BRIDGE VARIABLES w_q^A

The relations of the bridge variables $(\bar{\mathrm{w}}_{qA}, \mathrm{w}_q^A)$ [obeying (99) and thus providing a complex representation for $SO(16)$ valued matrix] with SO(9) vector frame (95) read

$$\bar{\mathrm{w}}_{qA}(S^T\gamma^I S)_{qp}\bar{\mathrm{w}}_{pB} = U_I\mathcal{U}_{AB}, \quad \mathrm{w}_q^A(S^T\gamma^I S)_{qp}\mathrm{w}_p^B = \bar{U}_I\bar{\mathcal{U}}^{AB},$$
$$\bar{\mathrm{w}}_{qA}(S^T\gamma^I_{qp}S)\mathrm{w}_p^B = iU_I^{\check{J}}(\sigma^{\check{J}}\bar{\mathcal{U}})_A{}^B, \tag{C1}$$

$$U_I(S^T\gamma^I S)_{qp} = 2\bar{\mathrm{w}}_{qA}\bar{\mathcal{U}}^{AB}\bar{\mathrm{w}}_{pB}, \quad \bar{U}_I(S^T\gamma^I S)_{qp} = 2\mathrm{w}_q^A\mathcal{U}_{AB}\mathrm{w}_p^B,$$
$$U_I^{\check{J}}(S^T\gamma^I S)_{qp} = 2i\mathrm{w}^A_{(q}(\sigma^{\check{J}}\bar{\mathcal{U}})_A{}^B\bar{\mathrm{w}}_{p)B}. \tag{C2}$$

Here $\sigma^{\check{J}}_{AB} = -\sigma^{\check{J}}_{BA}$ and

$$\tilde{\sigma}^{\check{J}AB} = -(\sigma^{\check{J}}_{AB})^* = (\sigma^{\check{J}}_{BA})^\dagger \tag{C3}$$

are obtained from $SO(7)$ Clebsch-Gordan coefficients $\sigma^{\check{J}}_{\tilde{A}\tilde{B}} = -\sigma^{\check{J}}_{\tilde{B}\tilde{A}}$ by $SU(8)$ transformations in its antifundamental and fundamental representation, respectively. They obey

$$\sigma^{\check{J}}_{AC}\tilde{\sigma}^{\check{I}CB} + \sigma^{\check{I}}_{AC}\tilde{\sigma}^{\check{J}CB} = 2\delta^{\check{I}\check{J}}\delta_A{}^B;. \tag{C4}$$

The constraints for the $Spin(9)$ spinor frame variables $(\bar{\tilde{w}}_{q\tilde{A}}, \tilde{w}_q^{\tilde{A}})$, presented in [59], can be obtained from (C1) and (C2) by setting $S^{qp} = \delta^{qp}$, $\mathcal{U}^{AB} \mapsto \delta_{\tilde{A}\tilde{B}}$, $\bar{\mathcal{U}}^{AB} \mapsto \delta_{\tilde{A}\tilde{B}}$ and $\tilde{\sigma}^{\check{J}AB} \mapsto -\sigma^{\check{J}}_{\tilde{A}\tilde{B}}$.

The action of the covariant derivatives (96) on the bridge variables,

$$\mathbb{D}^{\check{I}}\bar{\mathrm{w}}_{qA} = -\frac{i}{2}\mathrm{w}_q^B\sigma^{\check{I}}_{BA} = (\bar{\mathbb{D}}^{\check{I}}\mathrm{w}_q^A)^*,$$
$$\mathbb{D}^{\check{I}}\mathrm{w}_q^A = -\frac{i}{2}\bar{\mathrm{w}}_{qB}\tilde{\sigma}^{\check{I}BA} = (\bar{\mathbb{D}}^{\check{I}}\bar{\mathrm{w}}_{qA})^*, \quad \text{etc.},$$

can be reproduced using the $SO(16)$ rotations and $SU(8)$ transformations from their action on $Spin(9)$ variables, which can be found in [59].

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