

Color force acting on a quark in the pion and nucleon

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In the operator product expansion (OPE) of hard scattering amplitudes, the twist-3 operators describe local colored Lorentz forces acting on a quark, thereby providing a measure of the strength of the gluon fields. Its value is directly accessible from the nucleon twist-3 polarized g_2 -parton distribution function. In the semiclassical (instanton-based) QCD vacuum models, the leading nonperturbative contribution stems from correlated instanton-antiinstanton pairs, or molecules. We analyze the magnitude of the color force on a struck quark in light hadrons (pion and nucleon), in the context of the instanton liquid model (ILM). We derive explicitly the pertinent form factors associated with the color Lorentz force and show that they are intimately related to the pertinent hadronic gravitational and transversity form factors. Using the ILM enhanced by molecules, we detail the ensuing colored force distribution in the transverse plane for the luminal pions and nucleons. The results for the nucleons are in good agreement with those recently reported from a lattice collaboration.

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I. INTRODUCTION

Deep inelastic scattering (DIS) is the main process by which the quark and gluon (parton) substructure of the nucleon can be quantitatively addressed. This is usually captured in terms of pertinent matrix elements of gauge invariant and traceless operators, organized in the OPE as a *twist expansion*, a series in inverse powers of the momentum transfer $1/Q^2$ of certain light-cone current-current correlators. The leading *twist-2* operators are bilinear in the quark (or gluon) fields. When extracted from experimental data, they tell us about the parton distribution functions (PDFs), the densities of gluons, as well as various quarks and antiquarks in the target.

Higher twist operators are of higher dimensions, including higher powers of quark and gluon fields that carry information about the partonic correlations. The general theory for those operators was carried out in the early 1980s in [1] for unpolarized targets, and in [2] for polarized ones. They provide the QCD corrections to the partonic sum rules, such as the Bjorken and the Ellis-Jaffe sum rule. A

more recent study in [3] is also devoted to the twist-3 contributions for other (e.g., momentum) sum rules.

The most interesting correlations between partons are those stemming from a polarized target. While the structure function $g_1(x, Q^2)$ starts with the usual twist-2 operators, the structure function $g_2(x, Q^2)$ starts with twist 3. Since in experiments the structure functions can be separated *kinematically*, this fact offers the most direct access to the higher twist physics. For a transversely polarized nucleon, at 90° to the incoming momentum, $g_1(x, Q^2)$ vanishes and the remaining DIS amplitude is purely twist 3. The pertinent physics is related to the local operator¹

$$O_{\bar{q}Gq} = ig(\bar{q}\gamma^\rho G^{\mu\nu} q) \quad (1)$$

given by the value of the gauge field strength on the struck quark. (The color indices are not explicitly shown but assumed here and below.)

The magnitude of this effect has been discussed in [4]. It was pointed out that in the large- N_c limit, the forces on the u and d quarks in the proton should be equal in magnitude but opposite in sign. The suggested magnitude was of the order of

$$F_u = -F_d \sim 0.1 \text{ GeV/fm}. \quad (2)$$

¹Although the operator carries three open indices ρ, μ, ν , the last pair is antisymmetrized, and the total spin is 2 and not 3.

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Note that it is an order of magnitude smaller than the confining “string tension” force

$$F_{\bar{Q}Q} = \sigma \approx 1 \text{ GeV/fm} \approx 0.2 \text{ GeV}^2. \quad (3)$$

A. Twist-3 force in the instanton liquid model (ILM)

In the late 1970s to early 1980s studies of semiclassical pseudoparticles—instantons and anti-instantons—had led to describing the QCD vacuum as an “instanton liquid” [5]. The vacuum is very inhomogeneous, with four-quark operators of the type $LR + RL$ [here left-handed current means $L = \bar{q}(1 + \gamma_5)q$, and right-handed with the opposite sign in front of γ_5] strongly enhanced through fermionic zero modes, of a single pseudoparticle. Unfortunately, the operator (1) is of different type, $LL + RR$, therefore it is not enhanced in a single pseudoparticle, and thus it does not appear in the leading order in the pseudoparticle density. The evaluation of matrix elements of such operators has been postponed in earlier literature, in favor of discussion of the “most enhanced” effects.

A single instanton has a probability proportional to very small product of light quark (Higgs-induced) masses $m_u m_d m_s / \Lambda_{\text{QCD}}^3 \sim 10^{-4}$. Therefore, an ideal gas of independent pseudoparticles would be extremely dilute and thus irrelevant. Fortunately, the QCD vacuum is not an ideal gas but rather a “liquid” [5], with strong correlations mediated by light quark exchanges. As a result, small Higgs-induced “Lagrangian” masses are substituted by much larger effective quark masses. The $SU(N_f)$ chiral symmetry is broken by a nonzero quark condensate. In the ILM, the density and typical size of the pseudoparticles are

$$\frac{N}{V_4} = n_{I+\bar{I}} \approx 1 \text{ fm}^{-4}, \quad \rho \sim \frac{1}{3} \text{ fm} \quad (4)$$

In spite of the smallness of the diluteness parameter,

$$\frac{N\rho^4}{V_4 N_c} \approx \left(\frac{1}{3}\right)^5 \ll 1 \quad (5)$$

the ensemble properties are not expandable in simple Taylor series in the packing fraction (diluteness). For example, (for two light quark flavors) a constituent quark mass scales as a square root of the packing fraction,

$$M\rho \sim \sqrt{\frac{N\rho^4}{V_4}} \quad (6)$$

as it follows from Bethe-Salpeter equation summing an infinite number of quark loop diagrams.

Derivation of effective action S_{eff} for constituent quarks in the mean field approach [6] complemented this mass by a specific form factor $\mathcal{F}(k)$

$$S_{\text{eff}} = \int \frac{d^4 k}{(2\pi)^4} \psi^\dagger(k) [\not{k} - iM\mathcal{F}(k)] \psi(k) \quad (7)$$

related to quark zero mode. The form factor describes dependence of the constituent quark mass on the momentum scale under investigation. Further studies of quark propagators were completed using numerical simulations of the instanton ensembles, see review [7].

To our knowledge, the first attempts to theoretically quantify twist-3 and twist-4 matrix elements have been carried out in [8]. They have used a version of the ILM for the vacuum structure, and a chiral soliton (large- N_c realization of ILM) for the nucleon. Their main conclusion is that the twist-3 matrix elements are nonzero but still suppressed by a power of the diluteness parameter.

B. Lattice studies

We will not go into the technicalities and long history of lattice gauge field simulations, and many of the key achievements. It is sufficient to note that current lattice simulations are able to work with fermions as light as the quarks in the real world, reproducing major parts of hadronic phenomenology quite well. This includes the nucleon mass, form factors, PDFs and even generalized parton distributions (GPDs).

Recently, there have been new attempts to quantify the values of higher twist operators [9], in particular, the recent work by the Adelaide group [10], where numerical evaluation of the twist-3 operators in the nucleon was quoted well inside the (statistical) error bars. Remarkably, their results

$$F_u \approx 3 \text{ GeV/fm}, \quad F_d = 0. \pm 0.05 \text{ GeV/fm} \quad (8)$$

are much larger than the one suggested earlier in (2). The forces on u and d are surprisingly different. Furthermore, as clearly shown in Fig. 5 of that paper, it is normal to both the directions of the nucleon’s motion (z) and spin (x) being localized inside a small sphere of radius only 0.2–0.3 fm. The vanishing force on the d quark brings up a (perhaps simplistic) explanation: if the d quark is always locked into the spin-zero ud “good diquark,” there will be no polarization-sensitive effects associated with it.

The present analysis is carried out within the ILM, which provides a semiclassical description of the QCD vacuum. As such, it does not incorporate confinement explicitly and relies on phenomenological input for the instanton ensemble, in particular the average instanton size ρ and density $n_{I+\bar{I}} \equiv N/V_4$. While these parameters are constrained by phenomenology and lattice studies, the results retain a degree of model dependence.

Furthermore, the semiclassical treatment is not controlled by a strict small expansion parameter. Although the packing fraction $n_{I+\bar{I}}\rho^4/N_c$ is numerically small and allows for a systematic expansion for the many-body description, key observables are not analytic in this

parameter. As a result, the present calculation should be viewed as providing robust qualitative trends and order-of-magnitude estimates, rather than precision predictions.

In addition, observables sensitive to short-distance dynamics, such as the color Lorentz force, inherit a non-trivial dependence on the instanton size and ensemble characteristics, which propagates to the resulting matrix elements, as we will discuss below.

C. Outline of this paper

We develop a theoretical framework for the analysis of pseudoparticles and their pairs (or instanton–anti-instanton molecules) connected by light quarks. We first review their emergence in the semiclassical treatment of the QCD vacuum and discuss how these correlations modify the gauge field topology compared to the uncorrelated instanton liquid.

In Sec. II we detail the structure of the molecular configurations using a ratio ansatz, and construct the explicit gauge field configurations, allowing for an estimate of the color force. The overlap of the quark zero modes in these correlated backgrounds is analyzed, leading to quantitative estimates for the overlap integral $T_{\bar{I}I}$ that encodes the effective range of the quark pairing between the instanton and anti-instanton. We then apply Monte Carlo techniques to evaluate the average color-electric field acting on a propagating quark in the molecular ensemble, and from this, determine the corresponding color Lorentz force. Our calculations show that the typical magnitude of this force is $F \sim 2\text{--}3$ GeV/fm, consistent with lattice extractions and well above the strength expected from earlier QCD estimates.

In Sec. III, we show how these semiclassical configurations contribute to the twist-3 parton distribution functions, focusing on the decomposition of $g_T(x) = g_1(x) + g_2(x)$ into its Wandzura-Wilczek and genuine twist-3 parts. Through this connection, the color Lorentz force is directly related to the expectation value of the local operator $\bar{q}gG^{+i}\gamma^+q$, thus establishing a bridge between nonperturbative vacuum dynamics and experimentally measurable structure functions.

In Sec. IV, we show how the emergent momentum-dependent form factors provide explicit parametrizations of the quark-gluon interaction strength across a range of momentum transfers, revealing that the molecular configurations dominate certain nonperturbative effects.

In the next Secs. V–VII we extend the analysis to a polarized quark, pion, and nucleon, respectively. While the pion—being spinless—exhibits a vanishing d_2^π and no net transverse Lorentz force, the nucleon’s internal structure supports large color forces, consistent with the observed twist-3 moments. We explicitly show that the color Lorentz forces in the light hadrons are tied to the hadronic gravitational and transversity form factors. Our conclusions are presented in Sec. VIII. A number of Appendixes are added to complement the derivations in the main text.

II. INSTANTON-ANTI-INSTANTON “MOLECULES”

A. Generalities and estimates

Instantons are tunneling events between topologically distinct gauge field configurations. In a theory with massless quarks, an amplitude for an isolated tunneling event vanishes due to fermionic zero modes. For light quarks beyond chiral limit, it is proportional to the product of the light quark masses $m_u m_d m_s \rho^3$, which is nonzero but numerically negligibly small.

Therefore, there are only two ways in which QCD instantons can produce significant observable effects. One of them is the dominant one described by the original ILM where the instanton ensemble *collectively* breaks chiral symmetry, and trade the small “Lagrangian” quark masses of order few MeV by “constituent” ones of order $\sim \mathcal{O}(400 \text{ MeV})$.

The other is that few instantons and anti-instantons together can produce a cluster with zero topological charge. If so, the amplitude is nonzero even for vanishing quark masses. The simplest cluster of this kind is an instanton–anti-instanton ($I\bar{I}$) pair, or “molecule.” It can be considered as an “incomplete tunneling” event, in which the quantum path wanders into the classically forbidden area and then returns back. The light quarks must propagate in loops between I and $\bar{I}$, which provide the correct chirality flips, see the sketch in Fig. 4.

Before going through the history of this mechanism and its technical details, let us present simple (but naïve) estimates of the fields and forces involved, to make connection to the numbers mentioned in the preceding section. The instanton field squared at distance r from its center is

$$(gG_{\mu\nu}^a(r))^2 = \frac{192\rho^4}{(\rho^2 + r^2)^4} \quad (9)$$

so at the origin ($r = 0$) and $\rho = 1/3$ fm the rms field is numerically $G_{r.m.s.} \sim 4.4 \text{ GeV}^2$. The lattice estimates [11] of the density of $I\bar{I}$ molecules are an order of magnitude larger than that of ILM instantons (4) $n_{\text{mol}}\rho^4 \sim 0.1$. A very crude estimate of a field acting on a quark thus produces a value

$$F \sim (n_{\text{mol}}\rho^4)G_{r.m.s.} \sim 0.44 \text{ GeV}^2 \sim 2.2 \text{ GeV/fm} \quad (10)$$

comparable to the lattice results (8), even twice *larger* than the $\bar{Q}Q$ force in quarkonia $\sigma \sim 1 \text{ GeV/fm}$. Note that this is a crude estimate, but more detailed calculations will be reported below. Here we just remind the reader that in our paper [12] the confining quark–antiquark potential was also derived from the same “molecular” contribution.

B. Historical comments

The overlapping $I\bar{I}$ configurations were first considered in the framework of the famous quantum mechanical problem of a double-well potential. In [13], by keeping certain points on the path fixed and minimizing the action, one finds that the “gradient flow” produces a certain set of configurations, which are *conditional* minima of the action (in all variables except along the flow gradient). This construction was developed in parallel to previously known concept in complex analysis known as “Lefschetz thimbles,” particular lines connecting all extrema of a function or functional on a complex plane. The $I\bar{I}$ streamline connects well-separated I and $\bar{I}$ (the extremum at $R_{I\bar{I}} \rightarrow \infty$) to the zero field “perturbative vacuum.”

In gauge theory, the derivation of the gradient flow equation was pioneered in [14]. The solution was first found for large distances (small overlap) of $I\bar{I}$, and then at all distances in [15], via the special conformal transformation of I and $\bar{I}$ into a cocentering configuration.

The first applications of the “molecular configurations” were in finite temperature QCD [16]. Indeed, in the QGP phase above the chiral phase transition there is no quark condensate, and “molecules” are the only leftovers of the instanton ensemble.

The fact that the $I\bar{I}$ thimble goes all the way to zero (perturbative) fields was the main difficulty in the studies of instanton ensembles. Numerical simulations [7] treated it “by brute force”: sufficiently close instanton–anti-instanton ($I\bar{I}$) pairs were excluded through the introduction of an artificial repulsive core, thereby removing configurations deemed insufficiently semiclassical. It is fair to say that the quantitative theory of $I\bar{I}$ thimbles still remains undeveloped.

The issue has been studied on the lattice, with the advent of the gradient flow method a decade ago. The density of $I\bar{I}$ correlated pairs was quantified by [11]. With increasing flow time τ , their study shows how $I\bar{I}$ pairs get annihilated, leading to a dilute instanton ensemble and once more confirming the ILM parameters (4). The important new observation was the density of correlated pairs or “molecules” at finite τ . The extrapolation to zero flow time ($\tau \rightarrow 0$) is $n_{\text{mol}} \sim 10 \text{ fm}^{-4}$, about an order of magnitude larger than the instanton density in ILM.

The contribution of the correlated $I\bar{I}$ pairs also resurfaced a few years ago in several applications, where the nonperturbative gauge fields (rather than their fermionic zero modes) are directly involved. The first was our study of the pion form factors in the semihard regime [17]. The second was our study of the heavy quark static and spin-dependent potentials [12]. In both analyses, the results were in agreement with phenomenology, after the contribution of the $I\bar{I}$ “molecules” was included.

The application of the $I\bar{I}$ molecules to the quarkonium central and spin-dependent potentials was worked out in

our recent paper [18]. It bridges the gap between the vacuum and hadronic structure by focusing on the inter-quark potentials in heavy quarkonia. In this case, the basic theory is well known. The central potential is related to the expectation value of the Wilson loop, while the spin-dependent potentials follow by dressing the loop with magnetic (or electric) field strengths. These calculations are methodologically quite close to those to be reported below. For the spin-dependent potentials, one has *two* gauge field strength insertions on the temporal Wilson line, while the matrix element of the operator (1) contains *one* gauge field strength on a lightlike Wilson line.

C. Field configurations

We now proceed to the space-time shape of the $I\bar{I}$ configurations. Since the conformal mapping leads to rather complicated expressions to implement, we will use a simpler (but we think still qualitatively representative) ansatz to describe them, a variant of the so called “ratio ansatz”

$$A^{\mu a}(x) = \frac{\bar{\eta}^{a\mu\nu} y_I^\nu \rho^2 / Y_I^2 + \eta^{a\mu\nu} y_A^\nu \rho^2 / Y_A^2}{1 + \rho^2 / Y_A + \rho^2 / Y_I} \quad (11)$$

where I, A stand for instanton and anti-instanton. Their centers are located at $y_{I,A}^i = x^i, i = 1, 2, 3$ and $y_I^4 = x^4 - R/2, y_A^4 = x^4 + R/2$, with $Y_{I,A}$ referring to their squared lengths, i.e., $Y_{I,A} = (y_{I,A}^\mu)^2$. Near one of the centers (e.g. Y_I small and Y_A large), the dominant contribution in the numerator is the first term, and with the leading terms in the denominator, yields the familiar field of an instanton in singular gauge.

We obtain lengthy but manageable expressions for the field strength and its squares, though they remain too long to present here. The distributions of the action density $(G_{\mu\nu}^a)^2$ and the corresponding fields were shown in [18]. However, in small separations $R < 0.5$, it does not go to zero but displays a small repulsive core, in contrast to streamline configurations.

The qualitative shape of the field $G_{\mu\nu}^a$ depends on the direction of the vector $\vec{R}_{I\bar{I}}$ between the two centers. If it is directed along the time axes, then the electric fields of I and $\bar{I}$ are of opposite sign and tend to cancel, while the magnetic fields are of the same sign and are doubled. However, in the calculations, one has to take $\vec{R}_{I\bar{I}}$ to be homogeneously distributed over the four-sphere. This produces a nontrivial relation between electric-induced and magnetic-induced forces.

D. Quark zero modes and propagators

As shown by ’t Hooft [19], in the instanton case the chirality flip is described via the fermionic zero modes. As a result, an instanton should be considered as an

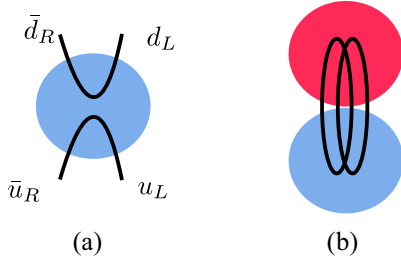


FIG. 1. (a) Instanton as an effective 't Hooft operator with four external lines; (b) quark propagation in the $I\bar{I}$ molecule. Both figures assume two light quark flavors, $N_f = 2$.

effective operator with $2N_f$ external quark lines, see Fig. 1(a).

The $I\bar{I}$ molecule is a configuration with zero topological charge, therefore its amplitude is always nonzero. The quark part of the functional determinant includes in this case loop diagrams proportional to the power of the so called “zero mode overlap” $\sim |T_{I\bar{I}}|^{2N_f}$, representing quarks traveling between an instanton and anti-instanton, as illustrated in Fig. 1(b). The analytic expression for this “hopping amplitude” is

$$T_{I\bar{I}} = \int d^4x \phi^\dagger(x - z_{\bar{I}}) i \not{D} \phi(x - z_I) \\ = \text{Tr}(U_I \tau_\mu^- U_{\bar{I}}^\dagger) \frac{R_\mu}{R} \frac{dT(R)}{dR} \quad (12)$$

where $R = z_I - z_{\bar{I}}$ and

$$T(R) = \frac{1}{2\pi^2 R} \int_0^\infty dp p^2 \frac{|\phi(p)|^2}{4\pi^2 \rho^2} J_1(pR) \quad (13)$$

and expressed via the Fourier transform

$$\phi(k) = \pi \rho^2 \frac{d}{dx} (I_0(x) K_0(x) - I_1(x) K_1(x))_{x=\frac{1}{2}k\rho} \quad (14)$$

of the fermion zero mode

$$\phi(x) = \frac{\rho}{\pi \sqrt{x^2} (x^2 + \rho^2)^{3/2}}. \quad (15)$$

Since $M(R)$ is an integral including many Bessel functions, we evaluated it numerically and found a simple parametrization of the result, see Figs. 2 and 3.

The amplitude of the $I\bar{I}$ molecule contains the overlap integral in the power $2N_f$. At large distances $R \gg \rho$ is decreasing very strongly. As a result, the distance between the centers is strongly restricted. As shown in the lower plot, after the overlap $|T_{I\bar{I}}|^2$ is multiplied by the 4D volume element R^3 , we find a rather sharp peak at $R/\rho \approx 1$. Therefore, we would focus on the $I\bar{I}$ molecules with a distance between the centers $R = \rho$.

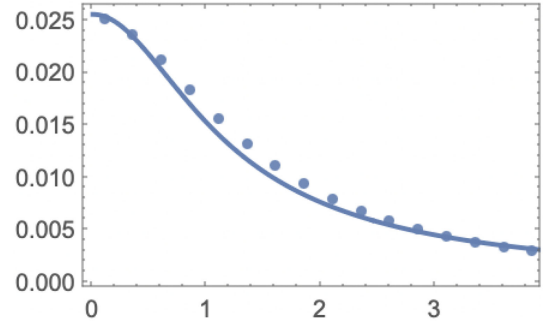


FIG. 2. Points show the numerical results for the dimensionless combination $\rho^2 T(r)$ versus r/ρ . The line is our approximate parametrization $0.0257/(r^2 + 1)^{0.75}$.

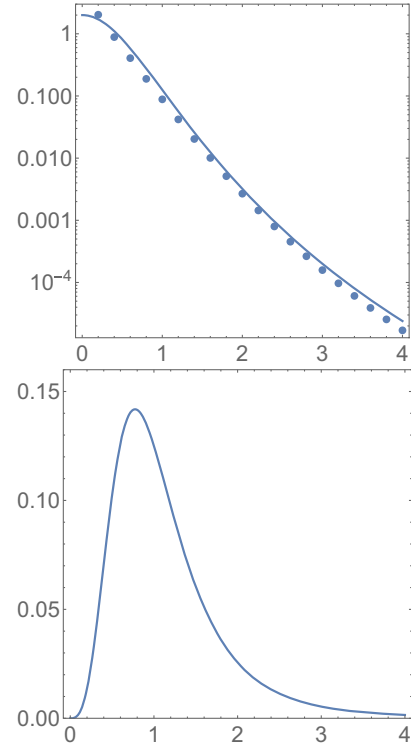


FIG. 3. Points in the upper plot show the numerical results for the zero mode overlap integral $|T_{I\bar{I}}(R)|^2$ versus R/ρ . The line is our approximate parametrization $2/(R^2 + 1)^4$. The lower plot shows the same amplitude times R^3 from the 4D radial integral. It displays a sharp peak at $R/\rho \approx 1$.

The average of the operator in question on the fields of the $I\bar{I}$ molecule is schematically shown in Fig. 4 (upper). The black dot corresponds to the location of the operator (1), where it picks up both the local field strength (shown) and the corresponding quark zero modes (not shown). It should then be integrated along the quark path (the vertical line). The orientation of the molecule needs to be averaged over the 4D orientation $\vec{R}_{I\bar{I}}$ (shown) and the location (not shown) of the molecule relative to the quark. An example of

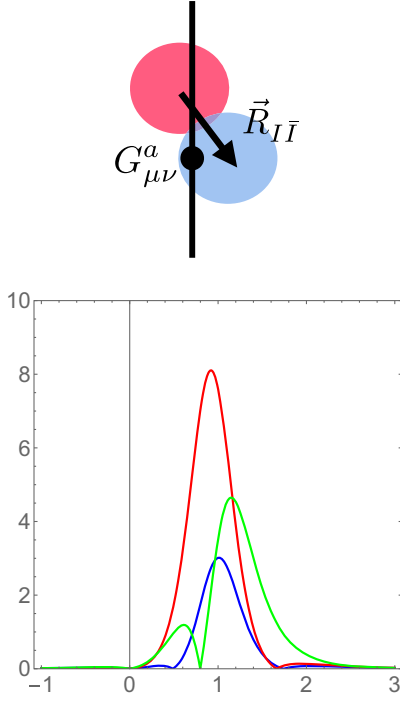


FIG. 4. Upper figure is a sketch of a molecular configuration pierced by a quark path in the time direction. The lower figure is a random example of how three components of the field ($m = 1, 2, 3$, blue, red, green, respectively) depend on time along the line.

the (absolute values) of the electric field components $E_m = (gG_{4m}^a \hat{r}^a)$ along the line (fourth coordinate) are shown in Fig. 4 (lower).

Using a Monte-Carlo method with 4D-random orientation of the vector $R_{I\bar{I}}$ and random shift of the molecule center, we generated thousands of such configurations of the fields and evaluated the integrated kick on a quark

$$\left\langle \int E d\tau \right\rangle^2 = \left(\int d\tau E_1 \right)^2 + \left(\int d\tau E_2 \right)^2 + \left(\int d\tau E_3 \right)^2 \approx [(4.15 \pm 0.21)/\rho]^2. \quad (16)$$

The spatial shift and temporal integration are selected accordingly with the ensemble measure normalized by the four volume. The distances between the molecules scale as the inverse fourth root of their dimensionless 4D density $n_{mol}\rho^4$. The force, defined by the integrated kick per time, thus reads

$$F = (n_{mol}\rho^4)^{\frac{1}{4}} \left\langle \int E d\tau \right\rangle \frac{1}{\rho^2}. \quad (17)$$

Using $n_{mol}\rho^4 \approx 0.10-0.15$ from [11] and $\rho = 0.3$ fm [5], we obtain an estimate for the force

$$F \approx (2-3) \text{ GeV/fm}. \quad (18)$$

The unstruck virtual quark loop connecting I and $\bar{I}$ contributes an additional factor $|T_{I\bar{I}}(R)|^2$, which is not shown here. We now proceed to a more refined analysis of this force in the low lying hadrons, e.g., pion and nucleon.

III. TWIST-3 PDF

As mentioned earlier, in DIS the twist-3 contribution describes the average Lorentz force acting on a quark in the nucleon. This contribution is accessible in the DIS process on a polarized nucleon target. In contrast to the structure function $g_1(x, Q^2)$ which is leading twist-2 and polarization independent, the structure function $g_2(x, Q^2)$ is twist-3 and polarization dependent [1,2]. We now proceed to its quantitative description in the QCD instanton vacuum, using the general framework developed in [20] that includes individual pseudoparticles (instantons, anti-instantons) and their molecular configurations (instanton-anti-instanton). We will focus on this average force acting on the emerging quark, the pion, and the nucleon.

The DIS process on a polarized target splits into longitudinal and transverse contributions

$$g_1(x)s_L = \int_{-\infty}^{\infty} \frac{d\xi^-}{4\pi} e^{i\xi^- p^+ x} \times \langle ps | \bar{q}(0) \gamma^+ \gamma_5 W(0, \xi^-) q(\xi^-) | ps \rangle \quad (19)$$

$$\frac{n_N}{p^+} g_T(x)s_{\perp} = \int_{-\infty}^{\infty} \frac{d\xi^-}{4\pi} e^{i\xi^- p^+ x} \times \langle ps | \bar{q}(0) \gamma^{\perp} \gamma_5 W(0, \xi^-) q(\xi^-) | ps \rangle. \quad (20)$$

The transverse distribution is the sum of g_2 and g_1

$$g_T(x) = g_1(x) + g_2(x) \quad (21)$$

a measure of the transverse spin. We now recall that g_2 satisfies the Burkhardt-Cottingham sum rule [21]

$$\int dx g_2(x, Q^2) = 0 \quad (22)$$

connecting the twist-2 and twist-3 parton distributions. Since the structure functions g_1, g_2 can be separated kinematically in experiments, this observation allows for the description of higher twist effects initially developed in [2,22–24].

For a transversely polarized nucleon $g_1(x, Q^2)$ vanishes and the remaining DIS amplitude is purely g_2 of twist 3. By analogy with the twist-2 PDFs $f_1(x), g_1(x), h_1(x)$, the twist-3 PDFs are referred to as $e(x), g_T(x), h_L(x)$. In particular, the twist-3 PDF $g_T(x)$ can be expressed as a sum of the Wandzura-Wilczek (WW) term, a piece that is determined entirely in terms of twist-2 helicity PDF $g_1(x)$, and an interaction dependent dynamical twist-3 term $\bar{g}_2(x)$, which involves quark-gluon correlations [25]

$$g_T(x, Q^2) = g_T^{WW}(x, Q^2) + \bar{g}_2(x, Q^2). \quad (23)$$

The first contribution is the twist-2 Wandzura-Wilczek fixed by [26]

$$g_T^{WW}(x, Q^2) \equiv \int_x^1 dy \frac{g_1(y, Q^2)}{y}. \quad (24)$$

After subtraction of this twist-2 contribution, the second Mellin moment of $\bar{g}_2(x)$ is related to the Lorentz color force matrix element [9,25]

$$\frac{d_2}{3} = \int_0^1 dx x^2 \bar{g}_2(x). \quad (25)$$

More specifically, the Lorentz force is defined by the matrix element

$$d_2 \epsilon_{\perp ij} S^j = \frac{\langle PS | \bar{q}(0) \gamma^+ g G^{+i}(0) q(0) | PS \rangle}{2m_N(P^+)^2}. \quad (26)$$

This result can be seen to follow from the identity [25]

$$\begin{aligned} & 2S^x m_N(P^+)^2 \int_0^1 dx x^2 g_T(x) \\ &= -\langle ps | \bar{q}(0) \gamma^x \gamma^5 (\vec{D}^+)^2 q(0) | ps \rangle \\ &= 2S^x m_N(P^+)^2 \int_{-1}^1 dx x^2 g_T^{WW}(x) \\ &\quad - \frac{1}{3} \langle ps | \bar{q}(0) \gamma^+ g G^{+y}(0) q(0) | ps \rangle. \end{aligned} \quad (27)$$

For a polarized nucleon along the x direction with light-cone momentum P^+ along the z direction and struck by a transverse probe $q_\perp$, the average color Lorentz force

density on a transverse plane is [10]

$$F_{q/h}^i(b) = i \int \frac{d^2 q_\perp}{(2\pi)^2} e^{-iq_\perp \cdot b} \frac{\langle h(p') | \bar{q} \gamma^+ i g G^{+i} q | h(p) \rangle}{\sqrt{2} \bar{p}^+}. \quad (28)$$

The presence of $\bar{q} \gamma^+ q$ in the operator means that this force is proportional to the quark density [27]

$$\rho_{q/h}(b) = \int \frac{d^2 q_\perp}{(2\pi)^2} e^{-iq_\perp \cdot b} \frac{\langle h(p') | \bar{q} \gamma^+ q | h(p) \rangle}{\sqrt{2} \bar{p}^+}, \quad (29)$$

which is a measure of the electromagnetic form factor in the transverse plane.

We note that the force involves the quark current with a single γ_μ which is chiral even ($LL + RR$). Since the instanton-induced operators are chiral odd ($LR + RL$), their contribution to the force is *a priori* suppressed, except for their nonzero mode contributions which are chiral even. This brings about the role and contribution of the molecular instanton-anti-instanton configurations to the force as we discussed earlier, although formally suppressed by an extra power of the packing fraction.

IV. EMERGENT FORM FACTORS AND THE COLOR LORENTZ FORCE

The Lorentz force contributions from individual instantons or anti-instantons are illustrated in Fig. 5, and the molecular contributions illustrated in Fig. 6. The details of their analytic forms are given in Appendix A. More specifically, the nonforward amplitude of the twist-3 operator can be deduced from the four-fermion contribution, as illustrated in Fig. 5, with the result

$$\begin{aligned} \langle h' | i g \bar{\psi} G_{\mu\nu} \gamma_\sigma \psi | h \rangle &= \frac{1}{4(N_c^2 - 1)} \frac{n_{I+\bar{I}}}{2} \left(\frac{4\pi^2 \rho^2}{m^*} \right) i \int \frac{d^4 k}{(2\pi)^4} 8G(\rho k) \left(\frac{k_\lambda k_\nu}{k^2} - \frac{1}{4} g_{\lambda\nu} \right) \\ &\quad \times \int d^4 x e^{-i(q+k)x} \langle h' | \bar{\psi}(x) \gamma_\sigma \lambda^A \psi(x) \bar{\psi}(0) \sigma_{\mu\lambda} \lambda^A \psi(0) | h \rangle \\ &\quad - \frac{n_{mol}}{4(N_c^2 - 1)^2} \gamma_{I\bar{I}} i t_{\mu\nu\rho\lambda\alpha\beta} \int \frac{d^4 k}{(2\pi)^4} \rho^2 \frac{k_\rho k_\lambda}{k^2} G(\rho k) \\ &\quad \times \int d^4 x e^{-i(q+k)x} \langle h' | \bar{\psi}(x) \gamma_\sigma \lambda^A \psi(x) \bar{\psi}(0) i \gamma_{(\alpha} \gamma^5 \vec{\partial}_{\beta)} \lambda^A \psi(0) | h \rangle. \end{aligned} \quad (30)$$

Here, each emergent quark carries a nonlocal form factor reflecting on its origin as a quark zero mode. In momentum space, this form factor is related to the Fourier transform of the zero mode in (14),

$$\mathcal{F}(k) = \left| \frac{\phi(k)}{2\pi\rho} \right|^2. \quad (31)$$

The Fourier transform of the instanton field profile gives

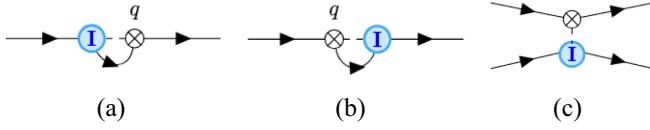


FIG. 5. Single instanton/anti-instanton vertices with the insertion of the color Lorentz operator (crossed circle): (a), (b) along a fermion line and (c) with a pair of fermion lines. The latter is suppressed by $1/N_c$ compared to the former.

$$G(k) = \frac{4\pi^2}{k^2} \left(1 - \frac{16}{k^2} + \frac{k^2}{2} K_2(k) + 2kK_3(k) \right) \quad (32)$$

where $K_{2,3}$ are modified Bessel functions of the second kind. More details are given in Appendix B. The emerging instanton–anti-instanton coefficient is estimated as

$$\gamma_{II} = \frac{2\pi^2}{n_{\text{mol}}} \left(\frac{n_{I+\bar{I}}}{2} \right)^2 \int_0^\infty dR R^3 \left| \frac{T_{II}}{m^*} \right|^{2N_f} \left(\frac{4\pi^2 \rho^2}{|T_{II}|} \right)^2 \times \left[\frac{-1}{4} R \frac{dT(R)}{dR} \right] \quad (33)$$

where the molecule density $n_{\text{mol}} = 7.248 \text{ fm}^{-4}$ is obtained after summing over all the three-flavors in the molecular pairing, as detailed in Appendix A.

We note that the derivation above relies on a local approximation for the quark-gluon operator. This approximation is justified by the strong localization of the instanton profile and the restriction on the quark separation,

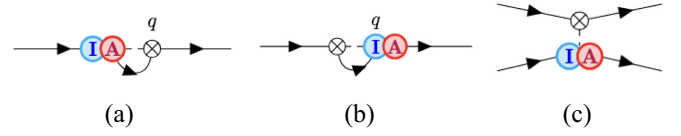


FIG. 6. Molecular pair of instanton–anti-instanton with the insertion of the color Lorentz operator (crossed circle): (a), (b) along a fermion line and (c) with a pair of fermion lines. The latter is suppressed by $1/N_c$ compared to the former.

$|x| \lesssim \rho$, as follows from the structure of the zero modes and the instanton-induced interaction. However, this simplification neglects nonlocal contributions inherent in the full expression (30), and therefore contains an additional source of systematic uncertainty in the evaluation of the color-force operator.

For the quark-gluon operator inside a hadron, the resulting effective quark operators are usually related to a four point correlation (two quark current and two hadronic source). Since the instanton profile is highly localized, the separation between the two-quark source in (30) is controlled by $|x| \lesssim \rho \ll \sqrt[4]{V_4/N}$. Thus, we can further approximate the hopping quark propagator in Figs. 5(a) and 5(b) for a single instanton, and 6(a) and 6(b) for an instanton–anti-instanton pair. The vertices in Figs. 5(c) and 6(c) will be neglected as they contribute to higher power of $1/N_c$.

With this in mind, the contribution to the (nonforward) matrix element of Fig. 5 is

$$\begin{aligned} \langle h' | i g \bar{\psi} \gamma_\sigma G_{\mu\nu} \psi | h \rangle_{I+\bar{I}} = & - \left(\frac{n_{I+\bar{I}}}{2} \right) \frac{1}{2N_c} \left(\frac{4\pi^2 \rho^2}{m^*} \right) \beta_{\bar{q}Gq,1}^{(+)}(\rho q) (g_{\mu\sigma} q_\nu - g_{\nu\sigma} q_\mu) \langle h' | \bar{\psi} \psi | h \rangle \\ & + \left(\frac{n_{I+\bar{I}}}{2} \right) \frac{1}{2N_c} \left(\frac{4\pi^2 \rho^2}{m^*} \right) \beta_{\bar{q}Gq,1}^{(+)}(\rho q) \epsilon_{\mu\nu\sigma\alpha} q_\alpha \langle h' | \bar{\psi} i \gamma^5 \psi | h \rangle \\ & - \left(\frac{n_{I+\bar{I}}}{2} \right) \frac{1}{2N_c} \left(\frac{4\pi^2 \rho^2}{m^*} \right) \rho^2 \beta_{\bar{q}Gq,2}^{(+)}(\rho q) 2i \epsilon_{[\mu\lambda\sigma\rho]} \left(q_\lambda q_\nu - \frac{1}{4} g_{\lambda\nu} q^2 \right) m^* \langle h' | \bar{\psi} \gamma_\rho \gamma^5 \psi | h \rangle \end{aligned} \quad (34)$$

where two new instanton form factors are defined as

$$\beta_{\bar{q}Gq,1}^{(+)}(q) = \frac{1}{q} \int_0^\infty dx \frac{16}{(x^2+1)^2} \frac{J_2(qx)}{qx} K_D(x), \quad (35)$$

$$\beta_{\bar{q}Gq,2}^{(+)}(q) = \frac{1}{q} \int_0^\infty dx \frac{16x^2}{(x^2+1)^2} \frac{J_3(qx)}{q^2 x^2} K_m(x) \quad (36)$$

with the quark zero mode induced modification during the quark propagation

$$K_D(x) = \frac{x^3}{(1+x^2)^{3/2}} \quad (37)$$

and

$$K_m(x) = \int_0^\infty dk x J_1(kx) \sqrt{\mathcal{F}(k)}. \quad (38)$$

At zero momentum transfer, the values of those two instanton form factors are $\beta_{\bar{q}Gq,1}^{(+)}(0) = \frac{2}{5}$ and $\beta_{\bar{q}Gq,2}^{(+)}(q \rightarrow 0) = -\frac{1}{3} \ln q$. The “molecular” contribution from Fig. 6 is

$$\begin{aligned}
\langle h' | i g \bar{\psi} \gamma_\sigma G_{\mu\nu} \psi | h \rangle_{I\bar{I}} = & -\frac{n_{mol} \gamma_{I\bar{I}}}{2N_c(N_c^2 - 1)} \beta_{\bar{q}Gq,1}^{(+)}(\rho q) (g_{\mu\sigma} g_{\nu\alpha} - g_{\nu\sigma} g_{\mu\alpha}) q_\beta \langle h' | \bar{\psi} \left(\gamma_{(\alpha} i \overleftrightarrow{\partial}_{\beta)} - \frac{1}{4} g_{\alpha\beta} i \overleftrightarrow{\not{D}} \right) \psi | h \rangle \\
& -\frac{n_{mol} \gamma_{I\bar{I}}}{2N_c(N_c^2 - 1)} \beta_{\bar{q}Gq,2}^{(+)}(\rho q) (g_{\mu\alpha} q_\nu - g_{\nu\alpha} q_\mu) \langle h' | \bar{\psi} \left(\gamma_{(\alpha} i \overleftrightarrow{\partial}_{\sigma)} - \frac{1}{4} g_{\alpha\sigma} i \overleftrightarrow{\not{D}} \right) \psi | h \rangle \\
& +\frac{n_{mol} \gamma_{I\bar{I}}}{2N_c(N_c^2 - 1)} \frac{1}{\rho^2} \left[\frac{1}{4} \rho^2 Q^2 \beta_{\bar{q}Gq,2}^{(+)}(\rho q) - \beta_{\bar{q}Gq,3}^{(+)}(\rho q) \right] i \epsilon_{\mu\nu\sigma\rho} \langle h' | \bar{\psi} \gamma_\rho \gamma^5 \psi | h \rangle \\
& -\frac{n_{mol} \gamma_{I\bar{I}}}{2N_c(N_c^2 - 1)} \left[\frac{1}{4} \beta_{\bar{q}Gq,2}^{(+)}(\rho q) - \beta_{\bar{q}Gq,4}^{(+)}(\rho q) \right] i \epsilon_{\mu\nu\sigma\lambda} q_\lambda q_\rho \langle h' | \bar{\psi} \gamma_\rho \gamma^5 \psi | h \rangle \\
& -\frac{n_{mol} \gamma_{I\bar{I}}}{2N_c(N_c^2 - 1)} \left[\frac{3}{4} \beta_{\bar{q}Gq,2}^{(+)}(\rho q) - \beta_{\bar{q}Gq,4}^{(+)}(\rho q) \right] i \epsilon_{\mu\nu\lambda\rho} q_\lambda q_\sigma \langle h' | \bar{\psi} \gamma_\rho \gamma^5 \psi | h \rangle \\
& -\frac{n_{mol} \gamma_{I\bar{I}}}{2N_c(N_c^2 - 1)} \beta_{\bar{q}Gq,5}^{(+)}(\rho q) m \epsilon_{\mu\nu\sigma\lambda} q_\lambda \langle h' | \bar{\psi} i \gamma^5 \psi | h \rangle \\
& -\frac{n_{mol} \gamma_{I\bar{I}}}{2N_c(N_c^2 - 1)} \rho^2 \beta_{\bar{q}Gq,6}^{(+)}(\rho q) i m \epsilon_{\mu\nu\alpha\lambda} q_\lambda q_\beta \langle h' | \bar{\psi} \sigma_{(\alpha} \gamma^5 \overleftrightarrow{\partial}_{\beta)} \psi | h \rangle \\
& +\frac{n_{mol} \gamma_{I\bar{I}}}{2N_c(N_c^2 - 1)} \beta_{\bar{q}Gq,7}^{(+)}(\rho q) i m \epsilon_{\mu\nu\alpha\lambda} \langle h' | \bar{\psi} \sigma_{\alpha\lambda} \gamma^5 \overleftrightarrow{\partial}_\lambda \psi | h \rangle
\end{aligned} \tag{39}$$

where the new “molecular form factors” $\beta^{(+-)}$ are defined in Appendix C.

V. DIS ON A SINGLE QUARK: COLOR FORCE

To analyze the effect in a nucleon we start with the simplest model, assuming that the ud quark pair form a spin-zero (“good scalar”) diquark and therefore do not contribute to the color force. The color force is then only active on the unpaired u quark. For (nonforward) quark states, with incoming momentum p and spin s , and outgoing momentum p' and spin s , covariance implies

$$\langle p' s | \bar{\psi} i g G_{\mu\nu} \gamma_\sigma \psi | p s \rangle = \bar{u}_s(p') \Gamma_{\mu\nu\sigma}^{(q)}(p', p) u_s(p) \tag{40}$$

where $u_s(p)$ stands for the in- and out-going (constituent) quark 4 spinor. In the forward limit, the contribution from

the single instanton vanishes as those contributions are proportional to the momentum transfer Q . The single instanton only contributes to the matrix element when the quark is kicked by nonzero Q . Therefore, near $Q = 0$ the molecular contribution is dominant. We now start the evaluation of the vertex function for a single quark using the fields of the instanton–anti-instanton molecules in QCD vacuum

$$\begin{aligned}
\Gamma_{\mu\nu\sigma}^{(q)}(p', p) \approx & -\frac{n_{mol}}{2N_c(N_c^2 - 1)} \gamma_{I\bar{I}} \rho^2 t_{\mu\nu\rho\lambda(\alpha\beta)} (\Gamma_{\rho\lambda\alpha\beta\sigma}(p') \\
& + \bar{\Gamma}_{\rho\lambda\alpha\beta\sigma}(p))
\end{aligned} \tag{41}$$

with

$$\begin{aligned}
\Gamma_{\rho\lambda\alpha\beta\sigma}(p) = & i \int \frac{d^4 k}{(2\pi)^4} \frac{(2p - k)_\beta k_\rho k_\lambda}{2k^2} G_{IA}(k) \sqrt{\mathcal{F}(p) \mathcal{F}(p - k)} \gamma_\alpha \gamma^5 S(p - k) \gamma_\sigma \\
= & -(g_{\alpha\delta} \gamma_\sigma \gamma^5 + g_{\sigma\delta} \gamma_\alpha \gamma^5 - g_{\alpha\sigma} \gamma_\delta \gamma^5 - i \epsilon_{\sigma\alpha\gamma\delta} \gamma_\gamma) \left[\int \frac{d^4 k}{(2\pi)^4} \frac{(2p - k)_\beta k_\rho k_\lambda (p - k)_\delta}{2k^2} \frac{G_{IA}(k) \sqrt{\mathcal{F}(p) \mathcal{F}(p - k)}}{(p - k)^2 - M^2} \right] \\
& + (g_{\sigma\alpha} + i \sigma_{\sigma\alpha}) \gamma^5 M \left[\int \frac{d^4 k}{(2\pi)^4} \frac{(2p - k)_\beta k_\rho k_\lambda}{2k^2} \frac{G_{IA}(k) \sqrt{\mathcal{F}(p) \mathcal{F}(p - k)}}{(p - k)^2 - M^2} \right]
\end{aligned} \tag{42}$$

and

$$\begin{aligned}
\bar{\Gamma}_{\rho\lambda\alpha\beta\sigma}(p) &= i \int \frac{d^4k}{(2\pi)^4} \frac{(2p-k)_\beta k_\rho k_\lambda}{2k^2} G_{IA}(k) \sqrt{\mathcal{F}(p)\mathcal{F}(p-k)} \gamma_\sigma S(p-k) \gamma_\alpha \gamma^5 \\
&= -(g_{\alpha\delta} \gamma_\sigma \gamma^5 + g_{\sigma\delta} \gamma_\alpha \gamma^5 - g_{\alpha\sigma} \gamma_\delta \gamma^5 + i\epsilon_{\sigma\alpha\gamma\delta} \gamma_\gamma) \left[\int \frac{d^4k}{(2\pi)^4} \frac{(2p-k)_\beta k_\rho k_\lambda (p-k)_\delta}{2k^2} \frac{G_{IA}(k) \sqrt{\mathcal{F}(p)\mathcal{F}(p-k)}}{(p-k)^2 - M^2} \right] \\
&\quad - (g_{\sigma\alpha} - i\sigma_{\sigma\alpha}) \gamma^5 M \left[\int \frac{d^4k}{(2\pi)^4} \frac{(2p-k)_\beta k_\rho k_\lambda}{2k^2} \frac{G_{IA}(k) \sqrt{\mathcal{F}(p)\mathcal{F}(p-k)}}{(p-k)^2 - M^2} \right]. \tag{43}
\end{aligned}$$

We used the effective quark propagator $S(k) = \frac{i(\not{k}+M)}{k^2-M^2}$.

A. Forward limit

Since these expressions are involved, let us evaluate them first at zero momentum transfer, following the decomposition [28],

$$\begin{aligned}
\langle ps | \bar{\psi} i g G^{\mu\nu} \gamma^\sigma \psi | ps \rangle &= 2i d_2^q M \epsilon^{\mu\nu\alpha\beta} s_\alpha p_\beta p^\sigma \\
&\quad + \frac{2i}{3} (f_2^q + d_2^q) M^3 \epsilon^{\mu\nu\sigma\alpha} s_\alpha. \tag{44}
\end{aligned}$$

Here d_2^q is related to the twist-3 moment on the light cone [28]. It represents a measure of the quark-gluon correlations and higher-order effects with spin structure. The parameter f_2^q is a twist-4 and spin-1 matrix element. This matrix element is a higher-twist correction to the spin-dependent deep inelastic scattering structure function, which describes the nucleon's spin structure, and is related to the twist-4 moment.

After averaging over quark spin ("spin independent"), the contribution vanishes

$$\langle ps | \bar{\psi} i g G_{\mu\nu} \gamma_\sigma \psi | ps \rangle_{\text{spin-indep}} = 0 \tag{45}$$

leaving only the spin contribution (proportional to quark spin four vector s_μ)

$$\begin{aligned}
&\langle ps | \bar{\psi} i g G_{\mu\nu} \gamma_\sigma \psi | ps \rangle \\
&\approx \frac{n_{mol}}{2N_c(N_c^2-1)} \gamma_{II} \rho^2 t_{\mu\nu\rho\lambda\alpha\beta} 4M \\
&\quad \times \left[\int \frac{d^4k}{(2\pi)^4} \frac{(2p-k)_\beta k_\rho k_\lambda}{2k^2} \frac{G_{IA}(k) \sqrt{\mathcal{F}(p-k)}}{(p-k)^2 - M^2} \right. \\
&\quad \left. \times (2p_\sigma s_\alpha + s \cdot k g_{\sigma\alpha} - k_\sigma s_\alpha - k_\alpha s_\sigma) \right] \\
&= \frac{n_{mol}}{N_c(N_c^2-1)} \gamma_{II} (M \mathcal{I}_1 2i \epsilon_{\mu\nu\alpha\beta} s^\alpha p^\beta p_\sigma + M^3 \mathcal{I}_2 2i \epsilon_{\mu\nu\sigma\alpha} s^\alpha). \tag{46}
\end{aligned}$$

The integrals defined in (46) simplify after the Lorentz symmetry reduction,

$$\begin{aligned}
\mathcal{I}_1 &\approx 2\rho^2 \int \frac{d^4k}{(2\pi)^4} \frac{G_{IA}(k) \sqrt{\mathcal{F}(k)}}{(p-k)^2 + M^2} \left(\frac{16(k \cdot p)^3}{3k^2 p^4} - \frac{32(k \cdot p)^2}{3k^2 p^2} \right. \\
&\quad \left. - \frac{8(k \cdot p)^2}{3p^4} + \frac{2k^2}{3p^2} + \frac{8(k \cdot p)}{3p^2} + \frac{8}{3} \right), \\
\mathcal{I}_2 &\approx 2\rho^2 \int \frac{d^4k}{(2\pi)^4} \frac{G_{IA}(k) \sqrt{\mathcal{F}(k)}}{(p-k)^2 + M^2} \left(\frac{8(k \cdot p)^3}{3k^2 p^4} - \frac{4(k \cdot p)^2}{3p^4} \right. \\
&\quad \left. + \frac{4k^2}{3p^2} - \frac{8(k \cdot p)}{3p^2} \right) \tag{47}
\end{aligned}$$

with the values $\mathcal{I}_1 \sim 0.1306$ and $\mathcal{I}_2 \sim -3.90$. Using the canonical constituent mass $M = 395$ MeV and the enhanced ILM with molecules with $n_{mol} \gamma_{II} = 176.42$ at the low resolution $\mu \sim 1/\rho$, and assuming that the unpaired quark carries the whole momentum, we obtain

$$\begin{aligned}
d_2^q &= \frac{n_{mol}}{N_c(N_c^2-1)} \gamma_{II} \mathcal{I}_1 = 0.96, \\
\frac{1}{3} (f_2^q + d_2^q) &= \frac{n_{mol}}{N_c(N_c^2-1)} \gamma_{II} \mathcal{I}_2 = -28.52 \tag{48}
\end{aligned}$$

which are to be compared to,

$$\begin{aligned}
d_2^q &= \frac{n_{mol} \gamma_{II}}{2N_c(N_c^2-1)} \beta_{qGq,7}^{(+)}(0) = 0.83, \\
\frac{1}{3} (f_2^q + d_2^q) &= \frac{-n_{mol} \gamma_{II}}{2N_c(N_c^2-1)} \frac{\beta_{qGq,3}^{(+)}(0)}{\rho^2 M^2} = -28.39 \tag{49}
\end{aligned}$$

using the short distance approximation as detailed in Appendix B. In this case, the results are similar, a measure of the accuracy of the approximation. Using the results (48), the ensuing average Lorentz force on a constituent quark produced by the instanton molecules, is

$$F^y = 2M^2 d_2^q = \frac{1.54 \text{ GeV}}{\text{fm}} d_2^q \sim 1.47 \frac{\text{GeV}}{\text{fm}} \tag{50}$$

which is still larger than the value of the string tension.

VI. DIS ON A PION: COLOR FORCE

The estimate of the effect on a nucleon in the previous section was done in a schematic quark-diquark model. For more accurate estimates, one would need some realistic LF

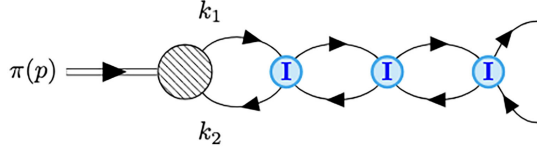


FIG. 7. Pion at low resolution $\mu \lesssim 1/\rho$, traveling through the QCD instanton vacuum.

nucleon wave functions. Fortunately, we will show below that this can be bypassed by using pertinent form factors.

To illustrate these ideas, let us start with a simpler case, that of deep inelastic scattering on a pion, as its description in the instanton vacuum is well established. The idea of how the pion emerges solely from the short distance 't Hooft $\bar{q}q$ attraction is diagrammatically illustrated in Fig. 7.

The nonforward amplitude for the twist-3 operator in a pion target is constrained by intrinsic parity and Hermiticity. Its Lorentz covariant form is characterized by two invariant form factors,

$$\begin{aligned} \langle \pi(p') | \bar{\psi} i g G^{\mu\nu} \gamma^\sigma \psi | \pi(p) \rangle \\ = 2(\bar{p}^\mu q^\nu - \bar{p}^\nu q^\mu) \bar{p}^\sigma \Phi_{\pi,1}^q(Q^2) \\ + 2m_\pi^2 (q^\mu g^{\nu\sigma} - q^\nu g^{\mu\sigma}) \Phi_{\pi,2}^q(Q^2) \end{aligned} \quad (51)$$

where

$$\Phi_{\pi,1}^q(Q^2) = -\frac{n_{mol}}{2N_c(N_c^2 - 1)} \gamma_{II} \beta_{\bar{q}Gq,2}^{(+-)}(\rho q) A_\pi^q(Q^2) \quad (52)$$

$$\begin{aligned} \Phi_{\pi,2}^q(Q^2) = & \left(\frac{n_{I+I}}{2} \right) \frac{1}{2N_c} \left(\frac{4\pi^2 \rho^2}{m^*} \right) \beta_{\bar{q}Gq,1}^{(+)}(\rho q) \frac{\sigma_\pi^q(Q^2)}{m} \\ & - \frac{n_{mol}}{8N_c(N_c^2 - 1)} \gamma_{II} \left[\beta_{\bar{q}Gq,1}^{(+-)}(\rho q) + \beta_{\bar{q}Gq,2}^{(+-)}(\rho q) \right] \\ & \times \left(1 + \frac{Q^2}{4m_\pi^2} \right) A_\pi^q(Q^2) \\ & - \frac{n_{mol}}{8N_c(N_c^2 - 1)} \gamma_{II} \left[\beta_{\bar{q}Gq,1}^{(+-)}(\rho q) - \frac{1}{3} \beta_{\bar{q}Gq,2}^{(+-)}(\rho q) \right] \\ & \times \frac{3Q^2}{4m_\pi^2} D_\pi^q(Q^2). \end{aligned} \quad (53)$$

Here σ_π^q is the quark scalar form factor for each flavor q in the pion, and A_π^q and D_π^q are the quark gravitational form factors for each flavor q in the pion. Their detailed definitions are given in Appendix E.

At zero momentum transfer, the forward matrix element of the color Lorentz force vanishes

$$\langle \pi(p) | \bar{\psi} i g G^{\mu\nu} \gamma^\sigma \psi | \pi(p) \rangle = 0. \quad (54)$$

This is expected, since the pion does not carry spin. However, the off-forward matrix element—the form factor

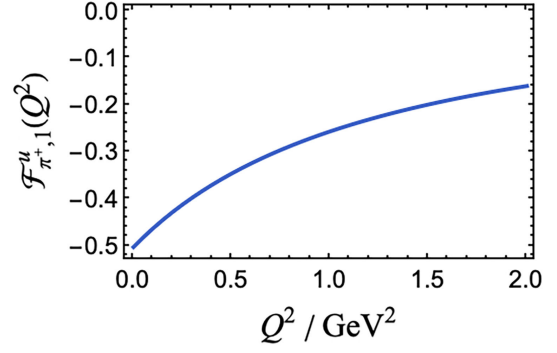


FIG. 8. Emergent form factor $\mathcal{F}_{\pi,1}^q(Q^2)$ induced by a color Lorentz operator in the pion in the ILM enhanced by “molecular” $I\bar{I}$ pairs.

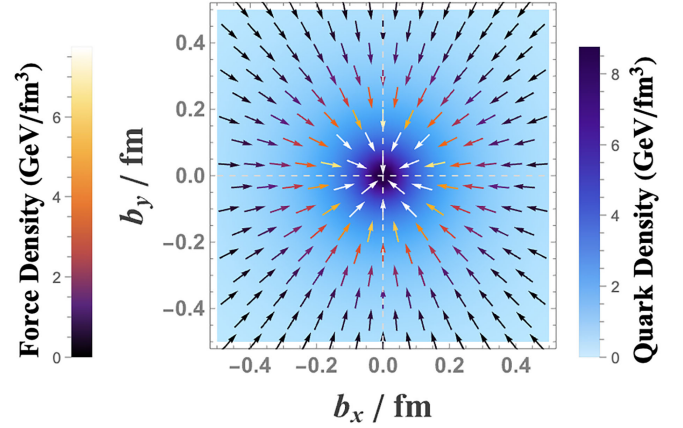


FIG. 9. Transverse field distribution of the color Lorentz force in an unpolarized up quark (arrows), along with the up quark density distribution (heat map), in impact parameter space for a pion in the ILM enhanced by molecular pairs.

of the color Lorentz force—is nonvanishing. Indeed, in the Drell-Yan frames with $q^+ = 0$, a collection of frames related by light-front boosts, we specialize to the Breit frame with $p_\perp = 0$, where the pion momenta have light-front components

$$\begin{aligned} p^\mu &= \left(p^+, -\frac{1}{2} q_\perp, \frac{m_\pi^2 + \frac{1}{4} q_\perp^2}{2p^+} \right), \\ p'^\mu &= \left(p^+, +\frac{1}{2} q_\perp, \frac{m_\pi^2 + \frac{1}{4} q_\perp^2}{2p^+} \right). \end{aligned} \quad (55)$$

The color force form factor in momentum space, is then

$$\langle \pi(p') | \bar{\psi} \gamma^+ i g G^{+i} \psi | \pi(p) \rangle = 2(p^+)^2 q^i \mathcal{F}_{\pi,1}^q(Q^2) \quad (56)$$

where

$$\mathcal{F}_{\pi,1}^q(Q^2) = \Phi_{\pi,1}^q(Q^2). \quad (57)$$

In transverse coordinate, it is given by (56)

$$F_{q/\pi}^i(b) = \frac{2b^i p^+}{2\pi b} \int_0^\infty dQ Q^2 \mathcal{F}_{\pi,1}^q(Q^2) J_1(Qb). \quad (58)$$

The behavior of the color force form factor in the pion (56) is shown in Fig. 8 for a range of Q^2 . We used the ILM parameters $n_{\text{mol}} = 7.248 \text{ fm}^{-4}$ and the physical pion mass $m_\pi = 140 \text{ MeV}$. The final result is evolved from $\mu = 1/\rho \approx 650 \text{ MeV}$ to 2 GeV for possible comparison to future lattice simulations. Note that the vanishing at $Q = 0$ of (56) is caused by the extra factor of Q^i . In Fig. 9 we show the distribution of the color-Lorentz force acting on an unpolarized up quark in the transverse plane (indicated by the vector field), superimposed on the up quark density distribution in impact parameter space in a pion. The quark density we get from the pion electromagnetic form factor,

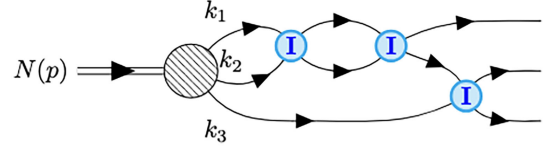


FIG. 10. Nucleon at low resolution $\mu \lesssim 1/\rho$, traveling through the QCD instanton vacuum.

using a vector meson dominance model with the vector meson mass $m_\rho = 791 \text{ MeV}$ [29,30].

VII. DIS ON THE NUCLEON: COLOR FORCE

The color Lorentz force on a quark in a nucleon can be carried out in the same spirit. The idea of how the nucleon emerges in ILM is diagrammatically illustrated in Fig. 10. Parity invariance, time reversal symmetry and Lorentz covariance, imply that the matrix element of the Lorentz force in the nucleon is characterized by eight form factors,

$$\begin{aligned} & \langle N(p', s') | \bar{\psi} i g G^{\mu\nu} \gamma^\sigma \psi | N(p, s) \rangle \\ &= \bar{u}_{s'}(p') \left\{ (\bar{p}^\mu q^\nu - \bar{p}^\nu q^\mu) \frac{\bar{p}^\sigma}{m_N} \Phi_{N,1}^q(Q^2) + m_N (q^\mu g^{\sigma\nu} - q^\nu g^{\sigma\mu}) \Phi_{N,2}^q(Q^2) \right. \\ &+ m_N i \sigma^{\mu\nu} \bar{p}^\sigma \Phi_{N,3}^q(Q^2) + m_N^2 i \epsilon^{\mu\nu\sigma\lambda} \gamma_\lambda \gamma^5 \Phi_{N,4}^q(Q^2) + m_N i \epsilon^{\mu\nu\sigma\lambda} q_\lambda \gamma^5 \Phi_{N,5}^q(Q^2) \\ &+ \frac{i \sigma^{\mu\alpha} \bar{p}^\nu - i \sigma^{\nu\alpha} \bar{p}^\mu}{2m_N} q_\alpha q^\sigma \Phi_{N,6}^q(Q^2) + (\bar{p}^\mu q^\nu - \bar{p}^\nu q^\mu) \frac{i \sigma^{\sigma\lambda} q_\lambda}{2m_N} \Phi_{N,7}^q(Q^2) + \left. \frac{i \sigma^{\mu\alpha} q^\nu - i \sigma^{\nu\alpha} q^\mu}{2m_N} q_\alpha \bar{p}^\sigma \Phi_{N,8}^q(Q^2) \right\} u_s(p). \quad (59) \end{aligned}$$

With the help of the short distance approximation from (34) and (39), each form factor can be expressed in terms of more standard nucleon form factors, as detailed in Appendix B,

$$\Phi_{N,1}^q(Q^2) = -\frac{n_{\text{mol}} \gamma \bar{\Gamma}}{2N_c(N_c^2 - 1)} \beta_{\bar{q}Gq,2}^{(+)}(\rho q) A_N^q(Q^2) + \frac{n_{\text{mol}} \gamma \bar{\Gamma}}{2N_c(N_c^2 - 1)} \beta_{\bar{q}Gq,7}^{(+)}(\rho q) \frac{m}{m_N} \left[\tilde{A}_T^q(Q^2) + \frac{1}{2} B_T^q(Q^2) \right], \quad (60)$$

$$\begin{aligned} \Phi_{N,2}^q(Q^2) &= \left(\frac{n_{I+\bar{I}}}{2} \right) \frac{1}{2N_c} \left(\frac{4\pi^2 \rho^2}{m^*} \right) \beta_{\bar{q}Gq,1}^{(+)}(\rho q) \frac{\sigma_N^q(Q^2)}{m} \\ &- \frac{n_{\text{mol}} \gamma \bar{\Gamma}}{8N_c(N_c^2 - 1)} \left[\beta_{\bar{q}Gq,1}^{(+)}(\rho q) + \beta_{\bar{q}Gq,2}^{(+)}(\rho q) \right] \left[\left(1 + \frac{Q^2}{4m_N^2} \right) A_N^q(Q^2) - \frac{Q^2}{2m_N^2} J_N^q(Q^2) \right] \\ &- \frac{n_{\text{mol}} \gamma \bar{\Gamma}}{8N_c(N_c^2 - 1)} \left[\beta_{\bar{q}Gq,1}^{(+)}(\rho q) - \frac{1}{3} \beta_{\bar{q}Gq,2}^{(+)}(\rho q) \right] \frac{3Q^2}{4m_N^2} D_N^q(Q^2) \\ &+ \frac{n_{\text{mol}} \gamma \bar{\Gamma}}{2N_c(N_c^2 - 1)} \beta_{\bar{q}Gq,7}^{(+)}(\rho q) \frac{m}{m_N} \left[\frac{1}{2} A_T^q(Q^2) + \frac{1}{2} B_T^q(Q^2) + \left(1 + \frac{Q^2}{4m_N^2} \right) \tilde{A}_T^q(Q^2) + 2C_T^q(Q^2) \right], \quad (61) \end{aligned}$$

$$\Phi_{N,3}^q(Q^2) = \frac{n_{\text{mol}} \gamma \bar{\Gamma}}{2N_c(N_c^2 - 1)} \frac{m}{m_N} \left[\beta_{\bar{q}Gq,7}^{(+)}(\rho q) - \frac{1}{2} \rho^2 Q^2 \beta_{\bar{q}Gq,6}^{(+)}(\rho q) \right] A_T^q(Q^2) \quad (62)$$

$$\begin{aligned} \Phi_{N,4}^q(Q^2) &= -\left(\frac{n_{I+\bar{I}}}{2} \right) \frac{1}{2N_c} \left(\frac{4\pi^2 \rho^2}{m^*} \right) \frac{\rho^2 Q^2}{2} \beta_{\bar{q}Gq,2}^{(+)}(\rho q) \frac{m}{m_N^2} G_A^q(Q^2) \\ &- \frac{n_{\text{mol}} \gamma \bar{\Gamma}}{2N_c(N_c^2 - 1)} \frac{1}{\rho^2 m_N^2} \left[\beta_{\bar{q}Gq,3}^{(+)}(\rho q) - \frac{1}{4} \rho^2 Q^2 \beta_{\bar{q}Gq,2}^{(+)}(\rho q) \right] G_A^q(Q^2), \quad (63) \end{aligned}$$

$$\begin{aligned}\Phi_{N,5}^q(Q^2) = & \left(\frac{n_{I+\bar{I}}}{2}\right) \frac{1}{2N_c} \left(\frac{4\pi^2\rho^2}{m^*}\right) \left[\beta_{\bar{q}Gq,1}^{(+)}(\rho q) \frac{\tilde{G}_P^q(Q^2)}{m} - \frac{1}{4}\rho^2 Q^2 \beta_{\bar{q}Gq,2}^{(+)}(\rho q) \frac{m}{m_N^2} G_P^q(Q^2) \right] \\ & + \frac{n_{mol}\gamma I\bar{I}}{N_c(N_c^2-1)} \left[\beta_{\bar{q}Gq,4}^{(+)}(\rho q) - \frac{1}{4}\beta_{\bar{q}Gq,2}^{(+)}(\rho q) \right] G_A^q(Q^2) \\ & + \frac{n_{mol}\gamma I\bar{I}}{2N_c(N_c^2-1)} \frac{1}{2\rho^2 m_N^2} \left[\frac{1}{2}\rho^2 Q^2 \beta_{\bar{q}Gq,2}^{(+)}(\rho q) - \beta_{\bar{q}Gq,3}^{(+)}(\rho q) - \rho^2 Q^2 \beta_{\bar{q}Gq,4}^{(+)}(\rho q) \right] G_P^q(Q^2) \\ & - \frac{n_{mol}\gamma I\bar{I}}{2N_c(N_c^2-1)} \beta_{\bar{q}Gq,5}^{(+)}(\rho q) \tilde{G}_P^q(Q^2)\end{aligned}\quad (64)$$

$$+ \frac{n_{mol}\gamma I\bar{I}}{2N_c(N_c^2-1)} \beta_{\bar{q}Gq,6}^{(+)}(\rho q) \frac{m}{m_N} \left[\rho^2 m_N^2 D_T^q(Q^2) - \frac{1}{2}\rho^2 Q^2 \tilde{D}_T^q(Q^2) \right], \quad (65)$$

$$\begin{aligned}\Phi_{N,6}^q(Q^2) = & -\frac{n_{mol}\gamma I\bar{I}}{N_c(N_c^2-1)} \left[\frac{3}{4}\beta_{\bar{q}Gq,2}^{(+)}(\rho q) - \beta_{\bar{q}Gq,4}^{(+)}(\rho q) \right] G_A^q(Q^2) \\ & + \frac{n_{mol}\gamma I\bar{I}}{2N_c(N_c^2-1)} \left[\beta_{\bar{q}Gq,7}^{(+)}(\rho q) + \frac{1}{2}\rho^2 Q^2 \beta_{\bar{q}Gq,6}^{(+)}(\rho q) \right] \frac{m}{m_N} \tilde{B}_T^q(Q^2)\end{aligned}\quad (66)$$

$$- \frac{n_{mol}\gamma I\bar{I}}{2N_c(N_c^2-1)} \beta_{\bar{q}Gq,6}^{(+)}(\rho q) \rho^2 m m_N D_T^q(Q^2), \quad (67)$$

$$\begin{aligned}\Phi_{N,7}^q(Q^2) = & -\left(\frac{n_{I+\bar{I}}}{2}\right) \frac{1}{N_c} \left(\frac{4\pi^2\rho^2}{m^*}\right) \beta_{\bar{q}Gq,2}^{(+)}(\rho q) \rho^2 m G_A^q(Q^2) \\ & - \frac{n_{mol}\gamma I\bar{I}}{2N_c(N_c^2-1)} \beta_{\bar{q}Gq,2}^{(+)}(\rho q) J_N^q(Q^2),\end{aligned}\quad (68)$$

$$\begin{aligned}\Phi_{N,8}^q(Q^2) = & \left(\frac{n_{I+\bar{I}}}{2}\right) \frac{1}{N_c} \left(\frac{4\pi^2\rho^2}{m^*}\right) \beta_{\bar{q}Gq,2}^{(+)}(\rho q) \rho^2 m G_A^q(Q^2) \\ & - \frac{n_{mol}\gamma I\bar{I}}{2N_c(N_c^2-1)} \beta_{\bar{q}Gq,2}^{(+)}(\rho q) J_N^q(Q^2) \\ & + \frac{n_{mol}\gamma I\bar{I}}{2N_c(N_c^2-1)} \beta_{\bar{q}Gq,7}^{(+)}(\rho q) \frac{m}{2m_N} B_T^q(Q^2).\end{aligned}\quad (69)$$

A. Parameter d_2 of the nucleons

At zero momentum transfer, the single instanton contribution is suppressed. The zero momentum transfer receives contributions from molecular configurations. $\Phi_{3,N}^q(0)$ is equal to $d_{2,N}^q$,

$$d_{2,N}^q = \frac{n_{mol}\gamma I\bar{I}}{2N_c(N_c^2-1)} \beta_{\bar{q}Gq,7}^{(+)}(0) \frac{m}{m_N} A_T^q(0). \quad (70)$$

If we assume the evolution does not mix flavors at one-loop, the one-loop evolution equation is [31]

$$d_2(\mu) = \left(\frac{\alpha_s(\mu')}{\alpha_s(\mu)} \right)^{\frac{\gamma_{d_2}}{\beta_0}} d_2(\mu') \quad (71)$$

where $\beta_0 = \frac{11}{3}N_c - \frac{2}{3}N_f$ and the one-loop anomalous dimension is defined by

$$\gamma_{d_2} = 3N_c - \frac{1}{6} \left(N_c - \frac{1}{N_c} \right). \quad (72)$$

The total proton and neutron d_2 can be constructed from $d_{2,N}^q$ using [31]

$$d_2^p = \left(\frac{2}{3} \right)^2 d_{2,N}^u + \left(-\frac{1}{3} \right)^2 d_{2,N}^d, \quad (73)$$

$$d_2^n = \left(-\frac{1}{3} \right)^2 d_{2,N}^u + \left(\frac{2}{3} \right)^2 d_{2,N}^d. \quad (74)$$

To compare our results with the lattice and global analyses, we evolve our ILM result from $\mu = 1/\rho \approx 650$ MeV to $\mu = 2$ GeV. The one-loop perturbative correction gives around

$$d_2(\mu = 2 \text{ GeV}) \approx 0.48 d_2(\mu = 1/\rho).$$

The final result is shown in Table I with our result using $n_{mol} = 7.248 \text{ fm}^{-4}$ and the physical pion mass $m_\pi = 140$ MeV,

TABLE I. ILM calculation with $n_{mol} = 7.248 \text{ fm}^{-4}$ and a physical pion mass $m_\pi = 140 \text{ MeV}$, evolved to 2 GeV by (71). Our results are compared to the lattice calculation in [36] at $\mu = 2.24 \text{ GeV}$, the recent lattice calculation from the QCDSF collaboration in [10] at a renormalization scale of 2 GeV and a heavy pion mass $m_\pi = 450 \text{ MeV}$, and the RQCD lattice collaboration in [31] at a renormalization scale of 2.24 GeV. Our results are also compared to the experimental fit from the E143 collaboration [37] at $\mu = 2.24 \text{ GeV}$.

	ILM	Göckeler <i>et al.</i> [36]	QCDSF [10]	RQCD [31]	E143 [37]
$d_{2,N}^u$	0.0255	0.010(12)	0.079(18)	0.025(4)(12)	0.02(4)
$d_{2,N}^d$	-0.00496	-0.0056(50)	-0.007(6)	-0.0081(25)(138)	0.02(4)
d_2^p	0.0108	0.004(5)	0.046(7)(16)	0.0105(68)	0.0122(106)
d_2^n	0.000636	-0.001(3)	0.023(5)(8)	-0.0009(70)	0.0106(443)

corresponding to current quark mass $m(\mu = 1/\rho) = 10 \text{ MeV}$, which has been suggested in many low energy vacuum-based models [7,29,32–35]. Note that in the result of the QCDSF collaboration in Table I, there is a mismatch between the d_2^p and d_2^n obtained from d_2^u and d_2^d by (73) presented in their plot, and the direct value of d_2^p and d_2^n they present in [10].

We note that the one-loop evolution from the low renormalization point $\mu \sim 1/\rho \approx 650 \text{ MeV}$ to higher scales introduces sizable uncertainties and scheme dependence, particularly below 1 GeV where perturbation theory is marginal. Therefore, the evolution presented here should be regarded as qualitative and illustrative. A quantitatively reliable matching to higher scales would require non-perturbative renormalization or lattice input as outlined in [38], which is still under development and lies beyond the scope of the present work.

B. Color force FFs of the nucleons

To evaluate the color force form factors (FFs) in the nucleon, we specialize again to the Breit frames with $q^+ = 0$ and $p_\perp = 0$ for the nucleon. The form factors defined on the light front are

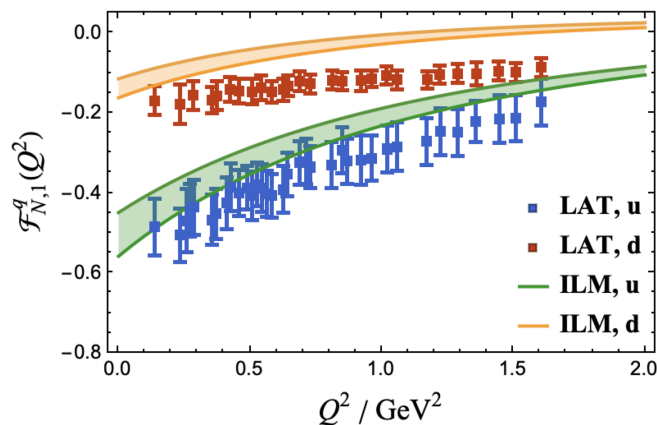


FIG. 11. Color force form factors $\mathcal{F}_{N,1}^q(Q^2)$ from ILM with parameters $n_{mol} = 7.248 \text{ fm}^{-4}$ and current quark mass $m = 10\text{--}58 \text{ MeV}$ ($m_\pi = 140\text{--}338 \text{ MeV}$) showing with bands, evolved to 2 GeV compared to the lattice calculation with pion mass 450 MeV in [10].

$$\begin{aligned} & \langle N(p's) | \bar{\psi} \gamma^+ i g G^{+i} \psi | N(ps) \rangle \\ &= \bar{u}_s(p') \left[\gamma^+ q^i \mathcal{F}_{N,1}^q(Q^2) + m_N i \sigma^{+i} \mathcal{F}_{N,2}^q(Q^2) \right. \\ & \quad \left. - \frac{i \sigma^{+j} q_j q^i}{m_N} \mathcal{F}_{N,3}^q(Q^2) \right] p^+ u_s(p) \end{aligned} \quad (75)$$

with

$$\begin{aligned} \mathcal{F}_{N,1}^q(Q^2) &= \Phi_{N,1}^q(Q^2), \\ \mathcal{F}_{N,2}^q(Q^2) &= \Phi_{N,3}^q(Q^2) \end{aligned} \quad (76)$$

and

$$\mathcal{F}_{N,3}^q(Q^2) = -\frac{1}{2} \left[\Phi_{N,7}^q(Q^2) + \Phi_{N,8}^q(Q^2) - \Phi_{N,1}^q(Q^2) \right]. \quad (77)$$

Note that all $\mathcal{F}_{N,1,2,3}^q(Q^2)$ are proportional to n_{mol} . The single instanton contribution in $\Phi_{N,7}$ and $\Phi_{N,8}$ cancel each other. Thus, even though in general the off-forward matrix element of $\bar{\psi} \gamma_\sigma G_{\mu\nu} \psi$ receives contribution from single instantons, the leading twist-3 contribution only receives contribution from $\bar{I}\bar{I}$ molecules.

The color force form factor $\mathcal{F}_{N,1}^q(Q^2)$ is related to the unpolarized nucleon gravitational form factor A_N^q and transversity gravitational form factor $2\bar{A}_T^q + B_T^q$. Its behavior is shown in Fig. 11, using the ILM with molecule density $n_{mol} = 7.248 \text{ fm}^{-4}$.

TABLE II. Numerical chiral relation among pion mass m_π and quark mass m at different energy scale with pion decay constant $f_\pi = 91 \text{ MeV}$ and quark condensate $\langle \bar{\psi} \psi \rangle = \langle \bar{u}u \rangle + \langle \bar{d}d \rangle = (300 \text{ MeV})^3$.

$m(\mu = 2 \text{ GeV})$	$m(\mu = 1/\rho)$	$m_\pi \text{ (MeV)}$
3 MeV	10 MeV	140
7.8 MeV	26 MeV	226
9.6 MeV	32 MeV	250
17.4 MeV	58 MeV	337

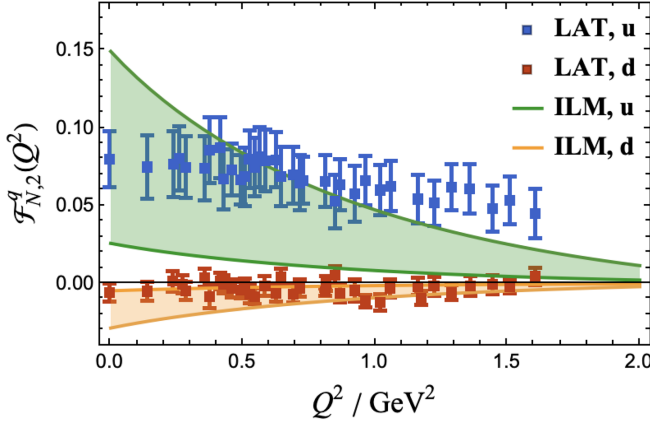


FIG. 12. Color Lorentz force form factors $\mathcal{F}_{N,2}^q(Q^2)$ using the ILM with parameters $n_{\text{mol}} = 7.248 \text{ fm}^{-4}$ and current quark mass $m = 10\text{--}58 \text{ MeV}$ ($m_\pi = 140\text{--}338 \text{ MeV}$) showing with bands, evolved to 2 GeV and compared to the lattice calculation with pion mass 450 MeV in [10].

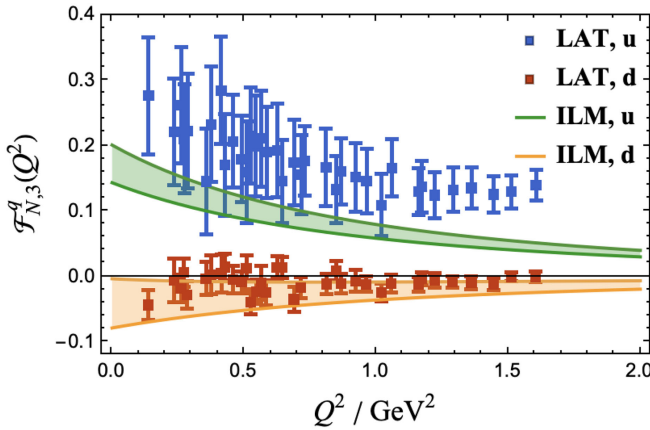


FIG. 13. Color Lorentz force form factors $\mathcal{F}_{N,3}^q(Q^2)$ from the ILM with parameters $n_{\text{mol}} = 7.248 \text{ fm}^{-4}$ and current quark mass $m = 10\text{--}58 \text{ MeV}$ ($m_\pi = 140\text{--}338 \text{ MeV}$) showing with bands, evolved to 2 GeV and compared to the lattice calculation with pion mass 450 MeV in [10].

In order to facilitate comparison with lattice results obtained at heavier pion masses ($m_\pi = 450 \text{ MeV}$), we adjust the quark mass m in the ILM using the chiral relation $m_\pi \propto \sqrt{m}$. The explicit numerical relation can be found in Table II. Under the assumption that the other ILM parameters (n_{mol} , γ_{II}) and the nucleon mass m_N are only weakly dependent on the quark mass m , this procedure provides an effective matching between the model parameters and the lattice setup. For completeness, we also present results over a range of the quark mass $m = 10 \text{ MeV}$ ($m_\pi = 140 \text{ MeV}$) and $m = 58 \text{ MeV}$ ($m_\pi = 338 \text{ MeV}$), which allows for a comparison of the quark-mass sensitivity. The plots suggest strong quark mass dependence in the second color force form factor

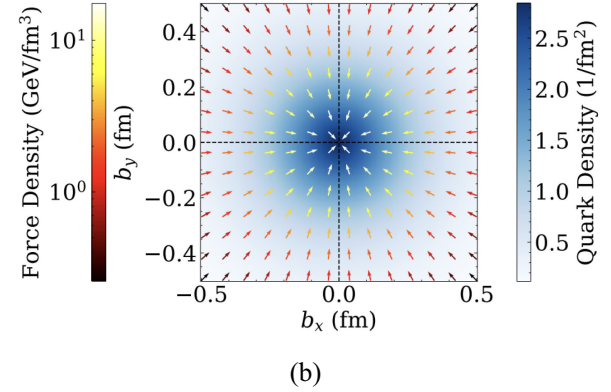
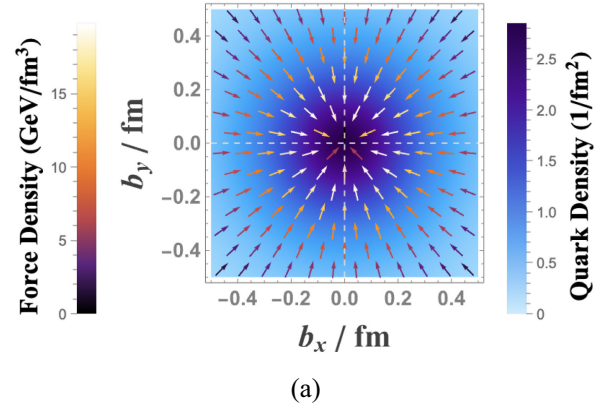


FIG. 14. (a) Transverse field distribution of the color Lorentz force in an unpolarized up quark (arrows), along with the up quark density distribution (heat map), for an unpolarized proton in the enhanced ILM. The up quark density follows from the proton Dirac and Pauli form factors from the phenomenological fit in [39]. The results are compared to the lattice results (b) in [10].

$\mathcal{F}_{N,2}^q(Q^2)$, which is opposite to the lattice observation in [31].

The results are evolved to 2 GeV, and compared to the lattice calculation with pion mass 450 MeV in [10]. Both the u - and d -flavor components of the Lorentz force are in good agreement with the reported lattice results, over a relatively broad range of Q^2 .

The color force form factor $\mathcal{F}_{N,2}^q(Q^2)$ is fully related to the transversity gravitational form factor A_T^q . Its behavior is shown in Fig. 12 using the ILM parameters. The comparison to the recently reported lattice results in [10] is fair. Note that the smallness of this form factor originates from its suppression by the quark mass m and in the ILM enhanced by molecules.

The color force form factor $\mathcal{F}_{N,3}^q(Q^2)$ is a combination of gravitational form factor B_N^q and transversity gravitational form factor $\tilde{A}_T^q$. This form factor is displayed in Fig. 13 using the ILM enhanced by molecules, and compared to the recent lattice results [10]. Again the comparison is satisfactory. Since $B_N^u + B_N^d \approx 0$, the asymmetry between u

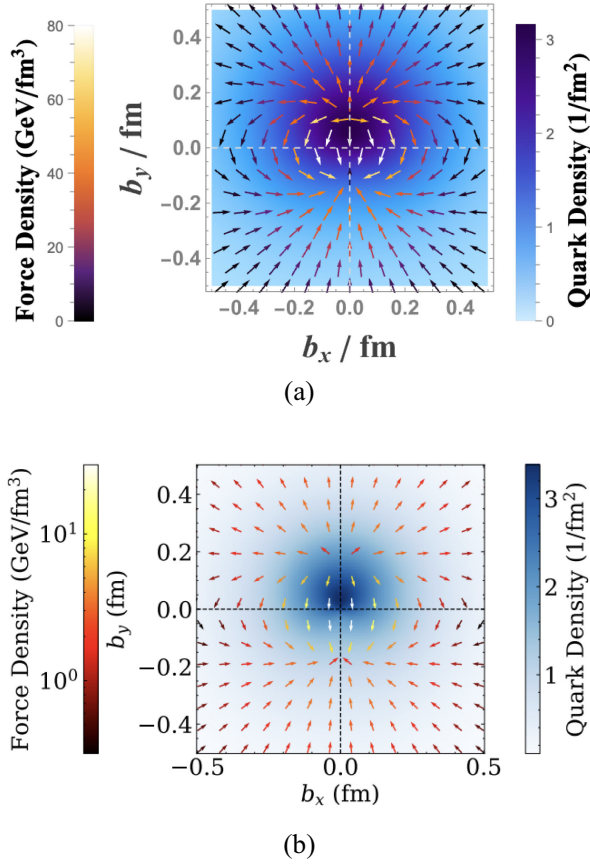


FIG. 15. (a) Transverse field distribution of the color Lorentz force in an unpolarized up quark (arrows), along with the up quark density distribution (heat map), for a proton transversely polarized in the $\hat{x}$ direction in the enhanced ILM. The up quark density follows from the proton Dirac and Pauli form factors from the phenomenological fit in [39]. The results are compared to the lattice results (b) in [10].

and d arises from the nonvanishing isoscalar transversity GPD moment $\tilde{A}_T^{u+d}(0)$.

In Figs. 14 and 15, we plot the color force density on a transverse plane at physical pion mass ($m_\pi = 140$ MeV). The result is also compared to the lattice calculation in [10] at a heavy pion mass ($m_\pi = 450$ MeV).

VIII. SUMMARY

We have developed a comprehensive and unified treatment of the twist-3 color Lorentz force in pions and nucleons, within the framework of the instanton liquid model and its “molecular” $\bar{I}\bar{I}$ extension.

The color Lorentz force arises naturally from the non-perturbative QCD vacuum fluctuations of the gauge field, including correlated instanton–anti-instanton pairs. By combining semiclassical strength and spatial structure of these fields with the operator structure of higher-twist quark-gluon correlators, we have shown that molecular

correlations provide an important mechanism behind the large higher-twist effects observed in polarized DIS, and confirmed in recent lattice simulations. This agrees with our previous derivation of the confining potential in quarkonia, from the same setting.

One new result of this study is the explicit construction and analysis of the gauge field structure of the molecular ensemble, and its role in the quark-colored force. By employing the “ratio ansatz” for the correlated instanton–anti-instanton pair, we derived explicit expressions for the color-electric and color-magnetic field components and computed their spatial profiles. The overlap of the quark zero modes across the molecular pair was analyzed in detail, revealing a strong delocalization effect that enhances the color force. Using Monte Carlo sampling over molecular orientations and separations, we obtained a robust estimate of the average color Lorentz force, $F \approx 2\text{--}3$ GeV/fm, acting on a single quark. This result indicates that the nonperturbative color forces are comparable to, or can even exceed the magnitude of the confining string tension.

Our analysis shows a direct connection between these microscopic nonperturbative fields and the experimentally accessible twist-3 observables. The color Lorentz force operator $\bar{q}gG^{+i}\gamma^+q$ —whose expectation value defines the twist-3 matrix element d_2 —is shown to acquire contributions from the molecular component of the instanton liquid. The induced form factors encode the nonlocal structure of this coupling and provide a unified description of the quark-gluon interaction over a broad range of momentum transfers. In particular, the molecular contributions dominate at low Q^2 , leading to enhanced color forces consistent with phenomenological extractions from polarized structure functions. These findings confirm that the twist-3 sector is a direct manifestation of the underlying topological fields in the QCD vacuum. Remarkably, the emergent color Lorentz force form factors are shown to be intimately related to the hadronic gravitational and transversity form factors, offering additional insights to the nature of the mass and force distributions in hadrons. However, one has to keep in mind that the relations shown in (34) and (39) are built on the local approximation of the original relation in (30), which involves a more complicated nonlocal two-current structure (see Appendix B).

From a phenomenological perspective, the implications of our results are as follows. For a constituent quark, the transverse color Lorentz force reaches $F_y \sim 1.5$ GeV/fm, reproducing the scale of the force inferred from lattice evaluations of d_2 and phenomenological analysis of $g_2(x, Q^2)$. In contrast, the pion, as a spinless bound state, exhibits a vanishing d_2^p , consistent with the absence of a net transverse Lorentz force. This clear dichotomy between spin-1/2 and spin-0 systems demonstrates that the twist-3 color force is intrinsically spin-dependent and directly tied to the internal color-magnetic structure of the hadron. Moreover, the ability of the instanton-molecule framework

to simultaneously describe both systems underscores its universality as a source of nonperturbative QCD dynamics.

Conceptually, this study shows how topological excitations of the QCD vacuum—once considered peripheral to the hadron structure—directly govern measurable high-energy observables. The emergence of strong, localized color forces from correlated instanton configurations provides a compelling microscopic picture that complements and extends conventional models based on confinement and gluon exchange. The instanton-molecule framework offers a natural bridge between the semiclassical field picture and the partonic description of hadrons, thereby unifying aspects of nonperturbative vacuum structure and experimental phenomenology.

The results also suggest new directions for future research. Extending the present formalism to GPDs and transverse-momentum-dependent distributions would enable mapping of the spatial and dynamical structure of color forces in both position and momentum space. Lattice simulations that isolate instanton and molecular contributions could provide stringent tests of the predicted force magnitudes and their dependence on quark flavor and spin.

The present analysis is subject to several limitations inherent to the ILM framework. First, confinement is not explicitly incorporated, and the results depend on phenomenological input for the instanton ensemble, including the typical instanton size and density. Second, the semiclassical treatment itself is not systematically controlled by a small parameter although the many-body expansion in our current framework is, and therefore the quantitative predictions should be interpreted at the level of order-of-magnitude estimates. Third, the use of a local approximation for the quark-gluon operator neglects nonlocal contributions that may become relevant at larger distances or lower momenta. Finally, the perturbative evolution from low scales introduces additional uncertainties. These notwithstanding, the framework captures essential nonperturbative features of the QCD vacuum and provides a coherent and physically transparent mechanism for the emergence of sizable color Lorentz forces, in qualitative agreement with recent lattice observations.

In summary, the instanton–anti-instanton molecular component of the QCD vacuum is an important source of the nonperturbative color Lorentz forces responsible for twist-3 phenomena in light hadrons. By relating the semiclassical field theory to measurable quantities, we have provided both qualitative understanding and quantitative predictions that connect the topology of the QCD vacuum to hadronic observables. The framework presented here not only enhances our comprehension of the QCD vacuum but also lays the foundation for a broader, unified picture of nonperturbative dynamics in strong interaction physics, ranging from hadronic spectroscopy to partonic physics on the light front.

ACKNOWLEDGMENTS

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DATA AVAILABILITY

The data are not publicly available. The data are available from the authors upon reasonable request.

APPENDIX A: EMERGENT INSTANTON INDUCED INTERACTIONS

To help organize the many-body calculus in the ILM enhanced by molecules, we introduce the effective vertices induced by single pseudoparticles and molecule, and we use the $1/N_c$ counting rules to organize the contributions. For that, consider the ILM vacuum partition function

$$\begin{aligned} Z_{\text{ILM}} &= \int \mathcal{D}\psi \mathcal{D}\psi^\dagger \frac{1}{N_+! N_-!} \prod_{l=1}^{N_+ + N_-} \\ &\times \left(\int d^4 z_l dU_l \int d\rho_l n(\rho_l) \rho_l^{N_f} \Theta_l(z_l) \right) \\ &\times \exp \left(\int d^4 x \psi^\dagger i \not{D} \psi \right) \\ &\rightarrow \int \mathcal{D}\psi \mathcal{D}\psi^\dagger \exp \left[- \int d^4 x (-\psi^\dagger i \not{D} \psi + \mathcal{L}_{\text{inst}} \right. \\ &\quad \left. + \mathcal{L}_{\text{mol}} + \dots) \right]. \end{aligned} \quad (\text{A1})$$

The molecular contributions follow from $\mathcal{L}_{\text{mol}}$, and the single pseudoparticle contributions from $\mathcal{L}_{\text{inst}}$, the detailed expression of which can be found in [32].

The molecular interactions between an instanton–anti-instanton pair mediated by light quarks, follow from

$$\begin{aligned} \mathcal{L}_{\text{mol}} &= \int d\rho_l d\rho_{\bar{l}} n(\rho_l) n(\rho_{\bar{l}}) \rho_l^{N_f} \rho_{\bar{l}}^{N_f} \\ &\times \int d^4 x d^4 y R \Theta_{\bar{l}l}(z_l, z_{\bar{l}}) \end{aligned} \quad (\text{A2})$$

where each instanton–anti-instanton pair induced vertex is defined by

$$\Theta_{\bar{l}l}(z_l, z_{\bar{l}}) = [\Theta_l(z_l) \Theta_{\bar{l}}(z_{\bar{l}})]_{\text{conn}}, \quad (\text{A3})$$

with all possible contractions. Some contractions are illustrated in Fig. 16 where the instanton vertex Θ_l is defined as

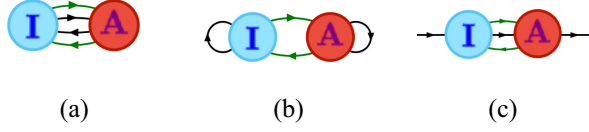


FIG. 16. Feynman diagrams for the vertices induced by a close pair of instanton (I) and anti-instanton (A): (a) the vacuum tunneling rate of a fully connected molecule, (b) its flavor reduced vacuum tunneling rate, and (c) the one-body (two-Fermi) vertex induced by an instanton-anti-instanton pair.

$$\Theta_I = (4\pi^2\rho^2)^{N_f} \times \prod_{q=1}^{N_f} \left(\frac{m_q}{4\pi^2\rho^2} - \frac{1}{8} \bar{q}_R U_I \tau_\mu^- \tau_\nu^+ \gamma_\mu \gamma_\nu U_I^\dagger q_L \right). \quad (\text{A4})$$

Here q denotes the quark flavor with current mass m_q , and L, R represents the chirality of the quark fields. A similar vertex holds for the anti-instantons, differing only by the interchange between $\tau_\mu^- \leftrightarrow \tau_\mu^+$ and $q_L \leftrightarrow q_R$.

The molecular tunneling rate can be obtained by resumming all closed diagrams such as in Figs. 16(a) and 16(b). The dominant contribution is from Fig. 16(a), due to the strong attraction induced by fermion exchange. The molecular tunneling rate is approximately of the form

$$\int d\rho_I d\rho_{\bar{I}} n(\rho_I) n(\rho_{\bar{I}}) \rho_I^{N_f} \rho_{\bar{I}}^{N_f} \int d^4 R du |T_{I\bar{I}}(u, R)|^{2N_f} = \left[\frac{n_{I+\bar{I}}}{2} \left(\frac{4\pi^2\rho^2}{m^*} \right)^{N_f} \right]^2 \left(\frac{|T_{I\bar{I}}|}{4\pi^2\rho^2} \right)^{2N_f} \quad (\text{A5})$$

where the quark hopping across the instanton-anti-instanton pair yields the hopping integral $T_{I\bar{I}}$ (the quark zero mode overlap between the pair) defined in (12).

The resulting determinantal mass is determined by

$$m^* = m - \frac{2\pi^2\rho^2}{N_c} \langle \bar{q}q \rangle \quad (\text{A6})$$

which characterizes the leading contribution in the ensemble average of the fermionic determinant with a measure of the unquenched the gauge configurations [32,34],

$$\left\langle \rho^{NN_f} \prod_f \text{Det}(\not{D} + m)_{\text{ZM}} \right\rangle = (\rho m^*)^{NN_f}. \quad (\text{A7})$$

For completeness, we present the numerical relation among the quark mass m , pion mass m_π , and determinantal mass m^* in Table II.

As shown in Fig. 16(c), the leading $1/N_c$ contribution to the color-singlet molecular vertex, is given by

$$\Theta_{I\bar{I}}^{\text{sin}}(z_I, z_{\bar{I}}) = \frac{1}{2N_c} |T_{I\bar{I}}|^{2N_f} \left(\frac{1}{|T_{I\bar{I}}|^2} \frac{iR_\mu}{R} \frac{dT(R)}{dR} \right) \times (4\pi^2\rho^2)^2 \left[\text{tr}_c(\tau_\mu^+ u) \text{tr}_c(\tau_\nu^- u^\dagger) \bar{\psi}_R(z_I) \gamma_\nu \psi_R(z_{\bar{I}}) - \text{tr}_c(\tau_\mu^- u^\dagger) \text{tr}_c(\tau_\nu^+ u) \bar{\psi}_L(z_{\bar{I}}) \gamma_\nu \psi_L(z_I) \right] + \dots \quad (\text{A8})$$

and the contribution to the color-octet vertex, is given by

$$\Theta_{I\bar{I}}^{\text{oct}}(z_I, z_{\bar{I}}) = \frac{1}{4} |T_{I\bar{I}}|^{2N_f} \left(\frac{1}{|T_{I\bar{I}}|^2} \frac{iR_\mu}{R} \frac{dT(R)}{dR} \right) \times (4\pi^2\rho^2)^2 \left[\text{tr}_c(\tau_\mu^+ u) \text{tr}_c(\tau_\nu^- u^\dagger \lambda^A) \bar{\psi}_R(z_I) \times \lambda^A \gamma_\nu \psi_R(z_{\bar{I}}) - \text{tr}_c(\tau_\mu^- u^\dagger) \text{tr}_c(\tau_\nu^+ \lambda^A u) \times \bar{\psi}_L(z_{\bar{I}}) \lambda^A \gamma_\nu \psi_L(z_I) \right] + \dots \quad (\text{A9})$$

where the Gell-Mann matrices are normalized to $\text{tr}_c(\lambda^A \lambda^B) = 2\delta^{AB}$. If we insert the vertices in (A8) and (A9) into (A2) and evaluate the integral in a similar way to (A5), we can simplify the integral to a typical coupling strength $\gamma_{I\bar{I}}$ for those one-body molecular vertices, which is defined by

$$\int d\rho_I d\rho_A n(\rho_I) n(\rho_A) \rho_I^{N_f} \rho_A^{N_f} \times \int du d^4 R \left[|T_{I\bar{I}}|^{2N_f} \frac{(4\pi^2\rho^2)^2}{|T_{I\bar{I}}|^2} \left(\frac{-1}{4} R \frac{dT(R)}{dR} \right) \right] \rightarrow n_{\text{mol}} \gamma_{I\bar{I}} \quad (\text{A10})$$

with the molecular density n_{mol} as defined in (A5). The value of $\gamma_{I\bar{I}}$ is obtained using determinantal mass 67.55 MeV in Eq. (33), yielding 311.13 fm^4 .

APPENDIX B: LORENTZ FORCE OPERATOR

In the ILM enhanced by molecules, the Lorentz force operator $g\bar{\psi} G^{\mu\nu} \gamma^\sigma \psi$ involves color-octet vertices, and follows by averaging over the individual pseudoparticles and their molecular pairs. To evaluate the Lorentz force produced by single instantons and $I\bar{I}$ molecules on a light quark, we decompose the field strength for a $I + \bar{I}$ pseudoparticle pair as the sum of the self-dual single field strengths plus a non-self-dual interaction term,

$$G_{\mu\nu}^a[I\bar{I}] = G_{\mu\nu}^a[A_{\bar{I}}] + G_{\mu\nu}^a[A_I] + G_{\mu\nu}^a[A_I, A_{\bar{I}}]. \quad (\text{B1})$$

1. Single instanton

The single instanton contribution to the Lorentz force operator reads

$$\begin{aligned}
g\bar{\psi}(x)G_{\mu\nu}(x)\gamma_\sigma\psi(x) &= \int d\rho n(\rho)\rho^{N_f} \int d^4z \int dU \bar{\psi}(x) \frac{\lambda^A}{2} G_{\mu\nu}^A(x) \gamma_\alpha \psi(x) [\Theta_I(z) + \Theta_{\bar{I}}(z)] \\
&= -\frac{1}{4(N_c^2 - 1)} \frac{n_{I+\bar{I}}}{2} \left(\frac{4\pi^2 \rho^2}{m^*} \right) \int d^4z \frac{8\rho^2}{[(x-z)^2 + \rho^2]^2 (x-z)^2} \left[\frac{(x-z)_\lambda (x-z)_\nu}{(x-z)^2} - \frac{1}{4} \delta_{\lambda\nu} \right] \\
&\quad \times \bar{\psi}(z) \sigma_{\mu\lambda} \lambda^A \psi(z) \bar{\psi}(x) \gamma_\sigma \lambda^A \psi(x) - (\mu \leftrightarrow \nu)
\end{aligned} \tag{B2}$$

where the Gell-Mann matrices here are normalized to $\text{tr}_c(\lambda^A \lambda^B) = 2\delta^{AB}$. Carrying the color average and the size integration, yield

$$\begin{aligned}
g\bar{\psi}G_{\mu\nu}\gamma_\sigma\psi(x) &= -\frac{1}{4(N_c^2 - 1)} \frac{n_{I+\bar{I}}}{2} \left(\frac{4\pi^2 \rho^2}{m^*} \right) \int d^4z \frac{8\rho^2}{[(x-z)^2 + \rho^2]^2 (x-z)^2} \\
&\quad \times \left(\frac{(x-z)_\lambda (x-z)_\nu}{(x-z)^2} - \frac{1}{4} g_{\lambda\nu} \right) \bar{\psi}(x) \gamma_\sigma \lambda^A \psi(x) \bar{\psi}(z) \sigma_{\mu\lambda} \lambda^A \psi(z) - (\mu \leftrightarrow \nu).
\end{aligned} \tag{B3}$$

2. Instanton pairs

Since we are interested in the Lorentz color force, the color-octet vertices are relevant. The operator $g\bar{\psi}G_{\mu\nu}\gamma_\sigma\psi$ in the instanton vacuum molecules, receives a dominant contribution from Fig. 16(c) in the form

$$\begin{aligned}
&\int d\rho_I d\rho_A n(\rho_I) n(\rho_{\bar{I}}) \rho_I^{N_f} \rho_A^{N_f} \int d^4z du d^4R \bar{\psi}(x) G_{\mu\nu}(x) \gamma_\alpha \psi(x) \Theta_{I\bar{I}}^{\text{oct}}(z_I, z_{\bar{I}}) \\
&= \int d^4z d^4R \left[\frac{n_{I+\bar{I}}}{2} \left(\frac{4\pi^2 \rho^2}{m^*} \right)^{N_f} \right]^2 \left(\frac{|T_{I\bar{I}}|}{4\pi^2 \rho^2} \right)^{2N_f} \left[\frac{(4\pi^2 \rho^2)^2}{|T_{I\bar{I}}|^2} \frac{iR_\beta}{R} \frac{dT(R)}{dR} \right] \\
&\quad \times \frac{1}{4(N_c^2 - 1)^2} \frac{\epsilon^{abc} (\bar{\eta}_{\mu\rho}^b \eta_{\nu\lambda}^d - \bar{\eta}_{\nu\rho}^b \eta_{\mu\lambda}^d) \rho^4 (x - z_I)_\rho (x - z_{\bar{I}})_\lambda}{[(x - z_I)^2 + \rho^2]^2 [(x - z_{\bar{I}})^2 + \rho^2]^2 (x - z_{\bar{I}})^2} \bar{\psi}(x) \gamma_\sigma \lambda^A \psi(x) \\
&\quad \times \left[\text{tr}_c(\tau_\alpha^- \tau^d \tau_\beta^+ \tau^c \tau^a) \bar{\psi}(z_I) \gamma_\alpha \frac{1 + \gamma^5}{2} \lambda^A \psi(z_{\bar{I}}) - \text{tr}_c(\tau_\alpha^+ \tau^a \tau_\beta^- \tau^c \tau^d) \bar{\psi}(z_{\bar{I}}) \gamma_\alpha \frac{1 - \gamma^5}{2} \lambda^A \psi(z_I) \right].
\end{aligned} \tag{B4}$$

Here $\bar{\eta}_{\mu\nu}^a$ and $\eta_{\mu\nu}^a$ are the 't Hooft symbols for the instanton and anti-instanton in singular gauge, and τ^a an $N_c \times N_c$ matrix with 2×2 Pauli matrices embedded in the upper left corner.

In general, the effective quark operators induced by the higher order instanton clusters are nonlocal in the quark fields $\bar{\psi}(z_I)\psi(z_{\bar{I}})$. However, as a result of the highly localized nature of the instanton profiles, the effective range for the instanton vertex is constrained within the instanton size $R = |z_I - z_{\bar{I}}| \sim \rho \ll \sqrt[4]{V_4/N}$. Hence we can approximate the nonlocal quark operators with local quark operators by localizing the instanton cluster profiles. This reduction will be referred to as the local approximation, as originally suggested in [20]. The contribution to those matrix elements in leading order, comes from those clusters. Therefore, in the local approximation

$$\bar{\psi}(z_I)\psi(z_{\bar{I}}) \simeq \bar{\psi}(z)\psi(z) - R_\mu \bar{\psi}(z) \overleftrightarrow{\partial}_\mu \psi(z) + \dots$$

where z is the center of the pseudoparticle pair, and R is their relative distance.

With this in mind, (B4) can be written as

$$\begin{aligned}
&g\bar{\psi}(x)G_{\mu\nu}(x)\gamma_\sigma\psi(x) \\
&= \frac{1}{4(N_c^2 - 1)^2} \int d^4z d^4R \left(\frac{n_{I+\bar{I}}}{2} \right)^2 \left(\frac{|T_{IA}|^2}{(m^*)^2} \right)^{N_f} \left(\frac{4\pi^2 \rho^2}{|T_{IA}|} \right)^2 \\
&\quad \times \left[\frac{-1}{4} R \frac{dT(R)}{dR} \right] \\
&\quad \times t_{\mu\nu\rho\lambda\alpha\beta} \frac{\rho^4 (x - z)_\rho (x - z)_\lambda}{[(x - z)^2 + \rho^2]^2 (x - z)^4} \\
&\quad \times \bar{\psi}(x) \gamma_\sigma \lambda^A \psi(x) \bar{\psi}(z) i\gamma_{(\alpha} \gamma^5 \overleftrightarrow{\partial}_{\beta)} \lambda^A \psi(z)
\end{aligned} \tag{B5}$$

where $(\dots)$ bracketing the Lorentz indices refers to full symmetrization, and

$$\bar{\psi} i \overleftrightarrow{\partial}_\mu \psi = \frac{1}{2} \bar{\psi} (i \vec{\partial}_\mu - i \overleftarrow{\partial}_\mu) \psi.$$

The color factor is defined as

$$t_{\mu\nu\rho\lambda\alpha\beta} = 2i(\bar{\eta}_{\mu\rho}^b \eta_{\nu\lambda}^d - \bar{\eta}_{\nu\rho}^b \eta_{\mu\lambda}^d) \text{tr}(\tau_\alpha^- \tau^d \tau_\beta^+ \tau^b) \tag{B6}$$

where the last two indices are symmetric $t_{\mu\nu\rho\lambda(\alpha\beta)} = t_{\mu\nu\rho\lambda\beta\alpha}$, and the middle two indices are also symmetric but traceless. Finally, the Fourier transform is then

$$\begin{aligned} \frac{1}{V} \int d^4x e^{iqx} g\bar{\psi}(x) G_{\mu\nu}(x) \gamma_\sigma \psi(x) &= \frac{1}{4(N_c^2 - 1)} \frac{n_{I+\bar{I}}}{2} \left(\frac{4\pi^2 \rho^2}{m^*} \right) \int \frac{d^4k}{(2\pi)^4} 8G_I(k) \left(\frac{k_\lambda k_\nu}{k^2} - \frac{1}{4} g_{\lambda\nu} \right) \\ &\times \int d^4x e^{i(q-k)x} \bar{\psi}(x) \gamma_\sigma \lambda^A \psi(x) \int d^4z e^{ikz} \bar{\psi}(z) \sigma_{\mu\lambda} \lambda^A \psi(z) \\ &- \frac{n_{\text{mol}}}{4(N_c^2 - 1)^2} \gamma_{I\bar{I}} t_{\mu\nu\rho\lambda\alpha\beta} \int \frac{d^4k}{(2\pi)^4} \rho^2 \frac{k_\rho k_\lambda}{k^2} G_{IA}(k) \\ &\times \int d^4x e^{i(q-k)x} \bar{\psi}(x) \gamma_\sigma \lambda^A \psi(x) \int d^4z e^{ikz} \bar{\psi}(z) i\gamma_{(\alpha} \gamma^5 \overleftrightarrow{\partial}_{\beta)} \lambda^A \psi(z) \end{aligned} \quad (\text{B7})$$

where the instanton field profile $G(k)$ in momentum space is defined by

$$\begin{aligned} G_I(k) &= \frac{4\pi^2}{k^2} \left(\frac{8}{k^2} - \frac{k^2}{2} K_2(k) - kK_3(k) \right) \text{ and} \\ G_{IA}(k) &= \frac{4\pi^2}{k^2} \left(1 - \frac{16}{k^2} + \frac{k^2}{2} K_2(k) + 2kK_3(k) \right) \end{aligned} \quad (\text{B8})$$

The final result (B7) is illustrated in Fig. 17. The Lorentz operator (crossed dot) takes the form of a product of two color-octet currents, analogous to a one-gluon exchange between quarks. The function $G(k)$ plays the role of an effective gluon propagator. The evaluation of its nucleon matrix element is therefore closely related to the calculation of gluon-exchange corrections to the nucleon mass [8,40].

a. Short distance approximation

As we noted earlier, the pseudoparticle profile is highly localized. The separation between two quark sources in (B7), is controlled by $|x - z| \lesssim \rho \ll \sqrt[4]{V_4/N}$, hence the approximation

$$\begin{aligned} \sqrt{\mathcal{F}(i\rho\partial)} S(x) &\equiv \langle \psi(x) \sqrt{\mathcal{F}(i\rho\partial)} \bar{\psi}(0) \rangle \\ &\simeq \frac{i\not{x}}{2\pi^2 x^4} K_D(x/\rho) + \frac{m}{4\pi^2 x^2} K_m(x/\rho) \\ &+ \mathcal{O}(m^2). \end{aligned} \quad (\text{B9})$$

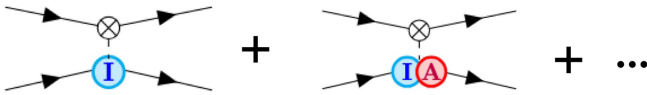


FIG. 17. Feynman diagrams for the Lorentz force operator (crossed dot) in the ILM enhanced by molecules. The quark lines connected to the crossed dot at the top of each diagram are located at x while the quark lines at the bottom of each diagram are connected to the blob of the classical pseudoparticle profile centered at z . The dashed lines indicate all the gauge fields in the operator share the classical pseudoparticle configuration.

In the ILM the quark propagator $S(x)$ is defined as

$$S(x) = \frac{i\not{x}}{2\pi^2 x^4} + \frac{m}{4\pi^2 x^2} + \mathcal{O}(m^2) \quad (\text{B10})$$

with the distortions induced by the instanton zero mode profiles

$$\begin{aligned} K_D(x) &= \frac{1}{2} \int_0^\infty dk k x^2 J_2(kx) \sqrt{\mathcal{F}(k)} = \frac{x^3}{(1+x^2)^{3/2}}, \\ K_m(x) &= \int_0^\infty dk x J_1(kx) \sqrt{\mathcal{F}(k)}. \end{aligned} \quad (\text{B11})$$

In this short distance approximation, the effective Lorentz force operator can be further reduced to a sum of local fermionic operators induced by the zero modes in the ensemble,

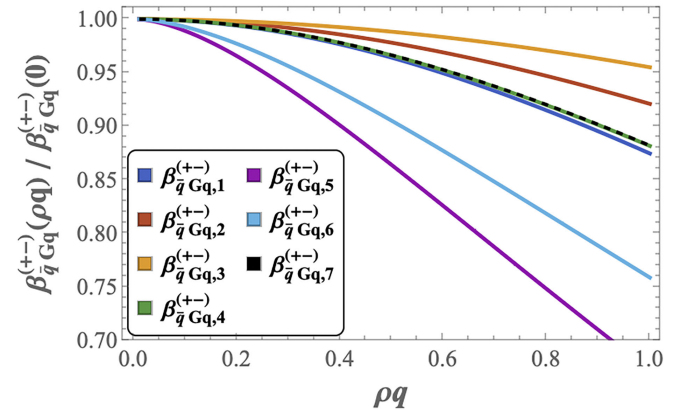


FIG. 18. Emergent form factors $\beta_{qGq}^{(+)}$ associated to the color Lorentz operator in a molecular (instanton-anti-instanton) pair.

$$\begin{aligned}
& g\bar{\psi}\gamma_\sigma G_{\mu\nu}\psi(x) \\
&= -\left(\frac{n_{I+\bar{I}}}{2}\right)\frac{1}{2N_c}\left(\frac{4\pi^2\rho^2}{m^*}\right)\int d^4z\left[F_{\bar{q}Gq,1,\mu\nu\sigma}^{(+)}(x-z)\bar{\psi}(z)\frac{1-\gamma^5}{2}\psi(z)+F_{\bar{q}Gq,2,\mu\nu\sigma\rho}^{(+)}(x-z)m\bar{\psi}(z)\gamma_\rho\gamma^5\psi(z)\right] \\
&\quad -\left(\frac{n_{I+\bar{I}}}{2}\right)\frac{1}{2N_c}\left(\frac{4\pi^2\rho^2}{m^*}\right)\int d^4z\left[F_{\bar{q}Gq,1,\mu\nu\sigma}^{(-)}(x-z)\bar{\psi}(z)\frac{1+\gamma^5}{2}\psi(z)+F_{\bar{q}Gq,2,\mu\nu\sigma\rho}^{(-)}(x-z)m\bar{\psi}(z)\gamma_\rho\gamma^5\psi(z)\right] \\
&\quad +\left(\frac{n_{I+\bar{I}}}{2}\right)^2\frac{1}{2N_c(N_c^2-1)}\int d^4R\left(\frac{|T_{IA}|^2}{(m^*)^2}\right)^{N_f}\frac{(4\pi^2\rho^2)^2}{|T_{IA}|^2}\left[\frac{-1}{4}R\frac{dT(R)}{dR}\right] \\
&\quad \times\int d^4z\left[F_{\bar{q}Gq,1,\mu\nu\sigma\alpha\beta}^{(+-)}(x-z)\bar{\psi}(z)\gamma_\alpha i\overleftrightarrow{\partial}_\beta\psi(z)+F_{\bar{q}Gq,2,\mu\nu\sigma\rho}^{(+-)}(x-z)\bar{\psi}(z)\gamma_\rho\gamma^5\psi(z)\right. \\
&\quad \left.+F_{\bar{q}Gq,3,\mu\nu\sigma\alpha\beta}^{(+-)}(x-z)m\bar{\psi}(z)\sigma_{\alpha\beta}\gamma^5\overleftrightarrow{\partial}_\beta\psi(z)+F_{\bar{q}Gq,4,\mu\nu\sigma}^{(+-)}(x-z)m\bar{\psi}(z)i\gamma^5\psi(z)\right]
\end{aligned} \tag{B12}$$

with

$$F_{\bar{q}Gq,1,\mu\nu\sigma}^{(\pm)}(x)=\frac{8\rho^2}{(x^2+\rho^2)^2}K_D(x/\rho)\frac{g_{\mu\sigma}x_\nu-g_{\nu\sigma}x_\mu\pm i\epsilon_{\mu\nu\sigma\alpha}x_\alpha}{2\pi^2x^4}, \tag{B13}$$

$$F_{\bar{q}Gq,2,\mu\nu\sigma\rho}^{(\pm)}(x)=\pm\frac{4\rho^2}{(x^2+\rho^2)^2}K_m(x/\rho)\left[\frac{i(g_{\mu\rho}g_{\lambda\sigma}-g_{\lambda\rho}g_{\mu\sigma})\pm\epsilon_{\mu\lambda\sigma\rho}}{2\pi^2x^2}\left(\frac{x_\lambda x_\nu}{x^2}-\frac{1}{4}g_{\lambda\nu}\right)-(\mu\leftrightarrow\nu)\right], \tag{B14}$$

$$F_{\bar{q}Gq,1,\mu\nu\sigma\alpha\beta}^{(+-)}(x)=\frac{\rho^4}{(x^2+\rho^2)^2x^2}K_D(x/\rho)\frac{x_\rho x_\lambda}{x^2}t_{\mu\nu\rho\lambda(\gamma\beta)}\frac{\epsilon_{\alpha\sigma\delta\gamma}x_\delta}{2\pi^2x^4}, \tag{B15}$$

$$\begin{aligned}
F_{\bar{q}Gq,2,\mu\nu\sigma\rho}^{(+-)}(x) &= t_{\mu\nu\gamma\lambda\alpha\beta}\left[\frac{1}{2}\partial_\beta\left(\frac{\rho^4}{(x^2+\rho^2)^2x^2}\frac{x_\gamma x_\lambda x_\delta}{x^2}\right)-\frac{\rho^4}{(x^2+\rho^2)^2x^2}\frac{x_\gamma x_\lambda}{x^2}\partial_\beta\left(\frac{x_\delta}{x^4}\right)\right. \\
&\quad \left.-\frac{1}{2}\frac{\rho^4 x_\rho x_\lambda x_\mu}{(x^2+\rho^2)^2x^8}\partial_\beta\ln K_D(x/\rho)\right]K_D(x/\rho)\frac{g_{\sigma\delta}g_{\alpha\rho}-g_{\sigma\alpha}g_{\delta\rho}+g_{\sigma\rho}g_{\delta\alpha}}{2\pi^2} \\
&= \frac{1}{\pi^2}\left[-\frac{24\rho^4}{x^8(x^2+\rho^2)^2}\left(K_D(x/\rho)-\frac{1}{6}x\frac{d}{dx}K_D(x/\rho)\right)+\frac{16\rho^4}{x^6(x^2+\rho^2)^3}K_D(x/\rho)\right]x_{[\rho}\epsilon_{\sigma]\mu\nu\lambda}x_\lambda,
\end{aligned} \tag{B16}$$

$$F_{\bar{q}Gq,3,\mu\nu\sigma\alpha\beta}^{(+-)}(x)=t_{\mu\nu\rho\lambda\alpha\beta}\frac{\rho^4}{(x^2+\rho^2)^2x^2}\frac{x_\rho x_\lambda}{x^2}\frac{1}{4\pi^2x^2}K_m(x/\rho)-t_{\mu\nu\rho\lambda\alpha\gamma}\frac{\rho^4}{(x^2+\rho^2)^2x^2}\frac{x_\rho x_\lambda x_\beta}{x^2}\frac{1}{4\pi^2}\partial_\gamma\left(\frac{K_m(x/\rho)}{x^2}\right) \tag{B17}$$

$$\begin{aligned}
F_{\bar{q}Gq,4,\mu\nu\sigma}^{(+-)}(x) &= \frac{1}{2\pi^2}t_{\mu\nu\rho\lambda\sigma\beta}\left[\frac{1}{2}\partial_\beta\left(\frac{\rho^4}{(x^2+\rho^2)^2x^4}\frac{x_\rho x_\lambda}{x^2}\right)-\frac{\rho^4}{(x^2+\rho^2)^2x^2}\frac{x_\rho x_\lambda}{x^2}\partial_\beta\left(\frac{1}{x^2}\right)\right. \\
&\quad \left.-\frac{1}{2}\frac{\rho^4}{(x^2+\rho^2)^2x^4}\frac{x_\rho x_\lambda}{x^2}\partial_\beta\ln K_m(x/\rho)\right]K_m(x/\rho) \\
&= \frac{1}{2\pi^2}\left[\frac{16\rho^4}{x^6(x^2+\rho^2)^2}\left(K_m(x/\rho)-\frac{1}{4}x\frac{d}{dx}K_m(x/\rho)\right)-\frac{16\rho^4}{x^4(x^2+\rho^2)^3}K_m(x/\rho)\right]\epsilon_{\mu\nu\sigma\lambda}x_\lambda
\end{aligned} \tag{B18}$$

with the help of the identity [41],

$$\bar{\psi}\gamma^{[\mu}i\overleftrightarrow{\partial}^{\nu]}\psi=\frac{1}{4}\epsilon^{\mu\nu\rho\sigma}\partial_\rho(\bar{\psi}\gamma_\sigma\gamma_5\psi).$$

APPENDIX C: MOLECULAR FORM FACTORS

The molecular form factors entering (34) are

$$\beta_{\bar{q}Gq,1}^{(+)}(q) = \frac{1}{q} \int_0^\infty dx \frac{16}{(x^2+1)^2 x^2} \times \left(\frac{3J_3(qx)}{q^2 x^2} - \frac{J_4(qx)}{qx} \right) K_D(x), \quad (C1)$$

$$\beta_{\bar{q}Gq,2}^{(+)}(q) = \frac{1}{q} \int_0^\infty dx \frac{16}{(x^2+1)^2 x^2} \frac{J_3(qx)}{q^2 x^2} K_D(x), \quad (C2)$$

$$\beta_{\bar{q}Gq,3}^{(+)}(q) = \frac{1}{q} \int_0^\infty dx \left[\frac{128}{x^4(x^2+1)^2} \left(K_D(x) - \frac{1}{4} x K'_D(x) \right) \times \frac{J_2(qx)}{qx} - \frac{16}{(x^2+1)^2 x^4} K_D(x) J_3(qx) \right], \quad (C3)$$

$$\beta_{\bar{q}Gq,4}^{(+)}(q) = \frac{1}{q} \int_0^\infty dx \left[\frac{64}{x^2(x^2+1)^2} \left(K_D(x) - \frac{1}{4} x K'_D(x) \right) \times \frac{J_3(qx)}{q^2 x^2} - \frac{8}{(x^2+1)^2 x^2} K_D(x) \times \left(\frac{J_4(qx)}{qx} - \frac{2J_3(qx)}{q^2 x^2} \right) \right], \quad (C4)$$

$$\beta_{\bar{q}Gq,5}^{(+)}(q) = \frac{1}{q} \int_0^\infty dx \left[\frac{16}{x^2(x^2+1)^2} \left(K_m(x) - \frac{1}{2} x K'_m(x) \right) \times \frac{J_2(qx)}{qx} - \frac{4}{(x^2+1)^2} K_m(x) J_3(qx) \right], \quad (C5)$$

$$\beta_{\bar{q}Gq,6}^{(+)}(q) = \frac{1}{q} \int_0^\infty dx \frac{16}{(x^2+1)^2} \frac{J_3(qx)}{q^2 x^2} K_m(x), \quad (C6)$$

$$\beta_{\bar{q}Gq,7}^{(+)}(q) = \frac{1}{q} \int_0^\infty dx \left[\frac{16}{x^2(x^2+1)^2} \left(K_m(x) - \frac{1}{2} x K'_m(x) \right) \times \frac{J_2(qx)}{qx} \right]. \quad (C7)$$

as illustrated in Fig. 18 at zero momentum transfer, their values are

$$\beta_{\bar{q}Gq,1}^{(+)}(0) = \frac{2}{15}, \quad \beta_{\bar{q}Gq,2}^{(+)}(0) = 0.0444, \quad (C8)$$

$$\beta_{\bar{q}Gq,3}^{(+)}(0) = 3.048, \quad \beta_{\bar{q}Gq,4}^{(+)}(0) = 0.1460, \quad (C9)$$

$$\beta_{\bar{q}Gq,5}^{(+)}(0) = 0.2331, \quad \beta_{\bar{q}Gq,6}^{(+)}(0) = 0.0935, \quad \beta_{\bar{q}Gq,7}^{(+)}(0) = 0.2262. \quad (C10)$$

APPENDIX D: OFF-FORWARD DIRAC STRUCTURE

For a spin-1/2 particle with mass m , symmetries such as Poincare covariance, parity, and Hermiticity allow for the determination of the off-forward tensor structures of any matrix element carrying Lorentz indices $\{\gamma^\mu, \gamma^\mu \gamma^5, \bar{p}^\mu, q^\mu, g^{\mu\nu}, i\sigma^{\mu\nu}, \epsilon^{\mu\nu\rho\lambda}\}$ alongside the Dirac structure $\{1, \not{p}, \not{q}, [\not{p}, \not{q}]\}$. The linear combinations of those can be used to establish the Lorentz structure of the baryon matrix elements and form factors. Those Lorentz structures can be further simplified using the identities

$$\bar{u}_{s'}(p') \not{p} u_s(p) = m_h \bar{u}_{s'}(p') u_s(p), \quad (D1)$$

$$\bar{u}_{s'}(p') \not{q} u_s(p) = 0. \quad (D2)$$

Between on-shell baryon spinors $\bar{u}_s(p')$ and $u_s(p')$, not all allowed combinations are linearly independent. The following identities

$$\bar{u}_{s'}(p') \gamma^\mu u_s(p) = \bar{u}_{s'}(p') \left(\frac{\bar{p}^\mu}{m_h} + \frac{i\sigma^{\mu\nu} q_\nu}{2m_h} \right) u_s(p), \quad (D3)$$

$$\bar{u}_{s'}(p') \gamma^\mu \gamma^5 u_s(p) = \bar{u}_{s'}(p') \left(\frac{i\sigma^{\mu\nu} \gamma^5 \bar{p}_\nu}{m_h} + \frac{q^\mu \gamma^5}{2m_h} \right) u_s(p), \quad (D4)$$

$$\bar{u}_{s'}(p') (\gamma^\mu \bar{p}^\nu - \gamma^\nu \bar{p}^\mu) u_s(p) = \frac{i}{2} \epsilon^{\mu\nu\rho\lambda} \bar{u}_{s'}(p') q_\rho \gamma_\lambda \gamma^5 u_s(p), \quad (D5)$$

$$\bar{u}_{s'}(p') i\sigma^{\mu\nu} u_s(p) = \bar{u}_{s'}(p') \left(\frac{\gamma^\mu q^\nu - \gamma^\nu q^\mu}{2m_h} - \frac{i}{m_h} \epsilon^{\mu\nu\rho\sigma} \bar{p}_\rho \gamma_\sigma \gamma^5 \right) u_s(p) \quad (D6)$$

can be used to simplify. Note that in the forward limit,

$$\bar{u}_s(p) \gamma^\mu u_s(p) = 2p^\mu, \quad \bar{u}_s(p) \gamma^\mu \gamma^5 u_s(p) = 2m_h S^\mu, \quad \bar{u}_s(p) \sigma^{\mu\nu} u_s(p) = 2\epsilon^{\mu\nu\rho\lambda} S_\rho p_\lambda \quad (D7)$$

with $\epsilon^{0123} = 1$. The Schouten identity of the Levi-Civita symbol also simplifies the Lorentz decomposition of the form factors. Given arbitrary Lorentz covariant four-vector $\{a^\mu, b^\mu, c^\mu\}$, the identity reads

$$a^\mu \epsilon^{\nu\sigma\rho\lambda} b_\rho c_\lambda - a^\nu \epsilon^{\mu\sigma\rho\lambda} b_\rho c_\lambda + a^\sigma \epsilon^{\mu\nu\rho\lambda} b_\rho c_\lambda = (a \cdot b) \epsilon^{\mu\sigma\gamma} c_\gamma - (a \cdot c) \epsilon^{\mu\nu\sigma\gamma} b_\gamma. \quad (D8)$$

Some useful identities of the Dirac $\sigma^{\mu\nu}$ are also useful in the process of simplifying the Lorentz structures, such as

the relation between $\sigma^{\mu\nu}$ and $\sigma^{\mu\nu}\gamma^5$ through the Levi-Civita symbol

$$\frac{i}{2}\epsilon^{\mu\nu\rho\lambda}\sigma_{\rho\lambda} = \sigma_{\mu\nu}\gamma^5 \quad (\text{D9})$$

and the identities involving three gamma matrices

$$\begin{aligned} \sigma^{\mu\nu}\gamma^\rho &= ig^{\nu\rho}\gamma^\mu - ig^{\mu\rho}\gamma^\nu - \epsilon^{\mu\nu\rho\sigma}\gamma_\sigma\gamma_5, \\ \gamma^\rho\sigma^{\mu\nu} &= ig^{\mu\rho}\gamma^\nu - ig^{\nu\rho}\gamma^\mu - \epsilon^{\mu\nu\rho\sigma}\gamma_\sigma\gamma_5. \end{aligned}$$

APPENDIX E: HADRONIC FORM FACTORS

In an instanton vacuum, the presence of pseudoparticles modifies the pointlike quark and gluon operators at low momentum transfer Q^2 characterized by $Q \lesssim 1/\rho$, [8,20,42]. As a result, the color force operator in (B3) and (B4) can be mapped to various hadronic quark form factors: the scalar form factor σ_h^q , the energy-momentum tensor form factor A_h^q and D_h^q for the unpolarized part, and the axial form factors G_A^q , the pseudoscalar form factors G_P^q and $\tilde{G}_P^q$, and the transversity form factors $A_{h,T}^q$ for the spin-dependent process.

In this regime ($Q \lesssim 1/\rho$), the instantons and anti-instanton act collectively, inducing effective glueball-quark or meson-quark interactions. The form factors are mostly described in the form of meson or glueball exchanges, characterized by hadronic parameters, through their masses or hadronic couplings [17,32].

In order to evaluate the color force form factor, here we present the details of the definition of each form factor, and its pertinent form in the ILM.

1. Pion form factors

To evaluate the $\Phi_{\pi,1}^q$ and $\Phi_{\pi,2}^q$ form factors, we need the quark scalar form factor σ_π^q and energy-momentum tensor form factors A_π^q and D_π^q . For pions, the quark scalar form factor is defined by

$$\langle \pi' | m \bar{\psi} \psi | \pi \rangle = 2m_\pi^2 \sigma_\pi(Q^2). \quad (\text{E1})$$

The traceless energy momentum tensor form factor for the pion is defined as

$$\begin{aligned} &\langle \pi' | \bar{\psi} \left(\gamma_{(\mu} i \overleftrightarrow{\partial}_{\nu)} - \frac{1}{4} g_{\mu\nu} i \overleftrightarrow{\not{D}} \right) \psi | \pi \rangle \\ &= 2A_\pi^q(Q^2) \left(\bar{p}_\mu \bar{p}_\nu - \frac{1}{4} g_{\mu\nu} \bar{p}^2 \right) \\ &+ \frac{1}{2} D_\pi^q(Q^2) \left(q_\mu q_\nu - \frac{1}{4} g_{\mu\nu} q^2 \right). \end{aligned} \quad (\text{E2})$$

These form factors are also related to the generalized form factors obtained by the second moment of unpolarized pion GPD

$$\int_{-1}^1 dx x H^{q/\pi}(x, \xi, t = -Q^2) = A_\pi^q(Q^2) + \xi^2 D_\pi^q(Q^2). \quad (\text{E3})$$

As we are only interested in the twist-3 contribution or the $\Phi_{\pi,1}^q$ form factor in this work, we only need to model the energy momentum tensor form factor A_π^q . It can be parametrized by a monopole form

$$A_\pi^{u+d}(Q^2) = \frac{0.481}{1 + Q^2/1.262^2} \quad (\text{E4})$$

with the lattice monopole fitting parameters from [43], assuming

$$A_\pi^{u-d}(Q^2) = 0 \quad (\text{E5})$$

thanks to isospin symmetry.

2. Nucleon form factors

To estimate the nucleon $\Phi_{N,1}^q$ and $\Phi_{N,2}^q$ form factors, we need the quark scalar form factor σ_N^q and the energy-momentum tensor form factors A_N^q , J_N^q , and D_N^q both in the isoscalar and isovector channels. For the nucleon, the quark scalar form factor (sigma term) is defined as

$$\langle N' | m \bar{\psi} \psi | N \rangle = m_N \sigma_N(Q^2) \bar{u}_{s'}(p') u_s(p). \quad (\text{E6})$$

The nucleon traceless energy momentum tensor form factor is defined as

$$\begin{aligned} &\langle N' | \bar{\psi} \left(\gamma_{(\mu} i \overleftrightarrow{\partial}_{\nu)} - \frac{1}{4} g_{\mu\nu} i \overleftrightarrow{\not{D}} \right) \psi | N \rangle \\ &= \bar{u}_{s'}(p') \left[A_N^q(Q^2) \frac{1}{m_N} \left(\bar{p}^\mu \bar{p}^\nu - \frac{1}{4} g_{\mu\nu} \bar{p}^2 \right) \right. \\ &+ J_N^q(Q^2) \frac{1}{2m_N} \left(2i \bar{p}^{(\mu} \sigma^{\nu)\alpha} q_\alpha - \frac{1}{4} g_{\mu\nu} q^2 \right) \\ &+ D_N^q(Q^2) \frac{1}{4m_N} \left(q^\mu q^\nu - \frac{1}{4} g^{\mu\nu} q^2 \right) \left. \right] u_s(p) \end{aligned} \quad (\text{E7})$$

where the angular momentum form factor is defined by $J_N^q = \frac{1}{2}(A_N^q + B_N^q)$. Those form factors are also related to the generalized form factors derived from the unpolarized nucleon GPD $H^{q/N}$ and $E^{q/N}$ with skewness $2\xi = -q^+/\bar{p}^+$ [44],

$$\int_{-1}^1 dx x H^{q/N}(x, \xi, t = -Q^2) = A_N^q(Q^2) + \xi^2 D_N^q(Q^2), \quad (\text{E8})$$

$$\int_{-1}^1 dx x E^{q/N}(x, \xi, t = -Q^2) = B_N^q(Q^2) - \xi^2 D_N^q(Q^2). \quad (\text{E9})$$

To evaluate nucleon $\Phi_{N,4-8}^q$ form factors, we also need the quark axial form factor G_A^q , and the pseudoscalar form factors $\tilde{G}_P^q$ and G_P^q both in isoscalar and isovector channels. The pseudoscalar form factor is defined as

$$\langle N' | m \bar{\psi} \gamma^5 \psi | N \rangle = m_N \tilde{G}_P^q(Q^2) \bar{u}_{s'}(p') \gamma^5 u_s(p). \quad (\text{E10})$$

The axial form factor and the induced pseudoscalar form factors, are defined as

$$\begin{aligned} \langle N' | \bar{\psi} \gamma_\mu \gamma^5 \psi | N \rangle \\ = \bar{u}_{s'}(p') [\gamma_\mu \gamma^5 G_A^q(Q^2) + \frac{q_\mu \gamma^5}{2m_N} G_P^q(Q^2)] u_s(p). \end{aligned} \quad (\text{E11})$$

However, these form factors $G_{A,P}^q$ and $\tilde{G}_P^q$ do not contribute to the twist-3 combination. To evaluate the color force form factors $\Phi_{N,1-3}^q$, $\Phi_{N,5}^q$, $\Phi_{N,6}^q$, and $\Phi_{N,8}^q$, we also need the seven tensor quark gravitational form factors A_T^q , $\tilde{A}_T^q$, B_T^q , $\tilde{B}_T^q$, C_T^q , D_T^q , and $\tilde{D}_T^q$ [45], while only the first four contribute to the twist-3 transversity gravitational form factors, which correspond to the second moment of the transversity GPD defined by [45,46].

$$\begin{aligned} \langle N' | \bar{\psi} \sigma_{\mu\nu} \overleftrightarrow{\partial}_\rho \psi | N \rangle \\ = \bar{u}_{s'}(p') \left\{ -i \sigma^{\mu\nu} \bar{p}^\rho A_T^q - \frac{\bar{p}^\mu q^\nu - \bar{p}^\nu q^\mu}{m_N^2} \bar{p}^\rho \tilde{A}_T^q \right. \\ \left. - \frac{\gamma^\mu q^\nu - \gamma^\nu q^\mu}{2m_N} \bar{p}^\rho B_T^q - \frac{\gamma^\mu \bar{p}^\nu - \gamma^\nu \bar{p}^\mu}{m_N} q^\rho \tilde{B}_T^q \right. \\ \left. - (g^{\mu\rho} q^\nu - g^{\nu\rho} q^\mu) C_T^q - i m_N \epsilon^{\mu\nu\rho\lambda} \gamma_\lambda \gamma^5 D_T^q \right. \\ \left. - i \epsilon^{\mu\nu\rho\lambda} q_\lambda \gamma^5 \tilde{D}_T^q \right\} u_s(p). \end{aligned} \quad (\text{E12})$$

Their relations to the nucleon transversity GPD in [47] are defined as

$$\int_{-1}^1 dx x H_T^{q/N}(x, \xi, t = -Q^2) = A_T^q(Q^2), \quad (\text{E13})$$

$$\int_{-1}^1 dx x E_T^{q/N}(x, \xi, t = -Q^2) = B_T^q(Q^2), \quad (\text{E14})$$

$$\int_{-1}^1 dx x \tilde{H}_T^{q/N}(x, \xi, t = -Q^2) = \tilde{A}_T^q(Q^2), \quad (\text{E15})$$

$$\int_{-1}^1 dx x \tilde{E}_T^{q/N}(x, \xi, t = -Q^2) = -2\xi \tilde{B}_T^q(Q^2). \quad (\text{E16})$$

Using the equation of motion [41,45],

$$\bar{\psi} i \sigma^{\lambda\mu} \gamma_5 i \overleftrightarrow{\partial}_\mu \psi = 2M \bar{\psi} \gamma^\lambda \gamma_5 \psi + i \partial^\lambda (\bar{\psi} \gamma_5 \psi) \quad (\text{E17})$$

and

$$\epsilon_{\lambda\mu\alpha} \bar{\psi} i \sigma^{\lambda\mu} \gamma_5 i \overleftrightarrow{\partial}^\alpha \psi = 2 \partial_\alpha (\bar{\psi} \gamma_5 \psi), \quad (\text{E18})$$

those tensor quark gravitational form factors can be further constrained by the relations

$$\begin{aligned} \frac{2M}{m_N} G_A^q = A_T^q - \frac{Q^2}{m_N^2} \tilde{A}_T^q - \frac{Q^2}{4m_N^2} B_T^q + \frac{Q^2}{2m_N^2} \tilde{B}_T^q \\ - 2C_T^q - 3D_T^q \end{aligned} \quad (\text{E19})$$

$$\begin{aligned} \frac{2M}{m_N} G_P^q - \frac{2m_N}{m} \tilde{G}_P^q = -A_T^q + \frac{Q^2}{m_N^2} \tilde{A}_T^q - B_T^q \\ + 2\tilde{B}_T^q + 2C_T^q - 6\tilde{D}_T^q, \end{aligned} \quad (\text{E20})$$

$$-\frac{2m_N}{m} \sigma_N^q = A_T^q + 2\tilde{A}_T^q - \frac{Q^2}{m_N^2} \tilde{A}_T^q + B_T^q + 4C_T^q. \quad (\text{E21})$$

As we are only interested in the twist-3 contribution in the Breit frame $q^+ = 0$, we will only focus on the $\Phi_{N,1}^q$, $\Phi_{N,3}^q$, $\Phi_{N,7}^q$, and $\Phi_{N,8}^q$ form factors in this work. In order to evaluate these form factors, we parametrize the energy momentum tensor form factors A_N^q and B_N^q by dipoles. Their scale dependence is also controlled by the renormalization group (RG) equation

$$A^{u-d}(Q^2, \mu) = \left(\frac{\alpha_s(\mu_0)}{\alpha_s(\mu)} \right)^{-\frac{8C_F}{3\beta_0}} A^{u-d}(Q^2, \mu_0) \quad (\text{E22})$$

and mixed equation in isoscalar sector

$$\begin{aligned} \mu \frac{d}{d\mu} A^{u+d}(\mu) &= \frac{\alpha_s}{4\pi} \left[-\frac{16}{3} C_F A^{u+d}(\mu) + \frac{8}{3} A^g(\mu) \right], \\ \mu \frac{d}{d\mu} A^g(\mu) &= \frac{\alpha_s}{4\pi} \left[\frac{16}{3} C_F A^{u+d}(\mu) - \frac{8}{3} A^g(\mu) \right]. \end{aligned} \quad (\text{E23})$$

At $\mu = 2$ GeV, the lattice fitted dipole forms read

$$A_N^u(Q^2) = \frac{0.4055}{(1 + Q^2/1.772^2)^2}, \quad (\text{E24})$$

$$A_N^d(Q^2) = \frac{0.1385}{(1 + Q^2/1.555^2)^2}, \quad (\text{E25})$$

$$B_N^u(Q^2) = \frac{0.171}{(1 + Q^2/1.535^2)^2}, \quad (\text{E26})$$

$$B_N^d(Q^2) = \frac{-0.124}{(1 + Q^2/1.657^2)^2} \quad (\text{E27})$$

where the parameter for the isovector channel is obtained by fitting the lattice result of the second moment of GPD in [48] with slightly heavier pion mass $m_\pi = 260$ MeV, and the parameter for the isoscalar channel is obtained by

fitting the lattice result of the gravitational form factor from [49].

For the transversity gravitational form factors, we also parametrize them by dipoles. Their scale dependence is also controlled by the RG equation,

$$A_T^q(Q^2, \mu) = \left(\frac{\alpha_s(\mu_0)}{\alpha_s(\mu)} \right)^{-\frac{13C_F}{3\beta_0}} A_T^q(Q^2, \mu_0). \quad (\text{E28})$$

At $\mu = 2$ GeV, the lattice fitted dipole forms read

$$A_T^u(Q^2) = \frac{0.268}{(1 + Q^2/2.312^2)^2}, \quad (\text{E29})$$

$$A_T^d(Q^2) = \frac{-0.052}{(1 + Q^2/2.448^2)^2}, \quad (\text{E30})$$

$$2\tilde{A}_T^{u+d}(t) + B_T^{u+d}(t) = \frac{0.680}{(1 + Q^2/1.736^2)^2}, \quad (\text{E31})$$

$$2\tilde{A}_T^{u-d}(t) + B_T^{u-d}(t) = \frac{0.267}{(1 + Q^2/1.395^2)^2}, \quad (\text{E32})$$

$$\tilde{A}_T^{u+d}(t) = \frac{0.576}{(1 + Q^2/1.517^2)^2}, \quad (\text{E33})$$

$$\tilde{A}_T^{u-d}(t) = \frac{-0.0764}{(1 + Q^2/1.517^2)^2}, \quad (\text{E34})$$

where the parameters for $A_T^{u,d}$ are obtained by fitting the lattice result from the QCDSF/UKQCD collaboration in [50], and the parameters for $\tilde{A}_T^{u+d}$ and B_T^{u+d} in the isoscalar channel are obtained by fitting the lattice result from the QCDSF/UKQCD collaboration [51] where they extrapolate the pion mass to $m_\pi = 140$ MeV. The parameters for $\tilde{A}_T^{u-d}$ and B_T^{u-d} in the isovector channel are obtained by fitting the lattice result from Cyprus lattice group [52]. Here we assume that the dipole mass for $\tilde{A}_T^{u+d}$ in (E33) is identical to that of $\tilde{A}_T^{u-d}$, due to the lack of available data constraining its momentum dependence.

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