Graviton-mediated entanglement due to light bending from a quantum rotor

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One of the key tests of the quantum nature of gravity is to test whether the virtual mediator of gravity between matter and photon gives rise to the quantum light-bending phenomenon. The off-shell degrees of freedom, involving the spin-2 and spin-0 components of graviton, reproduce the classical deviation of light rays, as well as have been predicted to generate entanglement between matter and photon. This paper explores the generation of entanglement due to the quantum gravitational interaction in an optomechanical setup with a quantum rotor and photon. The virtual exchange of a graviton provides entanglement between the photon degrees of freedom and the spatial position of the quantum rotor, with the rotational state affecting its magnitude. We analyze the case of a high spinning rotor, in an approximately classical state of angular momentum, and quantify its effect on the gravitationally induced entanglement between the photon and the position of the quantum rotor. We show that the difference in the linear entanglement entropies, of prograde-and-retrograde motion of the photon with respect to the quantum rotor, provide tangible observable consequences.

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I. INTRODUCTION

The pivotal moment for general relativity was the detection of the deflection of light during the solar eclipse in 1919 [1]. Einstein's prediction from general relativity matched the data, with the distinction that Newtonian and Nordström's gravity could not explain the correct deflection angle due to the Sun's gravitational potential. This momentous experimental result has paved the way into a quantum domain, where one would quantize gravity around the Minkowski background as a low-energy effective field theory, see Refs. [2–4]. In fact, in a perturbative quantum theory of gravity, it is possible to explain the angular deflection due to an off-shell graviton exchange between a massive object and a photon, see Ref. [3].

All these profound observations have led one group to propose an experiment to test the quantum nature of gravity in a lab first between two quantum superposed masses interacting via a graviton exchange: "spin entanglement witness for quantum gravity" [5,6], and also [7], and second via matter-graviton-photon interaction, while treating all three entities on par with quantum mechanics [8], see also Ref. [9]. The primary idea is rooted in entanglement: two quantum systems can entangle if there exists a quantum mediator [10,11]; in the quantum gravity induced entanglement protocol that entails an exchange of a graviton between the two quantum superposed nanoparticles [6], see theory papers [12–19], and for experimental implementation [6,20–26], for an experimental white paper, see Ref. [27].[1] There are also proposals in which two rotors are prepared in superpositions of rotational states, and hence of gravitational masses, so that their mutual interaction generates a genuinely relativistic entangling phase [32]. However, there are also challenges regarding rotation when considering a one-loop interferometer and the contrast loss. There are ways of mitigating these effects, see Refs. [26,33–35]. Also, the primary effect originates from the rotational-energy contribution $E_{\mathrm{rot}} \propto J^2$ arising from two rotational-energy superpositions [32].

[1] In fact, a single matter-wave interferometer depicts all the scopes of quantum gravity if gravity and matter are treated at par at a quantum level [28–30]. However, it is extremely hard to witness the entanglement between graviton and matter [30]. Hence, one requires an experimental setup where two of the interferometers are kept adjacent to each other, and the graviton-mediated entanglement is witnessed by building correlations between the two matter interferometers [6] or matter and photon degrees of freedom in [8,31].

By contrast, as we show in the following sections, our massive rotor–photon setup probes an interaction linear in the angular momentum J and the resulting prograde-retrograde[2] splitting of photon-position entanglement.

In [8], we studied the entanglement arising from the matter-photon interaction via graviton exchange, and in [31], we provided an entanglement witness protocol for this optomechanical experiment. The aim of this paper is to introduce rotation in the matter sector and to show how the linearised Hamiltonian naturally provides interactions among position, angular momentum, and photon degrees of freedom. To exemplify the importance of rotation, we consider a large angular momentum of a massive object and show that it is possible to witness an entanglement between prograde and retrograde motions of the matter with respect to the photon beam. Any rotating source also generates polarisation-changing terms that are relevant to helicity asymmetry and Skrotskii/Faraday rotation [37–39]. The present computation keeps the *polarization-diagonal* $\mathcal{O}(J)$ piece that fits directly into the modified optomechanical Hamiltonian. In previous works, a single-photon source has enabled the experimental exploration of multimode interference and entanglement within the context of quantum field theory in curved spacetime, see Refs. [40–43], a regime that can be thought of as an approximation where gravity is treated classically. However, in this paper, the aim will be to treat gravity at the quantum level along with the matter.

In this paper, we will not consider the effects of dephasing or decoherence. These effects have been partially taken into account in [8] as far as the matter sector is concerned. A white noise analysis for the photon has also been taken into account while preparing the witness to observe the entanglement in [31]. We will leave the details of how to induce rotation for a separate topic. Note that nanoparticle-controlled rotation has been performed in these experiments, see Ref. [8].

II. SCATTERING AMPLITUDE OF A QUANTUM ROTOR AND A PHOTON DUE TO GRAVITON EXCHANGE

Let us begin our discussion of the static Newtonian potential between matter and photon. The starting point is the tree-level amplitude between matter and photon mediated via an off-shell graviton. We use the metric signature: $(-,+,+,+)$, graviton coupling $f^2 = 8\pi G$, where G is Newton's constant. In this paper, we are working in *natural units* ($c = \hbar = 1$). We take k^μ and k'^μ are the incoming and outgoing photon four-momenta. The momentum transfer is given by: $q^\mu = k^\mu - k'^\mu$, small-grazing-angle approximation $q^2 \simeq 0$ in the numerator, with $1/(q^2 + i\epsilon)$ kept intact in the propagator, where ϵ is the usual Feynman regularization prescription. In the harmonic-gauge, the graviton propagator is given by [3,8]

$$D_{\mu\nu\alpha\beta}(q) = \frac{iP_{\mu\nu\alpha\beta}}{q^2 + i\epsilon}, \tag{1}$$

where q is the four-momentum of the off-shell propagating graviton, with the standard projection operator, which contains both spin-2 and spin-0 components of the graviton, see Refs. [10,44,45][3] [3,4],

$$P_{\mu\nu\alpha\beta} = \frac{1}{2}(\eta_{\mu\alpha}\eta_{\nu\beta} + \eta_{\nu\alpha}\eta_{\mu\beta} - \eta_{\mu\nu}\eta_{\alpha\beta}). \tag{2}$$

In the notation of [8], the static tree-level covariant amplitude is given by

$$S^{\text{cov}}_{fi} = (-i)^2 4 f^2 p'_\delta p_\sigma \frac{iP^{\delta\sigma}{}_{\mu\nu}}{q^2 + i\epsilon}\Big[k'_\alpha k^\mu \eta_\beta{}^\nu + k_\beta k'^\mu \eta_\alpha{}^\nu - \eta_{\alpha\beta} k'^\mu k^\nu - \frac{1}{2}\eta^{\mu\nu} k_\beta k'_\alpha\Big]\varepsilon'^{*\beta}\varepsilon^\alpha \delta^4(P_i), \tag{3}$$

which simplifies to

$$S^{\text{cov}}_{fi} = -2i f^2 p'_\delta p_\sigma \left(\frac{-2k'^\sigma k^\delta}{q^2 + i\epsilon}\right)\varepsilon'^* \cdot \varepsilon. \tag{4}$$

The Born prescription used in [8] maps this amplitude to the static potential,

$$V_0(r) = -\frac{2GM\omega}{r}\varepsilon'^* \cdot \varepsilon, \tag{5}$$

where M is the central mass, ω is the photon frequency, and $\varepsilon, \varepsilon'$ are the initial and final polarization vectors, see Ref. [8].[4] Here, note that $V_0(r)$ is a quantum operator by virtue of the position and the polarization, i.e., r, ϵ, see Refs. [8,11]. We now derive its leading correction linear in the source angular momentum $\mathbf{J}$. For a localized stationary spinning source, the appropriate external source is the standard pole-dipole stress tensor in the source's rest frame,

[2] This arises from the well-known *frame-dragging* due to a massive rotating object, in classical GR. See Ref. [36] for an astrophysical application of Lense-Thirring frame-dragging in the study of rapidly spinning white dwarfs.

[3] Note that in this paper the position, momentum, and angular momentum are all operators, i.e., $\hat{x}, \hat{p}, \hat{J}$, and so are the creation, annihilation operators, $\hat{a}, \hat{a}^\dagger, \ldots$. However, for brevity, we will not put the hat-sign in the expressions; instead, we will remind the readers about the c-numbered entities and operator-valued entities whenever there is any confusion.

[4] This potential yields the correct deflection angle as produced by classical general relativity, see the computation in [3].

$$T^{00}(\mathbf{x}) = M\delta^{(3)}(\mathbf{x}), \qquad T^{0i}(\mathbf{x}) = -\frac{1}{2}\epsilon^{ijk}J_j\partial_k\delta^{(3)}(\mathbf{x}),$$
$$T^{ij}(\mathbf{x}) = \mathcal{O}(J^2, \partial^2). \tag{6}$$

The first term is the mass monopole; the second term is the current dipole responsible for frame dragging. Here ϵ^{ijk} is the three-dimensional Levi-Civita symbol with $\epsilon^{123} = +1$. We define the momentum-space source tensor by

$$\Theta^{\mu\nu}_{(J)}(\mathbf{q}) = \int \mathrm{d}^3x \mathrm{e}^{-\mathrm{i}\mathbf{q}\cdot\mathbf{x}}T^{\mu\nu}(\mathbf{x}). \tag{7}$$

Using Eq. (6) gives

$$\Theta^{00}_{(J)}(\mathbf{q}) = M, \qquad \Theta^{0i}_{(J)}(\mathbf{q}) = \Theta^{i0}_{(J)}(\mathbf{q}) = \frac{\mathrm{i}}{2}\epsilon^{ijk}q_jJ_k,$$
$$\Theta^{ij}_{(J)}(\mathbf{q}) = \mathcal{O}(q^2, J^2). \tag{8}$$

This is the spinning-source replacement for the static source factor appearing in (4). The natural spinning-source extension of the tree-level covariant amplitude is given by (3) is given by,

$$S^{\mathrm{cov}}_{fi}[J] = (-\mathrm{i})^2 4f^2\Theta^{(J)}_{\delta\sigma}(\mathbf{q})\frac{\mathrm{i}P^{\delta\sigma}{}_{\mu\nu}}{q^2+\mathrm{i}\epsilon}[k'_\alpha k^\mu\eta_\beta{}^\nu + k_\beta k'^\mu\eta_\alpha{}^\nu$$
$$- \eta_{\alpha\beta}k'^\mu k^\nu - \frac{1}{2}\eta^{\mu\nu}k_\beta k'_\alpha]\varepsilon'^{*\beta}\varepsilon^\alpha\delta^4(P_i). \tag{9}$$

Using Eqs. (2) and (8) in Eq. (9), we immediately obtain,

$$S^{\mathrm{cov}}_{fi}[J] = -2\mathrm{i}f^2\Theta^{(J)}_{\delta\sigma}(\mathbf{q})\left(\frac{-2k'^\sigma k^\delta}{q^2+\mathrm{i}\epsilon}\right)\varepsilon'^*\cdot\varepsilon\delta^4(P_i) + S_{\mathrm{flip}}, \tag{10}$$

where S_{flip} denotes the polarization-changing $\mathcal{O}(J)$ terms [see Eqs. (12) and (15)]. We show further details in the next section and also show how S_{flip} can be set to zero depending on the direction of $\mathbf{J}$. Substituting Eq. (8) and using the eikonal regime $k'^\mu \simeq k^\mu = (\omega, \omega\mathbf{n})$, where $\mathbf{n} = \mathbf{k}/\omega$ is the photon propagation direction, yields the polarization-diagonal kernel

$$S^{\mathrm{cov}}_{fi,\mathrm{diag}}[J] = -2\mathrm{i}f^2\frac{-2}{q^2+\mathrm{i}\epsilon}\left[M\omega^2 + \frac{\mathrm{i}\omega}{2}(\mathbf{n}\times\mathbf{q})\cdot\mathbf{J}\right]$$
$$\times\varepsilon'^*\cdot\varepsilon\delta^4(P_i). \tag{11}$$

The new term is linear in $\mathbf{J}$, linear in $\mathbf{q}$, and odd under $\mathbf{n}\to-\mathbf{n}$.

For completeness, we now make explicit the quantity S_{flip} that appeared in Eq. (10). By definition, S_{flip} is the part of the spinning-source matrix element that is *not* proportional to the polarisation-preserving structure $\varepsilon'^*\cdot\varepsilon$. Starting from Eq. (9) and carrying out the same projector contraction as in Appendix A of [8], we get

$$S^{\mathrm{cov}}_{fi}[J] = S^{\mathrm{cov}}_{fi,\mathrm{diag}}[J] + S_{\mathrm{flip}}, \tag{12}$$

with

$$S_{\mathrm{flip}} = (-\mathrm{i})^2 4f^2\frac{\mathrm{i}}{q^2+\mathrm{i}\epsilon}\Theta^{(J)}_{\delta\sigma}(\mathbf{q})\Delta^{\delta\sigma}_{\mathrm{flip}}\delta^4(P_i), \tag{13}$$

where

$$\Delta^{\delta\sigma}_{\mathrm{flip}} = k^\delta\varepsilon'^{*\sigma}(k'\cdot\varepsilon) + k^\sigma\varepsilon'^{*\delta}(k'\cdot\varepsilon) + k'^\delta\varepsilon^\sigma(k\cdot\varepsilon'^*)$$
$$+ k'^\sigma\varepsilon^\delta(k\cdot\varepsilon'^*). \tag{14}$$

Equation (14) is the part of the projector-contracted amplitude that mixes polarization states. It vanishes identically for a static source, because then $\Theta^{0i} = 0$ and only the diagonal mass-monopole term survives.

For a stationary spinning source, the only components relevant at linear order in $\mathbf{J}$ are given by Eq. (8). Using $q^0 = 0$, $\varepsilon^0 = \varepsilon'^0 = 0$,[5] and keeping only the linear $\mathcal{O}(J)$ terms, Eq. (13) reduces to

$$S_{\mathrm{flip}} = (-\mathrm{i})^2 4f^2\frac{\mathrm{i}}{q^2+\mathrm{i}\epsilon}2\Theta^{(J)}_{0i}[\omega\varepsilon'^{*i}(k'\cdot\varepsilon)$$
$$+ \omega'\varepsilon^i(k\cdot\varepsilon'^*)]\delta^4(P_i), \tag{15}$$

where $\omega = k^0$ and $\omega' = k'^0$. We now specialize to the physically relevant case in which the photon propagates in a plane and the source spin is perpendicular to that plane. Choosing the photon plane to be the xy plane, we take

$$\mathbf{J} = J\mathbf{z}, \qquad \mathbf{p} = (p, 0, 0),$$
$$\mathbf{k} = (p\cos\theta, p\sin\theta, 0), \qquad \mathbf{q} = \mathbf{k} - \mathbf{p}. \tag{16}$$

Note that all the above entities are operator-valued ones. A convenient linear-polarization basis is given by,

$$\boldsymbol{\varepsilon}^{(1)}_\alpha = (0, 1, 0), \qquad \boldsymbol{\varepsilon}^{(2)}_\alpha = (0, 0, 1),$$
$$\boldsymbol{\varepsilon}^{(1)}_\beta = (-\sin\theta, \cos\theta, 0), \qquad \boldsymbol{\varepsilon}^{(2)}_\beta = (0, 0, 1). \tag{17}$$

Here, polarization (1) lies in the photon plane, while polarization (2) is orthogonal to that plane. With $\mathbf{J} = J\mathbf{z}$ and $\mathbf{q}$ confined to the xy plane, Eq. (8) gives

$$\Theta^{(J)}_{0i} = -\frac{\mathrm{i}J}{2}(q_y, -q_x, 0), \qquad \Longrightarrow \qquad \Theta^{(J)}_{03} = 0. \tag{18}$$

[5] $k^0 = k'^0 = 0$ is simply a convenient gauge choice for the physical external photons; on the other hand, $q^0 = (k - k')^0$ denotes the energy transfer in the scattering process, which is taken to be zero in order to extract the Born potential for a stationary (time-independent) source potential.

This simple observation is enough to show that the polarization-changing terms vanish. Indeed, consider the matrix element for the transition from the in-plane polarization (1) to the out-of-plane polarization (2),

$$S^{21}_{\rm flip} \propto \Theta^{(J)}_{0i}[\omega\varepsilon^{(2)i}_{\beta}(k'\cdot\varepsilon^{(1)}_{\alpha}) + \omega'\varepsilon^{(1)i}_{\alpha}(k\cdot\varepsilon^{(2)}_{\beta})]. \tag{19}$$

Since $\varepsilon^{(2)}_{\beta} = \mathbf{z}$, the first term is proportional to $\Theta^{(J)}_{03}$ and therefore vanishes by Eq. (18). The second term vanishes because k^{μ} lies in the xy plane whereas $\varepsilon^{(2)}_{\beta}$ points along $\mathbf{z}$, so

$$k\cdot\varepsilon^{(2)}_{\beta} = 0 \qquad \Rightarrow \qquad S^{21}_{\rm flip} = 0. \tag{20}$$

Likewise, for the transition from the out-of-plane polarization (2) to the in-plane polarization (1),

$$S^{12}_{\rm flip} \propto \Theta^{(J)}_{0i}[\omega\varepsilon^{(1)i}_{\beta}(k'\cdot\varepsilon^{(2)}_{\alpha}) + \omega'\varepsilon^{(2)i}_{\alpha}(k\cdot\varepsilon^{(1)}_{\beta})]. \tag{21}$$

Now $\varepsilon^{(2)}_{\alpha} = \mathbf{z}$, so the first term vanishes because k'^{μ} lies in the xy-plane and therefore

$$k'\cdot\varepsilon^{(2)}_{\alpha} = 0. \tag{22}$$

The second term is proportional to $\Theta^{(J)}_{03}$ and again vanishes by Eq. (18). Therefore,

$$S^{12}_{\rm flip} = 0. \tag{23}$$

Equations (20) and (23) show that, to linear order in $\mathbf{J}$,

$$\mathbf{J}\perp \text{photon plane} \quad\Longrightarrow\quad S_{\rm flip} = 0 \quad \text{at } \mathcal{O}(J). \tag{24}$$

Thus, when the source angular momentum is normal to the plane of photon propagation, the entire $\mathcal{O}(J)$ correction is polarization-diagonal and is already contained in Eq. (28). In particular, for the half-ring geometry used below, where the optical trajectory lies in the equatorial plane and $\mathbf{J} = J\mathbf{z}$, there is no additional polarisation-mixing contribution to the entanglement entropy at linear order in J.

As a cross-check, our analysis agrees with the small-angle polarisation matrix derived in [38]. The authors computed the scattering amplitude of a photon and a spinning mass. The small-angle assumption amounted to setting the momentum transfer between the initial and final photon 3-momenta ($|\mathbf{k}_i - \mathbf{k}_f| = 2p\sin\beta/2$, where β is the scattering angle between the incoming and outgoing photon). They obtained the scattering (amplitude) matrix,

$$A \propto \begin{bmatrix} \cos\frac{\beta}{2} - i2p\frac{J_z}{M}\sin\frac{\beta}{2} & -2p\frac{\tilde{J}}{M}\sin^2\frac{\beta}{2} \\ 2p\frac{\tilde{J}}{M}\sin^2\frac{\beta}{2} & \cos\frac{\beta}{2} - i2p\frac{J_z}{M}\sin\frac{\beta}{2} \end{bmatrix} \tag{25}$$

where J_z is the component of the angular momentum normal to the photon plane, which contributes only to the diagonal part of the amplitude, while the off-diagonal entries are controlled by the in-plane component of the angular momentum: $\tilde{J} = J_x\cos\frac{\beta}{2} + J_y\sin\frac{\beta}{2}$.

We now use the same Born prescription as in [8], that is, we compute the effective potential

$$V(\mathbf{d}) = \frac{1}{4M\omega}\int\frac{\mathrm{d}^3q}{(2\pi)^3}S^{\rm cov}_{fi}(\mathbf{q})\mathrm{e}^{\mathrm{i}\mathbf{q}\cdot\mathbf{d}}. \tag{26}$$

To align the overall normalization with Appendix A of [8], we require that the $J\to 0$ limit reproduce the static potential.[6] Applying Eq. (26) to Eq. (11) gives the leading polarization-diagonal potential

$$V^{\rm diag}_J(\mathbf{d},\mathbf{n}) = -\frac{2GM\omega}{d}\varepsilon'^{*}\cdot\varepsilon + 2G\omega\frac{\mathbf{J}\cdot(\mathbf{d}\times\mathbf{n})}{d^3}\varepsilon'^{*}\cdot\varepsilon + \mathcal{O}(J^2). \tag{28}$$

This is the main correction to the static potential. For the equatorial propagation around a source with $\mathbf{J} = J\mathbf{z}$, the $\mathcal{O}(J)$ term flips sign between corotating and counterrotating trajectories, consistent with the weak-field Kerr deflection splitting discussed in [37,38].

III. OPTOMECHANICAL COUPLING

Let us now repeat the half-ring reduction [8]. The optical path lies in the equatorial plane (see Fig. 1),

$$\begin{aligned} \mathbf{R}(\theta) &= (r\cos\theta, r\sin\theta, 0), \\ \mathbf{X} &= (\delta x, 0, 0), \\ \mathbf{d}(\theta) &= \mathbf{R}(\theta) - \mathbf{X}, \\ \mathbf{n}_{\sigma}(\theta) &= \sigma(-\sin\theta, \cos\theta, 0), \qquad \sigma = \pm 1, \end{aligned} \tag{29}$$

with $\theta\in[-\pi/2, \pi/2]$ parametrizing the half-ring of radius r and σ labeling the two circulation senses. The static expansion used in [8] is

$$\frac{1}{|\mathbf{d}(\theta)|} = \frac{1}{r} + \frac{\delta x\cos\theta}{r^2} + \mathcal{O}(\delta x^2). \tag{30}$$

[7] We now reproduce the static optomechanical interaction of [8]. Using $\mathbf{J} = J\mathbf{z}$, we have

[6]The relevant Fourier transforms are,

$$\int\frac{\mathrm{d}^3q}{(2\pi)^3}\frac{\mathrm{e}^{\mathrm{i}\mathbf{q}\cdot\mathbf{d}}}{\mathbf{q}^2} = \frac{1}{4\pi d}, \qquad \int\frac{\mathrm{d}^3q}{(2\pi)^3}\frac{\mathrm{i}q_i\mathrm{e}^{\mathrm{i}\mathbf{q}\cdot\mathbf{d}}}{\mathbf{q}^2} = -\frac{d_i}{4\pi d^3}. \tag{27}$$

[7]The half-ring integral turns $\cos\theta$ into the factor

$$\int_{-\pi/2}^{\pi/2}\cos\theta\mathrm{d}\theta = 2. \tag{31}$$

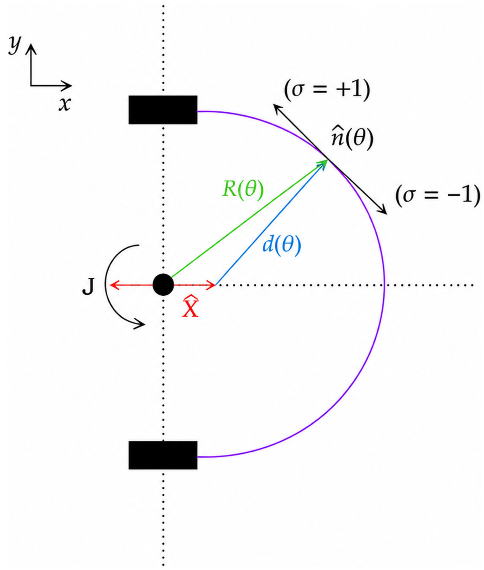

FIG. 1. Schematic of the setup showing the various quantities defined in Sec. III. At the centre there is a mass in a ground state initially $|0\rangle$, rotating counterclockwise with an angular momentum J. The optical beam (purple) lies in the XY plane, and the tangential direction senses ($\sigma = \pm 1$), e.g., the photon beam can prograde and retrograde with respect to the rotation of the mass at the center. We assume that the photon is in a coherent state $|\alpha\rangle$.

$$\mathbf{J}\cdot(\mathbf{d}(\theta)\times\mathbf{n}_\sigma(\theta)) = \sigma J(r-\delta x\cos\theta),$$
$$|\mathbf{d}(\theta)| = r-\delta x\cos\theta+\mathcal{O}(\delta x^2). \tag{32}$$

Therefore,

$$\frac{\mathbf{J}\cdot(\mathbf{d}\times\mathbf{n}_\sigma)}{|\mathbf{d}|^3} = \sigma\frac{J}{r^2}\left(1+2\frac{\delta x\cos\theta}{r}\right)+\mathcal{O}(\delta x^2, J^2). \tag{33}$$

Integrating the linear term over the half-ring again produces the factor 2, so the part linear in δx is

$$V_{J,\mathrm{lin}}^{(\sigma)} = +\frac{8\sigma GJ\omega}{r^3}\delta x. \tag{34}$$

Combining this with the static linear term from [8] gives the modified interaction

$$V_{\mathrm{int}}^{(J,\sigma)} = -\left(\frac{4GM\omega}{r^2}-\frac{8\sigma GJ\omega}{r^3}\right)\delta x a^\dagger a. \tag{35}$$

We now study the effective optomechanical coupling in presence of the rotation. Using the zero-point fluctuation operator,

$$\delta x = x_{\mathrm{zpf}}(b+b^\dagger), \qquad x_{\mathrm{zpf}} = \frac{1}{\sqrt{2M\omega_m}}, \tag{36}$$

we obtain

$$V_{\mathrm{int}}^{(J,\sigma)} = -g_{0,\sigma}^{(J)}(b+b^\dagger)a^\dagger a, \tag{37}$$

where

$$g_{0,\sigma}^{(J)} = \left(\frac{4GM\omega}{r^2}-\frac{8\sigma GJ\omega}{r^3}\right)x_{\mathrm{zpf}} = g_0\left(1-2\sigma\frac{J}{Mr}\right),$$
$$g_0 = \frac{4GM\omega}{r^2\sqrt{2M\omega_m}}, \tag{38}$$

g_0 is the static single-photon coupling. Therefore, the explicit linear correction due to the rotation becomes,

$$\delta g_{0,\sigma} = -2\sigma\frac{J}{Mr}g_0. \tag{39}$$

Thus, the rotating source splits the two circulation directions linearly in J, where $\sigma = \pm 1$. The sign of the σ label depends on which circulation direction is called "+" and on the orientation chosen for $\mathbf{J}$. The invariant statement is that the two directions split linearly in J. In the absence of J, we recover the results of [8].

IV. LINEARIZED QUANTUM HAMILTONIAN

Note that $\mathbf{r}$, δx, $\mathbf{J}$, $a_{\lambda'}^\dagger a_\lambda$ are all operator valued entities. In the polarization-diagonal sector, and for the geometry in which the spin is perpendicular to the photon plane, the polarization matrix is proportional to the identity operator. Therefore, the optical factor reduces to the photon number operator

$$N_a = \sum_\lambda a_\lambda^\dagger a_\lambda.$$

For a single occupied polarization mode, this is simply $N_a = a^\dagger a$. The polarization-diagonal potential is given by,

$$V_J = \left[-\frac{2GM\omega}{r}+2G\omega\frac{\mathbf{J}\cdot(\mathbf{r}\times\mathbf{n})}{r^3}\right]N_a. \tag{40}$$

Here, M is still treated as the rest mass entering the monopole part of the potential, while $\mathbf{J}$ is now a quantum angular-momentum operator.

A. Torsional oscillator representation of J_z

In the half-ring geometry used above, the optical path lies in the equatorial plane, and the spin is taken to be normal to that plane: $\mathbf{J} = J_z\mathbf{z}$. Thus, only the component J_z enters the $\mathcal{O}(J)$ polarization-diagonal correction. To write J_z in an oscillator basis, it is important to note that angular momentum is the *canonical momentum* conjugate to an angular coordinate. Therefore, the direct analog of $\delta x = x_{\mathrm{zpf}}(b+b^\dagger)$, is not $J_z \propto (c+c^\dagger)$, but rather a momentum-quadrature expansion. Let ϕ be a small angular displacement about the z axis and let δJ_z be the fluctuation of the angular momentum about a mean spin J_0,

$$J_z = J_0 + \delta J_z. \tag{41}$$

The canonical commutation relation is (in natural units),

$$[\phi, \delta J_z] = i. \tag{42}$$

For a torsional oscillator of moment of inertia I and torsional frequency Ω_J, the rotational Hamiltonian becomes,

$$H_J = \frac{(\delta J_z)^2}{2I} + \frac{1}{2} I \Omega_J^2 \phi^2. \tag{43}$$

Introducing a bosonic operator, c, satisfying $[c, c^\dagger] = 1$, we define

$$\phi = \phi_{\rm zpf}(c + c^\dagger), \qquad \phi_{\rm zpf} = \sqrt{\frac{1}{2I\Omega_J}}, \tag{44}$$

and

$$\delta J_z = i J_{\rm zpf}(c^\dagger - c), \qquad J_{\rm zpf} = \sqrt{\frac{I\Omega_J}{2}}. \tag{45}$$

These definitions obey: $\phi_{\rm zpf} J_{\rm zpf} = 1/2$, and therefore reproduce Eq. (42). With these definitions,

$$H_J = \Omega_J \left(c^\dagger c + \frac{1}{2} \right). \tag{46}$$

Thus, the angular momentum operator becomes,

$$J_z = J_0 + i J_{\rm zpf}(c^\dagger - c), \tag{47}$$

where the above expression becomes the angular-momentum analog of the mechanical displacement expansion $\delta x = x_{\rm zpf}(b + b^\dagger)$, with the crucial difference that J_z is a momentumlike quadrature.

B. Effective Hamiltonian

For a photon circulating in direction $\sigma = \pm 1$, the spin-dependent part of the potential contains

$$\frac{J_z \mathbf{d}(\theta) \times \mathbf{n}_\sigma(\theta) \cdot \mathbf{z}}{|\mathbf{d}(\theta)|^3} = \sigma \frac{J_z}{r^2} \left(1 + 2 \frac{\delta x \cos\theta}{r} \right) + \mathcal{O}(\delta x^2). \tag{48}$$

The term independent of δx gives a spin-dependent optical phase,

$$V_{J,0}^{(\sigma)} = \chi_\sigma J_z N_a, \tag{49}$$

where, with the same half-ring normalization used above,

$$\chi_\sigma = \frac{2\pi\sigma G\omega}{r^2}. \tag{50}$$

If J is treated as a classical number, this term is only a photon phase and can be removed. However, since J_z is a quantum operator and the spin state is not an eigenstate of J_z, Eq. (49) can entangle the optical mode with the rotational degree of freedom. The term linear in δx gives the modified optomechanical interaction

$$V_{\rm int}^{(J,\sigma)} = -\left(\frac{4GM\omega}{r^2} - \frac{8\sigma G\omega}{r^3} J_z \right) \delta x N_a. \tag{51}$$

Using Eq. (36), we may write

$$V_{\rm int}^{(J,\sigma)} = -g_{0,\sigma}^{(J)} (b + b^\dagger) N_a, \tag{52}$$

where the single-photon coupling is now itself an operator, where

$$g_{0,\sigma}^{(J)} = g_M - g_J J_z, \tag{53}$$

with

$$g_M = \frac{4GM\omega}{r^2} x_{\rm zpf}, \qquad g_J = \frac{8\sigma G\omega}{r^3} x_{\rm zpf}. \tag{54}$$

Substituting Eq. (47) gives

$$g_{0,\sigma}^{(J)} = g_M - g_J J_0 - i g_J J_{\rm zpf}(c^\dagger - c). \tag{55}$$

Therefore, the interaction becomes

$$\begin{aligned} V_{\rm int}^{(J,\sigma)} = &-(g_M - g_J J_0)(b + b^\dagger) N_a \\ &+ i g_J J_{\rm zpf}(c^\dagger - c)(b + b^\dagger) N_a. \end{aligned} \tag{56}$$

It is useful to define

$$g_{0,\sigma}^{\rm cl} = g_M - g_J J_0 = \left(\frac{4GM\omega}{r^2} - \frac{8\sigma G J_0 \omega}{r^3} \right) x_{\rm zpf}, \tag{57}$$

and

$$\lambda_{J,\sigma} = g_J J_{\rm zpf} = \frac{8\sigma G\omega}{r^3} x_{\rm zpf} J_{\rm zpf}. \tag{58}$$

Then

$$V_{\rm int}^{(J,\sigma)} = -g_{0,\sigma}^{\rm cl}(b + b^\dagger) N_a + i\lambda_{J,\sigma}(c^\dagger - c)(b + b^\dagger) N_a. \tag{59}$$

The first term is the semiclassical spinning-source optomechanical interaction. The second term is new: it couples the photon number to both the center-of-mass displacement quadrature and the angular-momentum quadrature of the spinning source. Including the displacement-independent spin-light term, the effective Hamiltonian becomes

$$H = \omega_m b^\dagger b + \Omega_J c^\dagger c + \chi_\sigma J_z N_a - g_{0,\sigma}^{\mathrm{cl}}(b + b^\dagger)N_a + i\lambda_{J,\sigma}(c^\dagger - c)(b + b^\dagger)N_a. \tag{60}$$

The zero-point term $\Omega_J/2$ has been omitted from Eq. (60) because it contributes only a global phase.

V. SPIN-EIGENSTATE AND CLASSICAL LIMITS

If the rotational degree of freedom is prepared in a state sharply peaked around J_0, or if one takes the formal classical limit $J_{\mathrm{zpf}} \to 0, J_z \to J_0$, then Eq. (59) reduces to

$$V_{\mathrm{int}}^{(J,\sigma)} \to -g_{0,\sigma}^{\mathrm{cl}}(b + b^\dagger)N_a, \tag{61}$$

which is precisely the semiclassical coupling used in the previous section. If, however, the spin state has appreciable quantum fluctuations in J_z, the coupling $g_{0,\sigma}^{(J)}$ is operator valued. The optical field can then become entangled not only with the center-of-mass oscillator, $(b, b^\dagger)$, but also with the rotational oscillator $(c, c^\dagger)$.

We now compute the photon–matter entanglement generated by the spin-corrected optomechanical Hamiltonian. In this section we specialize to the case in which the angular momentum is either classical, $J = J_0$, or the rotational degree of freedom is prepared in a sufficiently sharp eigenstate of J_z. In this limit, the coupling $g_{0,\sigma}^{(J)}$ is a c number and the calculation is a direct generalization of the nonrotating case. The relevant spin-corrected optomechanical interaction becomes,

$$V_{\mathrm{int}}^{(J,\sigma)} = -g_{0,\sigma}^{(J)}(b + b^\dagger)a^\dagger a, \tag{62}$$

where

$$g_{0,\sigma}^{(J)} = \left(\frac{4GM\omega}{r^2} - \frac{8\sigma GJ\omega}{r^3}\right)x_{\mathrm{zpf}} = g_0\left(1 - 2\sigma\frac{J}{Mr}\right). \tag{63}$$

Here $\sigma = \pm 1$ labels the two optical circulation directions. The static coupling is $g_0 = (4GM\omega/r^2)x_{\mathrm{zpf}}$ and $x_{\mathrm{zpf}} = 1/\sqrt{2M\omega_m}$ in the natural units. The full Hamiltonian for the center-of-mass oscillator and the optical mode then becomes, therefore

$$H_\sigma = \omega_m b^\dagger b - g_{0,\sigma}^{(J)}(b + b^\dagger)a^\dagger a. \tag{64}$$

For classical J, the displacement-independent part of the spin potential only gives an optical phase proportional to $a^\dagger a$, and hence does not contribute to the photon-matter entanglement. If J_z is kept as an operator, this statement no longer holds because the corresponding phase can entangle the optical mode with the rotational degree of freedom. The photon-number operator $N_a = a^\dagger a$ commutes with H_σ. Hence, the optical number operator evolves independently. We denote the photon-number indices by n and ℓ to avoid confusion with the source mass M, and define

$$G_\sigma^{(J)} = \frac{g_{0,\sigma}^{(J)}}{\omega_m}, \qquad t = \omega_m \tau, \qquad \Lambda = |\alpha|^2, \tag{65}$$

where ω_m is the mechanical frequency, τ is the laboratory interaction time, and α is the coherent-state amplitude of the optical mode.

VI. MODIFIED ENTANGLEMENT ENTROPY

Taking the initial state of the photon to be in a coherent state and the matter to be in a ground state, i.e.,

$$|\Psi(0)\rangle = |0\rangle_b \otimes |\alpha\rangle_a, \quad |\alpha\rangle_a = e^{-\Lambda/2}\sum_{n=0}^{\infty}\frac{\alpha^n}{\sqrt{n!}}|n\rangle_a, \tag{66}$$

we may write the evolved state under the Hamiltonian equation (64), given by as

$$|\Psi(t)\rangle = e^{-\Lambda/2}\sum_{n=0}^{\infty}\frac{\alpha^n}{\sqrt{n!}}|\phi_n(t)\rangle_b|n\rangle_a. \tag{67}$$

For a fixed photon number n, the oscillator evolves under Eq. (64), and by completing the square in the Hamiltonian equation (64), gives

$$H_{\sigma,n} = \omega_m(b^\dagger - G_\sigma^{(J)}n)(b - G_\sigma^{(J)}n) - \omega_m(G_\sigma^{(J)}n)^2. \tag{68}$$

Starting from the oscillator ground state, the mechanical state in the n-photon branch is therefore a coherent state,

$$|\phi_n(t)\rangle_b = \exp\left[i(G_\sigma^{(J)})^2 n^2(t - \sin t)\right]|G_\sigma^{(J)}n(1 - e^{-it})\rangle_b. \tag{69}$$

Thus, different photon-number sectors displace the mechanical oscillator by different amounts. This is the origin of the photon-matter entanglement. The reduced state of the mechanical oscillator is obtained by tracing over the optical mode,

$$\rho_b(t) = \mathrm{Tr}_a|\Psi(t)\rangle\langle\Psi(t)| = e^{-\Lambda}\sum_{n=0}^{\infty}\frac{\Lambda^n}{n!}|\phi_n(t)\rangle\langle\phi_n(t)|. \tag{70}$$

The linear entropy is defined as,

$$S_{J,\sigma}(t) = 1 - \mathrm{Tr}_b\rho_b^2(t). \tag{71}$$

Using the above form of ρ_b, the purity is

$$\mathrm{Tr}_b\rho_b^2(t) = e^{-2\Lambda}\sum_{n=0}^{\infty}\sum_{\ell=0}^{\infty}\frac{\Lambda^{n+\ell}}{n!\ell!}|\langle\phi_\ell(t)|\phi_n(t)\rangle|^2. \tag{72}$$

For coherent states,

$$|\langle\beta|\gamma\rangle|^2 = \exp[-|\beta-\gamma|^2]. \tag{73}$$

Here, the coherent-state displacement in the n-photon branch is

$$\beta_n(t) = G_\sigma^{(J)} n(1-e^{-it}). \tag{74}$$

Therefore,

$$\begin{aligned}|\beta_n(t)-\beta_\ell(t)|^2 &= (G_\sigma^{(J)})^2(n-\ell)^2|1-e^{-it}|^2\\ &= 2(G_\sigma^{(J)})^2(n-\ell)^2(1-\cos t).\end{aligned} \tag{75}$$

Thus, we get

$$|\langle\phi_\ell(t)|\phi_n(t)\rangle|^2 = \exp\left[2(G_\sigma^{(J)})^2(\ell-n)^2(\cos t-1)\right]. \tag{76}$$

The exact analog of Eq. (21) of [8] is then

$$\begin{aligned}S_{J,\sigma}(t) = 1 - e^{-2|\alpha|^2}\sum_{n=0}^{\infty}\sum_{\ell=0}^{\infty}\frac{|\alpha|^{2(n+\ell)}}{n!\ell!}\\ \times\exp\left[2(G_\sigma^{(J)})^2(\ell-n)^2(\cos t-1)\right].\end{aligned} \tag{77}$$

A. Gaussian approximation for large $|\boldsymbol{\alpha}|$

For a bright coherent optical field, $|\alpha|^2 = \Lambda \gg 1$, the Poisson distribution may be approximated by a Gaussian,

$$e^{-\Lambda}\frac{\Lambda^n}{n!} \simeq \frac{1}{\sqrt{2\pi\Lambda}}\exp\left[-\frac{(n-\Lambda)^2}{2\Lambda}\right]. \tag{78}$$

The double sum in the exact expression for $S_{J,\sigma}$ can then be approximated by a two-dimensional Gaussian integral. Since the overlap depends only on the photon-number difference $n-\ell$, one introduces

$$u = \frac{n+\ell}{2}, \qquad v = n-\ell.$$

The u integral is elementary, while v is distributed with variance 2Λ. This gives

$$\mathrm{Tr}\rho_b^2(t) \simeq \frac{1}{\sqrt{1+8(G_\sigma^{(J)})^2|\alpha|^2(1-\cos t)}}. \tag{79}$$

Using the same Gaussian approximation to the Poisson weights as in [8] yields

$$S_{J,\sigma}(t) \approx 1 - \frac{1}{\sqrt{1+8(G_\sigma^{(J)})^2|\alpha|^2(1-\cos t)}}. \tag{80}$$

In the weak-coupling regime

$$(G_\sigma^{(J)})^2|\alpha|^2 \ll 1,$$

this becomes

$$S_{J,\sigma}(t) \approx S_{\mathrm{max},J,\sigma}(1-\cos t), \qquad S_{\mathrm{max},J,\sigma} = 4\frac{(g_{0,\sigma}^{(J)})^2|\alpha|^2}{\omega_m^2}. \tag{81}$$

Comparing with the static (nonrotating) case gives

$$S_{\mathrm{max},J,\sigma} = S_{\mathrm{max},0}\left(1-2\sigma\frac{J}{Mr}\right)^2 \approx S_{\mathrm{max},0}\left(1-4\sigma\frac{J}{Mr}\right), \tag{82}$$

where

$$S_{\mathrm{max},0} = 4\frac{g_0^2|\alpha|^2}{\omega_m^2} \tag{83}$$

is the static result.

B. Short-time limit

Define the light-enhanced coupling $g_\sigma^{(J)} = g_{0,\sigma}^{(J)}|\alpha|$, and in the short-time regime $t = \omega_m\tau \ll 1$, we can reduce Eq. (80),

$$S_{J,\sigma}^{\mathrm{short}} = 2(g_\sigma^{(J)})^2\tau^2 = 2(g_{0,\sigma}^{(J)})^2|\alpha|^2\tau^2. \tag{84}$$

Using Eq. (63) in the limit, $t = \omega_m\tau \ll 1$, yields:

$$S_{J,\sigma}^{\mathrm{short}} = S_0^{\mathrm{short}}\left(1-2\sigma\frac{J}{Mr}\right)^2 \approx S_0^{\mathrm{short}}\left(1-4\sigma\frac{J}{Mr}\right), \tag{85}$$

where

$$S_0^{\mathrm{short}} = 2g_0^2|\alpha|^2\tau^2 = \frac{4G^2M\omega^2|\alpha|^2\tau^2}{r^4\omega_m}, \tag{86}$$

where the light intensity I can be related to the occupation number of photons $|\alpha|$, the electric field E_c can be related to the cavity volume $V_c = (\pi r)(\pi w_d^2)$, where w_d is the cavity waist, and r is the radial distance of the photon cavity from the massive object,

$$|\alpha|^2 = \frac{2I}{\epsilon_0 E_c^2}, \quad E_c = \sqrt{\frac{\omega}{2\epsilon_0 V_c}}, \quad |\alpha| = \sqrt{\frac{4IV_c}{\omega}}. \tag{87}$$

In [8], we set the following values for the mechanical oscillator, $M = 10$ kg, $\omega_m = 2\pi\times 150$ Hz [for these values and $\rho \sim 10^4\ \mathrm{kg\,m^{-3}}$, corresponding to radius $R \sim 6$ cm, $I = 10^{13}\ \mathrm{W\,cm^{-2}}$ at the optical wavelength $\lambda = 1\ \mu\mathrm{m}$ ($\omega = 2\pi/\lambda$)], while for the half-ring cavity we set the radius to $r = 25$ cm and the waist to $w_d = 6$ cm, such that the total power circulating in the cavity is ∼1 PW, which corresponds to $|\alpha| \sim 10^{13}$. One can obtain $S_0^{\mathrm{short}} \sim O(1)$ for $\tau \sim 1$ ms while squeezing the matter oscillator $\Delta x \sim x_{\mathrm{zpf}} e^{\xi} \sim 6$ cm, for $\xi \sim 41$, see Ref. [8]. Recently, a 10 kg mirror has been motionally cooled close to the quantum ground state [46,47]. Of course, squeezing all the way to 41 is an extremely challenging task, and many technical hurdles will need to be overcome. In addition, petawatt laser beams are available [48]. However, these are on-shot lasers, and running them continuously for 10 ms will also be a huge technical challenge.

However, with the rotation of the matter oscillator, there is an extra vantage that we can study the difference of short-time entanglement (maximum values) generated in the two cases (pro- and retrograde motion of the photons), which isolates the source spin directly,

$$S_{J,+}^{\mathrm{short}} - S_{J,-}^{\mathrm{short}} \approx -8 \frac{J}{Mr} S_0^{\mathrm{short}}. \tag{88}$$

The difference is therefore proportional to $J/(Mcr)$ (if we restore the factors of c). The angular momentum of any object depends on its breaking point, the stress it can handle. The highest stress the material can handle without breaking is given by $\sigma \sim \rho v^2$, while $J/(Mc) \sim \kappa R v/c$, where $\kappa = 2/5$ for a rigid sphere, and R is the radius of the sphere. Hence, the maximum rotational velocity for a sphere can be written as: $v_{\max} \sim \sqrt{\sigma_{\max}/\rho}$. For a dense object with $\rho \sim 2.2 \times 10^4\ \mathrm{Kg\,m^{-3}}$, such as iridium of mass, $M = 10$ kg, $R = 4.72$ cm, $\sigma_{\max} \sim 10^9$ Pa, which gives $v_{\max}/c \sim 10^{-5}$, hence $J/(Mcr) \sim O(10^{-7})$ for $r = 25$ cm (the c.m. distance between the spinning sphere and the laser beam). The current lab results for $(J/Mcr) \sim 12 \times 10^{-10}$ [49], and $\sim 10^{-6}$ [50], but for much smaller masses, while keeping $r = 25$ cm. Although in our case the difference between retrograde and prograde linear entropies is small, it could be detectable in future experiments if decoherence parameters are well controlled. Typically, for a small entanglement phase, there exists a relationship: $\phi_{\mathrm{ent}} \propto S_{J,\sigma}^{1/2}$ [10], which suggests that a small variation in the entanglement phase is a measurable quantity in a laboratory. In our case, the difference in the linear entropy will entail an entanglement phase difference of $\phi_{\mathrm{ent}} \sim \mathcal{O}(10^{-3})$.

Detecting such a small difference in the entanglement phase will depend on the witnessing scheme, and we may need to seek a new one for entanglement between a spinning sphere and a laser beam. Another important facet will be the estimation of decoherence in a massive matterwave setup [8,51–55], and optical setup [56,57]. Besides these sources of decoherence, there are other sources of dephasing due to random acceleration noise and gravity gradient noise [58–61], which need to be considered for such a precision-led experiment. However, these issues are beyond the current scope of the paper, as they depend on the choice of platform for creating a spatial superposition of matter and on the details of the witnessing mechanism.

In any case, our study provides the first-ever theoretical handle on rotational dynamics in the presence of a quantum gravitational interaction with a photon in a lab. Moreover, it will ascertain how a massless graviton can entangle a rotating quantum matter with a photon via the off-shell graviton exchange.

VII. CONCLUSION

The highlight of the paper is to show that by including the spinning object, at a quantum level, the scattering cross-section between the former and a photon via a massless graviton exchange yields a polarization-changing contribution, which is a known classical effect, see Refs. [37,38]. However, demonstrating this effect of rotation at a quantum level in a laboratory will be very interesting and require a gyroscopically stable rotor, see Refs. [26,50,62–64]. By imparting a classical angular momentum, the effect of spin can be witnessed in the entanglement entropy for the choice of prograde and retrograde motion of the photon with respect to the spinning object. The difference in the entanglement entropy is proportional to the angular momentum $\propto J/(Mcr)$. Further improvements in the observability of the entanglement can be made by squeezing the matter sector to enhance the superposition size. If we set $M = 100$ g (radius $R = 1$ cm and density $\rho = 10\ \mathrm{g\,cm^{-3}}$), we can generate unit entanglement $S_0^{\mathrm{short}} \sim \mathcal{O}(1)$ in $\tau = 10$ ms using the mechanical frequency $\omega_m = 2\pi \times 1$ Hz and setting the delocalization to $\Delta x = x_{\mathrm{zpf}} e^{\xi} \sim R \sim 1$ cm (corresponding to a squeezing parameter $\xi = 35$) while maintaining the same laser intensity. We also note that a small change in the linear entropy resulting from prograde and retrograde motion with respect to the spinning sphere is a measurable quantity in a laboratory. A rough estimation for a $M = 10$ kg highly dense sphere with a high breaking stress, such as in the case of iridium, can provide $S_{J,+}^{\mathrm{short}} - S_{J,-}^{\mathrm{short}} \sim 10^{-7}$. Although this is a small difference, it can be measured with an appropriate witnessing scheme. In the future, we will be keen to study the feasibility of the scheme in the experimental context and report a viable witnessing scheme.

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DATA AVAILABILITY

There are no publicly available research data or software supporting this manuscript. Requests for further information or data should be sent to the authors.

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