Letter

Chiral anomaly from anomalous spin hydrodynamics

Jay Armas [1,2,3,4,*] and Giorgos Batzios [1,2,†]

[1] *Institute for Theoretical Physics, University of Amsterdam, 1090 GL Amsterdam, The Netherlands*
[2] *Dutch Institute for Emergent Phenomena, The Netherlands*
[3] *Institute for Advanced Study, University of Amsterdam, Oude Turfmarkt 147, 1012 GC Amsterdam, The Netherlands*
[4] *Niels Bohr International Academy, The Niels Bohr Institute, University of Copenhagen, Blegdamsvej 17, DK-2100 Copenhagen Ø, Denmark*

(Received 22 June 2025; accepted 11 May 2026; published 6 July 2026)

We show that the low energy fluctuations of spinning black Dp branes are described by a theory of spin hydrodynamics on a spacetime $\mathbb{M}_{p+1} \times \mathbb{T}^{n+2}$ in which the fluid is flowing on $\mathbb{M}_{p+1}$ and spinning on $\mathbb{T}^{n+2}$. Focusing on the hydrodynamic regime of $\mathcal{N} = 4$ supersymmetric Yang-Mills theory, we provide a geometric interpretation of the R-current anomaly in terms of a gravitational anomaly from the ten-dimensional point of view. This follows from the holographic duality between a spinning fluid in ten dimensions and an anomalous chiral fluid in four dimensions. We comment on the relations between the theory of spin hydrodynamics introduced here and other theories of spin hydrodynamics in the context of heavy-ion collisions.

DOI: 10.1103/fwxg-kc7m

The chiral anomaly and spin hydrodynamics [1–4] constitute two, seemingly different, areas of intense research in both high-energy and condensed matter physics. In the context of electroweak interactions, the chiral anomaly is responsible for the rapid pion decay rate [5,6], while in heavy-ion collisions it is expected to give rise to novel transport properties of the quark-gluon plasma such as the chiral vortical and chiral magnetic effects [1,2]. At the same time, the observation of spin polarization of Λ hyperons in heavy-ion collisions suggests that spin degrees of freedom should be included in hydrodynamic descriptions [3]. In condensed matter physics, the chiral anomaly leads to unusual electronic properties such as negative magnetoresistance [7] whereas spin currents can be measured in liquid metals [8]. In this Letter, we describe a connection between chiral transport and spin hydrodynamics in the context of holography. More precisely, we uncover a duality between hydrodynamics with *transverse* spin in ten spacetime dimensions and an anomalous chiral fluid in four spacetime dimensions [9].

Many strongly coupled plasmas when sourced by background electromagnetic fields are expected to exhibit a chiral anomaly [10]. Indeed, upon gauging, $\mathcal{N} = 4$ supersymmetric Yang-Mills theory (SYM) has an R-current anomaly [11]. In the low-energy, the hydrodynamic regime of $\mathcal{N} = 4$ SYM, holography has revealed the existence of chiral transport determined by the presence of this $U(1)^3$ anomaly [12–14]. However all of these studies come from a 5D perspective, making use of dimensional reduction on the S^5, and consider fluctuations of 5D R-charged black holes [12–15]. Here we uplift the rich physics of chiral transport to 10D in which the low energy effective theory of spinning Dp-branes plays a crucial role.

We show that this low energy description is a theory of spin hydrodynamics on a spacetime $\mathbb{M}_{p+1} \times \mathbb{T}^{n+2}$ in which the fluid lives on the world volume $\mathbb{M}_{p+1}$ and spins on the transverse space $\mathbb{T}^{n+2}$. For the case of the near-horizon limit of the D3 brane, the transverse angular momenta (or spins) corresponding to the three transverse Cartan planes of rotation realize holographically the charges of the $SO(6)$ R-symmetry group of the dual field theory. After introducing the local current whose integral provides the global spin charges, which we call the spin current, we explain how the hydrodynamics of the R-charged black holes of [12–14] corresponds to the hydrodynamics of (anomalous) spinning fluids characterizing the long-wavelength fluctuations of spinning black D3 branes. This enables a geometrization of the chiral anomaly of the 4D theory in terms of a gravitational anomaly in 10D. Nevertheless, this is merely an artifact of imposing a specific dimensional reduction ansatz, and in the full 10D string theory picture this anomaly is absent. We argue that the precise geometric

*Contact author: j.armas@uva.nl
†Contact author: g.batzios@uva.nl

origin of the R-current anomaly should be understood via the coupling of the world volume fluid with the five-form flux, leading to the mapping of the chiral fluid in 4D to a (nonanomalous) spinning fluid with higher-form charge in 10D. We begin by looking at the thermodynamics and currents of spinning D3 branes in order to extract the low energy description of their dynamics.

Currents of spinning D3 branes from gravity. We consider asymptotically flat black brane solutions of type IIB supergravity with world volume $\mathbb{M}_{p+1}$ spinning on the transverse $\mathbb{T}^{n+2}$ space in $d = n + p + 3$ spacetime dimensions. It is always possible to introduce a coordinate $\boldsymbol{u} = 1/r$ such that when $\boldsymbol{u} \to 0$ and hence $r \to \infty$ a far away observer with respect to the black brane horizon sees a distribution of matter that admits a multipole expansion of the form [16]

$$\mathbf{T}^{\mu\nu} = T^{\mu\nu}(\sigma)\hat{\delta}(X) - \nabla_\rho(T^{\mu\nu\rho}(\sigma)\hat{\delta}(X)) + \cdots, \qquad (1)$$

where $T^{\mu\nu}(\sigma)$ is the monopole stress tensor, σ^a coordinates on $\mathbb{M}_{p+1}$, $T^{\mu\nu\rho}$ the dipole moment of the stress tensor, $\hat{\delta}(X) = (\sqrt{-\gamma}/\sqrt{-g})\delta^{(n+2)}(x^\alpha - X^\alpha)$ the weighted $(n+2)$-dimensional delta function where x^μ are coordinates on spacetime with metric $g_{\mu\nu}$, determinant g and ∇_μ is built from the Christoffel connection associated to $g_{\mu\nu}$. In addition, $X^\mu(\sigma)$ are a set of scalar mapping functions that determine the location of $\mathbb{M}_{p+1}$ in the spacetime $\mathcal{M}_d = \mathbb{M}_{p+1} \times \mathbb{T}^{n+2}$, while $\gamma_{ab} = g_{\mu\nu}\partial_a X^\mu \partial_b X^\nu$ is the induced metric on $\mathbb{M}_{p+1}$ and γ its determinant while a, b indices run over $\mathbb{M}_{p+1}$ (see the Supplemental Material [17]). The dots in (1) stand for additional quadrupole or higher corrections to the stress tensor which we will not deal with here.

The stress tensor (1) can be extracted from the metric of a black Dp brane by expanding it in powers of $\boldsymbol{u}$, in particular $g_{\mu\nu} = \eta_{\mu\nu} + h^{(M)}_{\mu\nu} + h^{(D)}_{\mu\nu} + \mathcal{O}(\boldsymbol{u}^{n+2})$, where $h^{(M)}_{\mu\nu}$ stands for the monopole contribution to the metric of order $\boldsymbol{u}^n$ and $h^{(D)}_{\mu\nu}$ for the dipole contribution of order $\boldsymbol{u}^{n+1}$. Contrary to cases in which dipole terms in (1) are viewed as perturbations, here we are interested in geometries in which monopole and dipole terms in (1) are taken to be of equal order. We can achieve this by rescaling $\boldsymbol{u} \to \lambda \boldsymbol{u}$ as well as $h^{(D)}_{\mu\nu} \to h^{(D)}_{\mu\nu}/\lambda$, and approaching the asymptotic region by sending $\lambda \to 0$ while keeping $\boldsymbol{u}$ fixed. We then extract the monopole and dipole contributions to the stress tensor using linearized gravity:

$$\nabla^2 \bar{h}^{(M)}_{ab} = -16\pi G T_{ab}\hat{\delta}(r), \qquad \nabla^2 \bar{h}^{(D)}_{ai} = 8\pi G \mathcal{S}_{ai}{}^{j}\partial_j \hat{\delta}(r), \qquad (2)$$

where we have introduced $\bar{h}_{\mu\nu} = h_{\mu\nu} - h\eta_{\mu\nu}/2$ with $h = \eta^{\mu\nu}h_{\mu\nu}$, and focused on the nontrivial spin components of $T^{aij} = \mathcal{S}^{aij}$, where $\mathcal{S}^{aij} = \mathcal{S}^{a[ij]}$ is the spin current and i, j indices run on $\mathbb{T}^{n+2}$. Adopting the conventions of [24] for the boosted spinning D3 brane, using (2) we obtain the monopole stress tensor and spin current

$$T^{ab} = (\varepsilon + P)u^a u^b + P\gamma^{ab}, \qquad \mathcal{S}_a{}^{ij} = \ell J^A \epsilon^{ij}_A u_a. \qquad (3)$$

Here T^{ab} takes the perfect fluid form with ε, P, and u^a the energy density, pressure, and fluid (boost) velocity, while J^A is the angular momentum density on the Cartan plane $A = 1, 2, 3$, ϵ^{ij}_A the Levi-Civita symbol on the plane A, and $\ell = 1/\lambda$. The thermodynamic quantities ε, P, J^A can be parametrized in terms of the horizon size r_0, rotation parameter l_A on the plane A, and constant D3 brane charge Q_3. The charge Q_3 is constant and hence only labels solutions (is not a dynamical degree of freedom [25]).

Effective theory and equations of motion. The description of long-wavelength fluctuations of spinning Dp branes can be done by extending the blackfold approach [25–28] to strongly spinning branes. As discussed in [28], at ideal order in the gradient expansion, part of type IIB supergravity equations yield a constraint equation which is simply the conservation of the stress tensor $\nabla_\mu \mathbf{T}^{\mu\nu} = 0$ in the absence of background fluxes. By integrating this equation using a Gaussian pillbox [16,29,30] one arrives at the world volume effective theory,

$$\nabla_a T^a{}_b = \mathcal{S}^a{}_{ij}\Omega_{ba}{}^{ij}, \qquad \tilde{\nabla}_a \mathcal{S}^a{}_{ij} = 0, \qquad (4)$$

where ∇_a is a covariant derivative compatible with both $g_{\mu\nu}$ and γ_{ab}, while $\tilde{\nabla}_a$ also acts on transverse i indices according to $\tilde{\nabla}_a V^i = \partial_a V^i + \omega_a{}^{ij}V_j$ for some arbitrary vector V^i and where we introduced the spin connection $\omega_a{}^{ij} = n^i_\mu \nabla_a n^{j\mu}$ with n^μ_j a set of normal vectors to $\mathbb{M}_{p+1}$ normalized such that $n^\mu_i n^j_\mu = \delta^j_i$. In turn $\Omega_{ab}{}^{ij}$ is the field strength (or outer curvature) associated with $\omega_a{}^{ij}$, namely $\Omega_{ab}{}^{ij} = \tilde{\nabla}_a \omega_b{}^{ij} - \tilde{\nabla}_b \omega_a{}^{ij} + 2\omega_{[b}{}^{ik}\omega_{a]k}{}^{j}$. Equations (4) are the equations of *transverse* spin hydrodynamics. Backgrounds that are rotating in $\mathbb{T}^{n+2}$ have a nontrivial outer curvature, thereby leading to a nonvanishing spin-curvature coupling in the first equation in (4). The second equation in (4) states that the spin current is conserved ensuring that transverse angular momentum charges are well defined [31]. Equations (4) can be written in the more common form of spin hydrodynamics as in [32–34], which we show in the Supplemental Material [35].

The effective theory (4) can be derived from an action principle on $\mathbb{M}_{p+1}$ where the stress tensor and spin current are defined via the variations

$$\delta S = \int_{\mathbb{M}_{p+1}} d^{p+1}\sigma \sqrt{-\gamma}\left(\frac{1}{2}T^{ab}\delta\gamma_{ab} + \mathcal{S}^a{}_{ij}\delta\omega_a{}^{ij}\right). \qquad (5)$$

Tangential diffeormorphism transformations along $\mathbb{M}_{p+1}$ give rise, using (5), to the first equation in (4), while $SO(n+2)$ rotations of the normal vectors $\delta n_i^\mu = M_i{}^j n_j^\mu$ for some antisymmetric matrix M^{ij} lead to the spin conservation equation in (4) since $\delta\omega_a{}^{ij} = -\tilde{\nabla}_a M^{ij}$. We mention that the effective theory (5) can be seen as a non-Abelian gauge theory for the spin connection $\omega_a{}^{ij}$. As we are formulating a gradient expansion, we must assign a gradient ordering to each operator and source in the theory. Here we are considering the situation in which the monopole stress tensor and the spin current are equally important, thus $T^{ab}, \gamma_{ab} \sim \mathcal{O}(1)$, and since $\omega_a{}^{ij} \sim \mathcal{O}(\partial)$ we must require $\mathcal{S}^a{}_{ij} \sim \mathcal{O}(\partial^{-1})$. This latter condition ensures that, contrary to [16,29,36–38] where dipole terms were taken to be corrections, both monopole and dipole terms in $\mathbf{T}^{\mu\nu}$ contribute equally at ideal order. Given the spin current in (3) this implies that $\ell \sim \mathcal{O}(\partial^{-1})$, and hence the parameter ℓ can be used as a bookkeeping parameter as in [39,40] for organizing the gradient expansion. We will now show that the form of the monopole stress tensor and spin current for spinning D3 branes in (3) is a special case of a more general theory of spin hydrodynamics described by Eq. (4).

Degrees of freedom and conserved charges. The starting point of any hydrodynamic theory, whenever possible, is to construct the equilibrium partition function [29,41–43] from which the equilibrium currents can be derived. This can be done by looking at the hydrostatic requirements for an action whose variation is (5), in particular

$$\begin{aligned}\delta_K\gamma_{ab} &= \pounds_K\gamma_{ab} = 0,\\ \delta_K\omega_a{}^{ij} &= \pounds_K\omega_a{}^{ij} - \tilde{\nabla}_a\Lambda_K^{ij} = 0,\end{aligned} \tag{6}$$

where K^a and Λ_K^{ij} are equilibrium parameters, and $\pounds_K$ denotes the Lie derivative along K^a. The invariance of the conditions (6) under tangential diffeomorphisms ξ^a and rotations M^{ij} requires the transformation properties for the equilibrium parameters $\delta_B K^a = \pounds_\xi K^a$ and $\delta_B\Lambda_K^{ij} = \xi^a\tilde{\nabla}_a\Lambda_K^{ij} - K^a\tilde{\nabla}_a M^{ij}$ where $B = (\xi^a, M^{ij})$. This allows us to introduce the thermal twist vector $\beta^a = u^a/T \sim \mathcal{O}(1)$ where $u^a = K^a/|K|$ is the unit normalized fluid velocity $u_a u^a = -1$ with $|K|$ the modulus of the Killing vector K^a and $T = T_0/|K|$, interpreted as the temperature of the fluid, where T_0 is a constant global temperature. It may appear that no other ideal scalar order can be built out of the geometric data since $\omega_a{}^{ij} \sim \mathcal{O}(\partial)$ and hence also $\Lambda_K^{ij} \sim \mathcal{O}(\partial)$. However, using the bookkeeping parameter $\ell \sim \mathcal{O}(\partial^{-1})$ we can introduce the spin chemical potential μ^{ij} according to

$$\frac{\mu^{ij}}{T} = \ell\left(-\Lambda_K^{ij} + \beta^a\omega_a{}^{ij}\right), \tag{7}$$

which is antisymmetric in the i, j indices and $\mu^{ij} \sim \mathcal{O}(1)$ as desired. The temperature and spin chemical potential constitute the basic ingredients of the equilibrium partition function

$$S_{\text{eq}} = \int_{\mathbb{M}_{p+1}} d^{p+1}\sigma\sqrt{-\gamma}P(T, \mu^{ij}), \tag{8}$$

with P the fluid pressure, and whose invariance under tangential diffeomorphisms and normal rotations yields (4). We may also extract the equilibrium stress tensor as in (1) together with the thermodynamic identities

$$\varepsilon + P = Ts + s_{ij}\mu^{ij}, \qquad dP = sdT + s_{ij}d\mu^{ij}, \tag{9}$$

where $s = \partial P/\partial T$ is the entropy density and $s_{ij} = \partial P/\partial\mu^{ij}$ the spin density, as well as the spin current

$$\mathcal{S}^a{}_{ij} = \ell s_{ij}u^a, \tag{10}$$

which satisfies $\mathcal{S}^a{}_{ij} \sim \mathcal{O}(\partial^{-1})$ by construction. The thermodynamic relations (9) also match those of the spinning D3 brane which confirm that the gradient ordering of (7) is appropriate since the angular momenta of spinning Dp branes are not a perturbative quantity in gravity. The common lore of hydrodynamics is to promote the equilibrium parameters K^a and Λ_K^{ij} to slowly varying functions out of equilibrium, allowing us to identify T, μ^{ij}, u^a as the dynamical degrees of freedom of spin hydrodynamics whose gradients correct the currents T^{ab} and $\mathcal{S}^a{}_{ij}$.

In equilibrium, Eq. (4) have two sets of conserved charges that can be obtained by integrating the conserved currents $T^a = T^{ab}k_b + \mathcal{S}^a{}_{ij}n^{i\mu}n^{j\nu}\nabla_\nu k_\mu$ and $\mathcal{S}^a{}_{ij}$ over a spatial slice of $\mathbb{M}_{p+1}$. The current T^a gives rise to a total conserved energy if the Killing vector $k^\mu = k^a\partial_a X^\mu$ is the generator of time translations, and to linear/angular momentum in $\mathbb{M}_{p+1}$ when k_a is the generator of spatial translations/rotations. In turn, the current $\mathcal{S}^a{}_{ij}$ leads to a set of spin charges S^{ij}. In the specific case in which the configuration is rigidly rotating in $\mathbb{T}^{n+2}$, as for spinning D3 branes, there is an associated set of transverse Killing vector fields ζ_μ^{ij} for each plane (i, j). The conserved current associated with ζ_μ^{ij} is obtained by integrating the conserved current $\mathbf{T}^{\mu\nu}\zeta_\nu^{ij}$ over a spatial slice [29,30,37], and now the spin charges S^{ij} have the physical interpretation of (transverse) angular momenta in $\mathbb{T}^{n+2}$.

Spin current as the holographic dual of the R current. AdS/CFT geometrizes the global $SO(6)$ R symmetry of the $\mathcal{N} = 4$ SYM theory by mapping in the decoupling limit the charges of the R-symmetry group to angular momenta of the D3 brane in the planes perpendicular to its world volume. As mentioned, the current whose integral generically gives the (transverse) angular momenta is the spin current $\mathcal{S}^a{}_{ij}$. This clearly implies a holographic relation between the (near-horizon) spin current of the spinning D3 brane and the 4D R current of strongly coupled $\mathcal{N}=4$ SYM.

This holographic relation can immediately be made precise once we specialize to thermal states whose gravity duals are spinning D3 branes with equal angular momenta in the three Cartan planes of rotation. Such solutions fit into a Kaluza-Klein (KK) ansatz which when inserted into the type IIB supergravity equations of motion leads to a Einstein-Maxwell-Chern-Simons 5D theory (see for instance [44]). With a suitable choice of coordinates, the spin current is contained in the 10D metric component mixing transverse angles with world volume directions. In the large r or equivalently near-boundary expansion, the spin current is extracted from the near-horizon geometry as the coefficient of the term scaling as $\sim\mu_i^2 d\phi_i/r^2$. Upon the above KK reduction, this coefficient precisely gives rise to the (three copies of) gauge-covariant 4D R current(s) $\mathcal{J}^A_a$, where the index $A=1,2,3$ labels the three copies of the R current, and it is left there for clarity. We can then identify the near-horizon spin current $\mathcal{S}^a{}_{ij}$ as the dual of the 4D R current of strongly coupled $\mathcal{N}=4$ SYM in the equally spinning case via $\mathcal{S}_a{}^{ij}\to\ell\mathcal{J}^A_a$. Note that for this natural identification to be possible, a treatment of spinning D-branes in the strongly spinning regime is essential.

Anomalous spin hydrodynamics. The duality between spin current and R current implies that the former must also be an anomalous current. Corrections to the effective theory governing the dynamics of black branes at a given order can arise due to three different sources as explained in [16,28]: hydrodynamic corrections to the currents $T^{\mu\nu}$ and $T^{\mu\nu\rho}$, multipole corrections to (1), and corrections either due to modifications of the asymptotic structure or the presence of nontrivial fluxes that lead to modifications of the conservation law for the stress tensor, that is $\nabla_\mu\mathbf{T}^{\mu\nu}=\mathbf{f}^\nu$ where $\mathbf{f}^\mu$ is a forcing function that admits a multipole expansion as in (1). Here we focus on the decoupling limit of the D3 brane within the KK ansatz of [44] for which both hydrodynamic corrections and corrections due to the presence of fluxes arise, in particular $\mathbf{f}^\mu=F^{\mu\nu_1\ldots\nu_4}\mathbf{J}_{\nu_1\ldots\nu_4}/4!$ where $F^{\mu\nu_1\ldots\nu_4}$ is the five-form flux and $\mathbf{J}_{\nu_1\ldots\nu_4}$ the D3-brane current [28]. In this limit, the presence of fluxes leads to anomalous contributions to the spin conservation equation (4) [14,45], such that from a 10D perspective,

$$\tilde{\nabla}_a\mathcal{S}^a{}_{ij}=\frac{\ell^3}{8}C_{ijklmn}\epsilon^{abcd}\Omega_{ab}{}^{kl}\Omega_{cd}{}^{mn},\tag{11}$$

where C_{ijklmn} is a constant matrix of anomaly coefficients, antisymmetric in each pair $(i,j),(k,l),(m,n)$ and symmetric under exchange of pairs, while ϵ^{abcd} is the Levi-Civita tensor in $\mathbb{M}_4$ and where ℓ^3 guarantees the correct order in the expansion. The relation between the anomaly term and the forcing function $\mathbf{f}^\mu$ is given in the Supplemental Material. We note that the five-form flux does not appear explicitly in (11) because the KK ansatz [44] for the flux is determined in terms of 10D metric components and its derivatives, effectively generating the anomaly. As with the usual chiral anomaly, the anomalous term in the conservation law (11) is generated using anomaly inflow (see, e.g., [43,46]) in which the currents in (5) should now be viewed as covariant currents acquiring modifications due to inflow while the partition function (8) receives nongauge invariant corrections (see, e.g., [47]).

The form of the rhs of (11), modifying the conservation law for $\mathbf{T}^{\mu\nu}$, makes it clear that it is a gravitational anomaly. Not only is $\Omega_{ab}{}^{kl}$ purely geometric, it is also given in terms of components of the Riemann tensor (see the Supplemental Material). This interpretation is also manifest by the nature of the conserved charges.

Given (11), it is straightforward to find the corrected constitutive equations by requiring the second law of thermodynamics $\nabla_a S^a\geq 0$, where $S^a=P\beta^a-T^{ab}\beta_b-\ell^{-1}\mu^{ij}\mathcal{S}^a{}_{ij}/T+S^a_{\mathrm{nc}}$ is the entropy current and S^a_{nc} the noncanonical contribution to the entropy current. Parametrizing the corrections to the currents as $T^{ab}=(\varepsilon+P)u^a u^b+P\gamma^{ab}+\mathcal{T}^{ab}$ and $\mathcal{S}^a{}_{ij}=\ell s_{ij}u^a+\Sigma^a{}_{ij}$, and using the first equation in (4) as well as (11) we can determine the corrections in the Landau frame ($u_b\mathcal{T}^{ab}=u_a\Sigma^a{}_{ij}=0$) to be

$$\begin{aligned}\mathcal{T}^{ab}&=-\eta\sigma^{ab}-\zeta\theta P^{ab}+\mathcal{O}(\partial^2)\\ \Sigma^a{}_{ij}&=-\ell\mathcal{D}_{ijkl}P^{ab}\left(\ell\beta^c\Omega_{cb}{}^{kl}+\tilde{\nabla}_b\left(\frac{\mu^{kl}}{T}\right)\right)\\ &\quad+\ell\xi_{ij}\varpi^a+\ell^2\xi^S_{ijkl}B^{akl}+\mathcal{O}(\partial),\end{aligned}\tag{12}$$

where $\eta,\zeta,\mathcal{D}_{ijkl}\geq 0$ are the shear viscosity, bulk viscosity, and spin diffusion coefficients respectively. We have also introduced the fluid shear tensor $\sigma^{ab}=P^{ac}P^{bd}(2\nabla_{(c}u_{d)}-P_{cd}\theta/p)$ and expansion $\theta=\nabla_a u^a$ with $P^{ab}=u^a u^b+\gamma^{ab}$ the projector orthogonal to u_a. In turn ξ_{ij} is an antisymmetric matrix of coefficients yielding a nonzero chiral vortical effect where $\varpi^a=\epsilon^{abcd}u_b\nabla_c u_d/2$ is the fluid vorticity, while ξ^S_{ijkl} is a matrix of coefficients antisymmetric in the pairs (i,j) and (k,l) responsible for a chiral spin effect (the analog of the chiral magnetic effect), where we have introduced the "magnetic" outer curvature $B^{akl}=\epsilon^{abcd}u_b\Omega_{cd}{}^{kl}/2$. An analogous computation to [14,48] sets

$$\xi_{ij}=\left.\frac{\partial\tilde{\xi}}{\partial(\mu^{ij}/T)}\right|_P,\qquad \xi^S_{ijkl}=\left.\frac{\partial\tilde{\xi}^S_{kl}}{\partial(\mu^{ij}/T)}\right|_P,\tag{13}$$

where $\tilde{\xi}$ and $\tilde{\xi}^S_{kl}$ are the coefficients appearing in $S^a_{\mathrm{nc}}=\tilde{\xi}\varpi^a+\ell\tilde{\xi}^S_{kl}B^{akl}+\mathcal{O}(\partial^2)$ and given by $\tilde{\xi}=C_{ijklmn}\mu^{ij}\mu^{kl}\mu^{mn}/3T$

and $\tilde{\xi}^S_{ij} = C_{ijklmn}\mu^{kl}\mu^{mn}/2T$, where we have neglected other potential contributions to $\tilde{\xi}$ and $\tilde{\xi}^S_{ij}$ [48] that are not relevant for the D3 brane.

This analysis matches precisely the hydrodynamics of R-charged black holes when the spin current is holographically identified as the R current. Turning on a background field strength in the 4D anomalous theory means in 10D supergravity turning on a background outer curvature. Starting from the first equation in (4) and (11) in the equally spinning case, and using the holographic relations $\mathcal{S}_a{}^{ij} \to \ell \mathcal{J}^A_a$, $2\ell\Omega_{ab}{}^{ij} \to F^A_{ab}$ where F^A_{ab} are the field strengths of the 4D theory, we arrive at the standard conservation laws of a theory with a chiral anomaly [14].

Discussion. Using spinning D3 branes as a guiding example, we have given a geometric interpretation of the chiral anomaly; in particular the chiral anomaly in 4D is seen as a gravitational anomaly in 10D. In the hydrodynamic regime of a Quantum Field Theory with chiral anomaly, for instance $\mathcal{N} = 4$ SYM, the dynamics of the boundary stress tensor and R current of anomalous R-charged fluids in 4D is mapped to the dynamics of the spacetime stress tensor (1) of anomalous spinning fluids in 10D. This point of view is valuable for studying instabilities of Dp branes. For instance, using the exact coefficients for the spin current (12) of the D3 brane given in the Supplemental Material, it is straightforward to perform a linearized analysis and find, directly in 10D, the hydrodynamic instability in the spin diffusion modes recently uncovered in [49] from a 5D perspective.

From a 10D string theory point of view, however, the low energy effective theory of spinning D3 branes is that of a spinning fluid carrying a higher-form D3-brane charge coupled to a nontrivial background five-form flux such that $\mathbf{f}^\mu = F^{\mu\nu_1\ldots\nu_4}\mathbf{J}_{\nu_1\ldots\nu_4}/4!$. Following the same procedure outlined around (4), leads to

$$\tilde{\nabla}_a \mathcal{S}^a{}_{ij} = \frac{1}{4!} J^{\mu_1\ldots\mu_4}{}_{[i} F_{j]\mu_1\ldots\mu_4}, \tag{14}$$

where $J^{\mu_1\ldots\mu_4 i}$ is the dipole contribution to the current $\mathbf{J}_{\nu_1\ldots\nu_4}$. In particular, the D3 brane at ideal order possesses a magnetic dipole moment $J^{abcij} \sim \ell\epsilon^{acbd}u_d s^{ij}$. We see that the coupling between dipole contributions and five-form flux (14) violates spin conservation, similarly to charged spinning point particles coupled to electromagnetic fields (see, e.g., [50]), and hence can effectively generate anomalous contributions for special classes of backgrounds [51]. We intend to develop a hydrodynamic theory of higher-form spinning fluids along the lines of [53] in order to trace the string theoretic origin of the chiral anomaly in holographic fluids.

In the context of heavy-ion collisions, there are various formulations of spin hydrodynamics [3,4,54], some of which instead have *intrinsic* spin [32–34], i.e., the spin lives on $\mathbb{M}_{p+1}$ rather than on $\mathbb{T}^{n+2}$, and other formulations that do not include spin degrees of freedom [55–57]. The formulation presented here is yet another form of spin hydrodynamics, but we expect various connections with theories with *intrinsic* spin [32–34] which will be explored elsewhere [58]. To note is the relation between the R current of $\mathcal{N} = 1$ theories and the *intrinsic* spin current [59]. Using the construction of NS5-D3 branes in Polchinski-Strassler as in [60], it would be relevant to understand if the spin current discussed in this Letter is related to the R current in $\mathcal{N} = 1$. It would be interesting to see if, for all spinning Dp branes, the *transverse* spin current is directly related to the *intrinsic* spin current, thus providing an easy method for extracting transport properties of spin hydrodynamics from holography.

Acknowledgments. We would like to acknowledge fruitful discussions with, or comments on an earlier draft from, D. Berenstein, J. Bhattacharya, J. de Boer, U. Gürsoy, T. Harmark, A. Jain, P. Kovtun, K. Landsteiner, N. Obers, K. Schalm, R. Singh, C. Vafa, and A. Zhiboedov. J. A. is partly supported by the Dutch Institute for Emergent Phenomena (DIEP) cluster at the University of Amsterdam via the DIEP program Foundations and Applications of Emergence (FAEME).

Data availability. The data supporting this study's findings are available within the article.

[1] X.-G. Huang, Electromagnetic fields and anomalous transports in heavy-ion collisions—A pedagogical review, Rep. Prog. Phys. **79**, 076302 (2016).
[2] D. E. Kharzeev, J. Liao, and P. Tribedy, Chiral magnetic effect in heavy ion collisions: The present and future, Int. J. Mod. Phys. E **33**, 2430007 (2024).
[3] F. Becattini, M. Buzzegoli, T. Niida, S. Pu, A.-H. Tang, and Q. Wang, Spin polarization in relativistic heavy-ion collisions, Int. J. Mod. Phys. E **33**, 2430006 (2024).
[4] W. Florkowski, Spin hydrodynamics, J. Subatomic Part. Cosmol. **3**, 100028 (2025).
[5] S. L. Adler, Axial-vector vertex in spinor electrodynamics, Phys. Rev. **177**, 2426 (1969).

[6] J. S. Bell and R. Jackiw, A pcac puzzle: $\pi^0 \to \gamma\gamma$ in the σ-model, Il Nuovo Cimento A (1965-1970) **60**, 47 (1969).
[7] N. P. Ong and S. Liang, Experimental signatures of the chiral anomaly in Dirac–Weyl semimetals, Nat. Rev. Phys. **3**, 394 (2021).
[8] R. Takahashi, M. Matsuo, M. Ono, K. Harii, H. Chudo, S. Okayasu, J. Ieda, S. Takahashi, S. Maekawa, and E. Saitoh, Spin hydrodynamic generation, Nat. Phys. **12**, 52 (2016).
[9] In fact this duality is present between anomalous chiral fluids in four spacetime dimensions and spinning fluids in six spacetime dimensions. However, since we want to highlight the string theory origin of these chiral fluids we consider spinning fluids in ten spacetime dimensions.
[10] K. Landsteiner, Notes on anomaly induced transport, Acta Phys. Pol. B **47**, 2617 (2016).
[11] E. Witten, Anti-de Sitter space and holography, Adv. Theor. Math. Phys. **2**, 253 (1998).
[12] J. Erdmenger, M. Haack, M. Kaminski, and A. Yarom, Fluid dynamics of R-charged black holes, J. High Energy Phys. 01 (2009) 055.
[13] N. Banerjee, J. Bhattacharya, S. Bhattacharyya, S. Dutta, R. Loganayagam, and P. Surowka, Hydrodynamics from charged black branes, J. High Energy Phys. 01 (2011) 094.
[14] D. T. Son and P. Surowka, Hydrodynamics with triangle anomalies, Phys. Rev. Lett. **103**, 191601 (2009).
[15] M. Cvetic and S. S. Gubser, Phases of r-charged black holes, spinning branes and strongly coupled gauge theories, J. High Energy Phys. 04 (1999) 024.
[16] J. Armas, J. Camps, T. Harmark, and N. A. Obers, The young modulus of black strings and the fine structure of blackfolds, J. High Energy Phys. 02 (2012) 110.
[17] See Supplemental Material at http://link.aps.org/supplemental/10.1103/fwxg-kc7m for details on the geometry of embedded spaces, modifications due to fluxes, as well as the D3-brane thermodynamics and explicit transport coefficients, which includes Refs. [18–23].
[18] J. Armas, J. Gath, and N. A. Obers, Black branes as piezoelectrics, Phys. Rev. Lett. **109**, 241101 (2012).
[19] J. Armas, J. Gath, and N. A. Obers, Electroelasticity of charged black branes, J. High Energy Phys. 10 (2013) 035.
[20] S. Bhattacharyya, R. Loganayagam, S. Minwalla, S. Nampuri, S. P. Trivedi, and S. R. Wadia, Forced fluid dynamics from gravity, J. High Energy Phys. 02 (2009) 018.
[21] S. Bhattacharyya, V. E. Hubeny, S. Minwalla, and M. Rangamani, Nonlinear fluid dynamics from gravity, J. High Energy Phys. 02 (2008) 045.
[22] N. Banerjee, J. Bhattacharya, S. Bhattacharyya, S. Dutta, R. Loganayagam, and P. Surówka, Hydrodynamics from charged black branes, J. High Energy Phys. 01 (2011) 094.
[23] E. Megias and F. Pena-Benitez, Holographic gravitational anomaly in first and second order hydrodynamics, J. High Energy Phys. 05 (2013) 115.
[24] T. Harmark and N. A. Obers, Thermodynamics of spinning branes and their dual field theories, J. High Energy Phys. 01 (2000) 008.
[25] R. Emparan, T. Harmark, V. Niarchos, and N. A. Obers, Blackfolds in supergravity and string theory, J. High Energy Phys. 08 (2011) 154.
[26] R. Emparan, T. Harmark, V. Niarchos, and N. A. Obers, World-volume effective theory for higher-dimensional black holes, Phys. Rev. Lett. **102**, 191301 (2009).
[27] R. Emparan, T. Harmark, V. Niarchos, and N. A. Obers, Essentials of blackfold dynamics, J. High Energy Phys. 03 (2010) 063.
[28] J. Armas, J. Gath, V. Niarchos, N. A. Obers, and A. V. Pedersen, Forced fluid dynamics from blackfolds in general supergravity backgrounds, J. High Energy Phys. 10 (2016) 154.
[29] J. Armas, How fluids bend: The elastic expansion for higher-dimensional black holes, J. High Energy Phys. 09 (2013) 073.
[30] J. Armas and J. Tarrio, On actions for (entangling) surfaces and DCFTs, J. High Energy Phys. 04 (2018) 100.
[31] There is also another equation that in general arises from $\nabla_\mu \mathbf{T}^{\mu\nu} = 0$ describing the transverse dynamics of the brane but it is not relevant for the purposes of this letter so we present it elsewhere.
[32] A. D. Gallegos, U. Gürsoy, and A. Yarom, Hydrodynamics of spin currents, SciPost Phys. **11**, 041 (2021).
[33] M. Hongo, X.-G. Huang, M. Kaminski, M. Stephanov, and H.-U. Yee, Relativistic spin hydrodynamics with torsion and linear response theory for spin relaxation, J. High Energy Phys. 11 (2021) 150.
[34] A. D. Gallegos, U. Gursoy, and A. Yarom, Hydrodynamics, spin currents and torsion, J. High Energy Phys. 05 (2023) 139.
[35] We note that in other formulations of hydrodynamics of spin [32–34] there is a Belinfante-Rosenfeld ambiguity in the definition of stress tensor and spin current which does not appear in the formulation presented here because the stress tensor T^{ab} is symmetric.
[36] J. Armas, (Non)-dissipative hydrodynamics on embedded surfaces, J. High Energy Phys. 09 (2014) 047.
[37] J. Armas and T. Harmark, Constraints on the effective fluid theory of stationary branes, J. High Energy Phys. 10 (2014) 063.
[38] J. Armas, T. Harmark, and N. A. Obers, Extremal black hole horizons, J. High Energy Phys. 03 (2018) 099.
[39] J. Armas, A. Jain, and R. Lier, Approximate symmetries, pseudo-Goldstones, and the second law of thermodynamics, Phys. Rev. D **108**, 086011 (2023).
[40] J. Armas and A. Jain, Approximate higher-form symmetries, topological defects, and dynamical phase transitions, Phys. Rev. D **109**, 045019 (2024).
[41] K. Jensen, M. Kaminski, P. Kovtun, R. Meyer, A. Ritz, and A. Yarom, Towards hydrodynamics without an entropy current, Phys. Rev. Lett. **109**, 101601 (2012).
[42] N. Banerjee, J. Bhattacharya, S. Bhattacharyya, S. Jain, S. Minwalla, and T. Sharma, Constraints on fluid dynamics from equilibrium partition functions, J. High Energy Phys. 09 (2012) 046.
[43] K. Jensen, R. Loganayagam, and A. Yarom, Anomaly inflow and thermal equilibrium, J. High Energy Phys. 05 (2014) 134.
[44] M. Cvetic, M. Duff, P. Hoxha, J. T. Liu, H. Lü, J. Lu, R. Martinez-Acosta, C. Pope, H. Sati, and T. Tran, Embedding ads black holes in ten and eleven dimensions, Nucl. Phys. **B558**, 96 (1999).

[45] J. Erdmenger, M. Rangamani, S. Steinfurt, and H. Zeller, Hydrodynamic regimes of spinning black D3-branes, J. High Energy Phys. 02 (2015) 026.

[46] K. Jensen, R. Loganayagam, and A. Yarom, Thermodynamics, gravitational anomalies and cones, J. High Energy Phys. 02 (2013) 088.

[47] M. Ammon, S. Grieninger, J. Hernandez, M. Kaminski, R. Koirala, J. Leiber, and J. Wu, Chiral hydrodynamics in strong external magnetic fields, J. High Energy Phys. 04 (2021) 078.

[48] Y. Neiman and Y. Oz, Relativistic hydrodynamics with general anomalous charges, J. High Energy Phys. 03 (2011) 023.

[49] L. Gladden, V. Ivo, P. Kovtun, and A. O. Starinets, Instability in $\mathcal{N} = 4$ supersymmetric Yang-Mills theory at finite density, Phys. Rev. D **111**, 086030 (2025).

[50] L. F. O. Costa, J. Natário, and M. Zilhao, Spacetime dynamics of spinning particles: Exact electromagnetic analogies, Phys. Rev. D **93**, 104006 (2016).

[51] This idea is similar to the one implemented in [52].

[52] R. Casero, E. Kiritsis, and A. Paredes, Chiral symmetry breaking as open string tachyon condensation, Nucl. Phys. **B787**, 98 (2007).

[53] J. Armas, J. Gath, A. Jain, and A. V. Pedersen, Dissipative hydrodynamics with higher-form symmetry, J. High Energy Phys. 05 (2018) 192.

[54] X.-G. Huang, An introduction to relativistic spin hydrodynamics, Nucl. Sci. Tech. **36**, 208 (2025).

[55] F. Becattini, V. Chandra, L. Del Zanna, and E. Grossi, Relativistic distribution function for particles with spin at local thermodynamical equilibrium, Ann. Phys. (N.Y.) **338**, 32 (2013).

[56] W. Florkowski, B. Friman, A. Jaiswal, and E. Speranza, Relativistic fluid dynamics with spin, Phys. Rev. C **97**, 041901(R) (2018).

[57] F. Becattini, M. Buzzegoli, G. Inghirami, I. Karpenko, and A. Palermo, Local polarization and isothermal local equilibrium in relativistic heavy ion collisions, Phys. Rev. Lett. **127**, 272302 (2021).

[58] One point of departure with theories with *intrinsic* spin is that in this formulation with *transverse* spin, coupling to the sources γ_{ab} and ${\omega_a}^{ij}$ directly in (5) and not to the tangent $\partial_a X^\mu$ and normal n_i^μ vectors, does not lead to a relation between the spin chemical potential μ^{ij} and the thermal vorticity $\partial_{[a} u_{b]}$ of the fluid in equilibrium.

[59] C. Cartwright, D. Gallegos, U. Gürsoy, R. Klein, and A. Yarom, A supersymmetric spin current, J. High Energy Phys. 08 (2025) 129.

[60] J. Armas, G. Batzios, and J. P. van der Schaar, Holographic duals of the $\mathcal{N} = 1^*$ gauge theory, J. High Energy Phys. 04 (2023) 021.