

Geometric realization of stress-tensor deformed field theory

Yun-Ze Li^{1,*}, Yunfei Xie^{1,†} and Song He^{2,1,3,‡}

¹*Center for Theoretical Physics and College of Physics, Jilin University, Changchun 130012, People's Republic of China*

²*Institute of Fundamental Physics and Quantum Technology and School of Physical Science and Technology, Ningbo University, Ningbo, Zhejiang 315211, China*

³*Max Planck Institute for Gravitational Physics (Albert Einstein Institute), Am Mühlenberg 1, 14476 Golm, Germany*



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We present a semiclassical framework in which stress-tensor deformations of a quantum field theory (QFT) reorganize into a gravitational action evaluated at a metric saddle. The deformed partition function can be written as a gravitational path integral evaluated at the saddle, establishing a direct link between stress-tensor flows and gravitational dynamics. Two complementary routes arise: (i) from gravitational actions such as Einstein and Palatini, which map to stress-tensor deformations of a seed QFT; and (ii) from deformed QFTs such as generalized Nambu-Goto and $T\bar{T}$ -like deformed models, which reconstruct the corresponding gravitational actions. Finally, in a free, massive scalar theory, we show that the one-loop effective action of the nonlocal deformation contains a local curvature term; its coefficient defines an induced Newton constant at a chosen renormalization scale, thereby demonstrating a bidirectional link between stress-tensor flows and classical gravity.

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Introduction. Understanding whether and how spacetime geometry can be encoded in quantum field theory (QFT) remains a central problem in theoretical physics [1]. Thermodynamic arguments suggest that Einstein's equations may reflect equilibrium conditions of quantum matter [2,3], while holographic dualities such as anti-de Sitter/conformal field theory (AdS/CFT) [4,5] provide a framework for reconstructing spacetime from large- N QFT data under specific boundary conditions, e.g., [6]. Nonholographic proposals connecting geometric structures directly to QFT degrees of freedom [7] aim to broaden this perspective beyond fixed asymptotics.

Stress-tensor deformations provide a concrete probe of such connections. In classical gravity, $T_{\mu\nu}$ sources curvature, whereas in QFT, deformations built from $T_{\mu\nu}$ encode how geometry influences quantum matter. In two dimensions, the solvable $T\bar{T}$ deformation admits a precise geometric interpretation in terms of random geometry or topological gravity [8–13]. Higher-dimensional generalizations have been

explored [14–16], and recent works have constructed geometric realizations of stress-tensor flows without invoking dynamical gravity [17,18]. These developments, however, primarily encode classical kinematical geometric features and do not yet incorporate propagating gravitational degrees of freedom.

This paper formulates a calculable semiclassical correspondence in which the stress-tensor deformations reorganize a QFT into a gravitational action evaluated at a metric saddle, providing a geometric representation of stress-tensor deformed QFTs without claiming to derive gravity as a fundamental emergent phenomenon. Two complementary viewpoints arise. Starting from Einstein or Palatini gravity, one derives the corresponding stress-tensor deformation of the seed QFT. Conversely, starting from deformed theories such as generalized Nambu-Goto and $T\bar{T}$ -like models, one reconstructs the associated gravitational action. Beyond the classical saddle, quantum corrections to a nonlocal stress-tensor deformation of massive scalar field theory produces a linearized Einstein action, showing that the classical metric saddle is the semiclassical limit of the same deformation and ensuring consistency between the classical and quantum levels. The correspondence is semiclassical and arises from a single QFT with a nonlocal stress-tensor deformation, distinguishing it from two-sector constructions [19], metric reformulations [16,20], and flows derived from holographic renormalization [21].

* Contact author: lyz21@mails.jlu.edu.cn

† Contact author: jieyf22@mails.jlu.edu.cn

‡ Contact author: hesong@nbu.edu.cn

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Assumptions and framework. The framework relates a stress-tensor-deformed QFT to a corresponding gravitational theory through the path integral, with the deformation controlled by a parameter λ . For simplicity, we mainly focus on the following Euclidean gravitational partition function is

$$\mathcal{Z}_{\text{grav}}^{(\lambda)} = \int \mathcal{D}\psi \mathcal{D}g e^{-\hat{S}[g, \psi] - S_{\text{grav}}^{(\lambda)}[g]}, \quad (1)$$

where $\hat{S}[g, \psi]$ is the seed theory action and ψ denotes matter fields. The analysis presented in the first part of this paper is conducted within a semiclassical framework. Evaluating the metric path integral at the saddle gives

$$\begin{aligned} \mathcal{Z}_{\text{grav}}^{(\lambda)} &= \int \mathcal{D}\psi \sum_{g^*} \mathcal{Z}_{\text{loops}}^{(\lambda)}[g^*] e^{-\hat{S}[g^*, \psi] - S_{\text{grav}}^{(\lambda)}[g^*]} \\ &= \int \mathcal{D}\psi \sum_{\hat{g}} \sum_{\alpha} \mathcal{Z}_{\text{loops}}^{(\lambda)}[g_{\alpha}^*] e^{-S_{\alpha}^{(\lambda)}[\hat{g}, \psi]}, \end{aligned} \quad (2)$$

where $g_{\alpha}^* \rightarrow \hat{g}$ as $\lambda \rightarrow 0$, and $\mathcal{Z}_{\text{loops}}^{(\lambda)}[g^*]$ denotes the fluctuation (loop) contribution around each saddle $g_{\alpha}^*(\lambda)$ with $g_{\alpha}^*(\lambda)$ and $\hat{g}$ satisfying

$$\begin{aligned} \frac{\delta}{\delta g_{\mu\nu}} [\hat{S}[g, \psi] + S_{\text{grav}}^{(\lambda)}[g]]|_{g=g_{\alpha}^*} &= 0 \\ \frac{\delta}{\delta g_{\mu\nu}} [\hat{S}[g, \psi] + S_{\text{grav}}^{(\lambda=0)}[g]]|_{g=\hat{g}} &= 0. \end{aligned} \quad (3)$$

In this paper, the interpretation (2) is examined at the classical level. The classical deformed field theory action is related to the gravitational action via

$$S_{\alpha}^{(\lambda)}[\hat{g}, \psi] = \hat{S}[g_{\alpha}^*, \psi] + S_{\text{grav}}^{(\lambda)}[g_{\alpha}^*], \quad (4)$$

This framework incorporates the random geometric interpretation of two-dimensional $T\bar{T}$ deformation [12,13]. The corresponding gravitational action is $S_{\text{grav}}^{(\delta\lambda)} = (1/8\delta\lambda) \int d^2x \sqrt{\hat{g}} \epsilon^{\mu\rho} \epsilon^{\nu\sigma} (g_{\mu\nu} - \hat{g}_{\mu\nu})(g_{\rho\sigma} - \hat{g}_{\rho\sigma})$, where $\epsilon^{\mu\nu}$ is the Levi-Civita symbol, and $\hat{g}$ will eventually emerge as the background metric. By solving the equations of motion (EOM) (3), the unique saddle is given by $g_{\mu\nu}^* = \hat{g}_{\mu\nu} + 2\delta\lambda \epsilon_{\mu\rho} \epsilon_{\nu\sigma} \hat{T}^{\rho\sigma}$, where $\hat{T}^{\rho\sigma} = \frac{-2}{\sqrt{\hat{g}}} \frac{\delta \hat{S}}{\delta \hat{g}_{\rho\sigma}}$. Substituting it into (4), the first-order $T\bar{T}$ deformed action is obtained. In this letter, the aforementioned framework is employed to investigate more general gravitational actions involving the Ricci curvature and featuring a dynamical metric. The construction provides a geometric representation of stress-tensor deformed QFTs, reproducing classical gravitational dynamics at the saddle.

Effective deformed field theory action: Within the framework above, once the seed theory $\hat{S}$ and gravitational action $S_{\text{grav}}^{(\lambda)}$ are specified, the deformed QFT action $S_{\alpha}^{(\lambda)}$ can, in principle, be obtained perturbatively from (4). However, determining the metric saddle point $g_{\alpha}^*(\lambda)$ nonperturbatively is generally intractable. Thus, it is useful to define an effective deformed action (EDA) $S_{\text{EDA}}^{(\lambda)}$ on a *deformed metric*, which incorporates the saddle g_{α}^* . The deformed metric is taken as a local function of λ and g_{α}^* (more generally, a functional of g_{α}^*). [22]

A natural choice is to take the metric saddle to be the deformed metric, in which case

$$\begin{aligned} S_{\text{EDA}}^{(\lambda)}[g_{\alpha}^*, \psi] &= S_{\alpha}^{(\lambda)}[\hat{g}, \psi] \\ &= \hat{S}[g_{\alpha}^*, \psi] + \int d^d x \mathcal{L}_{\text{st}}^{(\lambda)}(g_{\alpha}^{*\mu\nu}, \hat{T}_{\mu\nu}^*), \end{aligned} \quad (5)$$

where $\mathcal{L}_{\text{st}}^{(\lambda)}$ is a local function of the seed theory stress-tensor obtained by substituting the gravitational EOM into $S_{\text{grav}}^{(\lambda)}$ and eliminating terms associated with curvature tensors. Taking the total derivative of (4) with respect to λ and using Eq. (3) yields

$$\frac{d}{d\lambda} S_{\text{EDA}}^{(\lambda)}[g_{\alpha}^*, \psi] \equiv \frac{d}{d\lambda} S_{\alpha}^{(\lambda)}[\hat{g}, \psi] = \partial_{\lambda} S_{\text{grav}}^{(\lambda)}[g_{\alpha}^*]. \quad (6)$$

This formulation provides a direct correspondence between a deformed QFT on the background $\hat{g}$ and a classical gravitational theory. The deformation parameter λ appears solely in the gravitational action, and the flow equation (6) relates the action's variation to the underlying gravitational dynamics.

Linearized Einstein gravity as stress tensor deformed theory. As a preliminary application, but not limited to, we apply the framework to linearized Einstein gravity. The metric saddle and the induced nonlocal stress-tensor deformation are derived explicitly. The starting point is the seed theory $\hat{S}[g, \psi]$ coupled to the Einstein-Hilbert action,

$$S = \hat{S}[g, \psi] + \frac{l^2}{2\lambda} \int d^d x \sqrt{g} R. \quad (7)$$

The saddle-point equation for the metric yields the Einstein field equations with the field theory stress-tensor as source,

$$R_{\mu\nu}^* - \frac{1}{2} R^* g_{\mu\nu}^* = -\lambda l^{-2} \hat{T}_{\mu\nu}^*, \quad (8)$$

where the subscript α of the metric saddle has been omitted for notational simplicity.

Consider the expansion $g = \hat{g} + \lambda h$. To keep the saddle-point fluctuation h^* of order $O(\lambda^0)$ and preserve the validity

of the perturbative organization, we take the reference background $\hat{g}_{\mu\nu}$ to satisfy the vacuum Einstein equations, as spelled out in SM A [23] (A5). By employing the standard second-order expansion of the Einstein-Hilbert action [31,32], and including the gauge-fixing and ghost terms to handle diffeomorphism redundancy with following standard procedure [33,34] for Einstein gravities without cosmological constant [35], the solution for saddle point in terms of the graviton Green's function is obtained [37], $h_{\mu\nu}^* = -2l^{-2} \int d^d y \sqrt{\hat{g}} G_{\mu\nu\rho\sigma} (\hat{T}^{\rho\sigma} - \frac{1}{d-2} \hat{T} \hat{g}^{\rho\sigma}) + O(\lambda)$, with G satisfying the equation $[-\hat{g}^{\mu\rho} \hat{g}^{\nu\sigma} \hat{\square} - \hat{R}^{\mu\nu\rho\sigma}]_x G_{\rho\sigma\alpha\beta}(x, y) = \delta^{(d)}(x - y) \delta_\alpha^\mu \delta_\beta^\nu / \sqrt{\hat{g}}$ [38]. Substituting the saddle into the action yields the leading deformation of the QFT,

$$S^{(\lambda)} = \hat{S} + \frac{\lambda}{2l^2} \int d^d x d^d y \sqrt{\hat{g}(x)\hat{g}(y)} G_{\mu\nu\rho\sigma}(x, y) \times \hat{T}^{\mu\nu}(x) \left[\hat{T}^{\rho\sigma} - \frac{1}{d-2} \hat{T} \hat{g}^{\rho\sigma} \right](y) + O(\lambda^2). \quad (9)$$

A standard de Donder gauge-fixing of the metric fluctuation (including the associated ghost term) is employed [SM A [23] (A7)]. $G_{\mu\nu\rho\sigma}(x, y)$ is the Green's function of the resulting gauge-fixed spin-2 operator on the reference background $\hat{g}_{\mu\nu}$, with boundary conditions chosen according to the manifold under consideration. At the order relevant for (9), $G_{\mu\nu\rho\sigma}$ enters only through its first part when contracted with the conserved source $\hat{T}_{\mu\nu}$; gauge-dependent longitudinal components therefore do not affect the deformation. Similar nonlocal deformations were studied in [39]. For a flat background $\hat{g}_{\mu\nu} = \eta_{\mu\nu}$, the first-order deformation simplifies to

$$S_{[1]}^{(\lambda)} = \frac{\lambda}{2l^2} \int d^d x \hat{T}_{\mu\nu} \frac{1}{-\hat{\square}} \left(\hat{T}^{\mu\nu} - \frac{1}{d-2} \hat{T} \eta^{\mu\nu} \right). \quad (10)$$

The leading deformation organizes as a spin-2 channel exchange controlled by $\hat{\square}^{-1}$, providing a concrete check against the linearized Einstein response. The resulting deformation (10) is intrinsically nonlocal on a reference background $\hat{g}_{\mu\nu}$ and reproduces the linearized Einstein response for both scalar and Maxwell theories (SM A [23]). Higher-order corrections can be computed systematically in perturbation theory (SM A [23]); nonlocality is inevitable at every order in the deformation [40]. Moreover, for Gauss-Bonnet gravity, the leading-order deformation retains the same functional form as (9), with the reference metric $\hat{g}_{\mu\nu}$ replaced by the corresponding Gauss-Bonnet background. The same saddle-point analysis continues to apply; details are presented in SM A [23].

In the formulation of effective deformed action (5), substituting (8) into (7) yields $\mathcal{L}_{\text{st}}^{(\lambda)} = \frac{\sqrt{g^*}}{d-2} \hat{T}^*$. The effective stress-tensor is obtained by taking the functional derivative

of S_{EDA} with respect to the metric, $T_{\text{EDA}}^{(\lambda)\mu\nu} = \hat{T}^{\mu\nu} - \frac{1}{d-2} \hat{T} g^{\mu\nu} - \frac{2}{d-2} \frac{\partial \hat{T}}{\partial g_{\mu\nu}}$. For a massless free scalar seed, $\hat{\mathcal{L}} = \frac{\sqrt{g}}{2} \nabla^\mu \phi \nabla_\mu \phi$, one has $g_{\mu\nu} \frac{\partial \hat{T}}{\partial g_{\mu\nu}} = \hat{T}$. The trace of the seed theory stress-tensor $\hat{T}$ can be rewritten in terms of T_{EDA} . Combining this with (6) yields the total λ -derivative of the effective field action, $\frac{d}{d\lambda} S_{\text{EDA}}^{(\lambda)}[g^*, \psi] = \frac{1}{4\lambda} \int d^d x \sqrt{g^*} T_{\text{EDA}}^{(\lambda)*}$. In four-dimensional Maxwell theory the traceless stress tensor preserves conformal invariance, and the linearized Einstein-Hilbert deformation produces the nonlocal action (9) whose surviving transverse-traceless spin-2 exchange reproduces the standard graviton-mediated backreaction, demonstrating that the metric saddle responds nonlocally to the seed stress tensor and thereby links Einstein dynamics to stress-tensor flows extendable to broader theories.

Nonminimal Palatini gravity v.s. $T\bar{T}$ -like flow. Having established the framework and its realizations in Einstein gravity, we next address the inverse problem: at the classical (saddle) level, we reconstruct nonminimal, metric-affine gravitational actions directly from specified stress-tensor deformations of a seed QFT and verify the correspondence on representative cases via their flow equations. Our discussion is based on the Palatini formalism, in which the Riemann tensor $\mathcal{R}^\mu{}_{\nu\rho\sigma}$ and the Ricci tensor $\mathcal{R}_{\mu\nu}$ are constructed with the independent connection. Starting from a specific effective deformed action defined as $S_{\text{EDA}}^{(\lambda)}[g, \psi] = \int d^d x \sqrt{g} \mathcal{B}^{(\lambda)}(g^{\mu\nu}, X_{\mu\nu}, \psi)$, where the tensor $X_{\mu\nu} = \sum_{i,j} \mathcal{G}_{ij}(\psi) \nabla_\mu \phi^{(i)} \nabla_\nu \phi^{(j)}$ and $\mathcal{G}_{ij}$ is the target space metric. There are d independent invariants constructed from $X_{\mu\nu}$, expressed as $X_n = X_{\mu_1}^{\mu_n} X_{\mu_2}^{\mu_{n-1}} \cdots X_{\mu_n}^{\mu_1} = \text{tr}(X^n)$ for $n = 1, 2, \dots, d$. The Lagrangian $\mathcal{B}^{(\lambda)}$ is a local function of X_n and $\phi^{(i)}$, and the effective stress-tensor is given by $(T_{\text{EDA}}^{(\lambda)})_\nu^\mu = 2 \sum_{n=1}^d n (X^n)_\nu^\mu \partial_{X_n} \mathcal{B}^{(\lambda)} - \mathcal{B}^{(\lambda)} \delta_\nu^\mu$.

To derive the gravitational theory corresponding to $S_{\text{EDA}}^{(\lambda)}$, we postulate a candidate gravitational action, which is then verified through its equations of motion to check if it reproduces $\mathcal{B}^{(\lambda)}$. Here, we consider a general class of actions with nonminimal couplings between the metric, curvature, and matter fields. The gravitational action [41] can be written as

$$S_{\text{grav}}^{(\lambda)}[g, \psi] = \int d^d x \sqrt{g} \mathcal{A}^{(\lambda)}(g^{\mu\nu}, \mathcal{R}_{\mu\nu}, X_{\mu\nu}, \psi), \quad (11)$$

where $\mathcal{R}_{\mu\nu}$ is the Ricci tensor in the Palatini formalism. Variations of (11) with respect to the metric and the independent connection yield

$$\frac{\partial \mathcal{A}^{(\lambda)}}{\partial g^{\mu\nu}} - \frac{1}{2} \mathcal{A}^{(\lambda)} g_{\mu\nu} = 0, \quad (12a)$$

$$\bar{\nabla}_\sigma \left(\sqrt{g} \frac{\partial \mathcal{A}^{(\lambda)}}{\partial \mathcal{R}_{\mu\nu}} \right) = 0. \quad (12b)$$

A suitable gravitational Lagrangian $\mathcal{A}^{(\lambda)}$ yields the deformed field theory Lagrangian $\mathcal{B}^{(\lambda)}$ when evaluated at the metric saddle,

$$S_{\text{grav}}^{(\lambda)}[g^*, \psi] = \int d^d x \sqrt{g^*} \mathcal{B}^{(\lambda)}(g^{*\mu\nu}, X_{\mu\nu}, \psi). \quad (13)$$

The flow equation of the deformed field theory is given by $\partial_\lambda S_{\text{EDA}}^{(\lambda)}[g^*, \psi] = \int d^d x \sqrt{g^*} \partial_\lambda \mathcal{B}^{(\lambda)}(g^{*\mu\nu}, X_{\mu\nu}, \psi)$. Different choices of $\mathcal{A}^{(\lambda)}$ lead to different metric saddles g^* and hence distinct deformed actions on the reference background. Conversely, given the deformed field theory action on a reference background, the corresponding gravitational action can be reconstructed order by order in metric perturbations. As a realization, we show in SM C [23] that the generalized Nambu-Goto action arises from a specific Palatini Lagrangian with a nonminimal coupling.

Example: $T\bar{T}$ -like deformation in d dimensions: As a significant example, the d -dimensional (root-) $T\bar{T}$ -like deformation introduced in [17] is investigated, and the corresponding gravitational action is constructed. Suppose that X_ν^μ is diagonalizable, expressed as $X = U \text{diag}(\chi_1, \chi_2, \dots, \chi_d) U^{-1}$. The effective deformed Lagrangian is presented as follows:

$$\mathcal{B}^{(\lambda)} = \mathcal{B}_0 + \lambda^{1-\Sigma} l^\Delta \prod_{j=1}^d \left(\chi_j^{\frac{p_j}{2}} - \beta_j^{\frac{p_j}{2}} \right)^{\frac{1}{p_j}}, \quad (14)$$

where $\{\beta_j\}$ is the deformation parameters of the root- $T\bar{T}$ -like operator as discussed in [17], $\{p_j\}$ are the numbers that characterizing the deformation, $\Sigma = \sum_{j=1}^d 1/p_j$, and $\Delta = (2\Sigma + d - 4)/2$. $\mathcal{B}_0$ satisfies the differential equation $2\chi_j \partial_{\chi_j} \mathcal{B}_0 - \mathcal{B}_0 = 0$ and the solution is given by [42]. The effective stress-tensor can be diagonalized by the same matrix U , and its eigenvalues $\{\tau_j\}$ are given by $\tau_j^{(\lambda)} = 2\chi_j \partial_{\chi_j} \mathcal{B}^{(\lambda)} - \mathcal{B}^{(\lambda)}$. These eigenvalues can then be substituted into (14) to derive the flow equation

$$\partial_\lambda S_{\text{EDA}}^{(\lambda)} = \frac{(1-\Sigma)}{(l^\Delta b^{\frac{1}{\Sigma-1}})} \int d^d x \sqrt{g} \left(\prod_{j=1}^d (\tau_j^{(\lambda)})^{\frac{1}{p_j}} \right)^{\frac{1}{\Sigma-1}}, \quad (15)$$

where $b = \prod_{j=1}^d \beta_j$. In particular, setting $p_1 = p_2 = \dots = p_d = p$ reduces the deformation operator to $\mathcal{O}_{\text{st}} = (\det[(T_{\text{EDA}}^{(\lambda)})_\nu^\mu])^{\frac{1}{d-p}}$ [13, 15].

Next, a class of plausible gravitational actions is postulated that could reproduce the effective deformed action given in (14). The explicit form of the gravitational

Lagrangian $\mathcal{A}^{(\lambda)}$ is introduced as follows:

$$\mathcal{A}^{(\lambda)} = \mathcal{B}_0 + \lambda^{1-\Sigma} l^\Delta \prod_{j=1}^d \left(\chi_j^{\frac{p_j}{2}} - \left(\mathfrak{p}_j \chi_j^{\mathfrak{q}_j} + \mathfrak{s}_j r_j^{\mathfrak{q}_j} + \frac{\mathfrak{q}_j - 1}{\mathfrak{q}_j} \beta_j \right)^{\frac{p_j}{2}} \right)^{\frac{1}{p_j}}, \quad (16)$$

where $\{r_j\}$ are the eigenvalues of the Ricci tensor $\mathcal{R}_\nu^\mu$, assumed to be diagonalizable. $\mathfrak{p}_j$, $\mathfrak{q}_j$, and $\mathfrak{s}_j$ are arbitrary functions independent of χ and r . The EOMs for $\{r_j\}$ follow from Eq. (12a); a full derivation is provided in SM D [23]. Substituting them into (16) recovers the effective deformed action in (14). As a significant example, the two-dimensional case of this formula is explicitly analyzed in SM D [23], which corresponds to the $T\bar{T}$ flow [8–10, 18].

Nonlocal stress-tensor deformations and classical-quantum consistency. Beyond the classical saddle analysis, we examine the nonlocal stress-tensor deformation that underlies our framework and demonstrate its consistency at both classical and quantum levels. At the classical level, eliminating the metric perturbation $h_{\mu\nu}$ from the quadratic Einstein-Hilbert action yields a universal bilocal deformation,

$$\delta_\lambda S = \frac{\lambda}{2l^2} \int d^d x d^d y \sqrt{g(x)g(y)} T^{\mu\nu}(x) H_{\mu\nu\rho\sigma}(x, y) T^{\rho\sigma}(y), \quad (17)$$

where the kernel $H_{\mu\nu\rho\sigma}(x, y) = G_{\mu\nu\rho\sigma}(x, y) - 1/(d-2) G_{\mu\nu\alpha\beta}(x, y) g^{\alpha\beta}(y) g_{\rho\sigma}(y)$ and the Green's function satisfies $[-g^{\mu\rho} g^{\nu\sigma} \square - R^{\mu\rho\nu\sigma} - R^{\nu\rho\mu\sigma}]_x G_{\rho\sigma\alpha\beta}(x, y) = \delta(x-y) \delta_\alpha^\mu \delta_\beta^\nu$. Setting $g_{\mu\nu} \rightarrow \hat{g}_{\mu\nu}$ reproduces the leading nonlocal deformation (9) in linearized Einstein gravity.

To probe whether this semiclassical reorganization is compatible with quantum, we compute the leading one-loop contribution generated by the bilocal interaction (17) using the heat-kernel expansion (SM E [23]) [44–46]. Diffeomorphism covariance forces the induced local effective action into a derivative expansion, $\delta \ln \mathcal{Z}^{(\lambda)}[g] \propto \int \sqrt{g} (\alpha_0 + \alpha_1 R + O(\nabla^4))$, where $\mathcal{Z}^{(\lambda)}[g]$ is the first-order deformed partition function, and the operator content is universal renormalization invariant, but the coefficients $\alpha_{0,1}$ depend on the UV regulator and renormalization scheme through local counterterms. We therefore interpret α_0 and α_1 as the renormalized vacuum energy and Newton coupling at a chosen reference scale, fixed by a concrete renormalization prescription (SM E [23]). [47]. Implementing this prescription in the proper time cutoff scheme yields

$$\mathcal{Z}_{\text{scalar}}^{(\lambda)} = \mathcal{Z}_0 \left(1 - \frac{l^2}{2\lambda} \int d^d x \sqrt{g} [R + O(\lambda \nabla^4)] \right), \quad (18)$$

corresponding to the Einstein–Hilbert action supplemented by higher-derivative curvature terms. The effective Newton’s constant is given by $G_{\text{eff}} = \lambda/(8\pi l^2)$. The coefficients of local invariants (e.g., $\int \sqrt{g}$, $\int \sqrt{g}R$) are renormalization-scheme dependent and can be shifted by local counterterms. Consistency of the small- λ expansion requires $g|_{\lambda=0} = \hat{g}$, where $\hat{g}$ solves the Einstein equations, ensuring that the leading term reproduces the linearized gravitational action (7).

Classical-quantum consistency. Crucially, the classical bilocal kernel (17) and the quantum effective action (18) arise from the *same* nonlocal stress-tensor deformation. Classically, eliminating the metric yields a bilocal interaction governing linearized gravitational response; quantum mechanically, loop corrections to this interaction generate a local gravitational action with the correct tensorial structure. The metric saddle appearing in the classical analysis, therefore, corresponds to the semiclassical (tree-level) limit of the quantum deformation. This demonstrates a coherent, logically self-consistent connection between classical geometric reorganization and quantum gravitational dynamics within a single framework of stress tensor deformation.

Conclusion. We have formulated a semiclassical correspondence in which stress-tensor deformations of a QFT reorganize into a gravitational action evaluated at the metric saddle. In particular, our results comprise: (i) a universal bilocal kernel equal to the inverse graviton operator reproducing linearized Einstein response; (ii) an explicit inverse construction yielding Palatini actions from specified stress-tensor deformations; and (iii) a one-loop local curvature term generated by the same deformation, linking the classical saddle to its quantum completion. The deformation is intrinsically nonlocal, governed by the inverse graviton operator and encoding massless spin-2 exchange.

This nonlocal structure lies outside the locality assumptions of the Weinberg-Witten theorem [48] and remains

compatible with linearized Einstein dynamics. The non-local terms also generate quantum corrections to the effective action, offering a controlled setting in which aspects of quantum emergent gravity arise from stress-tensor deformations. In Lorentzian applications, the inverse operator in the kernel should be understood with a causal (linear-response) prescription, while a fully nonperturbative assessment of unitarity for the resulting nonlocal deformation lies beyond the scope of the present paper and will be addressed elsewhere.

Several directions follow naturally. (i) *Quantum gravity and holography:* The induced metric flows parallel holographic renormalization [49,50], suggesting routes toward emergent de Sitter or flat-space holography [51,52] and offering a complementary perspective on information-theoretic aspects of gravity [53,54]. (ii) *Spacetime gluing:* The consistent gluing of stress-tensor deformed QFTs suggests an analog for gluing spacetimes [39,55,56], where implementing junction conditions is typically nontrivial.

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- [22] Although this rewriting amounts to a saddle-level change of variables, it is operationally nontrivial: it recasts the intrinsically bilocal deformation into a covariant derivative expansion on the deformed geometry, yielding (i) a systematic perturbative control of subleading terms and (ii) a unified language for comparison with local stress-tensor flows of $T\bar{T}$ -type. Once the Green's-function prescription is fixed, scheme-dependent local counterterms are cleanly separated from the regulator-independent nonlocal kernel.
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- [42] The solution takes the form
- $$\mathcal{B}_0 = C(\phi) \prod_{j=1}^d \chi_j^{\frac{1}{2}} = C(\phi) \frac{\sqrt{\det(X_{\mu\nu})}}{\sqrt{\det(g_{\mu\nu})}}, \quad (19)$$
- which is analogous to the action of the Nambu-Goto string [43] up to a regular function $C(\phi)$.
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