

Would quantum entanglement be increased by anti-Unruh effect?

Taotao Li and Baocheng Zhang*

School of Mathematics and Physics, China University of Geosciences, Wuhan 430074, China

Li You

*State Key Laboratory of Low Dimensional Quantum Physics, Department of Physics,
Tsinghua University, Beijing 100084, China*



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We study the “anti-Unruh effect” for an entangled quantum state in reference to the counterintuitive cooling previously pointed out for an accelerated detector coupled to the vacuum. We show that quantum entanglement for an initially entangled (spacelike separated) bipartite state can be increased when either a detector attached to one particle is accelerated or both detectors attached to the two particles are in simultaneous accelerations. However, if the two particles (e.g. detectors for the bipartite system) are not initially entangled, entanglement cannot be created by the anti-Unruh effect. Thus, within a certain parameter regime, this work shows that the anti-Unruh effect can be viewed as an amplification mechanism for quantum entanglement.

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I. INTRODUCTION

It is widely accepted by now that an observer with uniform acceleration a in the Minkowski vacuum of a free quantum field would feel a thermal bath of particles at the temperature $T = \hbar a / (2\pi c k_B)$ [1]. This effect, discovered in 1976 by Unruh, indicates that the particle content of a quantum field is observer dependent. It has been since digested and extended to many different situations (see the review [2] and references therein). In one famous application to the Unruh-DeWitt detector [3], it is found that a quantum system consisting of a detector uniformly accelerating in Minkowski vacuum can sense the thermal emission and, thus, cause decoherence due to the coupling with the thermal field.

However, it was found recently that a particle detector in uniform acceleration coupled to the vacuum can cool down with increasing acceleration *under certain conditions*. This scenario is opposite to that given the celebrated Unruh effect, and has been appropriately named the anti-Unruh effect [4]. This initial discussion was based on a pointlike two-level system, and the transition probability was found to decrease with increasing acceleration rather than the expected nominal increasing dependence. Although the initial calculation is made in Ref. [4] for accelerated detectors coupled to a massless scalar field either in a periodic cavity or under a hard-IR momentum cutoff for the continuum, it is also shown to represent a general stationary mechanism that remains stable under the disturbance of

additional conditions, instead of being a sheer transient phenomenon [5]. Thus, like the Unruh effect, the anti-Unruh effect constitutes a significant breakthrough in our understanding.

Since the anti-Unruh effect can exist under a stationary state satisfying the Kubo-Martin-Schwinger (KMS) condition [6–8] and is independent of any kind of boundary conditions [4,5], how is it different from the Unruh effect? Although the physically essential reasons remain to be explored, some important elements, like the interaction time, the detector’s energy gap, the mass of the quantum field, etc., have been discussed carefully to distinguish the two situations in the earlier works [4,5]. For example, the anti-Unruh effect could appear when the interaction time-scale is far away from the timescale associated with the reciprocal of the detector’s energy gap. It is also noted that these discussions were made under the background of the Unruh-DeWitt detector, so a necessary step is to check whether the anti-Unruh effect can also be applied in other situations, i.e. whether it has any influence on quantum entanglement, and whether the influence is the same as or different from that of the Unruh effect.

The influence of the Unruh effect on quantum entanglement has been the subject of many studies [2]. A recent study finds that a maximally entangled quantum state in an inertial frame becomes less entangled to an observer in relative acceleration [9]. This degradation of entanglement as well as the possibility of its sudden death has also been investigated for spacelike separated observers with the same acceleration [10]. The results from these studies have been further extended to different situations [11–17],

* zhangbc.zhang@yahoo.com

and they help to establish the general conclusion that entanglement is also observer dependent. It decreases for accelerating observers, or accelerating observers only have partial access to the information encoded in the quantum entanglement. However, the real reason why the inertial observers would measure more (the maximal amount of) entanglement than others remains to be fully understood, since the notion of acceleration is always relative. In particular, the possibility of enhanced entanglement for accelerating observers is not ruled out. Indeed, several studies have concluded that the Unruh effect can actually lead to enhanced quantum entanglement by coupling one or two detectors into the local quantum fields even if they were spacelike separated [18–20]. Since no local quantum operations can increase the amount of entanglement between two parties of a quantum system [21], this entanglement enhancement is speculated to be extracted from the quantum entanglement of vacuum with which the accelerated detectors interacted, by a mechanism similar to entanglement swapping [22,23]. In particular, these phenomena of enhancement of entanglement did not represent a stationary mechanism.

When we attempt to discuss the influence of anti-Unruh effect on quantum entanglement between two detectors, it is expected that the phenomena of the enhancement of entanglement could appear in a case with the stationary state but has to avoid the influence of the vacuum entanglement. So, in this paper, we take a product state for the vacuum, but the results are still valid for the general entangled vacuum state. Thus, it establishes our problem for considering whether quantum entanglement between two spacelike separated parties can be enhanced or not when the anti-Unruh effect is enforced. The answer to this question could profoundly change our understanding of the anti-Unruh effect.

This paper is organized as follows. First, in Sec. II, we review the all important decoherence factors due to the anti-Unruh effect, instead of using the transition probability in earlier studies. This is followed in Sec. III by the discussions on entanglement enhancement due to the anti-Unruh effect for several different situations, where we take a product state for the vacuum in order to avoid the confusion with the mechanism of entanglement swapping. Finally we consider the case of an initial product state in Sec. IV, which is followed by the conclusion in Sec. V.

II. THE ANTI-UNRUH EFFECT

We first briefly review the anti-Unruh effect presented in Ref. [4] with the Unruh-DeWitt (UDW) model, but with the decoherence factor instead of the initial use of the transition probability and with the massive field instead of the initial massless field. Starting with the consideration that constitutes a scalar field ϕ interacting with a pointlike two-level quantum system, or a qubit (for short). It can be easily generalized to more complex situations such as a quantum

oscillator [24] as confirmed with KMS conditions for thermal equilibrium [5]. The ground $|g\rangle$ and excited $|e\rangle$ states of the qubit are separated by an energy gap Ω while experiencing accelerated motion in a vacuum cavity. The interaction Hamiltonian for this $(1+1)$ -dimension model is given by

$$H_I = \lambda \chi(\tau/\sigma) \mu(\tau) \phi(x(\tau)), \quad (1)$$

with λ the coupling strength. τ is the qubit's proper time along its trajectory $x(\tau)$, $\mu(\tau)$ is the qubit's monopole momentum, and $\chi(\tau/\sigma)$ is a switching function that is used to control the interaction time scale σ . $\phi(x(\tau))$ is the scalar field related to the vacuum. For a qubit accelerating in a vacuum cavity, the evolution of the total quantum state is determined perturbatively by the unitary operator which up to first order is given by,

$$U = I - i \int d\tau H(\tau) + O(\lambda^2). \quad (2)$$

Within the first-order approximation and in the interaction picture, this evolution is described by [4]

$$\begin{aligned} U|g\rangle|0\rangle &= C_0(|g\rangle|0\rangle - i\eta_0|e\rangle|1_k\rangle), \\ U|e\rangle|0\rangle &= C_1(|e\rangle|0\rangle + i\eta_1|g\rangle|1_k\rangle), \end{aligned} \quad (3)$$

where k denotes the mode of the $(1+1)$ -dimension scalar field with (bosonic) annihilation (creation) operator a_k ($a_k^\dagger$), $a_k|0\rangle = 0$ and $a_k^\dagger|0\rangle = |1_k\rangle$. $\eta_0 = \lambda \int dk I_{+,k}$ and $\eta_1 = \lambda \int dk I_{-,k}$ are related to the excitation and deexcitation probability of the qubit where $I_{\pm,k}$ is given as $I_{\pm,k} = \int_{-\infty}^{\infty} \chi(\tau/\sigma) \exp[\pm i\Omega\tau + i\omega t(\tau) - ikx(\tau)] d\tau / (\sqrt{4\pi\omega})$. $t(\tau) = a^{-1} \sinh(a\tau)$ and $x(\tau) = a^{-1}(\cosh(a\tau) - 1)$ is the trajectory of the accelerating qubit with acceleration a . $C_{0,1} = 1/\sqrt{1 + \eta_{0,1}^2}$ is the state normalization factor. It is worth pointing out that the periodic boundary conditions are not used for $I_{\pm,k}$, thus this review in essence generalizes the case considered in Ref. [4]. We consider massive field with e.g. $\omega = \sqrt{k^2 + m^2}$ as in Ref. [5] so that the anti-Unruh effect discussed here will not be constrained by the finite interaction time and its validity can be extended to situations where the detector is switched on adiabatically over an infinite long time. Without loss of generality, $m = 1$ is used for all numerical calculations.

For an initial qubit state $|\psi_i\rangle = (\alpha|g\rangle + \beta|e\rangle)|0\rangle$ with the complex amplitudes α and β satisfying the normalization $|\alpha|^2 + |\beta|^2 = 1$, its evolution according to Eq. (3), after interacting with the scalar field, leads to the state

$$|\psi_f\rangle = \alpha|g\rangle|\psi_0\rangle + \beta|e\rangle|\psi_1\rangle, \quad (4)$$

with $|\psi_0\rangle = C_0|0\rangle + i(\beta/\alpha)C_1\eta_1|1_k\rangle$ and $|\psi_1\rangle = C_1|0\rangle - i(\alpha/\beta)C_0\eta_0|1_k\rangle$. The loss of coherence for the qubit is

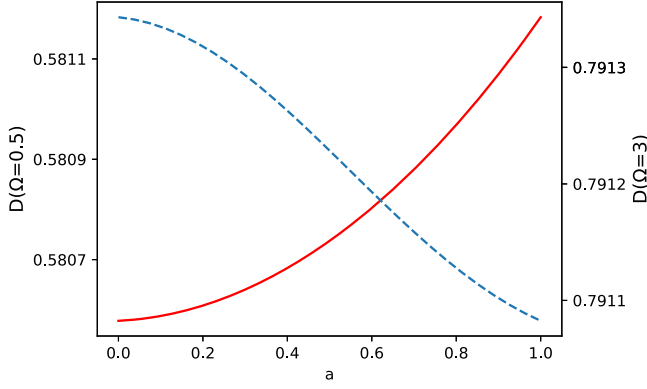


FIG. 1. The dependence of decoherence factor D on the acceleration a . The model parameters employed are $\lambda = 0.1$ and $\sigma = 0.4$. The red solid line denotes enhanced coherence with acceleration at $\Omega = 0.5$ (referenced to the left vertical axis), while the blue dashed line with respect to the right vertical axis is for decreased coherence with acceleration at $\Omega = 3$.

measured by the decoherence factor $D = |\langle \psi_0 | \psi_1 \rangle|$, which is related to the purity of the reduced qubit density matrix after evolution [25]. $D = 0$ means that the qubit state becomes completely mixed and loses coherence completely, while $D = 1$ means that the state remains pure. For any other value of D in between $\in (0, 1)$, the coherence of the qubit is partially lost. Figure 1 shows the dependence of D on a for our model with $\alpha = \beta = \sqrt{2}/2$ for the initial state. It explicitly shows that under suitable conditions, the qubit state coherence is enhanced, which implicates that accelerated motion can potentially purify a quantum state. One should of course remain cautious as in this example the initial coherence is not simply given by $D = 1$ for $a = 0$, due to the influence of the switching function [24,26]. However, the enhancement cannot be simply credited to the finite time interaction either. For significantly prolonged interaction times, this enhancement is observed to stay, however, as shown by the red solid line in Fig. 2. Although Figs. 1 and 2 correspond to different values of a and Ω because the switching function stops working when the interaction time becomes infinite, Fig. 2 firmly establishes that the anti-Unruh effect exists even for an infinite time interaction in this case.

The change to qubit coherence considered above can be extended to a many-body quantum state, e.g. in a tripartite state $|\psi_{3i}\rangle = (\alpha|g\rangle|g\rangle|g\rangle + \beta|e\rangle|e\rangle|e\rangle)$ which reduces to the famous GHZ state [27] for $\alpha = \beta = \sqrt{2}/2$. After evolution under the analogous interaction model, this state becomes $|\psi_{3f}\rangle = \alpha|g\rangle|g\rangle|g\rangle|\psi_0\rangle + \beta|e\rangle|e\rangle|e\rangle|\psi_1\rangle$, whose coherence is described by the same decoherence factor D and exhibits similar a -dependence. Furthermore, with two or three qubits (of the triqubit system) in acceleration simultaneously, the changes to the state coherence show similar behaviors which can be confirmed straightforwardly albeit after a little bit of more complicated calculations. In what follows, we will study another one question concerning the

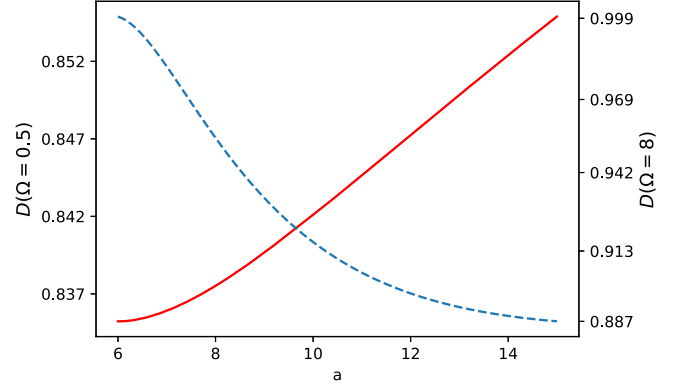


FIG. 2. The same as in Fig. 1 but for $\sigma \rightarrow \infty$. The blue dashed line now refers to $\Omega = 8$ for decreased coherence with acceleration.

change to quantum entanglement among different qubits when there are qubits coupled to the quantum vacuum. This is different from the change of the coherence which is for the whole state. We hope to understand whether this change of entanglement between two qubits has the similar phenomena to the coherence discussed here.

III. ENHANCEMENT OF ENTANGLEMENT

The previous section shows that single-qubit coherence can be enhanced by the anti-Unruh effect. We now take one step further and consider the influence of anti-Unruh effect on quantum entanglement [28]. This study is timely since entanglement is viewed as a resource for quantum information science. We will consider two causally separated qubits in an entangled state, where each qubit is independently accelerating in a vacuum cavity and assumes the same coupling with the scalar field in its respective (spatial) place by the same process presented in Eq. (3). Different from the steps reviewed above for treating the coherence factor for a single qubit, we will employ measures for quantum entanglement to discuss the influence on qubit entanglement due to the anti-Unruh effect in the following.

In the general scenario we consider, the initial two-qubit state is a pure one in flat spacetime. When one of the qubits is accelerating in a vacuum cavity, the bipartite state becomes mixed and can be measured by the decoherence factor. Concerning quantum entanglement, it is well understood for any bipartite pure state or mixed states of two qubits. There exist several established entanglement measures for mixed states of two qubits, which include the widely adopted concurrence [29], logarithmic negativity [30,31], and mutual information [21]. Bipartite mutual information is a measure of total correlation between the two subsystems of a quantum state, i.e. the sum of its quantum entanglement and classical correlation. So in this paper, we choose to adopt concurrence and logarithmic negativity as measures to the change of the bipartite entanglement due to acceleration.

The logarithmic negativity E_N is a nice measure for mixed state, although it fails to reproduce the entropy of entanglement of pure state like most other entanglement measures. It is defined as

$$E_N = \log_2(2N + 1), \quad (5)$$

and bounded by $0 \leq E_N \leq 1$, where the negativity N is computed according to $N = \sum_i (|\xi_i| - \xi_i)/2$ with ξ_i the i -th eigenvalue of the partial transpose of the bipartite state density matrix. Concurrence defined by

$$C(\rho) = \max\{0, \lambda_1 - \lambda_2 - \lambda_3 - \lambda_4\} \quad (6)$$

is also a widely used entanglement measure for bipartite mixed state, where $\lambda_1, \lambda_2, \lambda_3, \lambda_4$ are the eigenvalues of the Hermitian matrix $\sqrt{\sqrt{\rho}\tilde{\rho}\sqrt{\rho}}$ with $\tilde{\rho} = (\sigma_y \otimes \sigma_y)\rho^*$ ($\sigma_y \otimes \sigma_y$) the spin-flipped state of ρ , σ_y being the y-component Pauli matrix, and the eigenvalues listed in decreasing order.

We now consider the change to entanglement due to the anti-Unruh effect when qubits are in acceleration. The initial state is assumed to take the form

$$|\Psi_i\rangle = (\alpha|g\rangle_A|e\rangle_B + \beta|e\rangle_A|g\rangle_B)|0\rangle_A|0\rangle_B, \quad (7)$$

again with the complex coefficients satisfying $|\alpha|^2 + |\beta|^2 = 1$ and the vacuum as in a product state. Thus, the entanglement swapping between the vacuum and the entangled state of the two qubits far away from each other is excluded, and the contribution to the entanglement change due to the anti-Unruh effect can be singled out to study entanglement enhancement. In what follows, we will consider two different cases.

A. B -qubit in acceleration

Adopting the steps developed previously for a single qubit (B) accelerating in a vacuum cavity, we will treat the other qubit (A) as remaining stationary (in a vacuum cavity). The two vacuum cavities are assumed to be the same otherwise except that they are separated by a space-like distance. The accelerating B -qubit is coupled to the scalar field and evolves according to Eq. (3), the two-qubit quantum state thus becomes

$$\begin{aligned} |\Psi_f^{(A\bar{B})}\rangle &= (C_0\alpha|g\rangle_A|e\rangle_B + C_1\beta|e\rangle_A|g\rangle_B)|0\rangle_A|0\rangle_B \\ &\quad - i(C_1\alpha\eta_1|g\rangle_A|g\rangle_B + C_0\beta\eta_0|e\rangle_A|e\rangle_B)|0\rangle_A|1_k\rangle_B. \end{aligned} \quad (8)$$

Based on the understanding of the Unruh effect on a single qubit as studied earlier, the coupling with the scalar field clearly is capable of causing decoherence to the bipartite entangled state [9], consistent with the general influence of

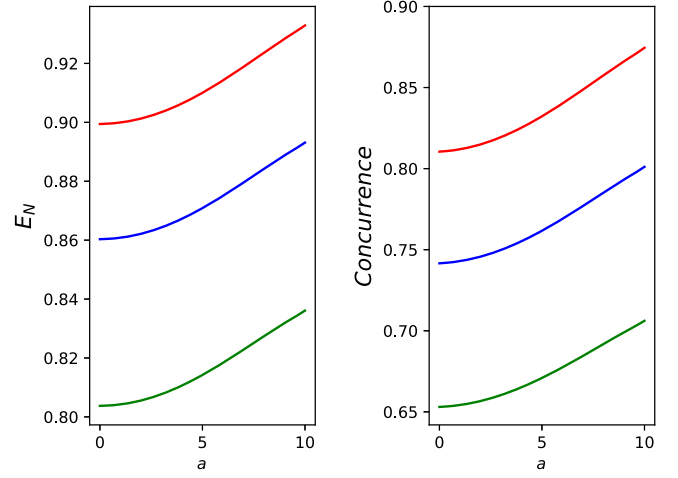


FIG. 3. The negativity (left panel) and the concurrence (right panel) as a function of a when one of the qubits is in acceleration. The parameters correspond to the cases of enhanced entanglement with $\lambda = 0.1$, $\sigma = 0.4$, and $\Omega = 0.5$. The different lines refer to different initial states: the red line for $\alpha = 1/\sqrt{2}$, the blue line for $\alpha = 1/\sqrt{3}$, and the green line for $\alpha = 1/2$.

environment on a quantum state [32]. On the other hand, if the single-qubit coherence can be enhanced by the anti-Unruh effect, one cannot be prevented from speculating that it is also possible to observe enhanced entanglement between the two qubits for an accelerating observer accompanying the accelerating B qubit. This is easily checked. We compute the logarithmic negativity according to Eq. (5) and concurrence given by Eq. (6) for the final bipartite quantum state, or the reduced density matrix ρ_{AB} obtained by tracing out the scalar field. The results are presented in Fig. 3 for different initial states. We see that when the interaction timescale σ differs significantly from the timescale associated with the detector gap $\sim \Omega^{-1}$, enhanced entanglement between the two qubits indeed appears, due to the anti-Unruh effect. In particular, this result also indicates that the phenomena of entanglement enhancement is independent of the exact form of the initial entanglement state, as selected results plotted illustrate different choices of $\alpha = 1/\sqrt{2}$, $1/\sqrt{3}$, and $1/2$.

One can simply check if the particle exchange symmetry remains true or not with the accelerated particle labeled by A instead. In contrast to the case considered above with the particle B being accelerated as in Fig. 3, in this case, the final two-qubit state does become slightly different although the final conclusion remains the same as before because the reduced two-qubit state observes particle exchange symmetry. For the symmetric initial state with $\alpha = \beta = 1/\sqrt{2}$, everything is symmetric during all intermediate steps. For other cases presented in Fig. 3, both the logarithmic negativity and concurrence are employed to check their respective dependence with the same parameters σ and Ω . The results are consistent with

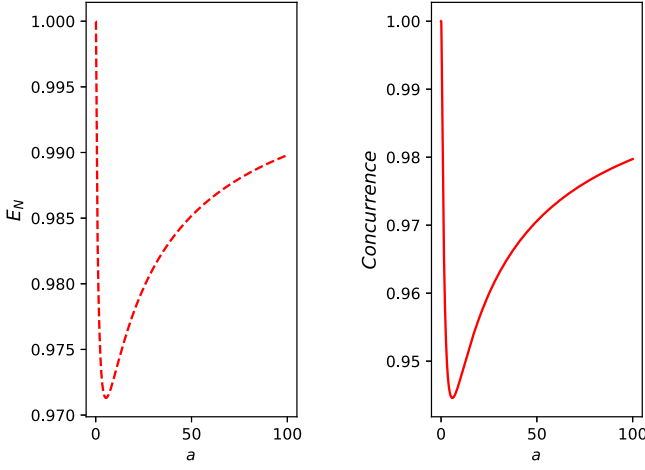


FIG. 4. The same as in Fig. 3 for the initial state corresponding to its red line but with the interaction time prolonged to infinity.

qubit exchange symmetry. In fact, it is easy to check that the acceleration of either qubit results in the same change to the bipartite entanglement, since the crucial elements η_0 and η_1 , which determine this change, appear in the same places in the reduced density matrix ρ_{AB} , although the constant parameters α and β have interchanged their roles in the reduced density matrix obtained from tracing out the scalar field modes from the final state.

An entangled two-qubit state, thus, cannot hold onto maximal entanglement when any one of the two qubits is accelerating. This can be understood from the view point of entanglement monogamy. If entanglement between two qubits reaches the maximum, then neither of the two qubits can be entangled with any third party unless the entanglement between the initial two qubits decreases. When any one qubit accelerates, it will interact with the vacuum. The entanglement between the accelerated qubit and the vacuum develops, and the qubit-qubit entanglement decreases. A subsequent question then arises: when the initial bipartite state is maximally entangled, how could its entanglement increased further by the anti-Unruh effect? In general, due to the presence of the switching function, for finite interaction times, even entanglement at $a = 0$ could change, e.g. become nonmaximal for the initial maximally entangled state (see the red line in Fig. 3). This leaves open the possibility for the discussed enhancement of entanglement. However, the effect of the switching function disappears when the interaction becomes infinite. Therefore, for an initial maximally entangled state, it seems that the anti-Unruh effect cannot enhance its entanglement. The real answer, of course, is more complicated than this. As seen in Fig. 4, the entanglement decreases rapidly at the beginning of the acceleration, but it starts to increase again with the larger acceleration later. Therefore, the anti-Unruh effect actually still works for infinite interaction time although only for a finite range of accelerations.

B. Two qubits with the same acceleration

The previous subsection considers the case of a two-qubit system when one of the qubits is in acceleration. It is found that the two-qubit entanglement can be either increased or decreased, depending on the different conditions. We now consider for some fixed conditions, e.g. those giving rise to enhanced entanglement when one of the qubits is in acceleration. What happens to the qubit-qubit entanglement if the other qubit assumes the same acceleration? This is an interesting question, since the follow-up observers can remain stationary, with respect to the two qubits simultaneously. For the Unruh effect, the answer to the above question is a simple resounding negative, because the acceleration causes only decoherence, as presented in Ref. [10]. But in the presence of the anti-Unruh effect, the situation becomes subtle and deserves a careful study.

In this case with the two qubits sharing the same acceleration, the initially entangled state would evolve into

$$\begin{aligned} |\Psi_f^{(2)}\rangle = & C_0 C_1 [(\alpha|g\rangle_A|e\rangle_B + \beta|e\rangle_A|g\rangle_B)|0\rangle_A|0\rangle_B \\ & - i(\alpha\eta_1|g\rangle_A|g\rangle_B + \beta\eta_0|e\rangle_A|e\rangle_B)|0\rangle_A|1_m\rangle_B \\ & - i(\beta\eta_1|g\rangle_A|g\rangle_B + \alpha\eta_0|e\rangle_A|e\rangle_B)|1_m\rangle_A|0\rangle_B \\ & + (\alpha\eta_0\eta_1|e\rangle_A|g\rangle_B + \beta\eta_0\eta_1|g\rangle_A|e\rangle_B)|1_m\rangle_A|1_m\rangle_B]. \end{aligned} \quad (9)$$

The logarithmic negativity and concurrence for the reduced density matrix ρ_{AB} can be analogously computed by first tracing out the scalar field modes. The results are presented in Fig. 5, which are largely similar to the results obtained in Fig. 3, and entanglement enhancement would occur which is seen to be independent of the initial entangled state chosen in this case.

Moreover, when the two qubits assume different accelerations, the final entangled state is found to take a similar form to that for two qubits with the same acceleration,

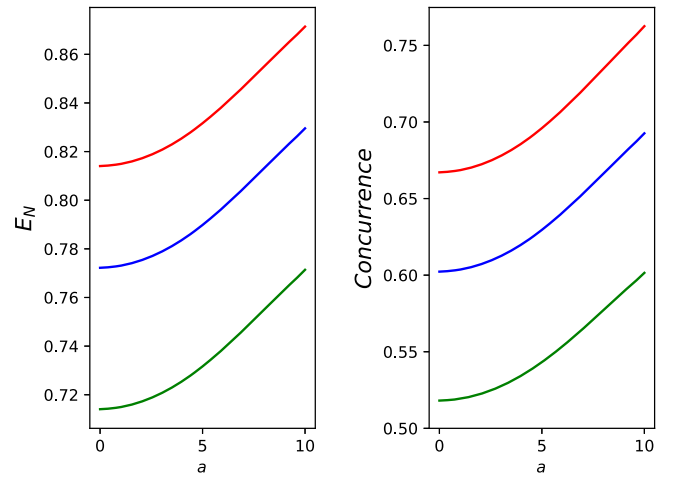


FIG. 5. The same as in Fig. 3 but for both qubits in simultaneous acceleration. The parameters correspond to enhanced entanglement at $\lambda = 0.1$, $\sigma = 0.4$, and $\Omega = 0.5$.

although the exact values for η_0 and η_1 depend on these accelerations. The same procedures as employed in the above can be used to compute the logarithmic negativity and concurrence for the reduced density matrix, and analogous figures can be obtained by fixing the acceleration for one qubit while varying the acceleration for the other qubit. Since nothing particularly new arises for this case, the detailed results will not be presented here.

IV. TWO-QUBIT PRODUCT STATE

Our discussions for the two-qubit example in the above section show that qubit-qubit entanglement can be increased when the anti-Unruh effect is taken into consideration. This section intends to show that the mechanism behind enhanced entanglement is more like an entanglement amplifier [33] when the two qubits share the same acceleration. It is not an entanglement generator since the initial two-qubit entanglement must be nonzero. This prompts us to study the simple question if one can make two qubits, which possess no correlation initially and are separated far away from each other, to become correlated when one or both of the two qubits accelerate? For this example, we take the initial state with $\alpha = 1$ and $\beta = 0$ from Eq. (7) and first consider the situation when the B qubit is in acceleration. Like the case considered earlier, the state then becomes

$$|\Psi_f^{(B)'}\rangle = C_1(|g\rangle_A|e\rangle_B|0\rangle_A|0\rangle_B - i\eta_1|g\rangle_A|g\rangle_B|0\rangle_A|1_k\rangle_B). \quad (10)$$

Tracing out the scalar field modes, we obtain the final state for the two qubits,

$$\begin{aligned} \rho'_{AB} &= |g\rangle_A\langle g|_A \otimes |C_1|^2(|e\rangle_B\langle e|_B + \eta_1^2|g\rangle_B\langle g|_B) \\ &= \rho_A \otimes \rho_B, \end{aligned} \quad (11)$$

obviously this is still a product state. The same result arises if qubit A is assumed to be in acceleration instead. Thus entanglement creation from accelerating one qubit is impossible.

What happens when both qubits are accelerated simultaneously? For simplicity, we still consider the same acceleration for the two qubits, and the initial product state $|\Psi_i'\rangle = |g\rangle_A|e\rangle_B|0\rangle_A|0\rangle_B$ then evolves into

$$\begin{aligned} |\Psi_f'\rangle &= C_0C_1(|g\rangle_A|e\rangle_B|0\rangle_A|0\rangle_B - i\eta_1|g\rangle_A|g\rangle_B|0\rangle_A|1_m\rangle_B \\ &\quad - i\eta_0|e\rangle_A|e\rangle_B|1_m\rangle_A|0\rangle_B - \eta_0\eta_1|e\rangle_A|g\rangle_B|1_m\rangle_A|1_m\rangle_B). \end{aligned} \quad (12)$$

Again, the reduced two-qubit density matrix is obtained by tracing out the scalar field modes, and we obtain

$$\rho'_{AB} = \rho_A \otimes \rho_B; \quad (13)$$

i.e. no entanglement is created.

Based on the perturbation treatment for the two-qubit evolution dynamics with accelerations, we thus find

entanglement cannot be created for an initial product state, but can be enhanced for any initial state with entanglement through the anti-Unruh effect. We believe that the main reason for this conclusion lies at our treatment of the vacuum as a product state. If the two qubits are also in a product state, they cannot influence each other when one or both qubits interact with the vacuum through accelerated motion since no channel can be used to transfer the influence. On the other hand, however, as discussed before [18–20], if the vacuum state is entangled, the initial product state for the two qubits can become entangled since the vacuum can provide the channel to facilitate the interchange of their respective information and coherence. When the initial two-qubit state is entangled, even if the vacuum state is not, the two qubits can still interchange their respective change in the coherence through the channel facilitated by their initial entanglement. When the two-qubit dynamics are the same, the change of their respective coherence will also be the same. Thus, the change of the two-qubit entanglement is essentially the same as the change of the single-qubit coherence, both can be derived from the interaction between the accelerated qubit(s) and the vacuum. This shows that the enhancement of the entanglement is indeed caused by the anti-Unruh effect recently discussed.

V. CONCLUSION

In this paper, we revisit the anti-Unruh effect for the single-qubit state from the change of the decoherence factor due to acceleration. We extend such a discussion to the two-qubit state and show that the enhanced entanglement between the two qubits can occur aided by the anti-Unruh effect, not only for the case when one of the qubits is in acceleration, but also for the case when both qubits are accelerating simultaneously. To avoid any possible misunderstanding that enhanced enhancement is derived from the exchange with the vacuum entanglement, we always model the vacuum in our calculation as in a product state. Thus, when the initial bipartite state is also taken as a product state, the acceleration of the qubit(s) cannot create any entanglement between the two qubits, even though the anti-Unruh effect remains present. This is reasonable, since the local effect caused by acceleration also requires the channel to transfer the influence to the place where the other qubit is located, no matter whether this channel is formed by the entanglement of the bipartite state itself or through the vacuum entanglement. Thus, we conclude that the anti-Unruh effect can only lead to entanglement enhancement for an initially entangled state, but cannot create entanglement out of a nonentangled state.

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