

Primordial equation of state transitions

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We revisit the physics of transitions from a general equation of state parameter to the final stage of slow-roll inflation. We show that it is unlikely for the modes comprising the cosmic microwave background to contain imprints from a preinflationary equation of state transition and still be consistent with observations. We accomplish this by considering observational consistency bounds on the amplitude of excitations resulting from such a transition. As a result, the physics which initially led to inflation likely cannot be probed with observations of the cosmic microwave background. Furthermore, we show that it is unlikely that equation of state transitions may explain the observed low multipole power suppression anomaly.

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I. INTRODUCTION

The paradigm of inflation provides a suitable framework for understanding the observed spectrum of cosmological perturbations [1]. Many questions remain regarding the origin of inflation [2–4] and its duration [5,6]. It has been proposed that the transition to inflation may explain observed anomalies in the cosmic microwave background (CMB) if the duration of inflation is not too long [7].

This paper revisits the generation of spectrum excitations due to the transition from an arbitrary equation of state parameter (w) to inflation. We emphasize the importance of using the proper matching conditions across the transition and show that previous studies have drawn incorrect conclusions when using improper matching conditions. By combining observational consistency bounds on the excitation amplitude with the proper matching conditions, we show that the cosmic microwave background likely does not contain imprints from the preinflationary Universe. Our study emphasizes three points:

- (1) Observation strongly bounds the amplitude of excitation.
- (2) The fractional change in w must be small if a w transition is observed.
- (3) It is unlikely that transitions explain the ($20 \lesssim l \lesssim 30$) anomaly.

The first point has been made before [8–11], but a more general formulation of the bound is presented in Sec. II. In Sec. III we analytically compute the excited spectrum for the case of an instant transition, allowing us to bound the pretransition equation of state parameter. In Sec. IV we discuss the excited spectrum for the case of a gradual transition which is modeled by a hyperbolic tangent function. In Sec. V we address the low power anomaly occurring for multipoles $20 \lesssim l \lesssim 30$ and show that the type

of models we discuss cannot explain the anomaly. In Sec. VI we conclude.

II. SINGLE FIELD INFLATION

A. Overview

In this section we review the basic equations of inflationary theory, emphasizing the appearance of $\dot{\epsilon}$ terms which play an important role in studying the enhanced spectrum that results from equation of state transitions. The simplest theory for inflation that is consistent with observational data is a minimally coupled scalar field with a Friedmann-Robertson-Walker metric [12],

$$S = \int \sqrt{-g} \left[\frac{1}{2} M_P^2 \dot{\phi}^2 - V \right],$$

$$ds^2 = -dt^2 + a^2 dx^2. \quad (2.1)$$

The Einstein field equations may be combined to obtain an expression relating the rate of Hubble parameter change directly to the sum of the energy density and the pressure of the contents in the Universe,

$$2M_P^2 \dot{H} = -(\rho + P). \quad (2.2)$$

For a single scalar field with negligible gradients we may write $\rho = K + V$ and $P = K - V$. We have introduced the kinetic energy of the inflaton field, $K = \dot{\phi}^2/2$. It is convenient to introduce three dimensionless parameters,

$$\epsilon_H = -\frac{\dot{H}}{H^2}, \quad \epsilon = 3\frac{K}{\rho} \quad \text{and} \quad w = \frac{P}{\rho} = -1 + \frac{2}{3}\epsilon. \quad (2.3)$$

It is clear from (2.2) that $\epsilon_H = \epsilon$. In order to obtain an accelerating geometry we require that the parameter ϵ_H be smaller than unity,

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$$\ddot{a} = aH^2(1 - \epsilon_H) > 0 \quad \text{if } \epsilon_H < 1. \quad (2.4)$$

Cosmological observables are obtained by computing correlation functions of gauge invariant fluctuations. The action for the comoving curvature perturbation is given by

$$S_{\mathcal{R}} = -M_P^2 \int \sqrt{-g} \epsilon (\partial \mathcal{R})^2. \quad (2.5)$$

The fields may be Fourier decomposed as

$$\begin{aligned} \hat{\mathcal{R}} &= \int \frac{d^3k}{(2\pi)^3} \hat{\mathcal{R}}_{\vec{k}} e^{i\vec{k}\cdot\vec{x}}, \\ \hat{\mathcal{R}}_{\vec{k}} &= \mathcal{R}_k \hat{a}_k^\dagger + \mathcal{R}_k^* \hat{a}_{-\vec{k}}, \\ [\hat{a}_k, \hat{a}_{k'}^\dagger] &= (2\pi)^3 \delta^3(\vec{k} - \vec{k}'). \end{aligned} \quad (2.6)$$

The resulting equation of motion for the mode function is¹

$$\ddot{\mathcal{R}}_k + \left(3H + \frac{\dot{\epsilon}}{\epsilon}\right) \dot{\mathcal{R}}_k + \frac{k^2}{a^2} \mathcal{R}_k = 0. \quad (2.7)$$

For the case of quasi-de Sitter expansion ($w \approx -1$, $\epsilon \ll 1$), the solution of lowest energy density is given by the Bunch-Davies solution

$$\mathcal{R}_{k,BD} = \frac{1}{\sqrt{2\epsilon}} \frac{H}{M_P \sqrt{2k^3}} \left(1 + i \frac{k}{aH}\right) e^{-ik/aH}. \quad (2.8)$$

The scalar power spectrum has been measured to great accuracy, while the scalar bispectrum and tensor power spectrum have not yet been detected. The scalar power spectrum is typically parametrized by an amplitude A_S and a scale dependence n_s ,

$$\begin{aligned} \langle \hat{\mathcal{R}}_{\vec{k}} \hat{\mathcal{R}}_{\vec{k}'} \rangle &= (2\pi)^3 \delta^3(\vec{k} + \vec{k}') P_{\mathcal{R}}(k), \\ \Delta_{\mathcal{R}}^2(k) &= \frac{k^3}{2\pi^2} P_{\mathcal{R}}(k) = A_S \left(\frac{k}{k_*}\right)^{n_s-1+\mathcal{O}(dn_s/dk)}. \end{aligned} \quad (2.9)$$

The observationally obtained values for A_S and n_s are given in Table I.

The predictions for the Bunch-Davies mode functions are

$$A_S = \frac{1}{8\pi^2 \epsilon} \frac{H^2}{M_P^2}, \quad n_s = 1 + 2\eta - 4\epsilon, \quad \eta = -\frac{\ddot{\phi}}{H\dot{\phi}}. \quad (2.10)$$

¹The absence of a friction term proportional to $\dot{\epsilon}$ in the equation of motion for the tensor mode functions is the reason tensor modes are not enhanced for equation of state transitions in the way that scalars are enhanced.

TABLE I. Cosmological parameters as measured by Planck TT + TE + EE + lowP [1,13–15]. The pivot scale used by the Planck experiment is $k_* = 0.05 \text{ Mpc}^{-1}$.

Scalar power spectrum amplitude	Scalar power spectrum tilt
$\ln(10^{10} A_S) = 3.094 \pm 0.034(1\sigma)$	$n_s = 0.9645 \pm 0.0049(1\sigma)$

B. Observables for general Bogoliubov parameters

The Bunch-Davies state is the solution of lowest energy density in quasi-de Sitter expansion. However, in general, the solution does not need to be the solution of lowest energy density. We write excited solutions as a Bogoliubov transformation of the Bunch-Davies solution

$$\begin{aligned} \mathcal{R}_{k,\text{excited}} &= \alpha_k \mathcal{R}_{k,BD} + \beta_k \mathcal{R}_{k,BD}^* \quad \text{and} \\ \hat{a}_{k,\text{excited}}^\dagger &= \alpha_k^* \hat{a}_{k,BD}^\dagger - \beta_k^* \hat{a}_{-\vec{k},BD}. \end{aligned} \quad (2.11)$$

Satisfying the canonical commutation relations requires that $|\alpha_k|^2 - |\beta_k|^2 = 1$.

Excited states change the cosmological parameters. In terms of the expressions corresponding to the Bunch-Davies solution previously discussed, the new parameter expressions are given by

$$A_S = A_{S,BD} |\alpha_k + \beta_k|^2, \quad n_s = n_{s,BD} + \frac{d \ln |\alpha_k + \beta_k|^2}{d \ln k}. \quad (2.12)$$

C. Bounds on excitation amplitude

We present the bounds arising from backreaction considerations for different functional forms of Bogoliubov excitations, extending the results of [9–11] to other functional forms, to show that the limits are similar. Bounds that arise from measurements of n_s and the observational limits on $f_{\text{NL}}^{\text{loc}}$ turn out to be weaker than the bounds that are obtained from backreaction considerations.

1. Bounds from backreaction considerations

The scalar power spectrum has been measured to deviate only slightly from scale invariance. This implies that the modes comprising the observable cosmic microwave background should not vary dramatically in amplitude across the approximately 3–4 decades of modes we observe today. Therefore, the modes we observe today should either be excited modes or Bunch-Davies modes. We compute the bounds on excitation parameters implied by this.

In order for all of the modes comprising the CMB to have exited the horizon during inflation, the highest- l mode must be at least $n \approx 3\text{--}4$ [16–18] decades shorter wavelength than the horizon size at the beginning of inflation. To obtain the most conservative bound from backreaction considerations we assume that modes with higher momentum than

the observable CMB modes are not excited. The highest momentum excited mode therefore satisfies

$$p_{UV} \geq 10^n H. \quad (2.13)$$

The energy density stored in the fluctuations due to excitations is obtained using adiabatic subtraction [19,20] and is given by

$$\langle \rho_{\mathcal{R}} \rangle = M_P^2 \epsilon \int_0^\infty \frac{d^3 k}{(2\pi)^3} \left[|\dot{\mathcal{R}}_{k,\text{excited}}|^2 - |\dot{\mathcal{R}}_{k,\text{BD}}|^2 + \frac{k^2}{a^2} (|\mathcal{R}_{k,\text{excited}}|^2 - |\mathcal{R}_{k,\text{BD}}|^2) \right]. \quad (2.14)$$

We consider different functional forms of scale dependence for the Bogoliubov parameter $\beta_k = |\beta|f_k$ shown in Table II.

The energy density stored in the fluctuations should remain subdominant to the kinetic energy of the inflaton field, $\epsilon M_P^2 H^2$, in order for $\epsilon_H = \epsilon_\phi$ to remain true. This backreaction bound may be written as

$$\langle \rho_{\mathcal{R}} \rangle < \epsilon M_P^2 H^2 = \frac{H^4}{8\pi^2 A_S} |\alpha_k + \beta_k|^2. \quad (2.15)$$

It is convenient to introduce a parameter which is the coefficient ratio for the leading energy density terms in Table II,

$$c_f = \begin{cases} 1 & f_k = \Theta(k_{UV} - k) \\ (2/3) & f_k = \exp[-k/k_{UV}] \\ 2 & f_k = \exp[-(k/k_{UV})^2]. \end{cases} \quad (2.16)$$

The backreaction bound gives us

$$p_{UV} \lesssim \frac{H |\alpha_k + \beta_k|^{1/2}}{A_S^{1/4} |\beta|^{1/2}} \times c_f^{1/4}. \quad (2.17)$$

Recalling that $|\alpha_k|^2 = 1 + |\beta_k|^2$, using (2.13) and (2.17) we obtain an upper bound for $|\beta|$ given by

TABLE II. The energy density for the different functional forms of $\beta_k = |\beta|f_k$ that are considered. The first form is that used in [8–11]. As emphasized in [21], the oscillations resulting from instant transitions are very rapid, so the effective β_k seen by experiments would appear nearly scale invariant. Bounds on the Gaussian form were considered in [22–25].

f_k	$\langle \rho_{\mathcal{R}} \rangle$
$\Theta(k_{UV} - k)$	$\frac{ \beta ^2}{8\pi^2} (p_{UV}^4 + H^2 p_{UV}^2)$
$\exp[-k/k_{UV}]$	$\frac{ \beta ^2}{16\pi^2} (3p_{UV}^4 + H^2 p_{UV}^2)$
$\exp[-(k/k_{UV})^2]$	$\frac{ \beta ^2}{16\pi^2} (p_{UV}^4 + H^2 p_{UV}^2)$

TABLE III. Bounds on $\beta_k = |\beta|f_k$ and $|\alpha_k + \beta_k|$ arising from backreaction considerations. We have taken $n = 3$ and $k = k_{UV}$ for the numerical estimates.

f_k	$ \beta $ UB	$ \alpha_k + \beta_k $ LB	$ \alpha_k + \beta_k $ UB
$\Theta(k_{UV} - k)$	0.022	0.97	1.022
$\exp[-k/k_{UV}]$	0.018	0.99	1.006
$\exp[-(k/k_{UV})^2]$	0.030	0.99	1.011

$$|\beta| \lesssim \frac{10^{-2n} A_S^{1/2} c_f^{1/2}}{\sqrt{1 - 2 \times 10^{-2n} A_S^{-1/2} c_f^{1/2} f_k}}. \quad (2.18)$$

We provide the numerical evaluation of the $|\beta|$ upper bound and the corresponding bounds on $|\alpha_k + \beta_k|$ in Table III.

2. Special case: Only $l \lesssim 30$ modes excited

For the lowest l cosmic variance limited modes ($l \lesssim 30$), one may not use scale invariance to bound the excitation amplitude. Instead, we compare the theoretical prediction with the observational error bar width [13] to obtain a conservative estimate of $0.5 \lesssim |\alpha_k + \beta_k|^2 \lesssim 2$.

III. EXCITATION MECHANISM: INSTANT TRANSITION

Although unrealistic, the idealized case of an instantaneous transition from one equation of state parameter value to a different value is useful since it allows for an analytical calculation of the excited spectrum. In the next section, the more realistic case of a gradual transition will be discussed. Figure 1 illustrates the transition for several different values of the equation of state parameter prior to the transition, w_0 .

A. Matching conditions

The instant transitions that we are considering effectively have a discontinuity in the slow-roll parameter ϵ . Therefore,

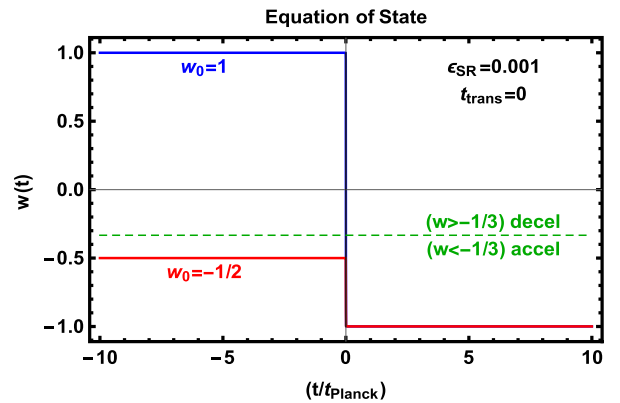


FIG. 1. Examples of instantaneous transitions. Though not explicitly plotted, we also have studied the transition which increases the equation of state parameter (see Table V).

one must be careful when determining which quantities related to $\mathcal{R}$ are continuous since the evolution of $\mathcal{R}$ depends explicitly on $\dot{\epsilon}$ according to (2.7). It was emphasized in [21,26] that the proper matching conditions are given by

$$[\mathcal{R}_k]_{\pm} = 0, \quad [\epsilon \dot{\mathcal{R}}_k]_{\pm} = 0, \quad (3.1)$$

where we have used the notation that $[\dots]_{\pm}$ denotes the change in a quantity across the transition. The origin of these conditions may be easily seen from the differential equation for scalar fluctuations (2.7), which can be rewritten as

$$\frac{d}{dt}(a^3 \epsilon \dot{\mathcal{R}}_k) = -k^2 a \epsilon \mathcal{R}_k. \quad (3.2)$$

The fluctuation mode function itself, $\mathcal{R}_k$, is continuous. If we note that $\epsilon = d(1/H)/dt$, we may time integrate both sides of the equation close to the transition to obtain the continuity condition on the derivative of the mode function,

$$[a^3 \epsilon \dot{\mathcal{R}}_k]_{\pm} = \lim_{\delta \rightarrow 0} \int_{(\Delta t)_t = -\delta}^{(\Delta t)_t = \delta} dt (-k^2 a \epsilon \mathcal{R}_k) = \left[-k^2 \frac{a}{H} \mathcal{R}_k \right]_{\pm} = 0. \quad (3.3)$$

We have introduced the notation $(\Delta t)_t = (t - t_{\text{trans}})$ to denote that time difference between the cosmic time and the time of transition.

Microscopically, the inflaton field will take on a uniform value² at the transition time $(\Delta t)_t = 0$. This implies, in the language of [21,26], that the transition is characterized by a spacetime hypersurface of constant field value, which directly yields (3.1). This is in contrast to the examples studied in [26] in which the transition was characterized by a surface of constant energy density; hence, the uniform density gauge is more appropriate since $\delta\rho = 0$ in that gauge. The continuity conditions (3.1) have been verified numerically by time evolving (2.7).

B. Observables: Allowed parameter space

Based on the matching conditions previously discussed, we would like to solve the following system of equations:

$$\begin{aligned} (\Delta t)_t = 0: & \alpha_{0,k} \mathcal{R}_{0,k} + \beta_{0,k} \mathcal{R}_{0,k}^* \\ & = \alpha_{f,k} \mathcal{R}_{\text{SR},k} + \beta_{f,k} \mathcal{R}_{\text{SR},k}^*, \\ (\Delta t)_t = 0: & \epsilon_0 (\alpha_{0,k} \dot{\mathcal{R}}_{0,k} + \beta_{0,k} \dot{\mathcal{R}}_{0,k}^*) \\ & = \epsilon_{\text{SR}} (\alpha_{f,k} \dot{\mathcal{R}}_{\text{SR},k} + \beta_{f,k} \dot{\mathcal{R}}_{\text{SR},k}^*). \end{aligned} \quad (3.4)$$

The explicit forms of the functions $\mathcal{R}_{0,k}$ and $\mathcal{R}_{\text{SR},k}$ are given in (3.8). The coefficients $\{\alpha_{0,k}, \beta_{0,k}\}$ account for the fact

that the spectrum may be excited prior to the transition to slow-roll inflation. The case of the lowest energy density vacuum state transitioning to inflation is given by the choice $\alpha_{0,k} = 1$ and $\beta_{0,k} = 0$.

In order to solve (3.4), we must find a solution to the scalar fluctuation mode equation (2.7) which is properly normalized. The normalization condition is given by the canonical commutation relation, which may be rewritten as a condition on the Wronskian of the scalar fluctuation mode function as follows:

$$2a^3 M_P^2 \epsilon (\dot{\mathcal{R}}_k \mathcal{R}_k^* - \mathcal{R}_k \dot{\mathcal{R}}_k^*) = i. \quad (3.5)$$

The background geometry evolution is given by

$$\begin{aligned} \epsilon &= \begin{cases} \epsilon_0 & (\Delta t)_t < 0 \\ \epsilon_{\text{SR}} & (\Delta t)_t > 0, \end{cases} \quad H(t) = H_t [1 + \epsilon H_t (\Delta t)_t]^{-1}, \\ a(t) &= a_t \left(\frac{H_t}{H(t)} \right)^{1/\epsilon}. \end{aligned} \quad (3.6)$$

We have defined H_t and a_t as the Hubble parameter and scale factor at the time of transition, $(\Delta t)_t = 0$. It is convenient to introduce the variables

$$\begin{aligned} (\Delta t)_t < 0: & \tilde{\tau}_0 = \int \frac{dt}{a} = \frac{1}{(-1 + \epsilon_0)} \frac{1}{a(t) H(t)}, \\ (\Delta t)_t > 0: & \tilde{\tau}_{\text{SR}} = \int \frac{dt}{a} = \frac{1}{(-1 + \epsilon_{\text{SR}})} \frac{1}{a(t) H(t)}. \end{aligned} \quad (3.7)$$

The properly normalized solution to the scalar equation of motion (2.7) for a constant equation of state is given as³

$$\begin{aligned} (\Delta t)_t < 0: & \mathcal{R}_{0,k}(t) = \sqrt{\frac{\pi}{2^3 \epsilon_0 M_P a(t)}} \sqrt{|\tilde{\tau}_0|} H_{\nu_0}^2[k|\tilde{\tau}_0|], \\ \nu_0 &= \frac{3(1 - w_0)}{2(1 + 3w_0)}, \\ (\Delta t)_t > 0: & \mathcal{R}_{\text{SR},k}(t) = \sqrt{\frac{\pi}{2^3 \epsilon_{\text{SR}} M_P a(t)}} \sqrt{|\tilde{\tau}_{\text{SR}}|} H_{\nu_{\text{SR}}}^2[k|\tilde{\tau}_{\text{SR}}|], \\ \nu_{\text{SR}} &= \frac{3(1 - w_{\text{SR}})}{2(1 + 3w_{\text{SR}})}. \end{aligned} \quad (3.8)$$

Employing the matching conditions (3.4) we are able to write the Bogoliubov parameters for an arbitrary choice of w_0 and initial excitat ion parameters α_0 and β_0 as

$$\begin{aligned} \alpha_{f,k} &= \sqrt{\frac{\epsilon_0}{\epsilon_{\text{SR}}}} \sqrt{\left| \frac{\epsilon_{\text{SR}} - 1}{\epsilon_0 - 1} \right|} \frac{\alpha_{\text{num}}}{\alpha_{\text{denom}}}, \\ \beta_{f,k} &= \alpha_{f,k} \quad \text{with} \quad H_{f(\nu_{\text{SR}})}^1 \leftrightarrow H_{f(\nu_{\text{SR}})}^2, \end{aligned} \quad (3.9)$$

²Note that $\delta\phi = 0$ in the comoving gauge [12].

³Note that $H_{\nu}^{1,2}(x \rightarrow \infty) \rightarrow e^{\pm ix}$.

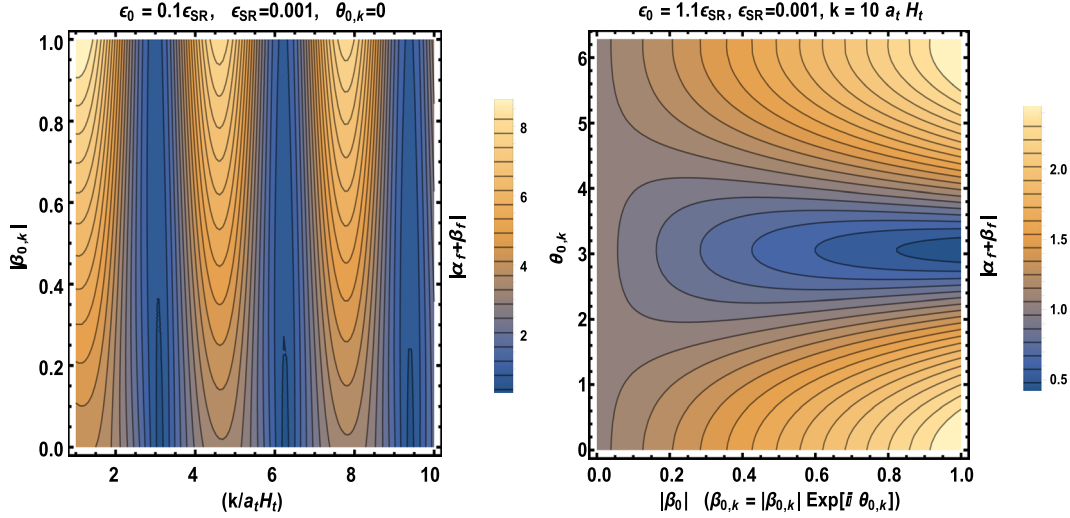


FIG. 2. Amplitude of excitations $|\alpha_{f,k} + \beta_{f,k}|$ after transition for an initially excited state with $\beta_{0,k} \neq 0$. We have taken the form of the excitation prior to the transition to be $\beta_{0,k} = |\beta_{0,k}| \exp(i\theta_{0,k})$ for arbitrary $\theta_{0,k}$ in the right plot and $\theta_{0,k} = 0$ in the left plot. These plots indicate that a larger $|\beta_{0,k}|$ typically leads to a larger deviation of $|\alpha_{f,k} + \beta_{f,k}|$ from unity. This justifies our choice of $\beta_{0,k} = 0$ to derive our bounds presented in Table IV.

$$\alpha_{\text{num}} = \left(\begin{aligned} & s_{\tau_0} H_{\nu_{\text{SR}}}^1(x_{\text{SR}}) \{ \alpha_0 [H_{\nu_0-1}^2(x_0) - H_{\nu_0+1}^2(x_0)] + \beta_0 [H_{\nu_0-1}^1(x_0) - H_{\nu_0+1}^1(x_0)] \} \\ & + [\alpha_0 H_{\nu_0}^2(x_0) + \beta_0 H_{\nu_0}^1(x_0)] \left\{ \frac{\epsilon_{\text{SR}}}{\epsilon_0} [H_{\nu_{\text{SR}}-1}^1(x_{\text{SR}}) - H_{\nu_{\text{SR}}+1}^1(x_{\text{SR}})] \right. \\ & \left. + \frac{a_t H_t}{k} H_{\nu_{\text{SR}}}^1(x_{\text{SR}}) \left[-2 \left(1 - \frac{\epsilon_{\text{SR}}}{\epsilon_0} \right) + s_{\tau_0} |\epsilon_0 - 1| + \frac{\epsilon_{\text{SR}}}{\epsilon_0} |\epsilon_{\text{SR}} - 1| \right] \right\} \end{aligned} \right), \quad (3.10)$$

$$\alpha_{\text{denom}} = \left(\begin{aligned} & H_{\nu_{\text{SR}}}^1(x_{\text{SR}}) [H_{\nu_{\text{SR}}+1}^2(x_{\text{SR}}) - H_{\nu_{\text{SR}}-1}^2(x_{\text{SR}})] \\ & - H_{\nu_{\text{SR}}}^2(x_{\text{SR}}) [H_{\nu_{\text{SR}}+1}^1(x_{\text{SR}}) - H_{\nu_{\text{SR}}-1}^1(x_{\text{SR}})] \end{aligned} \right). \quad (3.11)$$

Here we have defined $x_0 = |\epsilon_0 - 1|^{-1} (k/a_t H_t)$, $x_{\text{SR}} = |\epsilon_{\text{SR}} - 1|^{-1} (k/a_t H_t)$ and $s_{\tau_0} = \text{sign}(\tau_0)$. The expression for the Bogoliubov beta parameter after transition, $\beta_{f,k}$, is equal to the expression for $\alpha_{f,k}$ with the Hankel function “kind” changed for every Hankel function which depends on ν_{SR} .

A special case of interest is a transition for which $\alpha_0 = 1$ and $\beta_0 = 0$, corresponding to a transition from the state of lowest energy density prior to the transition. We will concentrate on this case for the remainder of the paper until Sec. V. The Bogoliubov parameters are oscillatory in nature, but in practice the oscillations cannot be resolved experimentally, and it is appropriate to approximate $\beta_{f,k} \approx \beta_f$.

To clarify what we mean when we state that the oscillations cannot be resolved experimentally, we compare the scale of oscillations to the binning scale used by the Planck experiment [14]. The baseline Plik likelihood bin sizes are $\Delta l = 1$ for $l < 30$, $\Delta l = 5$ for $30 \leq l \leq 99$, $\Delta l = 9$ for $100 \leq l \leq 1503$, $\Delta l = 17$ for $1504 \leq l \leq 2013$

and $\Delta l = 33$ for $2014 \leq l \leq 2508$. The oscillations in $\alpha_{f,k}$ and $\beta_{f,k}$ are controlled by $k\tau_{0,t}$, which results in several oscillations in a $\Delta l = 1$ window.

Having computed the excitation spectrum that results from an instant transition, we can translate the bounds obtained in the previous section into bounds on w_0 .

From our strongest bounds on $|\beta_f|$ summarized in Table III and Sec. II C 2, we may tabulate the largest allowed w_0 for a given ϵ_{SR} . Consider the effect of $\beta_0 = |\beta_0| \exp(i\theta_0)$. We see from Fig. 2 that $|\alpha_f + \beta_f|$ is maximal for $\{|\beta_0|, \theta_0\} = \{\text{large}, 0\}$ and minimal for $\{|\beta_0|, \theta_0\} = \{\text{large}, \pi\}$ for the parameter values specified on the plots. Since we have identified both an upper and lower bound on $|\alpha_f + \beta_f|$, we take the intermediate case of $\alpha_0 = 1$ and $\beta_0 = 0$. In Table IV we present the bounds on the fractional change of w and ϵ .

If the transition to inflation is well approximated as an instantaneous transition with an initial w_0 larger than is stated in Table IV, the modes which are excited cannot comprise our observable CMB. Note that modes which are super-Planckian at the time of transition should be described by the Bunch-Davies vacuum when their momenta redshift to become sub-Planckian in order for the stress-energy tensor to be renormalizable [21].

TABLE IV. The maximal allowed ϵ_0 for a given $\epsilon_{\text{SR}} < \epsilon_0$. We have reported the bounds for $\epsilon_{\text{SR}} = 10^{-3}$, though we have confirmed that the bound on the fractional change of ϵ is not numerically sensitive to this input. We have chosen $\alpha_0 = 1$ and $\beta_0 = 0$.

Observed multipoles excited	Relevant bound	$100 \frac{w_0 - w_{\text{SR}}}{w_{\text{SR}}} $	$100 \frac{\epsilon_0 - \epsilon_{\text{SR}}}{\epsilon_{\text{SR}}} $
$l \lesssim 30$	$0.71 \lesssim \alpha_{f,k} + \beta_{f,k} \lesssim 1.41$	$\lesssim 0.07$	$\lesssim 94$
$l > 30$	$0.97 < \alpha_{f,k} + \beta_{f,k} < 1.022$	$\lesssim 0.003$	$\lesssim 4.3$

TABLE V. The maximal allowed ϵ_0 for a given $\epsilon_{\text{SR}} > \epsilon_0$. We have reported the bounds for $\epsilon_{\text{SR}} = 10^{-3}$, though we have confirmed that the bound on the fractional change of ϵ is not numerically sensitive to this input. We have chosen $\alpha_0 = 1$ and $\beta_0 = 0$.

Observed multipoles excited	Relevant bound	$100 \frac{w_0 - w_{\text{SR}}}{w_{\text{SR}}} $	$100 \frac{\epsilon_0 - \epsilon_{\text{SR}}}{\epsilon_{\text{SR}}} $
$l \lesssim 30$	$0.71 \lesssim \alpha_{f,k} + \beta_{f,k} \lesssim 1.41$	$\lesssim 0.03$	$\lesssim 43$
$l > 30$	$0.97 < \alpha_{f,k} + \beta_{f,k} < 1.022$	$\lesssim 0.003$	$\lesssim 3.3$

We note, for completeness, that the bounds on the fractional change in ϵ are more restrictive if we consider a transition in which ϵ increases. The bounds are explicitly given in Table V. In Fig. 5 we demonstrate how the morphology of the spectrum changes depending on whether the step in ϵ is to a smaller or larger ϵ .

C. Comparison with previous work

There have been many previous studies analyzing equation of state transitions in single field inflation⁴ [6,7,9,21,29–32]. Some recent examples with which we could easily compare our matching criteria are [6,7,29–31]. The matching conditions used in those studies do not agree with the matching conditions presented in Eq. (3.1). We also note that studies which numerically evolve the Mukhanov-Sasaki equation without making approximations for $\{\dot{z}, \ddot{z}\}$ should yield the correct result if a proper step size is chosen so that the transition is sampled.

One of our conclusions is that only transitions from one inflationary phase to another are allowed to be imprinted on the observable CMB. A special case which has been studied previously includes steps in the inflationary potential which are modeled by a hyperbolic tangent of the field value [32]. We find that even for the most violent case of an instant transition with a step size $|\Delta V|/V_0 = \epsilon_0/3$, the fractional change in ϵ almost satisfies our least restrictive bound presented in Table V. To see this explicitly, note that the initial kinetic energy is given by $K_0 = |\Delta V|$ and therefore $K_f = 2K_0$ by energy conservation. The fractional change in $\epsilon \approx 3K/V$ is given by

$$\epsilon_0 = \frac{1}{2} \frac{\epsilon_{\text{SR}}}{(1 + \epsilon_{\text{SR}}/6)} \approx \frac{1}{2} \epsilon_{\text{SR}}, \quad \left| \frac{\epsilon_0 - \epsilon_{\text{SR}}}{\epsilon_{\text{SR}}} \right| \approx 0.5. \quad (3.12)$$

IV. EXCITATION MECHANISM: GRADUAL TRANSITION

A. Transition model

Our parametrization is of the form

$$w(t) = w_0 + \frac{1}{2} (w_{\text{SR}} - w_0) (1 + \tanh[\sigma(\Delta t)_t]),$$

$$w_{\text{SR}} = -1 + \frac{2}{3} \epsilon_{\text{SR}}. \quad (4.1)$$

Figure 3 illustrates the transition for two different values of w_0 and σ . The slow-roll parameter is explicitly given by

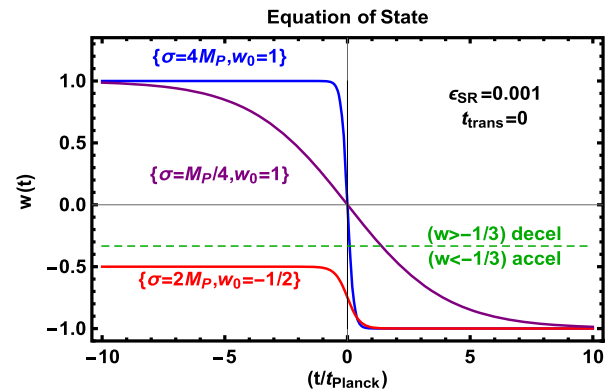


FIG. 3. The equation of state as a function of the cosmic time for gradual transitions parametrized by (4.1). Though not explicitly plotted, we also have studied transitions which increase the equation of state parameter (see Fig. 5).

⁴One may also consider equation of state transitions with multiple fields, such as discussed in the model of Cascade inflation [27,28], though these models are beyond the scope of this study.

$$\begin{aligned}\epsilon(t) &= \frac{3}{2}(1+w) \\ &= \frac{1}{2}(\epsilon_{SR} + \epsilon_0 + (\epsilon_{SR} - \epsilon_0) \tanh[\sigma(\Delta t)_t]).\end{aligned}\quad (4.2)$$

The sharpness of the transition is set by σ , (4.1). As we discuss below, a mode whose momentum is well below σ experiences a sudden transition, while a mode whose momenta is much greater than σ experiences an adiabatic transition.

B. Observables: Allowed parameter space

The gradual transition has three cases for modes depending on whether a mode experiences the transition as sudden, adiabatic or an intermediate case between the two. For an example of these three regimes, see Fig. 4. We

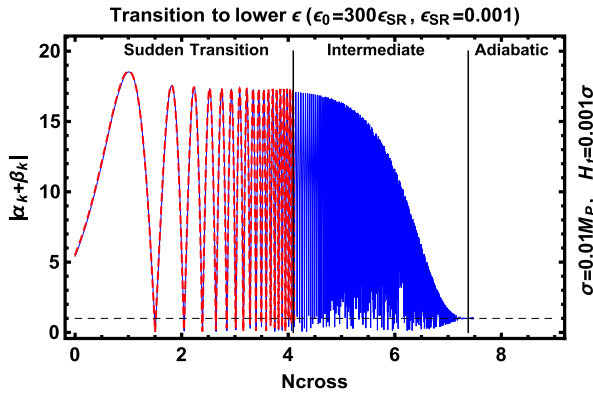


FIG. 4. A comparison of the spectrum enhancement obtained from the numerical solution (solid blue line) and the analytical formula (dashed red line) obtained from Eq. (3.9). The parameter $N_{\text{cross}} = \log(k/a_t H_t)$ is the number of e -folds from the time of the transition until the mode exits the horizon. For the sake of clarity, we end the analytically obtained line at 4 e -folds. The numerical solution stops at slightly over 7.5 e -folds. The separators between the three regimes of modes are approximately placed.

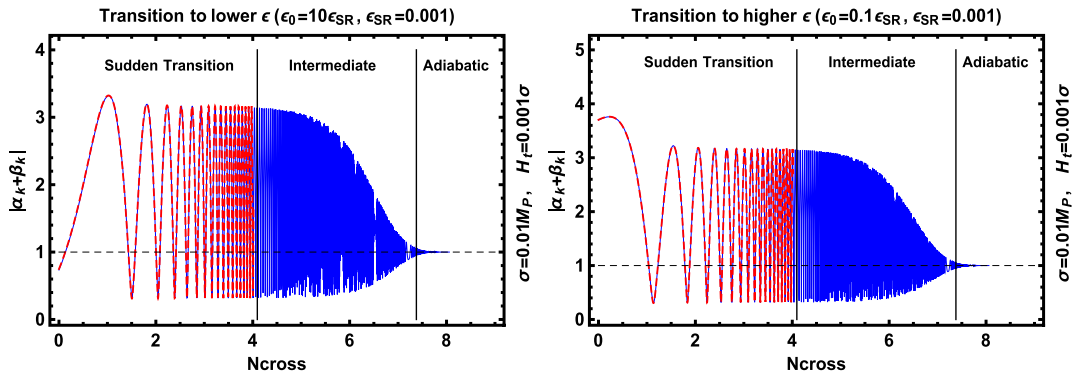


FIG. 5. Spectrum morphology comparison for a transition to a higher ϵ versus a transition to a lower ϵ . Note that a transition with $\epsilon_0 > \epsilon_{SR}$ tends to reach its first peak for higher momenta than the case of $\epsilon_0 < \epsilon_{SR}$. The separators between the three regimes of modes are approximately placed.

have explicitly compared the spectrum morphology for a case of transitioning to a lower ϵ to the case of transitioning to a higher ϵ in Fig. 5.

Comparing the $\dot{\epsilon}$ friction term and the frequency term in the equation of motion (2.7) provides an intuition for the three cases. This is most easily done by rescaling the curvature perturbation in order to eliminate the friction term altogether. The appropriate rescaling is given by

$$\mathcal{R}_k = (a^3 \epsilon)^{-1/2} \bar{\mathcal{R}}_k. \quad (4.3)$$

The resulting rescaled equation of motion is given by

$$\begin{aligned}\ddot{\bar{\mathcal{R}}}_k + \bar{\omega}_k^2 \bar{\mathcal{R}}_k &= 0, \\ \bar{\omega}_k^2 &= \left[\left(\frac{k}{a} \right)^2 - \left(\frac{9}{4} H^2 + \frac{3}{2} H \frac{\dot{\epsilon}}{\epsilon} - \frac{1}{4} \frac{\dot{\epsilon}^2}{\epsilon^2} \right) - \frac{1}{2} \left(3 \dot{H} + \frac{\ddot{\epsilon}}{\epsilon} \right) \right].\end{aligned}\quad (4.4)$$

Modes for which the term proportional to k dominates the effective frequency $\bar{\omega}_k$ tend to be adiabatic since they satisfy $|\dot{\bar{\omega}}_k/\bar{\omega}_k^2| \ll 1$ during the transition. Likewise, modes which satisfy $|\dot{\bar{\omega}}_k/\bar{\omega}_k^2| \gg 1$ during the transition tend to experience a sudden transition. We summarize these behaviors in Table VI.

From Figure 4 there are clearly two distinct cases in which we may observe excited modes and still obtain a nearly scale invariant spectrum:

- (1) We observe sudden/intermediate modes with an amplitude close to the maximum amplitude, in which case the fractional change in ϵ is strongly bounded (see Table IV).
- (2) We observe only intermediate modes at the low amplitude tail of the spectrum, in which case the fractional change in ϵ may have been large but the modes with a large amplitude are hidden outside of our horizon.

The second case may allow for large fractional change in ϵ compared to the bounds presented in Table IV, but it

TABLE VI. Three cases for a given momentum mode depending on how deep inside of the horizon it is during the transition.

Condition	Cases
$ \dot{\omega}_k/\bar{\omega}_k^2 \gg 1$	Sudden transition
$ \dot{\omega}_k/\bar{\omega}_k^2 \ll 1$	Intermediate modes
	Adiabatic

requires fine-tuning to ensure that the large amplitude modes are not observed. The fine-tuning becomes more concerning as the fractional change in ϵ increases because difference in amplitude between the hidden modes and the visible modes increases dramatically. Moreover, it is not clear if we would be able to infer ϵ_0 from the observation of the low amplitude tail modes.

V. IMPLICATIONS FOR LOW MULTIPOLE SCALAR POWER SPECTRUM SUPPRESSION

Observationally there is a suppression of power for multipoles of $20 \lesssim l \lesssim 30$ [14]. A brief discussion of the history associated with discovering and modeling this anomaly is contained in [6]. The observed power suppression is approximately given by

$$\frac{\Delta_{\mathcal{R}}^2|_{\text{expected } 20 < l < 30}}{\Delta_{\mathcal{R}}^2|_{\text{actual } 20 < l < 30}} \approx \frac{600}{1000} = 60\%. \quad (5.1)$$

To suppress the scalar power spectrum on large scales, one would need the scale dependence of $|\alpha_k + \beta_k|$ to suppress power for the relevant multipoles.

Based on our discussion in the previous section, there are two possibilities:

- (1) Sudden transition modes comprise the entire CMB, in which case the bounds from Table IV hold.
- (2) Only the lowest l modes are excited.

For the first case the largest relative suppression that may be obtained is

$$\frac{|\alpha_k + \beta_k|_{LB}^2}{|\alpha_k + \beta_k|_{UB}^2} = \left(\frac{0.97}{1.022}\right)^2 = 90\%. \quad (5.2)$$

This is an insufficient amount of suppression.

For the second case we note that the envelope of the excited spectrum typically monotonically decays from a large excitation amplitude to a smaller excitation amplitude

as is depicted in Fig. 4. Since the modes between $l \approx 10$ and $l \approx 20$ do not show a power suppression to the same extent that the $20 \lesssim l \lesssim 30$ modes do, we do not think that the transitions which we have studied are good candidates for explaining the power suppression anomaly. It may be possible to finely tune the pretransition excitation parameter $\beta_{0,k}$ (see Fig. 2) to obtain the desired spectrum, but it is not obvious what mechanism could give rise to such a selected excitation.

VI. CONCLUSIONS

We have revisited the physics of early Universe transitions to slow-roll inflation. The proper matching conditions must be used when determining the spectrum of excited fluctuations across an equation of state transition. A careful numerical study of the problem agrees with matching $\{\mathcal{R}, \epsilon\dot{\mathcal{R}}\}$ as opposed to $\{\mathcal{R}, \dot{\mathcal{R}}\}$. There are three regimes present in a gradual transition: modes which experience a sudden transition, adiabatic modes and modes which interpolate between those two regimes, which we call intermediate modes.

If the modes comprising the visible CMB contain imprints of the transition, we have shown that the pretransition Universe must likewise be an inflationary period. The only exception is if the cosmic variance limited modes are comprised of intermediate modes generated by a transition from a large w_0 . We have also argued that it is very unlikely that equation of state transitions can explain the low multipole power suppression observed in the CMB since it requires a very localized excitation in momentum space prior to the transition.

Our results state that the physics which preceded inflation is not likely to be imprinted on the observable CMB. This is a discouraging result from the perspective of using CMB observations to gain insight into the earliest stages of our Universe. However, it is encouraging since it allows us to interpret cosmological observations in the context of inflationary cosmology without having to worry about potential ambiguities introduced by preinflationary physics.

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