

# Search for $J^{PC} = 1^{-+}$ exotic state in $e^+e^-$ annihilation

Qian Wang\*

*Institut für Kernphysik, Institute for Advanced Simulation and Jülich Center for Hadron Physics,  
Forschungszentrum Jülich, D-52425 Jülich, Germany  
(Received 17 March 2014; published 10 June 2014)*

The  $P$ -wave and  $S$ -wave heavy-light mesons and their charge conjugates, i.e.,  $D_1(2420)\bar{D} + \text{c.c.}$ ,  $D_1(2420)\bar{D}^* + \text{c.c.}$ , and  $D_2(2460)\bar{D}^* + \text{c.c.}$ , can couple to states with vector quantum number  $J^{PC} = 1^{--}$  and exotic quantum number  $J^{PC} = 1^{-+}$  in a relative  $S$  wave. Near threshold, the heavy-light meson pair may form hadronic molecules due to the strong  $S$ -wave coupling, and the mysterious vector state  $Y(4260)$  could be such a state of the  $D_1(2420)\bar{D} + \text{c.c.}$  molecule. This implies the possible existence of its conjugate partner made of the same heavy-light mesons but with exotic quantum number  $J^{PC} = 1^{-+}$ . We evaluate the production rate of such exotic hadronic molecules and propose a direct experimental search for them in  $e^+e^-$  annihilation. Confirmation of such exotic states in experiment will certainly deepen our insights into strong QCD and the arrangement of multiquark degrees of freedom.

DOI: 10.1103/PhysRevD.89.114013

PACS numbers: 14.40.Rt, 13.20.Gd, 13.75.Lb

## I. INTRODUCTION

QCD allows for much richer and more complicated multiquark systems—namely, the “exotic” hadrons—beyond the simple quark model where mesons and baryons are color singlets made of  $q\bar{q}$  and  $qqq$ , respectively. This reflects the complexity related to structure formation in the strong interactions, which is still an unsolved question in fundamental science. Because of this, the discovery of exotic hadrons will apparently extend our knowledge of the strong force that binds the matter in our Universe.

Since the observation of  $X(3872)$  [1], there has been important experimental progress in the search for exotic hadrons. In particular, there have been more than 20 candidates for exotic hadrons observed in experiment which are tentatively named as “XYZ” states due to their unclear properties or the difficulties of accommodating them in the simple quark model (see, e.g., Refs. [2,3] for reviews of the “XYZ” states). Recently, the Belle Collaboration reported observations of charged bottomonium states,  $Z_b^{(\prime)\pm}$  [4,5], and last year BESIII reported their analogues in the charm sector, i.e., the charged charmonium states  $Z_c^{(\prime)\pm}$  [6–9]. The BESIII observation of the  $Z_c^\pm(3900)$  was soon confirmed by the Belle Collaboration [7] and a reanalysis based on the CLEO-c data [10].

The lowest open-charm thresholds in the vector sector with  $J^{PC} = 1^{--}$  are created by the  $S$ -wave heavy-light meson pairs, i.e.,  $D(D^*)$  and  $\bar{D}(\bar{D}^*)$ , in a  $P$  wave. Since it is much harder to form a  $P$ -wave molecule than an  $S$ -wave one, the study of the  $S$ -wave vector molecules with  $J^{PC} = 1^{--}$  would involve higher partial-wave charmed mesons, and one notices that the first  $S$ -wave open-charm threshold is the  $D_1D$ ,<sup>1</sup> which is located at 4.29 GeV and is close to the state  $Y(4260)$ . Eventually, due to the recent observation of

$Z_c(3900)$ , the role of the  $S$ -wave open-charm thresholds in vector-meson production via the  $e^+e^-$  annihilations has been carefully studied [11–13]. As pointed out in these works, the  $Y(4260)$  can be a manifestation of the  $S$ -wave  $D_1D$  threshold as its wave function might be dominated by a molecular  $D_1D$  component [11–13]. Near threshold, the  $S$ -wave production of a pair of heavy-light mesons should be more important than the  $P$ -wave production.

In the heavy-quark limit, the heavy-light mesons can be classified by their light-quark degrees of freedom, i.e., the total angular momentum  $s_l = l \pm \frac{1}{2}$  of the light quark with respect to the heavy quark. As a result, both  $s_l$  and the heavy-quark spin are conserved. So the  $S$ -wave heavy-light mesons ( $D, D^*$ ) can be denoted as a doublet  $\frac{1}{2}^-$ . Similarly, the  $P$ -wave heavy-light mesons can be classified into  $\frac{1}{2}^+$  and  $\frac{3}{2}^+$  multiplets according to their light degrees of freedom. Since the latter has a long enough lifetime to be viewed as an effective degree of freedom, we only consider the  $\frac{3}{2}^+$  multiplet which corresponds to  $(D_1(2420), D_2(2460))$  in the calculation. Although the production of  $\frac{3}{2}^+ + \text{anti-}\frac{1}{2}^-$  heavy-light meson pairs is suppressed in the heavy-quark limit [14], the heavy-quark spin-symmetry breaking effects in the charm sector can be significant, as discussed in Ref. [11]. The molecular prescription of the  $Y(4260)$  is consistent with its properties observed in experiment so far and provides a natural explanation for the production of the  $Z_c(3900)$ . Recognizing the importance of the  $S$ -wave thresholds, we will demonstrate in this work that the same  $S$ -wave thresholds can lead to the formation of exotic quantum numbers, i.e.,  $J^{PC} = 1^{-+}$ , based on the heavy-quark spin symmetry, and this eventually opens a doorway to a

\*q.wang@fz-juelich.de

<sup>1</sup>Here and in the following,  $D_1D$ ,  $D_1D^*$ , and  $D_2D^*$  mean  $D_1\bar{D} + \text{c.c.}$ ,  $D_1\bar{D}^* + \text{c.c.}$ , and  $D_2\bar{D}^* + \text{c.c.}$ , respectively.

dynamical study of hadronic molecules with exotic quantum numbers.

In what follows, we focus on the production of molecules with quantum number  $1^{--}$  composed of a  $P$ -wave heavy-light meson of  $\frac{3}{2}^+$  and an  $S$ -wave anti- $\frac{1}{2}^-$  meson. We analyze their correlation with the  $1^{--}$  molecular partners in nonrelativistic effective field theory and investigate the electromagnetic transitions from their  $1^{--}$  partners. This suggests a direct search for their signals at  $e^+e^-$  colliders. The details of the framework (Sec. II), the results and a discussion (Sec. III), and a summary (Sec. IV) will be presented in the following.

## II. THE FRAMEWORK

As studied in Ref. [13], the recently observed  $Z_c(3900)$  [6,7] can be explained naturally in the  $D_1D$  molecular picture for the  $Y(4260)$ , since a large number of  $D\bar{D}^* + \text{c.c.}$  meson pairs can be produced via  $D_1 \rightarrow D^*\pi$ . In a further study of the  $Y(4260)$  line shape in Ref. [15], we extended this scenario to include the  $D_1D$ ,  $D_1D^*$  and  $D_2D^*$  thresholds to explain its line shapes in the  $J/\psi\pi\pi$  and  $h_c\pi\pi$  channels. It means that if  $Y(4260)$  is the  $D_1D$  vector molecule, it is also possible that the other  $D_1D^*$  and  $D_2D^*$  vector molecules exist. In this work, these three vector molecules are denoted by  $Y$ ,  $Y'$ , and  $Y''$ , respectively. The Lagrangian for these three vector molecules to couple to their components reads

$$\begin{aligned} \mathcal{L}_Y = & \frac{y}{\sqrt{2}} Y^i (D_1^{i\dagger} \bar{D}^\dagger - D^\dagger \bar{D}_1^{i\dagger}) \\ & + i \frac{y'}{\sqrt{2}} \epsilon^{ijk} Y^i (\bar{D}_1^{k\dagger} D^{*j\dagger} - D_1^{k\dagger} \bar{D}^{*j\dagger}) \\ & + \frac{y''}{\sqrt{2}} Y''^i (\bar{D}_2^{ij\dagger} D^{*j\dagger} - D_2^{ij\dagger} \bar{D}^{*j\dagger}) + \text{H.c.}, \end{aligned} \quad (1)$$

where  $i, j, k$  are the indices for spin. This then implies the possible existence of the  $1^{--}$  molecules—denoted as  $X$ ,  $X'$ , and  $X''$ —as the charge-conjugate partners of the  $Y$ ,  $Y'$ , and  $Y''$ , respectively, with the relevant Lagrangian

$$\begin{aligned} \mathcal{L}_X = & \frac{x}{\sqrt{2}} X^i (D_1^{i\dagger} \bar{D}^\dagger + D^\dagger \bar{D}_1^{i\dagger}) \\ & + i \frac{x'}{\sqrt{2}} \epsilon^{ijk} X^i (\bar{D}_1^{k\dagger} D^{*j\dagger} + D_1^{k\dagger} \bar{D}^{*j\dagger}) \\ & + \frac{x''}{\sqrt{2}} X''^i (\bar{D}_2^{ij\dagger} D^{*j\dagger} + D_2^{ij\dagger} \bar{D}^{*j\dagger}) + \text{H.c.} \end{aligned} \quad (2)$$

The above couplings and their relative phases should, in principle, be determined by detailed dynamics. At this moment, since we lack such constraints on the couplings we then assume that these three states,  $X$ ,  $X'$ , and  $X''$ , are dominated by the  $D_1D$ ,  $D_1D^*$ , and  $D_2D^*$  components, respectively. Nevertheless, we expect that their masses are

not far away from their respective thresholds which would allow for the implementation of the Weinberg criterion [16–19]. It gives the probability  $1 - Z$  to find a bound state in the physical wave function, where  $Z \equiv [1 - \text{Re}(\Pi'(M^2))]$  is the renormalization constant with  $\Pi'(M^2)$  being the derivative of the self-energy at the physical mass  $M$ . For a pure bound state, i.e.,  $Z = 0$ , one obtains the effective coupling constant between the physical molecule and its components as [20]

$$x^2 = \frac{16\pi}{\mu} \sqrt{\frac{2e}{\mu}} [1 + \mathcal{O}(\sqrt{2\mu\epsilon}r)], \quad (3)$$

where  $\mu$  is the reduced mass,  $r$  is the range of the force, and  $\epsilon$  is the binding energy.

In the molecular scenario, the radiative transitions between the  $1^{--}$  and  $1^{--}$  states occur through their constituents, namely, the charmed heavy-light mesons. The corresponding effective Lagrangians for the radiative transitions between  $S$ -wave heavy-light mesons can be found in Ref. [20]. Doing the same analysis as in Ref. [20], we can express the radiative transition between two  $P$ -wave heavy-light mesons as

$$\mathcal{L}_{TT\gamma} = \frac{e\beta'}{2} \text{Tr}[T_a^{k\dagger} T_b^k \vec{\sigma} \cdot \vec{B} Q_{ab}] + \frac{eQ'}{2m_Q} \text{Tr}[T_a^{k\dagger} \vec{\sigma} \cdot \vec{B} T_a^k], \quad (4)$$

where  $T$  is the  $P$ -wave  $\frac{3}{2}^+$  multiplet,  $B^k \equiv \epsilon^{ijk} \partial^i A^j$  is the magnetic field,  $Q_{ab} \equiv \text{diag}(2/3, -1/3, -1/3)$  is the matrix of the light-quark charge, and  $Q'$  is the heavy-quark charge (in units of the proton charge  $e$ ). The parameters are taken to be the same as those in Ref. [20].

## III. RESULTS AND DISCUSSION

Since the spin structure of the  $\frac{3}{2}^+$  multiplet is more complicated than that of the  $\frac{1}{2}^-$  multiplet, there are more degrees of freedom for the  $\frac{3}{2}^+ + \frac{1}{2}^-$  system than for the  $\frac{1}{2}^+ + \frac{1}{2}^-$  system [21–23]. This means that in the  $\frac{3}{2}^+ + \frac{1}{2}^-$  system we should not expect all the potentials to be universal for all six molecular states. However, we can assume that the potentials do not vary dramatically due to the heavy-quark symmetry. Thus, the binding energies of these six molecules would be of the same order if they do exist as bound states. To simplify this issue, we assume they are the same. Using the results of the fit in Ref. [15] as a guide, we roughly estimate the binding energy  $\epsilon \sim 70$  MeV, which gives the masses for the corresponding  $D_1D^*$  and  $D_2D^*$  molecules,

$$M_{Y'(X')} \sim 4361 \text{ MeV}, \quad M_{Y''(X'')} \sim 4403 \text{ MeV}. \quad (5)$$

This makes  $Y(4360)$  and  $\psi(4415)$  good candidates for the  $D_1D^*$  and  $D_2D^*$  molecules. Since the width of  $D_2$  is twice as large as that of  $D_1$ , the signal of the  $D_2D^*$  threshold

effect near the  $Y''$  is not as significant as the other two thresholds [12]. To confirm or exclude the  $D_2D^*$  molecular interpretation of  $\psi(4415)$ , the angular distribution analysis of  $DD^*\pi$  and  $D^*D^*\pi$  at 4.415 GeV is needed.

In the vector sector the mass region of 4–4.7 GeV is far from well understood. Although the  $\psi(4040)$ ,  $\psi(4160)$ , and  $\psi(4415)$  are assigned as conventional  $3S$ ,  $2D$ , and  $4S$  vector charmonia, the  $Y(4260)$  and newly observed  $Y(4360)$  and  $Y(4660)$  cannot be well organized. Various explanations were proposed in the literature to explain the nature of these states. In Ref. [24], the  $Y(4360)$  and  $\psi(4415)$  were classified as conventional  $4S$  and  $3D$  vector charmonia in the constituent quark model, while in Ref. [25] they were interpreted as the canonical  $3^3D_1$  and  $5^3S_1$  states. Recently, Li and Voloshin argued that  $Y(4260)$  and  $Y(4360)$  are the mixing of spin-singlet and spin-triplet hadro-charmonia [26]. In Ref. [27] the  $Y(4660)$  was regarded as the  $\psi'f_0(980)$  molecule. In contrast, we propose that the  $Y(4260)$ ,  $Y(4360)$ , and  $\psi(4415)$  are most likely the  $D_1D$ ,  $D_1D^*$ , and  $D_2D^*$  vector molecules [15], which is consistent with the estimate of Eq. (5). However, their analogues in the bottom sector cannot be studied in  $e^+e^-$  colliders, as their production is highly suppressed in the heavy-quark limit [14].

The radiative transition between the molecular states actually offers a unique method to probe the nature of the hadronic molecules. It occurs through the constituents of the molecules, such as  $D^* \rightarrow D\gamma$  through the M1 transition, and  $D_1 \rightarrow D_1\gamma$  through the M1 transition in Eq. (4). The schematic diagram illustrating the radiative transition between the  $1^{--}$  and  $1^{-+}$  molecular states is shown in Fig. 1.  $[m_1m_2m_3]$  is used to denote different constituents. The mesons in the squared brackets are the intermediate mesons in the scalar three-point loop function as defined by Eq. (A.1) in Ref. [28]. There is only one Lorentz structure,  $\vec{e}_X \cdot \hat{k} \vec{e}_Y \cdot \vec{e}_Y - \vec{e}_Y \cdot \hat{k} \vec{e}_Y \cdot \vec{e}_X$ , where  $\vec{e}_X$ ,  $\vec{e}_Y$ , and  $\vec{e}_\gamma$  the polarization vectors of the corresponding  $X$ ,  $Y$ , and photon, respectively, and  $\hat{k}$  is the unit vector along the three-momentum of the photon.

The possible radiative transitions between the  $Y$ -type and  $X$ -type molecules at leading order are listed in Table I, while the transitions between  $Y$  and  $X''$ , and  $Y''$  and  $X$  can only happen when the next-to-leading-order diagrams are considered. As discussed in the previous paragraph, all the amplitudes have the same Lorentz structure associated with

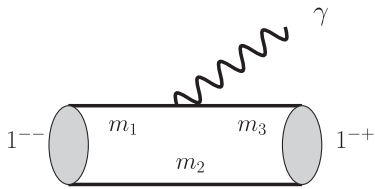


FIG. 1. Schematic diagram of the radiative transition between  $\frac{3}{2}^+ + \frac{1}{2}^-$  molecules with quantum numbers  $1^{--}$  and  $1^{-+}$ .  $m_1$ ,  $m_2$ , and  $m_3$  are the constituents of the corresponding molecules.

TABLE I. Amplitudes of the radiative transitions between different  $1^{--}$  and  $1^{-+}$  molecular states. The symbol “...” means that the transition is forbidden at leading order.

| $1^{--} \rightarrow 1^{-+}\gamma$ | $X$                                 | $X'$          | $X''$                      |
|-----------------------------------|-------------------------------------|---------------|----------------------------|
| $Y$                               | $[D_1DD_1]$                         | $[DD_1D^*]$   | ...                        |
| $Y'$                              | $[D^*D_1D][D_1D^*D_1], [D^*D_1D^*]$ |               | $[D_1D^*D_2]$              |
| $Y''$                             | ...                                 | $[D_2D^*D_1]$ | $[D_2D^*D_2], [D^*D_2D^*]$ |

a three-point scalar loop function. Due to the transverse property of the vector meson produced in  $e^+e^-$  colliders, the angular distribution of the process  $Y^{(\prime, \prime\prime)} \rightarrow X^{(\prime, \prime\prime)}\gamma$  can be extracted,

$$\frac{d\Gamma}{d\cos\theta} \propto 1 + \frac{3}{2}\sin^2\theta, \quad (6)$$

where  $\theta$  is the angle between the photon momentum and the beam axis in the overall c.m. frame. The equation

$$\sum_{\lambda=1,2} |\hat{k} \cdot \vec{e}_{Y^{(\prime, \prime\prime)}}^{(\lambda)}|^2 = \frac{1}{2}\sin^2\theta \quad (7)$$

is used to deduce Eq. (6), where the “—” denotes averaging the polarizations of the  $Y^{(\prime, \prime\prime)}$ . The angular distribution in Eq. (6) is very different from that of  $Y \rightarrow X(3872)\gamma$  [20], and can be easily detected by experiment with sufficiently large statistics.

Since the phase space of  $Y' \rightarrow X\gamma$  is twice as large as that of  $Y'' \rightarrow X'\gamma$ , it has the largest probability of being detected by experiment. Thus, we focus on the radiative decay of  $Y'$  to  $X$  in the following. With the width of  $D_1$  being taken into account, the partial width of  $Y' \rightarrow X\gamma$  with respect to the binding energy is shown in Fig. 2. Considering a binding energy of roughly 70 MeV, we obtain the partial width  $\Gamma(Y' \rightarrow X\gamma) \sim 70$  keV and the branching ratio  $\text{BR}(Y' \rightarrow X\gamma) \sim 10^{-3}$ , with  $\Gamma(Y(4360)) = 74$  MeV [29].

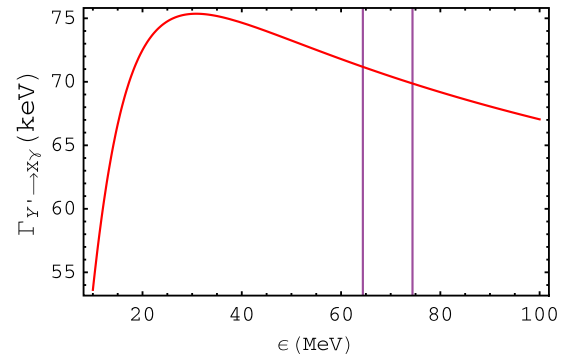


FIG. 2 (color online). The partial width of  $Y' \rightarrow X + \gamma$  as a function of the binding energy is shown. The two vertical lines are the lower and upper limits of the binding energy from the fit in Ref. [15].

Using Eq. (3) with the binding energy 70 MeV, we obtain the coupling constants in Eqs. (1) and (2),

$$y(x) = 2.35 \text{ GeV}^{\frac{1}{2}}, \quad y'(x') = 2.28 \text{ GeV}^{\frac{1}{2}}. \quad (8)$$

Following the same procedure as in Refs. [15,30], the cross section of the full process, i.e.,  $e^+e^- \rightarrow Y^{(\prime)} \rightarrow X\gamma$ , can be extracted as

$$\sigma(s) = (4\pi\alpha)^2 \left( g_{\gamma^* Y^{(\prime)}} \frac{M_{Y^{(\prime)}}^2}{s} \right)^2 (M_{Y^{(\prime)}} \Gamma_{Y^{(\prime)} \rightarrow X\gamma}) |G_{Y^{(\prime)}}(s)|^2,$$

where  $g_{\gamma^* Y^{(\prime)}}$  is the dimensionless coupling constant between the virtual photon and the vector state  $Y^{(\prime)}$ , and  $G_{Y^{(\prime)}}(s)$  is the corresponding propagator [31],

$$G_{Y^{(\prime)}}^{-1} = s - M_{Y^{(\prime)}}^2 + \hat{\Pi}(s) + iM_{Y^{(\prime)}} \Gamma_{Y^{(\prime)}}, \quad (9)$$

with  $\hat{\Pi}(s) = \Pi(s) - \text{Re}[\Pi(M_{Y^{(\prime)}}^2) + (s - M_{Y^{(\prime)}}^2) \partial_s \Pi(s)|_{s=M_{Y^{(\prime)}}^2}]$ . Here we assume that the constant widths of  $Y(Y(4260))$  and  $Y'(Y(4360))$  in Eq. (9) are the same, i.e.,  $\Gamma_{Y^{(\prime)}} = 40 \text{ MeV}$  [15]. Since both  $Y$  and  $Y'$  can decay to  $X\gamma$ , the relative strength will depend on the coupling constant  $\beta'$ . In the following, we assume that the coupling  $\beta' = \beta(1 \pm 0.5)$  with  $\beta^{-1} = 276 \text{ MeV}$  [20]. As a result, the cross section of  $e^+e^- \rightarrow Y^{(\prime)} \rightarrow X\gamma$  in the energy region [4.20, 4.50] GeV is shown in Fig. 3 and turns out to be nontrivial. As discussed before, the radiative transition between  $Y'$  and  $X$  is more important than that between  $Y$  and  $X$ , which makes the boundary from the uncertainty of  $\beta'$  negligible. Although the cross section of 0.1 pb around 4.36 GeV is smaller than those of  $J/\psi\pi\pi$  and  $h_c\pi\pi$ , which are roughly 70–80 pb, it is still larger than that of the isospin-violating process  $J/\psi\eta\pi^0$  [30]. Considering BESIII's luminosity of  $540 \text{ pb}^{-1}$  at 4.36 GeV, there should be 54 events when assuming that

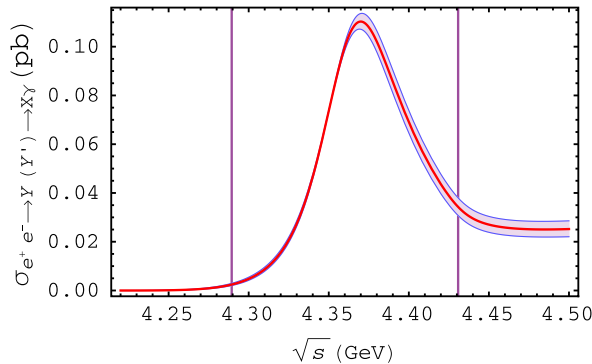


FIG. 3 (color online). The cross section of  $e^+e^- \rightarrow Y^{(\prime)} \rightarrow X\gamma$  in terms of the center-of-mass energy is illustrated. The vertical lines are the corresponding  $D_1D$  and  $D_1D^*$  thresholds. The boundary corresponds to the 50% uncertainty of the coupling constant  $\beta'$ .

TABLE II. The mainly hidden charm decay modes of the  $1^{--}$  and  $1^{+-}$  molecular states are listed.  $\mathcal{J}_{q,j}$  means the other light degrees of freedom with their corresponding quantum number  $j$  in the subindex.

| constituents | $D_1D$                       | $D_1D^*$                     | $D_2D^*$                     |
|--------------|------------------------------|------------------------------|------------------------------|
| $1^{--}$     | $J/\psi + \mathcal{J}_{q,S}$ | $J/\psi + \mathcal{J}_{q,V}$ | $J/\psi + \mathcal{J}_{q,S}$ |
| $1^{+-}$     | $\eta_c + \mathcal{J}_{q,A}$ | $\eta_c + \mathcal{J}_{q,T}$ | $\eta_c + \mathcal{J}_{q,A}$ |

the efficiency is 100%. It is still possible to detect the signals of these kinds of states, but for the angular distribution one would need larger statistics.

In the molecular scenario, i.e.,  $(\bar{Q}\Gamma^A q)(\bar{q}\Gamma^B Q)$  where  $Q$  is the heavy quark and  $q$  is the light quark, we can study their hidden charmonium decay patterns by using the Fierz transformation [32] as well as the 9-j symbol method [14]. Here we employ the Fierz transformation,

$$(\bar{u}_1 \Gamma^A u_2)(\bar{u}_3 \Gamma^B u_4) = \sum_{CD} C_{CD}^{AB} (\bar{u}_1 \Gamma^C u_4)(\bar{u}_3 \Gamma^D u_2), \quad (10)$$

with  $\Gamma^i = (\mathbf{1}, \gamma_\mu, \frac{i}{2}[\gamma_\mu, \gamma_\nu], \gamma_\mu \gamma_5, i\gamma_5)$  and  $C_{CD}^{AB} = \frac{1}{16} \text{Tr}[\Gamma^C \Gamma^A \Gamma^D \Gamma^B]$ , to study their hidden charm decay modes. The currents  $\bar{q}\gamma_\mu\gamma_5 c$ ,  $\bar{c}\gamma_\mu\gamma_5 q$ ,  $i\bar{q}\gamma_5 c$ , and  $i\bar{c}\gamma_5 q$  describe  $D_1$ ,  $\bar{D}_1$ ,  $D$ , and  $\bar{D}$ , respectively. When one analyzes their hidden charm decay patterns, the two currents of their constituents must be located at the same position. This is the reason why we can use the Fierz transformation to study their hidden charm decays qualitatively. As shown in Table II, we find that the dominant hidden charm decay modes of all the  $Y$ -type molecules are  $J/\psi$  plus some light mesons. This would explain why the  $Y(4260)$  is so far only well established in the  $J/\psi\pi\pi$  channel, while in the  $h_c\pi\pi$  channel there are only some indications of the  $Y(4260)$ . Meanwhile, the  $X$ -type molecules mainly decay into  $\eta_c$  plus some light mesons, which should be useful for future experimental detections.

#### IV. SUMMARY

In this work, we proposed a search for exotic hadronic molecules with  $J^{PC} = 1^{+-}$  in association with the  $S$ -wave thresholds opened by the charmed-meson pairs. Such states—which are closely correlated with their charge-conjugate partners of the  $1^{--}$  vector molecules in dynamics—can be probed in the radiative transitions between these two kinds of hadronic molecules. Numerical evaluations have been provided for their masses and decay patterns, which should be helpful for experimental studies at BESIII or future Super-B factories. In particular, we showed that the production of  $X$  via  $Y' \rightarrow X\gamma$  is nearly accessible at BESIII, for which the cross section of  $e^+e^- \rightarrow Y^{(\prime)} \rightarrow X\gamma$  is estimated as about 0.1 pb at 4.36 GeV. Given that there has been strong evidence for vector hadronic

molecules such as the  $Y(4260)$  in experiment, the search for their conjugate partners with exotic quantum number  $J^{PC} = 1^{-+}$  may provide smoking-gun evidence for the dynamic role played by the  $S$ -wave open-flavor thresholds in hadron spectroscopy. Whether or not such states can be confirmed by future experimental data, this would be an important indication of how strongly QCD has arranged the multi-quark degrees of freedom to construct hadronic matter.

## ACKNOWLEDGMENTS

Special thanks to Christoph Hanhart, Ulf-G. Meißner, and Qiang Zhao for useful comments and suggestions on improving this work, and to Feng-Kun Guo for useful discussions and for having benefited from the “AmpCalc.m” package written by him. Useful discussions with Wei Wang are also acknowledged. This work is supported by DFG and NSFC through funds provided to the Sino-German CRC 110 “Symmetries and the Emergence of Structure in QCD.”

- 
- [1] S. K. Choi *et al.* (Belle Collaboration), *Phys. Rev. Lett.* **91**, 262001 (2003).
  - [2] N. Brambilla *et al.*, [arXiv:1404.3723](#).
  - [3] S. L. Olsen, [arXiv:1403.1254](#).
  - [4] A. Bondar *et al.* (Belle Collaboration), *Phys. Rev. Lett.* **108**, 122001 (2012).
  - [5] I. Adachi *et al.* (Belle Collaboration), [arXiv:1209.6450](#).
  - [6] M. Ablikim *et al.* (BESIII Collaboration), *Phys. Rev. Lett.* **110**, 252001 (2013).
  - [7] Z. Q. Liu *et al.* (Belle Collaboration), *Phys. Rev. Lett.* **110**, 252002 (2013).
  - [8] M. Ablikim *et al.* (BESIII Collaboration), *Phys. Rev. Lett.* **112**, 132001 (2014).
  - [9] M. Ablikim *et al.* (BESIII Collaboration), *Phys. Rev. Lett.* **111**, 242001 (2013).
  - [10] T. Xiao, S. Dobbs, A. Tomaradze, and K. K. Seth, *Phys. Lett. B* **727**, 366 (2013).
  - [11] Q. Wang, M. Cleven, F.-K. Guo, C. Hanhart, U.-G. Meißner, X.-G. Wu, and Q. Zhao, *Phys. Rev. D* **89**, 034001 (2014).
  - [12] Q. Wang, C. Hanhart, and Q. Zhao, *Phys. Lett. B* **725**, 106 (2013).
  - [13] Q. Wang, C. Hanhart, and Q. Zhao, *Phys. Rev. Lett.* **111**, 132003 (2013).
  - [14] X. Li and M. B. Voloshin, *Phys. Rev. D* **88**, 034012 (2013).
  - [15] M. Cleven, Q. Wang, F.-K. Guo, C. Hanhart, U.-G. Meißner, and Q. Zhao, [arXiv:1310.2190](#).
  - [16] A. Salam, *Nuovo Cimento* **25**, 224 (1962).
  - [17] S. Weinberg, *Phys. Rev.* **130**, 776 (1963).
  - [18] K. Hayashi, M. Hirayama, T. Muta, N. Seto, and T. Shirafuji, *Fortschr. Phys.* **15**, 625 (1967).
  - [19] G. V. Efimov and M. A. Ivanov, *The Quark Confinement Model of Hadrons* (CRC, Boca Raton, FL, 1993).
  - [20] F.-K. Guo, C. Hanhart, U.-G. Meißner, Q. Wang, and Q. Zhao, *Phys. Lett. B* **725**, 127 (2013).
  - [21] F.-K. Guo, C. Hidalgo-Duque, J. Nieves, and M. P. Valderrama, *Phys. Rev. D* **88**, 054007 (2013).
  - [22] M. P. Valderrama, *Phys. Rev. D* **85**, 114037 (2012).
  - [23] J. Nieves and M. P. Valderrama, *Phys. Rev. D* **86**, 056004 (2012).
  - [24] J. Segovia, D. R. Entem, F. Fernandez, and E. Hernandez, *Int. J. Mod. Phys. E* **22**, 1330026 (2013).
  - [25] G.-J. Ding, J.-J. Zhu, and M.-L. Yan, *Phys. Rev. D* **77**, 014033 (2008).
  - [26] X. Li and M. B. Voloshin, *Mod. Phys. Lett. A* **29**, 1450060 (2014).
  - [27] F.-K. Guo, C. Hanhart, and U.-G. Meißner, *Phys. Lett. B* **665**, 26 (2008).
  - [28] M. Cleven, Q. Wang, F.-K. Guo, C. Hanhart, U.-G. Meißner, and Q. Zhao, *Phys. Rev. D* **87**, 074006 (2013).
  - [29] J. Beringer *et al.* (Particle Data Group Collaboration), *Phys. Rev. D* **86**, 010001 (2012).
  - [30] X.-G. Wu, C. Hanhart, Q. Wang, and Q. Zhao, *Phys. Rev. D* **89**, 054038 (2014).
  - [31] M. Cleven, F.-K. Guo, C. Hanhart, and U.-G. Meißner, *Eur. Phys. J. A* **47**, 120 (2011).
  - [32] A. E. Bondar, A. Garmash, A. I. Milstein, R. Mizuk, and M. B. Voloshin, *Phys. Rev. D* **84**, 054010 (2011).