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Tensor-induced B modes with no temperature fluctuations

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The recent indications for a tensor-to-scalar ratio $r \simeq 0.2$ from BICEP2 measurements of the cosmic microwave background (CMB) B-mode polarization present some tension with upper limits $r \lesssim 0.1$ from measurements of CMB temperature fluctuations. Here we point out that tensor perturbations can induce B modes in the CMB polarization without inducing *any* temperature fluctuations nor E-mode polarization whatsoever, but only at the expense of violating the Copernican principle. We present this mathematical possibility as a new ingredient for the model-builder's toolkit in case the tension between B modes and temperature fluctuations cannot be resolved with more conventional ideas.

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The BICEP2 experiment has received considerable attention for their claimed detection [1] of a large-angle curl component (B modes) [2,3] in the cosmic microwave background (CMB) polarization. The interpretation is that these B modes are due to gravitational waves produced during the time of inflation [4,5]. However, the tensor-to-scalar ratio $r \simeq 0.2$ indicated by BICEP2 CMB polarization is in $\sim 2\sigma$ conflict with upper bounds $r \lesssim 0.1$ from measurements of CMB temperature fluctuations [6,7]. The CMB upper limits can be relieved if the scalar spectral index is allowed to run [8], but the running required is unusually high for single-field slow-roll models. It may be worthwhile to explore explanations for this disagreement between the two values of r inferred in case the tension continues after the dust of the initial detection has settled.

Here we show that it is possible for a tensor fluctuation to produce a B mode in the CMB polarization without producing *any* temperature fluctuation nor E-mode polarization whatsoever. The argument is relatively straightforward given the total-angular-momentum (TAM) formalism we have developed in recent work [9]. The only catch is that, as we show, the tensor fluctuation required to do this violates the Copernican principe; i.e., it implies that we occupy a preferred location in the Universe.

We begin by reviewing how a polarization map is decomposed into E and B modes. The linear polarization in any direction $\hat{\mathbf{n}}$ on the sky is represented by Stokes parameters $Q(\hat{\mathbf{n}})$ and $U(\hat{\mathbf{n}})$. However, these quantities are components of a symmetric trace-free (STF) 2×2 tensor, or equivalently, spin-2 field, that lives on the surface of the sky, the two-sphere. The polarization can therefore be expanded in terms of tensor spherical harmonics, a complete set of basis functions for STF 2×2 tensors on the two-sphere. The linear polarization is specified at any point by the two Stokes parameters, and so two sets, E (for the curl-free part of the polarization field) and B (for the curl component) are required to provide a complete basis. The E and B tensor spherical harmonics can be written [2]

$$
\begin{aligned}
Y^{\rm E}_{(\ell m)AB}(\hat{\mathbf{n}}) &= \sqrt{\frac{2(\ell-2)!}{(\ell+2)!}}\left(-\nabla_A\nabla_B + \frac{1}{2}g_{AB}\nabla^C\nabla_C\right)Y_{(\ell m)}(\hat{\mathbf{n}}) \\
&\equiv -\sqrt{\frac{(\ell-2)!}{2(\ell+2)!}}W^{\rm E}_{AB}Y_{(\ell m)}(\hat{\mathbf{n}}), \\
Y^{\rm B}_{(\ell m)AB}(\hat{\mathbf{n}}) &= \sqrt{\frac{(\ell-2)!}{2(\ell+2)!}}(\epsilon_B{}^C\nabla_C\nabla_A + \epsilon_A{}^C\nabla_C\nabla_B)Y_{(\ell m)}(\hat{\mathbf{n}}) \\
&\equiv -\sqrt{\frac{(\ell-2)!}{2(\ell+2)!}}W^{\rm B}_{AB}Y_{(\ell m)}(\hat{\mathbf{n}}), \qquad (1)
\end{aligned}
$$

where here $\{A,B\} = \{\theta,\phi\}$, and ∇_A is a covariant derivative on the two-sphere, with metric $g_{AB} = \mathrm{diag}(1, \sin^2\theta)$ and antisymmetric tensor ϵ_{AB}. The last equality in each line defines the two orthogonal tensor-valued derivative operators $W^{\rm E}_{AB}$ and $W^{\rm B}_{AB}$ (written down in another form in Eq. (90) of Ref. [9]) that when applied to the usual scalar spherical harmonics $Y_{(\ell m)}(\hat{\mathbf{n}})$ give rise to the E and B tensor spherical harmonics. Thus, the two sets of tensor spherical harmonics can be obtained by applying two orthogonal STF 2×2 derivative operators to the scalar spherical harmonics. The presence of the antisymmetric tensor in the definition $Y^{\rm B}_{(\ell m)AB}(\hat{\mathbf{n}})$ indicates that the B mode and E mode have opposite parities for fixed ℓ and m. It is important to note that the derivative operators $W^{\rm E}_{AB}$ and $W^{\rm B}_{AB}$ commute with the total-angular-momentum operator and its third component. The tensor spherical harmonics are thus eigenstates of the total angular momentum with quantum numbers ℓ and m.

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We will now develop an analogous decomposition of *three*-dimensional transverse-traceless tensor fields, following the treatment of Ref. [9]. Before doing so, however, we review the standard treatment, in which the transverse-traceless tensor field is decomposed,

$$h_{ab}(\mathbf{x}) = \sum_{\mathbf{k},s} h_s(\mathbf{k})\hat{\varepsilon}^s_{ab}(\mathbf{k})e^{i\mathbf{k}\cdot\mathbf{x}}, \tag{2}$$

into Fourier components $h_s(\mathbf{k})$ of wave vector $\mathbf{k}$ and polarization s which can be $+$ or $\times$ (and $\hat{\varepsilon}^s_{ab}(\mathbf{k})$ are polarization tensors. Here $a, b, c, \ldots$ are *three*-dimensional spatial indices unlike *two*-dimensional indices $A, B, C, \cdots$. This decomposition into Fourier modes can be done because the plane waves $\hat{\varepsilon}^s_{ab}(\mathbf{k})e^{i\mathbf{k}\cdot\mathbf{x}}$ constitute a complete othonormal set of basis functions for a transverse-traceless tensor field. Power spectra $P_h(k)$ for these tensor perturbations (or gravitational waves) are given by

$$\langle h_s(\mathbf{k})h^*_{s'}(\mathbf{k}')\rangle = \delta_{ss'}(2\pi)^3\delta_D(\mathbf{k}-\mathbf{k}')\frac{P_h(k)}{4}. \tag{3}$$

In Ref. [9] we showed that transverse-traceless tensor fields can alternatively be expanded in transverse-tracless-tensor TAM waves. To see how this works, we begin by noting that a scalar function $\phi(\mathbf{x})$ in three dimensions can be expanded,

$$\phi(\mathbf{x}) = \int \frac{d^3k}{(2\pi)^3}\tilde{\phi}(\mathbf{k})\Psi^{\mathbf{k}}(\mathbf{x}), \tag{4}$$

where

$$\tilde{\phi}(\mathbf{k}) = \int d^3x\phi(\mathbf{x})[\Psi^{\mathbf{k}}(\mathbf{x})]^*, \tag{5}$$

in terms of scalar TAM waves,

$$\Psi^k_{(\ell m)}(\mathbf{x}) \equiv j_\ell(kr)Y_{(\ell m)}(\hat{\mathbf{n}}), \tag{6}$$

where $j_\ell(x)$ is the spherical Bessel function of the first kind. These TAM waves are eigenfunctions of the three-dimensional Laplacian ∇^2 with eigenvalue $-k^2$ and also eigenfunctions of the total angular momentum and its third component with quantum numbers ℓ and m, respectively. These scalar TAM waves constitute a complete orthonormal basis for scalar functions. The orthonormality relation for the basis functions is

$$\int d^3x[\Psi^k_{(\ell m)}(\mathbf{x})]^*\Psi^{k'}_{(\ell' m')}(\mathbf{x}) = \frac{\delta_{\ell\ell'}\delta_{mm'}}{16\pi^2}\frac{(2\pi)^3}{k^2}\delta_D(k-k'), \tag{7}$$

where δ_{ij} is the Kronecker delta. Completeness is demonstrated by

$$\sum_{\ell m}\int\frac{k^2dk}{(2\pi)^3}[4\pi i^\ell\Psi^k_{(\ell m)}(\mathbf{x})]^*[4\pi i^\ell\Psi^k_{(\ell m)}(\mathbf{x}')] = \delta_D(\mathbf{x}-\mathbf{x}'). \tag{8}$$

Just as the E/B tensor spherical harmonics were obtained by applying appropriately defined tensor-valued derivative operators, transverse-traceless-tensor TAM waves can be obtained by applying transverse-traceless-tensor–valued derivative operators to the scalar TAM waves. The appropriate derivative operators are

$$\begin{aligned} T^{\mathrm{B}}_{ab} &= K_{(a}M_{b)} + M_{(a}K_{b)} + 2D_{(a}K_{b)}, \\ T^{\mathrm{E}}_{ab} &= M_{(a}M_{b)} - K_{(a}K_{b)} + 2D_{(a}M_{b)}, \end{aligned} \tag{9}$$

where we define operators,

$$D_a \equiv \frac{i}{k}\nabla_a, \qquad K_a \equiv -iL_a, \qquad M_a \equiv \epsilon_{abc}D^bK^c, \tag{10}$$

for the subspace of $\nabla^2 = -k^2$, and $L_a = -i\epsilon_{abc}x^b\nabla^c$ is the orbital-angular-momentum operator. The two tensor operators T^{B}_{ab} and T^{E}_{ab} are orthogonal. They are also both transverse and traceless, and they both commute with the Laplacian and with the total angular momentum and its third component. Thus, the transverse-traceless-tensor TAM waves,

$$\begin{aligned} \Psi^{\mathrm{B},k}_{(\ell m)ab}(\mathbf{x}) &= -\sqrt{\frac{(\ell-2)!}{2(\ell+2)!}}T^B_{ab}\Psi^k_{(\ell m)}(\mathbf{x}), \\ \Psi^{\mathrm{E},k}_{(\ell m)ab}(\mathbf{x}) &= -\sqrt{\frac{(\ell-2)!}{2(\ell+2)!}}T^E_{ab}\Psi^k_{(\ell m)}(\mathbf{x}), \end{aligned} \tag{11}$$

constitute a complete basis for transverse-traceless tensor fields in three dimensions. Note that T^{E}_{ab} is even under a parity inversion, while T^{B}_{ab} is odd. The E and B tensor TAM waves thus have opposite parity for the same ℓ and m.

Thus, the most general three-dimensional transverse-traceless tensor field can be written

$$\begin{aligned} h_{ab}(\mathbf{x}) = \sum_{\ell m}\int\frac{k^2dk}{(2\pi)^3}4\pi i^\ell[&h^{\mathrm{E}}_{\ell m}(k)\Psi^{\mathrm{E},k}_{(\ell m)ab}(\mathbf{x}) \\ &+h^{\mathrm{B}}_{\ell m}(k)\Psi^{\mathrm{B},k}_{(\ell m)ab}(\mathbf{x})]. \end{aligned} \tag{12}$$

An alternative set of basis TAM waves can be derived in which the TAM waves have fixed helicity. These helicity $s = \pm 2$ TAM waves will be useful below and are related to the E/B TAM waves through $\Psi^{s=\pm 2,k}_{(\ell m)ab}(\mathbf{x}) = [\Psi^{\mathrm{E},k}_{(\ell m)ab} \pm i\Psi^{\mathrm{B},k}_{(\ell m)ab}]/\sqrt{2}$. They allow the most general transverse-traceless tensor field to be expanded,

$$h_{ab}(\mathbf{x}) = \sum_{\ell m}\sum_{s=\pm 2}\int \frac{k^2 dk}{(2\pi)^3} 4\pi i^\ell h^s_{\ell m}(k)\Psi^{s,k}_{(\ell m)ab}(\mathbf{x}). \quad (13)$$

In Ref. [9] it was proved for scalar fluctuations that scalar TAM waves with quantum numbers ℓ m contribute only to CMB (and any other observable) power spectra of the same ℓ. This is a consequence of the fact that the TAM waves and the scalar/vector/tensor spherical harmonics transform as representations of the rotation group of order ℓ. The same will be true for tensor TAM waves as well; i.e., tensor TAM waves of order ℓ will contribute only to observable power spectra of multipole moment ℓ. More importantly for our purposes, though, a similar statement applies to the E/B decomposition. The argument is based on the parity transformation, i.e., the dependence of the basis functions under a flip of the displacement vector $\mathbf{x} \to -\mathbf{x}$ with respect to the location of the observer. The E-mode tensor TAM wave, scalar spherical harmonic, and E-mode tensor spherical harmonic all have parity $(-1)^\ell$, while the B-mode tensor TAM wave and B-mode tensor spherical harmonic transform as $(-1)^{\ell+1}$. Since the physics (Thomson scattering and radiative transfer) involved in the generation of CMB fluctuations is parity conserving, the B-mode component of the tensor field gives rise to B-mode polarization but contributes nothing to the temperature fluctuation nor the E-mode polarization. The converse is true for the E-mode component of the tensor field. As an obvious corollary, if B-mode TAM-wave components have larger power than the E-mode components do, the observed B-mode polarization power spectrum may be larger than would be inferred from the temperature and E-mode polarization power spectra.

As we now demonstrate, an excess of B-mode power over E-mode power in the TAM-wave expansion requires a violation of the Copernican principle. This is because the decomposition of the tensor field into E and B modes is not translation invariant. This can be seen from the following physics argument: CMB polarization is generated by Thomson scattering of temperature fluctuations. B-mode polarization therefore requires that there be a temperature fluctuation at the surface of last scatter, even though, as argued above, a tensor B-mode TAM wave produces no temperature fluctuation for an observer at the origin. Thus, the decomposition into E- and B-mode tensor TAM waves is not translationally invariant.

More explicitly, one can show that a B-mode TAM wave defined with respect to a given origin has nonzero functional overlap with an E-mode TAM wave defined with respect to a different origin. To avoid technical complications, here we demonstrate for spin-one, transverse vector field; the statement is also true for a transverse-traceless tensor field. The vector analogue of Eq. (11) is simply

$$\Psi^{\mathrm{B},k}_{(\ell m)a}(\mathbf{x}) = [\ell(\ell+1)]^{-1/2}K_a(\mathbf{x})\Psi^k_{(\ell m)}(\mathbf{x}),$$
$$\Psi^{\mathrm{E},k}_{(\ell m)a}(\mathbf{x}) = [\ell(\ell+1)]^{-1/2}M_a(\mathbf{x})\Psi^k_{(\ell m)}(\mathbf{x}), \quad (14)$$

where the vector operators K_a and M_a explicitly depend on the choice of coordinate origin. We can compute the overlap between E- and B-mode TAM waves defined with respect to two different coordinate origins separated by a constant vector $\mathbf{d}$. Taking identical k, ℓ and m, for example,

$$\begin{aligned}&\int d^3x[\Psi^{\mathrm{B},k}_{(\ell m)a}(\mathbf{x}+\mathbf{d})]^*\Psi^{\mathrm{E},k,a}_{(\ell m)}(\mathbf{x})\\ &\quad\propto \int d^3x[K_a(\mathbf{x}+\mathbf{d})\Psi^k_{(\ell m)}(\mathbf{x}+\mathbf{d})]^* M^a(\mathbf{x})\Psi^k_{(\ell m)}(\mathbf{x})\\ &\quad= -\int d^3x[\Psi^k_{(\ell m)}(\mathbf{x}+\mathbf{d})]^*[K_a(\mathbf{x}+\mathbf{d})M^a(\mathbf{x})\Psi^k_{(\ell m)}(\mathbf{x})].\end{aligned} \quad (15)$$

Now the two operators act successively on the scalar TAM wave because of the anti-Hermitian condition $[K_a(\mathbf{x})]^\dagger = -K_a(\mathbf{x})$. Note that $K_a(\mathbf{x}+\mathbf{d})M^a(\mathbf{x})$ does not vanish if $\mathbf{d}\neq 0$, while $K_a(\mathbf{x})M^a(\mathbf{x}) = 0$ is true. Simple algebra gives

$$\begin{aligned}K_a(\mathbf{x}+\mathbf{d})M^a(\mathbf{x}) &= (K_a(\mathbf{x}) - \epsilon_{abc}d^b\nabla^c)M_a(\mathbf{x}),\\ &= -\epsilon_{abc}d^b\nabla^c M_a(\mathbf{x}) = ikd^b K_b(\mathbf{x}).\end{aligned} \quad (16)$$

If $\mathbf{d}$ is along the z direction so that $d^a K_a = -idL_z$, we have

$$\begin{aligned}&\int d^3x[\Psi^{\mathrm{B},k}_{(\ell m)a}(\mathbf{x}+\mathbf{d})]^*\Psi^{\mathrm{E},k,a}_{(\ell m)}(\mathbf{x})\\ &\quad\propto -\int d^3x[\Psi^k_{(\ell m)}(\mathbf{x}+\mathbf{d})]^* kdL_z\Psi^k_{(\ell m)}(\mathbf{x})\\ &\quad= -mkd\int d^3x[\Psi^k_{(\ell m)}(\mathbf{x}+\mathbf{d})]^*\Psi^k_{(\ell m)}(\mathbf{x}).\end{aligned} \quad (17)$$

The overlap between two scalar TAM waves of shifted coordinate origins is in general nonzero. This implies that even if the power of E-mode TAM waves might vanish when the *E/B* decomposition is performed with respect to one observer, the decomposition yields nonzero E-mode power with respect to another observer at a different location in the Universe.

We now show in a different way that statistical homogeneity requires the power in E and B TAM transverse-traceless-tensor TAM waves to be the same. We first relate the power spectra of TAM-wave coefficients to those of Fourier coefficients. Plane-wave tensor states of polarization $+$ and $\times$ can be added to construct helicity plane waves with Fourier amplitudes $h_s(\mathbf{k})$ labeled by the wave vector $\mathbf{k}$ and the helicity $s = \pm 2$. If statistical homogeneity (or translation invariance) is respected, then $\langle h_s(\mathbf{k})h^*_{s'}(\mathbf{k}')\rangle \propto \delta_D(\mathbf{k}-\mathbf{k}')$ (it is not required to be proportional to $\delta_{ss'}$

though). On the other hand, in terms of the TAM-wave coefficient,

$$\langle h^{\alpha}_{\ell m}(k)[h^{\beta}_{\ell' m'}(k')]^{*}\rangle = P_{\alpha\beta}(k)\delta_{\ell\ell'}\delta_{mm'}\frac{(2\pi)^3}{k^2}\delta_D(k-k'), \tag{18}$$

with $\alpha, \beta = \{E, B\}$. We have assumed diagonalization with respect to angular-momentum quantum numbers to preserve statistical isotropy. Note that parity invariance would require $P_{\rm EB}(k) = 0$. Transforming to the helicity TAM basis $h^{s=\pm 2}_{\ell m}(k) = [h^{\rm E}_{\ell m}(k) \mp i h^{\rm B}_{\ell m}(k)]/\sqrt{2}$, we have

$$\langle h^{s}_{\ell m}(k)[h^{s'}_{\ell' m'}(k')]^{*}\rangle = P_{ss'}(k)\delta_{\ell\ell'}\delta_{mm'}\frac{(2\pi)^3}{k^2}\delta_D(k-k'), \tag{19}$$

where the 2×2 matrix is given by

$$P_{ss'}(k) = \begin{pmatrix} P_{22}(k) & P_{2,-2}(k) \\ P_{-2,2}(k) & P_{-2,-2}(k) \end{pmatrix}, \tag{20}$$

with

$$\begin{aligned} P_{22}(k) &= \frac{1}{2}[P_{\rm EE}(k) + P_{\rm BB}(k)] - \mathfrak{Im} P_{\rm EB}(k), \\ P_{-2,-2}(k) &= \frac{1}{2}[P_{\rm EE}(k) + P_{\rm BB}(k)] + \mathfrak{Im} P_{\rm EB}(k), \\ P_{2,-2}(k) &= [P_{-2,2}(k)]^{*} \\ &= \frac{1}{2}[P_{\rm EE}(k) - P_{\rm BB}(k)] \mp i\mathfrak{Re} P_{\rm EB}(k). \end{aligned} \tag{21}$$

Now using the relation,

$$h_s(\mathbf{k}) = \sum_{\ell m}[{}_{-s}Y_{(\ell m)}(\hat{\mathbf{k}})]^{*} h^{s}_{\ell m}(k), \tag{22}$$

we find the power spectra for Fourier components,

$$\begin{aligned} \langle h_s(\mathbf{k})[h_{s'}(\mathbf{k}')]^{*}\rangle &= P_{ss'}(k)\frac{(2\pi)^3}{k^2}\delta_D(k-k') \\ &\quad \times \sum_{\ell m}[{}_{-s}Y_{(\ell m)}(\hat{\mathbf{k}}')]^{*}{}_{-s'}Y_{(\ell m)}(\hat{\mathbf{k}}). \end{aligned} \tag{23}$$

Only for $s = s'$ does the summation over ℓ, m add up to $\delta_D(\hat{\mathbf{k}} - \hat{\mathbf{k}}')$, so that the power spectrum $\langle h_s(\mathbf{k})[h_{s'}(\mathbf{k}')]^{*}\rangle \propto \delta_D(k-k')\delta_D(\hat{\mathbf{k}} - \hat{\mathbf{k}}')/k^2 = \delta_D(\mathbf{k} - \mathbf{k}')$. We therefore conclude that the Copernican principle requires $P_{2,-2}(k) = 0$; i.e. $P_{\rm EE}(k) = P_{\rm BB}(k)$ and the real part of $P_{\rm EB}(k)$ vanishes, whether or not the two-point statistics are parity invariant.

To conclude, the $\sim 2\sigma$ tension between the BICEP2 value for r and CMB upper limits is probably not sufficiently significant to warrant abandoning the Copernican principle. Still, the possibility discussed here to produce a larger B-mode signal, without inducing any temperature fluctuations or E-mode polarization, may prove to be useful should the $\sim 2\sigma$ disagreement continue to be established with greater significance. And perhaps this may be useful in an explanation that does not involve a violation of the Copernican principle. For example, perhaps a theory in which the amplitudes of the E- and B-mode tensor TAM waves have a highly non-Gaussian distribution. There may then be an *apparent* violation of the Copernican principle in our observable Universe that renders the B-mode TAM-wave amplitude larger, relative to the E-mode TAM-wave amplitude, than would be expected with a Gaussian distribution. We leave the construction of a model to implement this idea to future work.

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[1] P. A. R. Ade *et al.* (BICEP2 Collaboration), arXiv:1403.3985.

[2] M. Kamionkowski, A. Kosowsky and A. Stebbins, Phys. Rev. D **55**, 7368 (1997); M. Kamionkowski, A. Kosowsky and A. Stebbins, Phys. Rev. Lett. **78**, 2058 (1997).

[3] M. Zaldarriaga and U. Seljak, Phys. Rev. D **55**, 1830 (1997); U. Seljak and M. Zaldarriaga, Phys. Rev. Lett. **78**, 2054 (1997).

[4] A. H. Guth and S. Y. Pi, Phys. Rev. Lett. **49**, 1110 (1982); A. A. Starobinsky, Phys. Lett. **117B**, 175 (1982); J. M. Bardeen, P. J. Steinhardt and M. S. Turner, Phys. Rev. D **28**, 679 (1983).

[5] V. A. Rubakov, M. V. Sazhin and A. V. Veryaskin, Phys. Lett. **115B**, 189 (1982); R. Fabbri and M. d. Pollock, Phys. Lett. **125B**, 445 (1983); L. F. Abbott and M. B. Wise, Nucl. Phys. **B244**, 541 (1984); A. A. Starobinsky, JETP Lett. **30**, 682 (1979) [Pis'ma Zh. Eksp. Teor. Fiz. **30**, 719 (1979)].

[6] K. T. Story, C. L. Reichardt, Z. Hou, R. Keisler, K. A. Aird, B. A. Benson, L. E. Bleem, and J. E. Carlstrom *et al.*, Astrophys. J. **779**, 86 (2013).

[7] P. A. R. Ade *et al.* (Planck Collaboration), arXiv:1303.5076.

[8] P. A. R. Ade *et al.* (Planck Collaboration), arXiv:1303.5082.

[9] L. Dai, M. Kamionkowski and D. Jeong, Phys. Rev. D **86**, 125013 (2012).