

Charmless three-body decays of B_s mesons

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Branching fractions and CP -violating asymmetries of charmless $\bar{B}_s^0 \rightarrow PPP$ decays with $P = K, \pi$ are studied using a simple model based on the factorization approach. The penguin-dominated modes $\bar{B}_s^0 \rightarrow \bar{K}^0 K^+ \pi^-$ and $K^0 K^- \pi^+$ have the largest rates among the three-body B_s decays, and they are dominated by the $K_0^*(1430)$ resonances and nonresonant contributions. The branching fraction of $\bar{B}_s^0 \rightarrow K^0 K^+ K^-$ is predicted to be of order 1.4×10^{-6} , which is in the middle of the measured result $(0.2-3.4) \times 10^{-6}$ obtained by LHCb. We find sizable CP violation in $K^0 \pi^+ \pi^-$, $K^0 \pi^0 \pi^0$, $K^0 K^+ K^-$, and $K_S K_S K_S$ channels. Just as the B_u sector, the CP asymmetries of $\bar{B}_s^0 \rightarrow K^0 \pi^+ \pi^-$ and $\bar{B}_s^0 \rightarrow K^0 K^+ K^-$ have similar magnitudes but are opposite in sign. Several model-independent relations based on U -spin symmetry for CP asymmetries are derived. Although they cannot be tested by the present available data, they can be checked by the dynamical calculations. Since the U -spin symmetry that relates various matrix elements of scalar densities, for example, $\langle K^0 K | \bar{d}q | 0 \rangle$ and $\langle \bar{K}^0 \pi | \bar{s}q | 0 \rangle$, is badly broken, the derived U -spin relations are generally not well respected.

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I. INTRODUCTION

Recently LHCb has made the first observation of charmless three-body decays of the B_s^0 meson with a K_S^0 meson in the final state [1]. Specifically, LHCb has obtained the following branching fractions:

$$\begin{aligned} \mathcal{B}(B_s^0 \rightarrow K^0 K^\pm \pi^\mp) &= (73.6 \pm 5.7 \pm 6.9 \pm 3.0) \times 10^{-6}, \\ \mathcal{B}(B_s^0 \rightarrow \bar{K}^0 \pi^+ \pi^-) &= (14.3 \pm 2.8 \pm 1.8 \pm 0.6) \times 10^{-6}, \\ \mathcal{B}(B_s^0 \rightarrow \bar{K}^0 K^+ K^-) &\in [0.2; 3.4] \times 10^{-6}. \end{aligned} \quad (1.1)$$

The first observation of the decay modes $B_s \rightarrow K_S \pi^+ \pi^-$ and $B_s \rightarrow K_S K^\pm \pi^\mp$ is an important step at the LHCb toward extracting information on the mixing-induced CP -violating phase in the B_s^0 system and the weak phase γ from these decays.

In the charged B meson sector, LHCb recently found evidence of direct CP violation in charmless three-body decays: $B^+ \rightarrow \pi^+ \pi^+ \pi^-$, $B^+ \rightarrow K^+ K^+ K^-$, and $B^+ \rightarrow K^+ K^- \pi^+$ [2,3]. Moreover, LHCb has also observed large asymmetries in some localized regions of phase space. For example, the regional CP asymmetry in $B^+ \rightarrow \pi^+ \pi^+ \pi^-$ is of order 58% for $m_{\pi^+ \pi^-}^{\text{low}} < 0.4 \text{ GeV}^2$ and $m_{\pi^+ \pi^-}^{\text{high}} > 15 \text{ GeV}^2$. Hence, significant signatures of CP violation were found in the low mass regions devoid of most of the known resonances. LHCb also discovered interesting correlations of the CP violation between the decays, namely,

$$\begin{aligned} \mathcal{A}_{CP}(B^- \rightarrow \pi^- K^+ K^-) &\approx -\mathcal{A}_{CP}(B^- \rightarrow \pi^- \pi^+ \pi^-), \\ \mathcal{A}_{CP}(B^- \rightarrow K^- \pi^+ \pi^-) &\approx -\mathcal{A}_{CP}(B^- \rightarrow K^- K^+ K^-). \end{aligned} \quad (1.2)$$

It will be interesting to see if the same pattern of CP violation occurs in B_s^0 meson decays.

Contrary to B_u and B_d mesons, for three-body B_s decays, the $b \rightarrow sq\bar{q}$ penguin transitions contribute to the final states with an even number of kaons, namely, $KK\pi$, while the $b \rightarrow uq\bar{q}$ tree and $b \rightarrow dq\bar{q}$ penguin transitions contribute to final states with an odd number of kaons, e.g., $K\pi\pi$ and KKK . If the final-state pseudoscalar meson is restricted to be a kaon or a pion, then the allowed penguin-dominated three-body decays are $\bar{B}_s^0 \rightarrow K^0 K^- \pi^+$, $\bar{K}^0 K^+ \pi^-$, $K^+ K^- \pi^0$, and $K^0 \bar{K}^0 \pi^0$. Therefore, it is expected that the first two modes have the largest rates among three-body B_s decays. Tree-dominated decays are $\bar{B}_s^0 \rightarrow K^+ \pi^- \pi^0$, $K^0 \pi^+ \pi^-$, $K^0 \pi^0 \pi^0$, and $\bar{B}_s^0 \rightarrow K^+ K^- K^0$, $\bar{K}^0 K^0 K^0$.

This work is organized as follows. We outline the framework in Sec. II for the evaluation of the resonant and nonresonant contributions to three-body decays. Before proceeding to the numerical results, we discuss in Sec. III some model-independent flavor symmetry relations in which some of the $B_s \rightarrow PPP$ decays can be related to the $B_d \rightarrow PPP$ ones by U -spin symmetry. The calculated branching fractions and CP violation are presented in Sec. IV. Conclusions are given in Sec. V. Factorizable amplitudes of various $\bar{B}_s^0 \rightarrow PPP$ decays are shown in the Appendix.

II. FRAMEWORK

The evaluation of three-body decays of $B_{u,d}$ mesons has been studied in detail in [4]. We shall follow the same

framework based on the factorization approach to discuss the charmless three-body decays of the B_s meson. Here we shall recapitulate the main points.

Three-body decays of heavy mesons receive both resonant and nonresonant contributions. Consider the charmless three-body B_s decay $\bar{B}_s^0 \rightarrow P_1 P_2 P_3$. Under the factorization hypothesis, its decay amplitude consists of three distinct factorizable terms: (i) the current-induced process with a meson emission, $\langle \bar{B}_s^0 \rightarrow P_1 P_2 \rangle \times \langle 0 \rightarrow P_3 \rangle$, (ii) the transition process, $\langle \bar{B}_s^0 \rightarrow P_3 \rangle \times \langle 0 \rightarrow P_1 P_2 \rangle$, and (iii) the annihilation process, $\langle \bar{B}_s^0 \rightarrow 0 \rangle \times \langle 0 \rightarrow P_1 P_2 P_3 \rangle$, where $\langle A \rightarrow B \rangle$ denotes a $A \rightarrow B$ transition matrix element.

A. Nonresonant contributions

Dominant nonresonant contributions to tree-dominated three-body decays arise from the $b \rightarrow u$ tree transition

$\langle \bar{B}_s^0 \rightarrow P_1 P_2 \rangle \times \langle 0 \rightarrow P_3 \rangle$, which can be evaluated using heavy meson chiral perturbation theory (HMChPT) [5]. Take the decay $\bar{B}_s^0 \rightarrow K^0 K^- \pi^+$ as an example for illustration. Its factorizable amplitude is given in Eq. (A1). We first focus on nonresonant contributions. The $b \rightarrow u$ transition amplitude evaluated using HMChPT reads [5]

$$\begin{aligned} A_{\text{current-ind}}^{\text{HMChPT}} &\equiv \langle \pi^+(p_2) K^0(p_3) | (\bar{u}b)_{V-A} | \bar{B}_s^0 \rangle^{\text{NR}} \\ &\times \langle K^-(p_1) | (\bar{s}u)_{V-A} | 0 \rangle \\ &= -\frac{f_K}{2} [2m_1^2 r + (m_{B_s}^2 - s_{23} - m_1^2) \omega_+ \\ &\quad + (s_{12} - s_{13} - m_2^2 + m_3^2) \omega_-], \end{aligned} \quad (2.1)$$

with the form factors

$$\begin{aligned} \omega_+ &= -\frac{g f_{B^*} m_{B^*} \sqrt{m_{B_s} m_{B^*}}}{f_\pi^2 s_{12} - m_{B^*}^2} \left[1 - \frac{(p_{B_s} - p_3) \cdot p_3}{m_{B^*}^2} \right] + \frac{f_{B_s}}{2f_\pi^2}, \\ \omega_- &= \frac{g f_{B^*} m_{B^*} \sqrt{m_{B_s} m_{B^*}}}{f_\pi^2 s_{12} - m_{B^*}^2} \left[1 + \frac{(p_{B_s} - p_3) \cdot p_3}{m_{B^*}^2} \right], \\ r &= \frac{f_{B_s}}{2f_\pi^2} - \frac{f_{B_s}}{f_\pi^2} \frac{p_{B_s} \cdot (p_2 - p_3)}{(p_{B_s} - p_2 - p_3)^2 - m_{B_s}^2} + \frac{2g f_{B^*}}{f_\pi^2} \sqrt{\frac{m_{B_s}}{m_{B^*}}} \frac{(p_{B_s} - p_3) \cdot p_3}{s_{12} - m_{B^*}^2} \\ &\quad - \frac{4g^2 f_{B_s}}{f_\pi^2} \frac{m_{B_s} m_{B^*}}{(p_{B_s} - p_2 - p_3)^2 - m_{B_s}^2} \frac{p_3 \cdot p_2 - p_3 \cdot (p_{B_s} - p_3) p_2 \cdot (p_{B_s} - p_3) / m_{B_s}^2}{s_{12} - m_{B^*}^2}, \end{aligned} \quad (2.2)$$

where $s_{ij} \equiv (p_i + p_j)^2$. Likewise,

$$\begin{aligned} &\langle K^+(p_1) \pi^-(p_2) | (\bar{d}b)_{V-A} | \bar{B}_s^0 \rangle^{\text{NR}} \langle \bar{K}^0(p_3) | (\bar{s}d)_{V-A} | 0 \rangle \\ &= -\frac{f_K}{2} [2m_3^2 r' + (m_{B_s}^2 - s_{12} - m_3^2) \omega'_+ \\ &\quad + (s_{23} - s_{13} - m_2^2 + m_1^2) \omega'_-] \end{aligned} \quad (2.3)$$

for the decay $\bar{B}_s^0 \rightarrow \bar{K}^0 K^+ \pi^-$, where the form factors $\omega'_\pm$ and r' are obtained from $\omega_\pm$ and r , respectively, with the replacement $p_1 \leftrightarrow p_3$.

However, the predicted nonresonant rates due to the $B \rightarrow P_1 P_2$ transition alone already exceed the measured total branching fractions for the tree-dominated modes, e.g., $\bar{B}_d^0 \rightarrow \pi^- \pi^+ \pi^-$, $\bar{B}_d^0 \rightarrow \pi^- K^+ K^-$, and $\bar{B}_s^0 \rightarrow K^0 \pi^+ \pi^-$. For example, the nonresonant branching fraction of the last decay channel estimated using HMChPT is found to be of order 32×10^{-6} , which is even larger than the total branching fraction of order 14×10^{-6} [see Eq. (1.1)]. The issue has to do with the applicability of HMChPT. When it is applied to three-body decays, two of the final-state pseudoscalars have to be soft. If the soft meson result is assumed to be the same in the whole Dalitz plot, the decay rate will be greatly overestimated. To overcome this issue, we have proposed in [6] to parametrize the

momentum dependence of nonresonant amplitudes induced by the $b \rightarrow u$ transition in an exponential form

$$A_{\text{current-ind}} = A_{\text{current-ind}}^{\text{HMChPT}} e^{-\alpha_{\text{NR}} p_{B^*} \cdot (p_1 + p_2)} e^{i\phi_{12}}, \quad (2.4)$$

so that the HMChPT results are recovered in the soft pseudoscalar meson limit. The parameter $\alpha_{\text{NR}} = 0.081_{-0.009}^{+0.015} \text{ GeV}^{-2}$ [4] is fixed by the measured nonresonant rate in the tree-dominated decay $B^- \rightarrow \pi^+ \pi^- \pi^-$, where the nonresonant background due to the penguin diagram is suppressed by the smallness of penguin Wilson coefficients.

The other two types of nonresonant contributions to $\bar{B}_s^0 \rightarrow K^0 K^- \pi^+$ are

$$\begin{aligned} &\langle \pi^+(p_1) K^-(p_2) | (\bar{s}d)_{V-A} | 0 \rangle^{\text{NR}} \langle K^0(p_3) | (\bar{d}b)_{V-A} | \bar{B}_s^0 \rangle \\ &= F_1^{K\pi}(s_{12}) F_1^{B_s K}(s_{12}) \left[s_{23} - s_{13} - \frac{(m_{B_s}^2 - m_\pi^2)(m_K^2 - m_\pi^2)}{s_{12}} \right] \\ &\quad + F_0^{K\pi}(s_{12}) F_0^{B_s K}(s_{12}) \frac{(m_{B_s}^2 - m_\pi^2)(m_K^2 - m_\pi^2)}{s_{12}} \end{aligned} \quad (2.5)$$

and

$$\begin{aligned} & \langle \pi^+(p_1) K^-(p_2) | \bar{s} d | 0 \rangle^{\text{NR}} \langle K^0(p_3) | \bar{d} b | \bar{B}_s^0 \rangle \\ &= \frac{m_{B_s}^2 - m_K^2}{m_b - m_d} F_0^{B_s K}(s_{12}) \langle \pi^+(p_1) K^-(p_2) | \bar{s} d | 0 \rangle^{\text{NR}}. \end{aligned} \quad (2.6)$$

As stressed in [4,6], the matrix element of scalar densities $\langle M_1 M_2 | \bar{q}_1 q_2 | 0 \rangle$ must have a large nonresonant component in order to explain the large nonresonant signals observed in penguin-dominated three-body B decays. In other words, nonresonant contributions to penguin-dominated three-body B decays are also penguin dominated. From the study of $B_{u,d} \rightarrow KKK$ decays, it was found that [4]

$$\langle K^-(p_1) K^+(p_2) | \bar{s} s | 0 \rangle = \frac{v}{3} (3F_{\text{NR}} + 2F'_{\text{NR}}) + \sigma_{\text{NR}} e^{-\alpha s_{12}}, \quad (2.7)$$

where the parameters v , F_{NR} , F'_{NR} , and σ_{NR} are defined in [4] and $\alpha = (0.14 \pm 0.02) \text{ GeV}^{-2}$. However, if SU(3) symmetry is applied to the matrix element of scalar densities so that $\langle K^-(p_1) \pi^+(p_2) | \bar{s} d | 0 \rangle^{\text{NR}} = \langle K^-(p_1) K^+(p_2) | \bar{s} s | 0 \rangle^{\text{NR}}$, we found in [4] that the predicted $\mathcal{A}_{CP}(B^- \rightarrow \pi^- K^+ K^-)$ and $\mathcal{A}_{CP}(B^- \rightarrow K^- \pi^+ \pi^-)$ have

wrong signs when confronted with experiment. The correlations of the CP violation between the charged B decays shown in Eq. (1.2) have led to the conjecture that $\pi^+ \pi^- \leftrightarrow K^+ K^-$ rescattering may play an important role in the generation of the strong phase difference needed for such a violation to occur. It is thus plausible that a strong phase in $\langle K^- \pi^+ | \bar{s} d | 0 \rangle$ induced from final-state interactions might flip the sign of CP asymmetry. A fit to the data of $\bar{B}_d^0 \rightarrow K^- \pi^+ \pi^-$ yields [4]

$$\begin{aligned} & \langle K^-(p_1) \pi^+(p_2) | \bar{s} d | 0 \rangle^{\text{NR}} \\ & \approx \frac{v}{3} (3F_{\text{NR}} + 2F'_{\text{NR}}) + \sigma_{\text{NR}} e^{-\alpha s_{12}} e^{i\pi} \left(1 + 4 \frac{m_K^2 - m_\pi^2}{s_{12}} \right), \end{aligned} \quad (2.8)$$

with a strong phase of order 180° .

B. Resonant contributions

Resonant effects are commonly described in terms of the usual Breit-Wigner formalism. Contributions of vector meson and scalar resonances to the three-body and two-body matrix elements are given by [4]

$$\begin{aligned} \langle \pi^+(p_1) K^-(p_2) | (\bar{s} b)_{V-A} | \bar{B}^0 \rangle^R &= \sum_i \frac{g_{K_i^* \rightarrow K^- \pi^+}}{s_{12} - m_{K_i^*}^2 + im_{K_i^*} \Gamma_{K_i^*}} \sum_{\text{pol}} \epsilon^* \cdot (p_1 - p_2) \langle \bar{K}_i^{*0} | (\bar{s} b)_{V-A} | \bar{B}^0 \rangle \\ &\quad - \frac{g_{K_0^* \rightarrow K^- \pi^+}}{s_{12} - m_{K_0^*}^2 + im_{K_0^*} \Gamma_{K_0^*}} \langle \bar{K}_0^{*0} | (\bar{s} b)_{V-A} | \bar{B}^0 \rangle, \\ \langle \pi^+(p_1) K^-(p_2) | (\bar{s} d)_{V-A} | 0 \rangle^R &= \sum_i \frac{g_{K_i^* \rightarrow K^- \pi^+}}{s_{12} - m_{K_i^*}^2 + im_{K_i^*} \Gamma_{K_i^*}} \sum_{\text{pol}} \epsilon^* \cdot (p_1 - p_2) \langle \bar{K}_i^{*0} | (\bar{s} d)_{V-A} | 0 \rangle \\ &\quad - \frac{f_{K_0^*} g_{K_0^* \rightarrow K^- \pi^+}}{s_{12} - m_{K_0^*}^2 + im_{K_0^*} \Gamma_{K_0^*}} (p_1 + p_2)_\mu, \\ \langle \pi^+(p_1) K^-(p_2) | \bar{s} d | 0 \rangle^R &= - \frac{m_{K_0^*} \bar{f}_{K_0^*} g_{K_0^* \rightarrow K^- \pi^+}}{s_{12} - m_{K_0^*}^2 + im_{K_0^*} \Gamma_{K_0^*}}, \end{aligned} \quad (2.9)$$

with $K_i^* = K^*(892), K^*(1410), K^*(1680), \dots$, and $K_0^* = K_0^*(1430)$. In the above equations we have two different types of decay constants for the scale meson K_0^* defined by $\langle K_0^* | \bar{s} d | 0 \rangle = \bar{f}_{K_0^*} m_{K_0^*}$ and $\langle K_0^*(p) | \bar{s} \gamma_\mu d | 0 \rangle = f_{K_0^*} p_\mu$. They are related via

$$\bar{f}_{K_0^*} = \frac{m_{K_0^*}}{m_s(\mu) - m_d(\mu)} f_{K_0^*}. \quad (2.10)$$

Their values are given in [7].

III. U-SPIN SYMMETRY

Before proceeding to the numerical results in the next section, here we would like to discuss some

model-independent flavor symmetry relations in which some of the $B_s \rightarrow PPP$ decays can be related to $B_d \rightarrow PPP$ ones via U -spin symmetry. Hence these relations can be used to cross-check the dynamical calculations. As an example, the decay amplitudes of $\bar{B}_s^0 \rightarrow K^0 \pi^+ \pi^-$ and $\bar{B}_d^0 \rightarrow \bar{K}^0 K^+ K^-$ can be related to each other in the limit of U -spin symmetry:

$$\begin{aligned} A(\bar{B}_s^0 \rightarrow K^0 \pi^+ \pi^-) &= V_{ub}^* V_{ud} \langle K^0 \pi^+ \pi^- | O_d^u | \bar{B}_s^0 \rangle \\ &\quad + V_{cb}^* V_{cd} \langle K^0 \pi^+ \pi^- | O_d^c | \bar{B}_s^0 \rangle, \\ A(\bar{B}_d^0 \rightarrow \bar{K}^0 K^+ K^-) &= V_{ub}^* V_{us} \langle \bar{K}^0 K^+ K^- | O_s^u | \bar{B}_d^0 \rangle \\ &\quad + V_{cb}^* V_{cs} \langle \bar{K}^0 K^+ K^- | O_s^c | \bar{B}_d^0 \rangle, \end{aligned} \quad (3.1)$$

where the four-quark operator O_s is for the $b \rightarrow sq_1\bar{q}_2$ transition and O_d for the $b \rightarrow dq_1\bar{q}_2$ transition. The assumption of U -spin symmetry implies that under $d \leftrightarrow s$ transitions

$$\begin{aligned}\langle \bar{K}^0 K^+ K^- | O_s^u | \bar{B}_d^0 \rangle &= \langle K^0 \pi^+ \pi^- | O_d^u | \bar{B}_s^0 \rangle, \\ \langle \bar{K}^0 K^+ K^- | O_s^c | \bar{B}_d^0 \rangle &= \langle K^0 \pi^+ \pi^- | O_d^c | \bar{B}_s^0 \rangle,\end{aligned}\quad (3.2)$$

which can be checked from Eq. (A4) for $\bar{B}_s^0 \rightarrow K^0 \pi^+ \pi^-$ and Eq. (A4) of [6] for $\bar{B}_d^0 \rightarrow \bar{K}^0 K^+ K^-$. Using the relation for the Cabibbo-Kobayashi-Maskawa matrix [8]

$$\text{Im}(V_{ub}^* V_{ud} V_{cb} V_{cd}^*) = -\text{Im}(V_{ub}^* V_{us} V_{cb} V_{cs}^*), \quad (3.3)$$

it is straightforward to show that

$$\begin{aligned}&|A(\bar{B}_d^0 \rightarrow \bar{K}^0 K^+ K^-)|^2 - |A(B_d^0 \rightarrow K^0 K^- K^+)|^2 \\ &= |A(\bar{B}_s^0 \rightarrow K^0 \pi^+ \pi^-)|^2 - |A(B_s^0 \rightarrow \bar{K}^0 \pi^- \pi^+)|^2.\end{aligned}\quad (3.4)$$

Hence, U -spin symmetry leads to the relation

$$\frac{\mathcal{A}_{CP}(\bar{B}_s^0 \rightarrow K^0 \pi^+ \pi^-)}{\mathcal{A}_{CP}(\bar{B}_d^0 \rightarrow \bar{K}^0 K^+ K^-)} = -\frac{\Gamma(\bar{B}_d^0 \rightarrow \bar{K}^0 K^+ K^-)}{\Gamma(\bar{B}_s^0 \rightarrow K^0 \pi^+ \pi^-)}.\quad (3.5)$$

Therefore, we have the following U -spin symmetry relations [9]:

$$\begin{aligned}\mathcal{A}_{CP}(\bar{B}_s^0 \rightarrow K^0 K^- \pi^+) &= -\mathcal{A}_{CP}(\bar{B}_d^0 \rightarrow \bar{K}^0 K^+ \pi^-) \\ &\times \frac{\mathcal{B}(\bar{B}_d^0 \rightarrow \bar{K}^0 K^+ \pi^-) \tau(B_s^0)}{\mathcal{B}(\bar{B}_s^0 \rightarrow K^0 K^- \pi^+) \tau(B_d^0)}, \\ \mathcal{A}_{CP}(\bar{B}_s^0 \rightarrow \bar{K}^0 K^+ \pi^-) &= -\mathcal{A}_{CP}(\bar{B}_d^0 \rightarrow K^0 K^- \pi^+) \\ &\times \frac{\mathcal{B}(\bar{B}_d^0 \rightarrow K^0 K^- \pi^+) \tau(B_s^0)}{\mathcal{B}(\bar{B}_s^0 \rightarrow \bar{K}^0 K^+ \pi^-) \tau(B_d^0)}, \\ \mathcal{A}_{CP}(\bar{B}_s^0 \rightarrow K^0 \pi^+ \pi^-) &= -\mathcal{A}_{CP}(\bar{B}_d^0 \rightarrow \bar{K}^0 K^+ K^-) \\ &\times \frac{\mathcal{B}(\bar{B}_d^0 \rightarrow \bar{K}^0 K^+ K^-) \tau(B_s^0)}{\mathcal{B}(\bar{B}_s^0 \rightarrow K^0 \pi^+ \pi^-) \tau(B_d^0)}, \\ \mathcal{A}_{CP}(\bar{B}_s^0 \rightarrow K^0 K^+ K^-) &= -\mathcal{A}_{CP}(\bar{B}_d^0 \rightarrow \bar{K}^0 \pi^+ \pi^-) \\ &\times \frac{\mathcal{B}(\bar{B}_d^0 \rightarrow \bar{K}^0 \pi^+ \pi^-) \tau(B_s^0)}{\mathcal{B}(\bar{B}_s^0 \rightarrow K^0 K^+ K^-) \tau(B_d^0)}, \\ \mathcal{A}_{CP}(\bar{B}_s^0 \rightarrow K^0 K^0 \bar{K}^0) &= -\mathcal{A}_{CP}(\bar{B}_d^0 \rightarrow \bar{K}^0 \bar{K}^0 K^0) \\ &\times \frac{\mathcal{B}(\bar{B}_d^0 \rightarrow \bar{K}^0 \bar{K}^0 K^0) \tau(B_s^0)}{\mathcal{B}(\bar{B}_s^0 \rightarrow K^0 K^0 \bar{K}^0) \tau(B_d^0)}.\end{aligned}\quad (3.6)$$

Unlike the U -spin relation in two-body decays, namely [10],

$$\begin{aligned}\mathcal{A}_{CP}(\bar{B}_s^0 \rightarrow K^+ \pi^-) &= -\mathcal{A}_{CP}(\bar{B}_d^0 \rightarrow K^- \pi^+) \\ &\times \frac{\mathcal{B}(\bar{B}_d^0 \rightarrow K^- \pi^+) \tau(B_s^0)}{\mathcal{B}(\bar{B}_s^0 \rightarrow K^+ \pi^-) \tau(B_d^0)},\end{aligned}\quad (3.7)$$

which has been well tested with the recent LHCb measurement of CP violation in $\bar{B}_s^0 \rightarrow K^+ \pi^-$ [11], the relations in Eq. (3.6) cannot be checked by the present available data. Nevertheless, they can be tested by our dynamical calculations as shown below.

TABLE I. Branching fractions (in units of 10^{-6}) and direct CP asymmetries (in %) in $\bar{B}_s^0 \rightarrow PPP$ decays. Experimental results of branching fractions are taken from [1]. Theoretical errors correspond to the uncertainties in (i) α_{NR} , (ii) $F_0^{B_s K}$, σ_{NR} and $m_s(\mu) = (90 \pm 20)$ MeV at $\mu = 2.1$ GeV, and (iii) $\gamma = (69.7_{-2.8}^{+1.3})^\circ$.

Modes	$\mathcal{B}_{\text{theory}}$	$\mathcal{B}_{\text{expt}}$	$\mathcal{A}_{CP}$ (%)
$\bar{K}^0 K^+ \pi^-$	$35.3_{-0.2-9.8-0.0}^{+0.3+15.7+0.0}$		$-1.9_{-0.1-0.1-0.0}^{+0.1+0.1+0.0}$
$K^0 K^- \pi^+$	$36.7_{-0.2-9.0-0.1}^{+0.2+14.9+0.1}$		$4.6_{-0.3-1.1-0.1}^{+0.3+1.1+0.1}$
$(-)$			
$K^0 K^\mp \pi^\pm$	$72.0_{-0.2-13.3-0.1}^{+0.4+21.6+0.1}$	$73.6 \pm 5.7 \pm 6.9 \pm 3.0$	
$K^+ K^- \pi^0$	$19.1_{-0.0-4.5-0.0}^{+0.0+7.5+0.0}$		$3.3_{-0.0-1.5-0.1}^{+0.0+1.4+0.0}$
$K^0 \bar{K}^0 \pi^0$	$20.3_{-0.4-5.5-0.0}^{+0.3+8.7+0.0}$		$0.8_{-0.0-0.1-0.0}^{+0.0+0.1+0.0}$
$K^0 \pi^+ \pi^-$	$12.7_{-0.5-0.3-0.1}^{+0.5+0.5+0.1}$	$14.3 \pm 2.8 \pm 1.8 \pm 0.6$	$8.0_{-1.4-1.3-0.1}^{+0.9+1.1+0.0}$
$K^+ \pi^- \pi^0$	$16.9_{-0.4-1.9-0.0}^{+0.4+2.1+0.0}$		$0.6_{-0.5-0.3-0.0}^{+0.3+0.3+0.0}$
$K^0 \pi^0 \pi^0$	$0.48_{-0.01-0.06-0.01}^{+0.01+0.06+0.01}$		$-33.1_{-3.9-0.6-0.1}^{+5.6+0.6+0.1}$
$K^0 K^+ K^-$	$1.4_{-0.0-0.1-0.2}^{+0.0+0.3+0.2}$	$\in [0.2; 3.4]$	$-17.4_{-0.2-0.5-0.4}^{+0.1+0.7+0.4}$
$K_S K_S K_S$	$0.22_{-0.00-0.10-0.01}^{+0.00+0.07+0.01}$		$-13.4_{-0.1-0.4-0.2}^{+0.1+0.4+0.2}$

IV. BRANCHING FRACTIONS AND CP ASYMMETRIES

For numerical calculations, we shall use the input parameters given in [4] except the form factor $F_0^{B_s K}$. As discussed in [12], this form factor at $q^2 = 0$ calculated in the literature ranges from 0.23 to 0.31. We find that the form factor $F_0^{B_s K}(0) \approx 0.31$ gives a better description of the measured branching fractions of $\bar{B}_s^0 \rightarrow K^0 K^\mp \pi^\pm$ and $K^0 \pi^+ \pi^-$.

The calculated branching fractions and CP asymmetries are summarized in Table I. The theoretical errors shown there are from the uncertainties in (i) the parameter α_{NR} appearing in Eq. (2.7), which governs the momentum dependence of the nonresonant amplitude, (ii) the strange quark mass m_s for decay modes involving kaon(s), the form factor $F_0^{B_s K}$ with the uncertainty assigned to be 0.03, and the nonresonant parameter σ_{NR} given after Eq. (2.7), and (iii) the unitarity angle γ .

TABLE II. Predicted branching fractions (in units of 10^{-6}) of resonant and nonresonant (NR) contributions to $\bar{B}_s^0 \rightarrow \bar{K}^0 K^+ \pi^-$ and $K^0 K^- \pi^+$.

$\bar{B}_s^0 \rightarrow \bar{K}^0 K^+ \pi^-$		$\bar{B}_s^0 \rightarrow K^0 K^- \pi^+$	
$K^{*0} \bar{K}^0$	$1.5^{+0.0+2.4+0.0}_{-0.0-0.9-0.0}$	$\bar{K}^{*0} K^0$	$3.8^{+0.0+0.8+0.0}_{-0.0-0.7-0.0}$
$K^{*-} K^+$	$3.5^{+0.0+0.7+0.1}_{-0.0-0.7-0.1}$	$K^{*+} K^-$	$2.6^{+0.0+2.7+0.1}_{-0.0-1.1-0.1}$
$K_0^{*0}(1430) \bar{K}^0$	$0.6^{+0.0+0.9+0.0}_{-0.0-0.4-0.0}$	$\bar{K}_0^{*0}(1430) K^0$	$14.5^{+0.0+3.3+0.0}_{-0.0-2.9-0.0}$
$K_0^{*-}(1430) K^+$	$14.5^{+0.0+3.2+0.1}_{-0.0-2.9-0.1}$	$K_0^{*+}(1430) K^-$	$1.0^{+0.0+1.0+0.0}_{-0.0-0.4-0.0}$
NR	$23.8^{+0.2+9.9+0.0}_{-0.1-6.7-0.0}$	NR	$24.2^{+0.0+7.9+0.0}_{-0.0-5.1-0.0}$
Total	$35.3^{+0.3+15.7+0.0}_{-0.2-9.8-0.0}$	Total	$36.7^{+0.2+14.9+0.1}_{-0.2-9.0-0.1}$

A. Branching fractions

We see from Table I that the penguin-dominated modes $\bar{B}_s^0 \rightarrow K^0 K^- \pi^+$ and $\bar{K}^0 K^+ \pi^-$ have the largest rates among the three-body decays $\bar{B}_s^0 \rightarrow PPP$ with $P = K, \pi$. Resonant

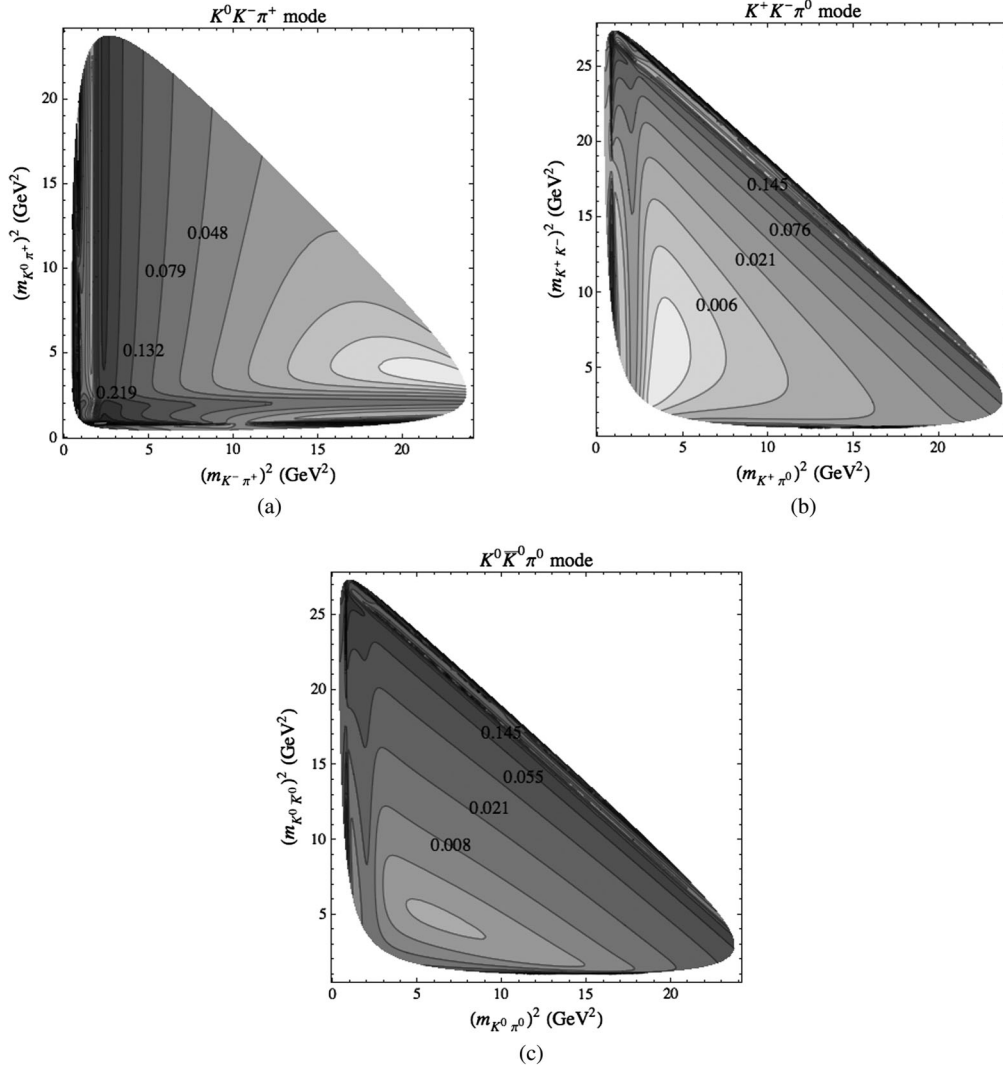


FIG. 1. Dalitz-plot distributions of penguin dominated $\bar{B}_s^0$ to (a) $K^0 K^- \pi^+$, (b) $K^+ K^- \pi^0$, and (c) $K^0 \bar{K}^0 \pi^0$ decays. The CP averaged differential rates are in units of 10^{-6} GeV^{-4} . Note that the contour lines are separated logarithmically.

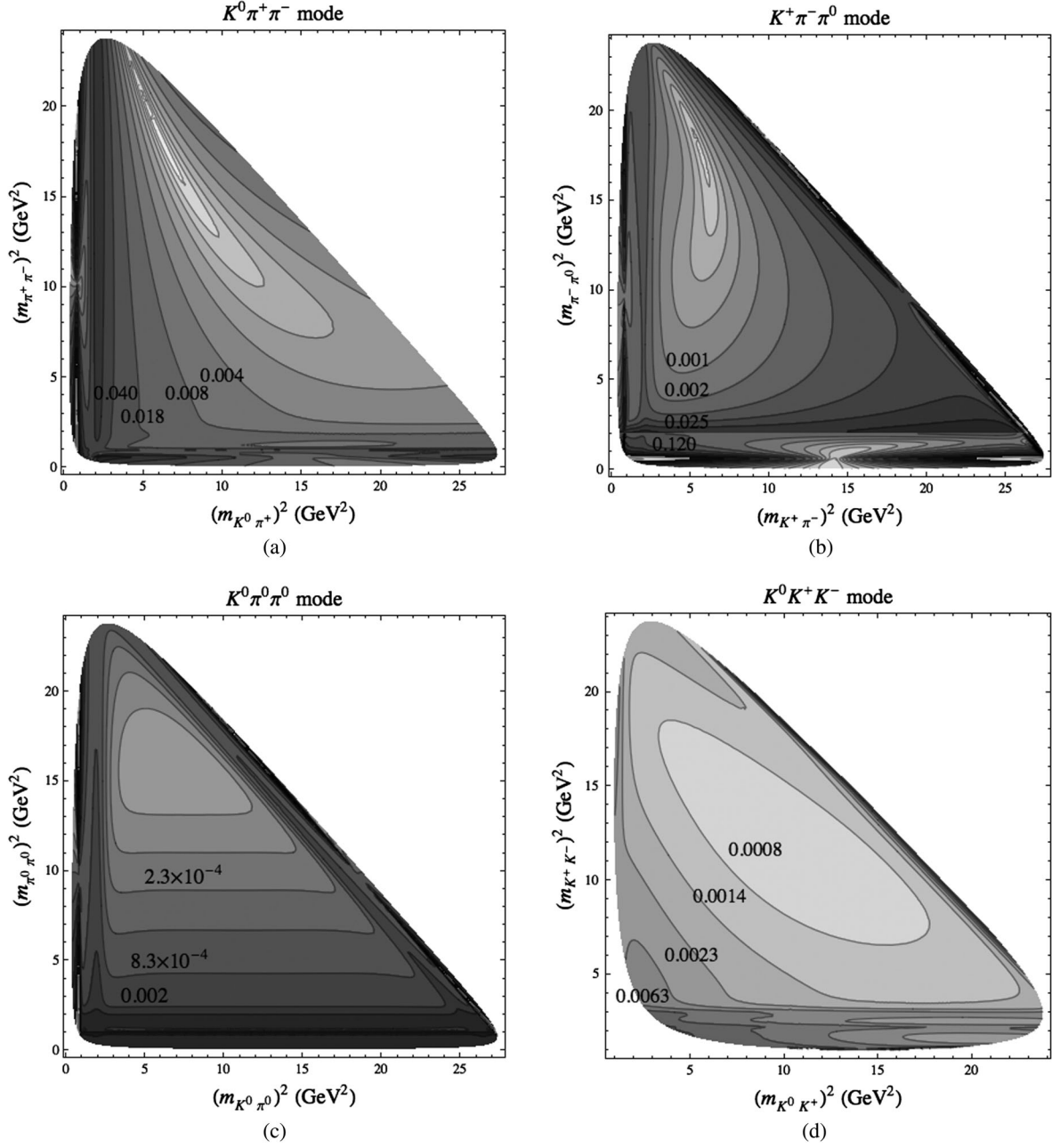


FIG. 2. Same as Fig. 1, but for tree-dominated $\bar{B}_s^0$ to (a) $K^0 \pi^+ \pi^-$, (b) $K^+ \pi^- \pi^0$, (c) $K^0 \pi^0 \pi^0$ and (d) $K^0 K^+ K^-$ decays.

and nonresonant contributions are shown in Table II.¹ It is evident that they are dominated by the $\bar{K}_0^{*0}(1430)$, $K_0^{*-}(1430)$ scalar resonances and nonresonant

¹Although in $\bar{B}_s^0 \rightarrow K_s K^\mp \pi^\pm$ decays, the rate of $\bar{B}_s^0 \rightarrow K_0^{*+}(1430) K^- \rightarrow K^0 K^- \pi^+$ is much smaller than that of $\bar{B}_s^0 \rightarrow K_0^{*-}(1430) K^+ \rightarrow \bar{K}^0 K^+ \pi^-$, it is the other way around in $\bar{B}_s^0 \rightarrow \pi^0 K^+ K^-$ decays: $\mathcal{B}(\bar{B}_s^0 \rightarrow K_0^{*+}(1430) K^- \rightarrow K^+ K^- \pi^0) \gg \mathcal{B}(\bar{B}_s^0 \rightarrow K_0^{*-}(1430) K^+ \rightarrow K^- K^+ \pi^0)$. Through the narrow width approximation, $\Gamma(B \rightarrow RP \rightarrow P_1 P_2 P) = \Gamma(B \rightarrow RP) \mathcal{B}(R \rightarrow P_1 P_2)$ with R being a resonance, it follows that the branching fraction of $\bar{B}_s^0 \rightarrow K_0^{*+}(1430) K^-$ is similar to that of $\bar{B}_s^0 \rightarrow K_0^{*-}(1430) K^+$.

contributions. The Dalitz-plot distribution shown in Fig. 1(a) also exhibits this feature. For the nonresonant contributions, we find if the nonresonant matrix element of scalar density given by Eq. (2.8) is applied to the B_s

meson, the total branching fraction of $\bar{B}_s^0 \rightarrow K^0 K^\mp \pi^\pm$ will be best accommodated with the parameter $\alpha \approx 0.10 \text{ GeV}^{-2}$.

For the channel $\bar{B}_s^0 \rightarrow K^0 K^+ K^-$, the current LHCb measurement of its branching fraction lies in the range $(0.2-3.4) \times 10^{-6}$, while our prediction 1.4×10^{-6} is in the middle. We see that both $\bar{B}_s^0 \rightarrow K^+ K^- \pi^0$ and $\bar{B}_s^0 \rightarrow K^0 \bar{K}^0 \pi^0$ have very similar rates, as it should be.

TABLE III. Direct CP asymmetries in $\bar{B}_s^0 \rightarrow PPP$ decays via U -spin symmetry. Theoretical predictions of branching fractions of $\bar{B}_d^0 \rightarrow \bar{K}^0 K^+ \pi^-$, $K^0 \pi^+ K^-$ and CP asymmetries for $\bar{B}_d^0 \rightarrow PPP$ are taken from [4], while branching fractions of $\bar{B}_d^0 \rightarrow \bar{K}^0 K^+ K^-$ and $\bar{K}^0 \pi^+ \pi^-$ are quoted from [13,14] and [1,15,16], respectively. Only the central values are given for CP asymmetries derived from U -spin symmetry.

Modes	$\mathcal{B}(10^{-6})$	$\mathcal{A}_{CP}(\%)$	Modes	$\mathcal{A}_{CP}(\%)$ (U -spin)	$\mathcal{A}_{CP}(\%)$ (theory)
$\bar{B}_d^0 \rightarrow \bar{K}^0 K^+ \pi^-^a$	$3.9^{+1.4}_{-0.9}$	$-9.4^{+1.7+1.5+0.0}_{-2.9-2.5-0.0}$	$\bar{B}_s^0 \rightarrow K^0 K^- \pi^+$	0.96	$4.6^{+0.3+1.1+0.1}_{-0.3-1.1-0.1}$
$\bar{B}_d^0 \rightarrow K^0 K^- \pi^+^a$	$0.8^{+1.4}_{-0.9}$	$-12.7^{+1.7+1.5+0.0}_{-2.9-2.5-0.0}$	$\bar{B}_s^0 \rightarrow \bar{K}^0 K^+ \pi^-$	0.28	$-1.9^{+0.1+0.1+0.0}_{-0.1-0.1-0.0}$
$\bar{B}_d^0 \rightarrow \bar{K}^0 K^+ K^-$	23.9 ± 1.0	$-5.5^{+1.4+0.5+0.1}_{-1.0-0.5-0.1}$	$\bar{B}_s^0 \rightarrow K^0 \pi^+ \pi^-$	10.1	$8.1^{+0.9+1.0+0.0}_{-0.9-0.9-0.0}$
$\bar{B}_d^0 \rightarrow \bar{K}^0 \pi^+ \pi^-$	49.6 ± 2.1	$-0.8^{+0.0+0.1+0.0}_{-0.0-0.1-0.0}$	$\bar{B}_s^0 \rightarrow K^0 K^+ K^-$	15.9	$-17.4^{+0.1+0.7+0.4}_{-0.2-0.5-0.4}$
$\bar{B}_d^0 \rightarrow K_S K_S K_S$	$6.19^{+1.62}_{-1.42}$	$0.7^{+0.0+0.0+0.0}_{-0.0-0.0-0.0}$	$\bar{B}_s^0 \rightarrow K_S K_S K_S$	-20.8	$-13.4^{+0.1+0.4+0.2}_{-0.1-0.4-0.2}$

^aBranching fractions and CP asymmetries of $\bar{B}_d^0 \rightarrow K^0 K^\pm \pi^\mp$ are calculated in [4], but not for the individual $\bar{K}^0 K^+ \pi^-$ and $K^0 K^- \pi^+$ modes. Here we have corrected some typos in the computer code for $\bar{B}_d^0 \rightarrow K^0 K^\pm \pi^\mp$ and updated the results.

As for the tree-dominated mode $\bar{B}_s^0 \rightarrow K^0 \pi^+ \pi^-$, our prediction of its rate is consistent with experiment within errors. See Fig. 2 for the Dalitz-plot distributions of tree-dominated modes.

B. Direct CP asymmetries

There are six U -spin relations exhibited in Eq. (3.6). They cannot be tested at present due to the lack of experimental data. Nevertheless, since the U -spin symmetry breaking is already included in the model calculations, we can check quantitatively how good the symmetry is. In Table III we show some of the direct CP asymmetries in B_s decays evaluated using the U -spin relations Eq. (3.6) and theoretical inputs for the branching fractions of $\bar{B}_s^0 \rightarrow PPP$ decays and CP asymmetries of $\bar{B}_d^0 \rightarrow PPP$. We see that in general $\mathcal{A}_{CP}$ obtained by U -spin symmetry are not in good agreement with the direct calculation based on factorization. In particular, the signs of the calculated CP rate asymmetries $\mathcal{A}_{CP}(\bar{B}_s^0 \rightarrow K^0 K^+ K^-)$ and $\mathcal{A}_{CP}(\bar{B}_s^0 \rightarrow \bar{K}^0 K^+ \pi^-)$ are opposite to what is expected from the U -spin symmetry argument. This has to do with sizable U -spin symmetry breaking in some matrix elements. For example, the matrix element $\langle \pi^- \bar{K}^0 | \bar{s} u | 0 \rangle$ appearing in the decay amplitude of $\bar{B}_s^0 \rightarrow \bar{K}^0 K^+ \pi^-$ should be the same as the one $\langle K^- K^0 | \bar{d} u | 0 \rangle$ in $\bar{B}_s^0 \rightarrow K^0 \pi^+ K^-$ under U -spin symmetry. However, from Eq. (2.7) we see that in reality the U -spin symmetry that relates the matrix elements of $\langle K^0 K | \bar{d} q | 0 \rangle$ to $\langle \bar{K}^0 \pi | \bar{s} q | 0 \rangle$ is badly broken. Therefore, it is not surprising that the U -spin relations are generally not well respected.

As mentioned in the Introduction, LHCb has observed interesting correlations of the CP violation between the three-body decays of charged B mesons given in Eq. (1.2). It will be interesting to see experimentally if the analog relation $\mathcal{A}_{CP}(\bar{B}_s^0 \rightarrow K^0 \pi^+ \pi^-) \approx -\mathcal{A}_{CP}(\bar{B}_s^0 \rightarrow K^0 K^+ K^-)$ also holds in the B_s sector. Our model calculation (see Table I) indeed shows an opposite sign of CP rate asymmetries between the above-mentioned two modes.

Besides inclusive CP violation, LHCb has also observed large asymmetries in localized regions of phase space [2,3].

Again, it is important to check if significant signatures of CP violation occur in three-body B_s decays in the low mass regions devoid of most of the known resonances. If the phase space of the decay $\bar{B}_s^0 \rightarrow K^+ K^- \pi^0$ is restricted to the mass region $m_{K^+ K^-}^2 < 1.5 \text{ GeV}^2$ as it has been done for the LHCb measurement of the regional CP violation in $B^- \rightarrow K^+ K^- \pi^-$, we find $\mathcal{A}_{CP}^{\text{region}}(\bar{B}_s^0 \rightarrow K^+ K^- \pi^0) = (29.6^{+0.5+1.4+0.1}_{-0.4-1.7-0.2})\%$ which is greater than the integrated inclusive CP asymmetry of order 0.9% (cf. Table I).

V. CONCLUSIONS

We have presented in this work a study of charmless three-body B_s decays within the framework of a simple model based on factorization. Our main results are

- (i) The penguin-dominated decays $\bar{B}_s^0 \rightarrow K^0 K^- \pi^+$ and $\bar{K}^0 K^+ \pi^-$ have the largest rates among the three-body decays $\bar{B}_s^0 \rightarrow PPP$ with P being a kaon or pion, and they are dominated by the $K_0^*(1430)$ resonances and nonresonant contributions. The decays $\bar{B}_s^0 \rightarrow K^+ K^- \pi^0$ and $K^0 \bar{K}^0 \pi^0$ have similar rates, and their branching fractions are of order 20×10^{-6} .
- (ii) The branching fraction of $\bar{B}_s^0 \rightarrow K^0 K^+ K^-$ is predicted to be 1.4×10^{-6} , which is in the middle of the measured region $(0.2-3.4) \times 10^{-6}$ obtained by LHCb.
- (iii) Several model-independent relations based on U -spin symmetry are derived. Although they cannot be tested by the present available data, they can be checked by the dynamical calculations as shown in Table III. Because the U -spin symmetry that relates the matrix elements of scalar densities, e.g., $\langle K \bar{K} | \bar{q}_1 q_2 | 0 \rangle$ and $\langle K \pi | \bar{s} q | 0 \rangle$, is badly broken, U -spin symmetry relations for CP violation in three-body B_s decays are generally not well respected.
- (iv) We found sizable CP asymmetries in $K^0 \pi^+ \pi^-$, $K^0 \pi^0 \pi^0$, $K^0 K^+ K^-$, and $K_S K_S K_S$ channels. Just as the B_u sector, the CP asymmetries of $\bar{B}_s^0 \rightarrow K^0 \pi^+ \pi^-$ and $\bar{B}_s^0 \rightarrow K^0 K^+ K^-$ have similar magnitudes but are opposite in sign.

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APPENDIX: Factorizable amplitudes of $B_s \rightarrow PPP$ decays

In the following we list the factorizable amplitudes of various $\bar{B}_s^0 \rightarrow PPP$ decays:

$$\begin{aligned}
\langle K^0 K^- \pi^+ | T_p | \bar{B}_s^0 \rangle &= \langle \pi^+ K^0 | (\bar{u}b)_{V-A} | \bar{B}_s^0 \rangle \langle K^- | (\bar{s}u)_{V-A} | 0 \rangle [a_1 \delta_{pu} + a_4^p + a_{10}^p - (a_6^p + a_8^p) r_\chi^K] \\
&\quad + \langle K^0 | (\bar{d}b)_{V-A} | \bar{B}_s^0 \rangle \langle \pi^+ K^- | (\bar{s}d)_{V-A} | 0 \rangle \left[a_4^p - \frac{1}{2} a_{10}^p \right] + \langle K^0 | \bar{d}b | \bar{B}_s^0 \rangle \langle \pi^+ K^- | \bar{s}d | 0 \rangle (-2a_6^p + a_8^p) \\
&\quad + \langle K^0 K^- \pi^+ | (\bar{u}u)_{V-A} | 0 \rangle \langle 0 | (\bar{s}b)_{V-A} | \bar{B}_s^0 \rangle (a_2 \delta_{pu} + a_3 + a_9) \\
&\quad + \langle K^0 K^- \pi^+ | (\bar{u}u)_{V+A} | 0 \rangle \langle 0 | (\bar{s}b)_{V-A} | \bar{B}_s^0 \rangle (a_5 + a_7) \\
&\quad + \langle K^0 K^- \pi^+ | \bar{s}(1 + \gamma_5)d | 0 \rangle \langle 0 | \bar{s}\gamma_5 b | \bar{B}_s^0 \rangle (2a_6^p - a_8^p), \tag{A1}
\end{aligned}$$

$$\begin{aligned}
\langle \bar{K}^0 K^+ \pi^- | T_p | \bar{B}_s^0 \rangle &= \langle \pi^- K^+ | (\bar{d}b)_{V-A} | \bar{B}_s^0 \rangle \langle \bar{K}^0 | (\bar{s}d)_{V-A} | 0 \rangle \left[a_4^p - \frac{1}{2} a_{10}^p - \left(a_6^p - \frac{1}{2} a_8^p \right) r_\chi^K \right] \\
&\quad + \langle K^+ | (\bar{u}b)_{V-A} | \bar{B}_s^0 \rangle \langle \pi^- \bar{K}^0 | (\bar{s}u)_{V-A} | 0 \rangle [a_1 \delta_{pu} + a_4^p + a_{10}^p] + \langle K^+ | \bar{u}b | \bar{B}_s^0 \rangle \langle \pi^- \bar{K}^0 | \bar{s}u | 0 \rangle (-2a_6^p - 2a_8^p) \\
&\quad + \langle \bar{K}^0 K^+ \pi^- | (\bar{u}u)_{V-A} | 0 \rangle \langle 0 | (\bar{s}b)_{V-A} | \bar{B}_s^0 \rangle (a_2 \delta_{pu} + a_3 + a_9) \\
&\quad + \langle \bar{K}^0 K^+ \pi^- | (\bar{u}u)_{V+A} | 0 \rangle \langle 0 | (\bar{s}b)_{V-A} | \bar{B}_s^0 \rangle (a_5 + a_7) + \langle \bar{K}^0 K^+ \pi^- | \bar{s}(1 + \gamma_5)d | 0 \rangle \langle 0 | \bar{s}\gamma_5 b | \bar{B}_s^0 \rangle (2a_6^p - a_8^p), \tag{A2}
\end{aligned}$$

$$\begin{aligned}
\langle \pi^0 K^+ K^- | T_p | \bar{B}_s^0 \rangle &= \langle K^+ \pi^0 | (\bar{u}b)_{V-A} | \bar{B}_s^0 \rangle \langle K^- | (\bar{s}u)_{V-A} | 0 \rangle [a_1 \delta_{pu} + a_4^p + a_{10}^p - (a_6^p + a_8^p) r_\chi^K] \\
&\quad + \langle K^+ K^- | (\bar{s}b)_{V-A} | \bar{B}_s^0 \rangle \langle \pi^0 | (\bar{u}u)_{V-A} | 0 \rangle [a_2 \delta_{pu} + a_3 - a_5 - a_7 + a_9] \\
&\quad + \langle K^+ K^- | (\bar{s}b)_{V-A} | \bar{B}_s^0 \rangle \langle \pi^0 | (\bar{d}d)_{V-A} | 0 \rangle \left[a_3 - a_5 + \frac{1}{2} (a_7 - a_9) \right] \\
&\quad + \langle K^+ | (\bar{u}b)_{V-A} | \bar{B}_s^0 \rangle \langle K^- \pi^0 | (\bar{s}u)_{V-A} | 0 \rangle [a_1 \delta_{pu} + a_4^p + a_{10}^p] + \langle K^+ | \bar{u}b | \bar{B}_s^0 \rangle \langle K^- \pi^0 | \bar{s}u | 0 \rangle (-2a_6^p - 2a_8^p) \\
&\quad + \langle \pi^0 K^+ K^- | (\bar{s}s)_{V-A} | 0 \rangle \langle 0 | (\bar{s}b)_{V-A} | \bar{B}_s^0 \rangle \left[a_3 + a_4^p - \frac{1}{2} (a_9 + a_{10}^p) \right] \\
&\quad + \langle \pi^0 K^+ K^- | (\bar{s}s)_{V+A} | 0 \rangle \langle 0 | (\bar{s}b)_{V-A} | \bar{B}_s^0 \rangle \left[a_5 - \frac{1}{2} a_7 \right], \tag{A3}
\end{aligned}$$

$$\begin{aligned}
\langle K^0 \pi^+ \pi^- | T_p | \bar{B}_s^0 \rangle &= \langle \pi^+ K^0 | (\bar{u}b)_{V-A} | \bar{B}_s^0 \rangle \langle \pi^- | (\bar{d}u)_{V-A} | 0 \rangle [a_1 \delta_{pu} + a_4^p + a_{10}^p - (a_6^p + a_8^p) r_\chi^K] \\
&\quad + \langle \pi^+ \pi^- | (\bar{s}b)_{V-A} | \bar{B}_s^0 \rangle \langle K^0 | (\bar{d}s)_{V-A} | 0 \rangle \left[a_4^p - \frac{1}{2} a_{10}^p - \left(a_6^p - \frac{1}{2} a_8^p \right) r_\chi^K \right] \\
&\quad + \langle K^0 | (\bar{d}b)_{V-A} | \bar{B}_s^0 \rangle \langle \pi^+ \pi^- | (\bar{u}u)_{V-A} | 0 \rangle (a_2 \delta_{pu} + a_3 + a_5 + a_7 + a_9) \\
&\quad + \langle K^0 | (\bar{d}b)_{V-A} | \bar{B}_s^0 \rangle \langle \pi^+ \pi^- | (\bar{d}d)_{V-A} | 0 \rangle \left[a_3 + a_4^p + a_5 - \frac{1}{2} (a_7 + a_9 + a_{10}^p) \right] \\
&\quad + \langle K^0 | (\bar{d}b)_{V-A} | \bar{B}_s^0 \rangle \langle \pi^+ \pi^- | (\bar{s}s)_{V-A} | 0 \rangle \left[a_3 + a_5 - \frac{1}{2} (a_7 + a_9) \right] + \langle K^0 | \bar{d}b | \bar{B}_s^0 \rangle \langle \pi^+ \pi^- | \bar{d}d | 0 \rangle (-2a_6^p + a_8^p) \\
&\quad + \langle \pi^+ \pi^- K^0 | (\bar{d}s)_{V-A} | 0 \rangle \langle 0 | (\bar{s}b)_{V-A} | \bar{B}_s^0 \rangle \left[a_4^p - \frac{1}{2} a_{10}^p \right] + \langle \pi^+ \pi^- K^0 | \bar{d}(1 + \gamma_5)s | 0 \rangle \langle 0 | \bar{s}\gamma_5 b | \bar{B}_s^0 \rangle (2a_6^p - a_8^p), \tag{A4}
\end{aligned}$$

$$\begin{aligned}
\langle K^0 K^+ K^- | T_p | \bar{B}_s^0 \rangle = & \langle K^+ K^- | (\bar{s}b)_{V-A} | \bar{B}_s^0 \rangle \langle K^0 | (\bar{d}s)_{V-A} | 0 \rangle \left[a_4^p - \frac{1}{2} a_{10}^p - \left(a_6^p - \frac{1}{2} a_8^p \right) r_\chi^K \right] \\
& + \langle K^0 | (\bar{d}b)_{V-A} | \bar{B}_s^0 \rangle \langle K^+ K^- | (\bar{u}u)_{V-A} | 0 \rangle [a_2 \delta_{pu} + a_3 + a_5 + a_7 + a_9] \\
& + \langle K^0 | (\bar{d}b)_{V-A} | \bar{B}_s^0 \rangle \langle K^+ K^- | (\bar{d}d)_{V-A} | 0 \rangle \left[a_3 + a_4^p + a_5 - \frac{1}{2} (a_7 + a_9 + a_{10}^p) \right] \\
& + \langle K^0 | (\bar{d}b)_{V-A} | \bar{B}_s^0 \rangle \langle K^+ K^- | (\bar{s}s)_{V-A} | 0 \rangle \left[a_3 + a_5 - \frac{1}{2} (a_7 + a_9) \right] \\
& + \langle K^+ | (\bar{u}b)_{V-A} | \bar{B}_s^0 \rangle \langle K^0 K^- | (\bar{d}u)_{V-A} | 0 \rangle [a_1 + a_4^p + a_{10}^p] + \langle K^0 | \bar{d}b | \bar{B}_s^0 \rangle \langle K^+ K^- | \bar{d}d | 0 \rangle (-2a_6^p + a_8^p) \\
& + \langle K^+ | \bar{u}b | \bar{B}_s^0 \rangle \langle K^0 K^- | \bar{d}u | 0 \rangle (-2a_6^p - 2a_8^p) + \langle K^0 K^+ K^- | (\bar{d}s)_{V-A} | 0 \rangle \langle 0 | (\bar{s}b)_{V-A} | \bar{B}_s^0 \rangle \left[a_4^p - \frac{1}{2} a_{10}^p \right] \\
& + \langle K^0 K^+ K^- | \bar{d}(1 + \gamma_5)s | 0 \rangle \langle 0 | \bar{s}\gamma_5 b | \bar{B}_s^0 \rangle (2a_6^p - a_8^p), \tag{A5}
\end{aligned}$$

$$\begin{aligned}
\langle \bar{K}^0 K^0 K^0 | T_p | \bar{B}_s^0 \rangle = & \langle \bar{K}^0 K^0 | (\bar{s}b)_{V-A} | \bar{B}_s^0 \rangle \langle K^0 | (\bar{d}s)_{V-A} | 0 \rangle \left(a_4^p - \frac{1}{2} a_{10}^p - \left(a_6^p - \frac{1}{2} a_8^p \right) r_\chi^K \right) \\
& + \langle K^0 | (\bar{d}b)_{V-A} | \bar{B}_s^0 \rangle \langle K^0 \bar{K}^0 | (\bar{u}u)_{V-A} | 0 \rangle [a_2 \delta_{pu} + a_3 + a_5 + a_7 + a_9] \\
& + \langle K^0 | (\bar{d}b)_{V-A} | \bar{B}_s^0 \rangle \langle K^0 \bar{K}^0 | (\bar{d}d)_{V-A} | 0 \rangle \left[a_3 + a_4^p + a_5 - \frac{1}{2} (a_7 + a_9 + a_{10}^p) \right] \\
& + \langle K^0 | (\bar{d}b)_{V-A} | \bar{B}_s^0 \rangle \langle K^0 \bar{K}^0 | (\bar{s}s)_{V-A} | 0 \rangle \left[a_3 + a_5 - \frac{1}{2} (a_7 + a_9) \right] \\
& + \langle K^0 | \bar{d}b | \bar{B}_s^0 \rangle \langle K^0 \bar{K}^0 | \bar{d}d | 0 \rangle (-2a_6^p + a_8^p) + \langle \bar{K}^0 K^0 K^0 | (\bar{d}s)_{V-A} | 0 \rangle \langle 0 | (\bar{s}b)_{V-A} | \bar{B}_s^0 \rangle \left[a_4^p - \frac{1}{2} a_{10}^p \right] \\
& + \langle \bar{K}^0 K^0 K^0 | \bar{d}(1 + \gamma_5)s | 0 \rangle \langle 0 | \bar{s}\gamma_5 b | \bar{B}_s^0 \rangle (2a_6^p - a_8^p), \tag{A6}
\end{aligned}$$

$$\begin{aligned}
\langle K^+ \pi^- \pi^0 | T_p | \bar{B}_s^0 \rangle = & \langle K^+ \pi^0 | (\bar{u}b)_{V-A} | \bar{B}_s^0 \rangle \langle \pi^- | (\bar{d}u)_{V-A} | 0 \rangle [a_1 \delta_{pu} + a_4^p + a_{10}^p - (a_6^p + a_8^p) r_\chi^\pi] \\
& + \langle \pi^- K^+ | (\bar{d}b)_{V-A} | \bar{B}_s^0 \rangle \langle \pi^0 | (\bar{u}u)_{V-A} | 0 \rangle [a_2 \delta_{pu} + a_3 - a_5 - a_7 + a_9] \\
& + \langle \pi^- K^+ | (\bar{d}b)_{V-A} | \bar{B}_s^0 \rangle \langle \pi^0 | (\bar{d}d)_{V-A} | 0 \rangle \left[a_3 - a_5 + \frac{1}{2} (a_7 - a_9) \right] \\
& + \langle K^+ | (\bar{u}b)_{V-A} | \bar{B}_s^0 \rangle \langle \pi^- \pi^0 | (\bar{d}u)_{V-A} | 0 \rangle [a_1 \delta_{pu} + a_4^p + a_{10}^p] \\
& + \langle K^+ | \bar{u}b | \bar{B}_s^0 \rangle \langle \pi^- \pi^0 | \bar{d}u | 0 \rangle (-2a_6^p - 2a_8^p) + \langle K^+ \pi^- \pi^0 | (\bar{d}s)_{V-A} | 0 \rangle \langle 0 | (\bar{s}b)_{V-A} | \bar{B}_s^0 \rangle \left[a_4^p - \frac{1}{2} a_{10}^p \right] \\
& + \langle K^+ \pi^- \pi^0 | \bar{d}(1 + \gamma_5)s | 0 \rangle \langle 0 | \bar{s}\gamma_5 b | \bar{B}_s^0 \rangle (2a_6^p - a_8^p), \tag{A7}
\end{aligned}$$

$$\begin{aligned}
\langle K^0 \pi^0 \pi^0 | T_p | \bar{B}_s^0 \rangle = & \langle K^0 \pi^0 | (\bar{d}b)_{V-A} | \bar{B}_s^0 \rangle \langle \pi^0 | (\bar{u}u)_{V-A} | 0 \rangle [a_2 \delta_{pu} + a_3 - a_5 - a_7 + a_9] \\
& \times \langle K^0 \pi^0 | (\bar{d}b)_{V-A} | \bar{B}_s^0 \rangle \langle \pi^0 | (\bar{d}d)_{V-A} | 0 \rangle \left[a_3 - a_5 + \frac{1}{2} (a_7 - a_9) \right] \\
& + \langle \pi^0 \pi^0 | (\bar{u}u)_{V-A} | 0 \rangle \langle K^0 | (\bar{d}b)_{V-A} | \bar{B}_s^0 \rangle [a_2 \delta_{pu} + a_3 + a_5 + a_7 + a_9] \\
& + \langle \pi^0 \pi^0 | (\bar{d}d)_{V-A} | 0 \rangle \langle K^0 | (\bar{d}b)_{V-A} | \bar{B}_s^0 \rangle \left[a_3 + a_4^p + a_5 - \frac{1}{2} (a_7 + a_9 + a_{10}^p) \right] \\
& + \langle \pi^0 \pi^0 | \bar{d}d | 0 \rangle \langle K^0 | \bar{d}b | \bar{B}_s^0 \rangle (-2a_6^p + a_8^p) + \langle K^0 \pi^0 \pi^0 | (\bar{d}s)_{V-A} | 0 \rangle \langle 0 | (\bar{s}b)_{V-A} | \bar{B}_s^0 \rangle \left[a_4^p - \frac{1}{2} a_{10}^p \right] \\
& + \langle K^0 \pi^0 \pi^0 | \bar{d}(1 + \gamma_5)s | 0 \rangle \langle 0 | \bar{s}\gamma_5 b | \bar{B}_s^0 \rangle (2a_6^p - a_8^p), \tag{A8}
\end{aligned}$$

$$\begin{aligned}
\langle \pi^0 K^0 \bar{K}^0 | T_p | \bar{B}_s^0 \rangle &= \langle K^0 \bar{K}^0 | (\bar{s}b)_{V-A} | \bar{B}_s^0 \rangle \langle \pi^0 | (\bar{u}u)_{V-A} | 0 \rangle [a_2 \delta_{pu} + a_3 - a_5 - a_7 + a_9] \\
&+ \langle K^0 \bar{K}^0 | (\bar{s}b)_{V-A} | \bar{B}_s^0 \rangle \langle \pi^0 | (\bar{d}d)_{V-A} | 0 \rangle \left[a_3 - a_5 + \frac{1}{2}(a_7 - a_9) \right] \\
&+ \langle \bar{K}^0 | (\bar{s}d)_{V-A} | 0 \rangle \langle K^0 \pi^0 | (\bar{d}b)_{V-A} | \bar{B}_s^0 \rangle \left[a_4^p - \frac{1}{2}a_{10}^p - \left(a_6^p - \frac{1}{2}a_8^p \right) r_{\chi}^K \right] \\
&+ \langle \bar{K}^0 \pi^0 | (\bar{s}d)_{V-A} | 0 \rangle \langle K^0 | (\bar{d}b)_{V-A} | \bar{B}_s^0 \rangle \left[a_4^p - \frac{1}{2}a_{10}^p \right] + \langle \bar{K}^0 \pi^0 | \bar{s}d | 0 \rangle \langle K^0 | \bar{d}b | \bar{B}_s^0 \rangle (-2a_6^p + a_8^p) \\
&+ \langle \pi^0 K^0 \bar{K}^0 | (\bar{s}s)_{V-A} | 0 \rangle \langle 0 | (\bar{s}b)_{V-A} | \bar{B}_s^0 \rangle \left[a_3 + a_4^p - \frac{1}{2}(a_9 + a_{10}^p) \right] \\
&+ \langle \pi^0 K^0 \bar{K}^0 | (\bar{s}s)_{V+A} | 0 \rangle \langle 0 | (\bar{s}b)_{V-A} | \bar{B}_s^0 \rangle \left[a_5 - \frac{1}{2}a_7 \right]. \tag{A9}
\end{aligned}$$

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