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Quark-hadron thermodynamics in a magnetic field

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Nonperturbative treatment of the quark-hadron transition at nonzero temperature T and chemical potential μ in the framework of the field correlator method is generalized to the case of nonzero magnetic field B. A compact form of the quark pressure for arbitrary B, μ, T is derived. As a result, the transition temperature is found as a function of B and μ, which depends only on the following parameters: the vacuum gluonic condensate G_2 and the field correlator $D_1^E(x)$, which defines the Polyakov loops and is known both analytically and on the lattice. A moderate (25%) decrease of $T_c(\mu = 0)$ for eB changing from zero to 1 GeV^2 is found. A sequence of transition curves in the (μ, T) plane is obtained for B in the same interval, monotonically decreasing in scale for growing B.

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I. INTRODUCTION

Strong magnetic fields (m.f.) are now a subject of numerous studies [1–8], since they can be present in different physical systems. Namely, in cosmology m.f. of the order of 10^{18} Gauss or higher can occur during strong and electroweak phase transitions [1,2]. In noncentral heavy ion collisions one can expect m.f. $O(10^{18} - 10^{21})$ Gauss [3–6], while in some classes of neutron stars m.f. can reach the magnitude of 10^{14} Gauss, or even more in the cental regions [7]. All this makes it necessary to study the effects of strong m.f. in all possible physical situations and using different methods (for a recent review see [8]).

One of most interesting aspects of strong m.f. is their influence on the QCD hadron-quark phase transition, which can occur both in astrophysics (neutron stars) and heavy ion experiments. On the theoretical side, many model QCD calculations have predicted the increase of the critical temperature T_c with growing B [9–35], and only a few obtained an opposite result [36,37] (see [38,39] for reviews and additional references). Recently, the lattice data of [40] with physical pion mass and extrapolated to a continuum have demonstrated the decreasing critical temperature as a function of B. This phenomenon was called the inverse magnetic catalysis, and further study of the dependence on the magnetic fields for the quark condensate and of its magnetic susceptibility was done in [41] and [42], respectively. It is our purpose in this paper to exploit the formalism of field correlators (FC) developed earlier for the QCD phase transition at zero m.f. [43–49] in order to study the same problem in the case of arbitrary m.f.

The advantage of the FC method is that it is based only on the fundamental QCD input: gluonic condensate $\langle G^2 \rangle$, string tension σ, $\alpha_s(q)$, and current quark masses. In contrast to [36], where the same basic principle [44] was used except that pions were elementary in CPTh, the FC method treats all hadrons, including pions, as $q\bar{q}$ or $3q$ systems, which allows us to consider high m.f. with $eB \gg m_\pi^2$, σ. As a result the critical temperature $T_c(B)$ decreases with the growing B as in the lattice data of [40].

Recently the FC method was successfully applied to the study of phase transition in neutron stars [50,51] without m.f. and to the case of strange quarks and strange matter in [52]. It is interesting to investigate the role of m.f. in these transitions, and our results below may be a reasonable starting point for this analysis.

The paper is organized as follows. In Sec. II the general FC formalism as applied to the hadron-quark transition is given, and in Sec. III the contribution of magnetic fields is explicitly taken into account. In Sec. IV the quark and hadron thermodynamic potentials are estimated at large m.f. and the corresponding transition temperature $T_c(B)$ is found. In Sec. V the case of nonzero chemical potential is treated and in Sec. VI a discussion of results and prospectives is presented.

II. GENERAL FORMALISM

We shall follow the ideas of [43–49] (see [53] for a review) and consider the low-temperature hadron phase as the hadron gas in the confining background vacuum field. The total free energy can be represented as

$$F = \varepsilon_{\text{vac}} V_3 + F_h, \tag{1}$$

where

$$\varepsilon_{\text{vac}} = \frac{\beta(\alpha_s)}{16\alpha_s} \langle G^a_{\mu\nu} G^a_{\mu\nu} \rangle + \sum_q m_q \langle \bar{q} q \rangle, \tag{2}$$

and F_h is the hadron free energy, which in the absence of a magnetic field, and treating hadrons as elementary, can be written as [54,55] $(\beta = 1/T)$

$$-F_{h/V_3} = \sum_i P_h^{(i)} = \sum_i \frac{g_i T}{2\pi^2}\int_0^\infty dp p^2 \eta \ln(1+\eta e^{-\beta E_i}), \quad (3)$$

where $\eta = -1$ or $+1$ for bosons or fermions, respectively, and in the relativistic case $E_i = \sqrt{\mathbf{p}^2 + m_i^2}$, while g_i is the spin-isospin multiplicity of hadron i. Taking the integral in (3), one can write $P_h^{(i)}$ as

$$P_h^{(i)} = \frac{g_i T^4}{2\pi^2}\sum_{n=1}^\infty (-\eta)^{n+1}\frac{(\beta m_i)^2}{n^2}K_2(n\beta m_i), \quad (4)$$

where K_2 is the McDonald function. As an example of another starting point, we present below the derivation of the quark pressure from the statistical sum in the form of the generating function with the proper time integration [45,47–49],

$$\frac{1}{T}F_q = \frac{1}{2}\ \ln\ \det(m_q^2 - \hat{D}^2)$$
$$= -\frac{1}{2}\mathrm{tr}\int_0^\infty \xi(s)\frac{ds}{s}e^{-sm_q^2 + s\hat{D}^2}. \quad (5)$$

The latter expression can be written as a path integral with the background field containing both electromagnetic $A_\mu^{(e)}(x)$ and color potential $A_\mu(x)$, [53,56–58],

$$\frac{1}{T}F_q(A, A^{(e)})$$
$$= -\frac{1}{2}\mathrm{tr}\int_0^\infty \xi(s)\frac{ds}{s}d^4x\overline{(Dz)}^w_{xx}e^{-K-sm_q^2}\langle W_\sigma(C_n)\rangle, \quad (6)$$

where $K = \frac{1}{4}\int_0^s (\frac{dz_\mu}{d\tau})^2 d\tau$,

$$W_\sigma(C_n) = P_F P_A \exp\left(ig\int_{C_n}A_\mu dz_\mu + ie\int_{C_n}A_\mu^{(e)}dz_\mu\right)$$
$$\times \exp\int_0^s (g\sigma_{\mu\nu}F_{\mu\nu} + e\sigma_{\mu\nu}F_{\mu\nu}^{(e)})d\tau, \quad (7)$$

and

$$\overline{(Dz)}^w_{xy} = \prod_{m=1}^n \frac{d^4\Delta z_k(m)}{(4\pi\varepsilon)^2}$$
$$\times \sum_{n=0,\pm 1,\pm 2}(-)^n\frac{d^4p}{(2\pi)^4}e^{ip_\mu(\sum \Delta z_\mu(m)-(x-y)-n\beta\delta_{\mu 4})}. \quad (8)$$

It was shown in [48,49], that $P_q^{(i)} \equiv P_q$ can be written as

$$P_q = 2N_c\int_0^\infty \frac{ds}{s}e^{-m_q^2 s}\sum_{n=1}^\infty (-)^{n+1}[S^{(n)}(s) + S^{(-n)}(s)], \quad (9)$$

where

$$S^{(n)}(s) = \int (\bar{D}z)^w_{\mathrm{on}}e^{-K}\frac{1}{N_c}\ \mathrm{tr}\ W_\sigma(C_n), \quad (10)$$

and in the case when only a one-particle contribution is retained,

$$S^{(n)}(s) = \frac{1}{16\pi^2 s^2}e^{-\frac{n^2\beta^2}{4s}-J_n^E}. \quad (11)$$

J_n^E defines the Polyakov loop configuration expressed via the field correlator $D_1(x)$ [48,49,59],

$$J_n^E = \frac{n\beta}{2}\int_0^{n\beta}d\nu\left(1-\frac{\nu}{n\beta}\right)\int_0^\infty \xi d\xi D_1^E(\sqrt{\nu^2+\xi^2}), \quad (12)$$

where $D_1^E(x)$ is the color-electric correlator, which stays nonzero above the deconfinement temperature.

The insertion of (11) into (9) yields

$$\frac{1}{T^4}P_q = \frac{N_c n_f}{4\pi^2}\sum_{n=1}^\infty \frac{(-)^{n+1}}{n^4}\int_0^\infty \frac{ds}{s^3}e^{-m_q^2 s - \frac{n^2\beta^2}{4s} - J_n^E}$$
$$= \frac{4N_c n_f}{\pi^2}\sum_{n=1}^\infty \frac{(-)^{n+1}}{n^4}\varphi_q^{(n)}L^n,$$
$$\varphi_q^{(n)} = \frac{n^2 m_q^2}{2T^2}K_2\left(\frac{m_q n}{T}\right), \quad (13)$$

where $L^n = e^{-J_n^E}$.

At this point it is convenient to give one more representation of P_q; namely, using [58] one can extract in $z_4(\tau)$ the fluctuating part $\tilde{z}_4(\tau)$,

$$z_4(\tau) = \bar{z}_4(\tau) + z_4(\tau), \qquad \bar{z}_4(\tau) = 2\omega\tau = t_E,$$
$$s = \frac{T_4}{2\omega}, \qquad T_4 = n\beta, \quad (14)$$

$$P_q = 2N_c n_f\int_0^\infty \frac{d\omega}{\omega}\sqrt{\frac{\omega}{2\pi}}\sum_{n=1}^\infty \frac{(-)^{n+1}}{\sqrt{n\beta}}\int D^3 z e^{-K(w)-J_n^E}, \quad (15)$$

$$K(\omega) = \int_0^{n\beta}dt_E\left(\frac{\omega}{2}+\frac{m^2}{2\omega}+\frac{\omega}{2}\left(\frac{dz}{dt_E}\right)^2\right), \qquad m_q \equiv m. \quad (16)$$

In this way we obtain the form, equivalent to (13),

$$P_q = \frac{N_c n_f}{\pi^2\beta^2}\sum_{n=1}^\infty \frac{(-)^{n+1}}{n^2}\int_0^\infty \omega d\omega e^{-(\frac{m^2}{2\omega}+\frac{\omega}{2})n\beta - J_n^E}. \quad (17)$$

Neglecting J_n^E and for $m_q = 0$, one obtains the standard result,

$$P_q = \frac{4N_c n_f T^4}{\pi^2}\sum_{n=1}^{\infty}\frac{(-)^{n+1}}{n^4} = \frac{7N_c n_f \pi^2}{180}T^4, \tag{18}$$

where we have used

$$\int_0^\infty \omega d\omega e^{-(\frac{m^2}{2\omega}+\frac{\omega}{2})n\beta} = 2m^2 K_2(mn\beta). \tag{19}$$

For the following it will be useful to keep in (15) the d^3p integration, contained in D^3z, which yields

$$P_q = \frac{n_f N_c}{\sqrt{\pi}}\int\frac{d^3p}{(2\pi)^3}\sum_{n=1}^{\infty}(-)^{n+1}\sqrt{\frac{2}{n\beta}}\int_0^\infty\frac{d\omega}{\sqrt{\omega}}e^{-(\frac{m^2+\mathbf{p}^2}{2\omega}+\frac{\omega}{2})n\beta}. \tag{20}$$

One can see that (20) coincides with (4), when $g_i = 4N_c n_f$. Finally, one obtains from (4) or (18) the pressure for gluons,

$$P_g = (N_c^2-1)\frac{2T^4}{\pi^2}\sum_{n=1}^{\infty}\frac{\langle\Omega^n\rangle + \langle\Omega^{*n}\rangle}{2n^4}, \tag{21}$$

where $\Omega^n = \exp(ig\int_0^{n\beta}A_4 dz_4)$, and Ω is the adjoint Polyakov loop.

III. QUARK AND HADRON THERMODYNAMICS IN A MAGNETIC FIELD

We discuss here the one-particle thermodynamics in a constant homogeneous m.f. $\mathbf{B}$ along the z axis, in which case one should replace E_i in (3) by the well-known expression [55], which in the relativistic case has the form

$$E_{n_\perp}^\sigma(B) = \sqrt{p_z^2 + (2n_\perp + 1 - \bar{\sigma})|e_q|B + m_q^2}, \quad \bar{\sigma} \equiv \frac{e_q}{|e_q|}\sigma_z, \qquad \sigma_z = \pm 1. \tag{22}$$

We also take into account that the phase space of an isolated quark in m.f. is changed as follows [54],

$$\frac{V_3 d^3p}{(2\pi)^3} \to \frac{dp_z}{2\pi}\frac{|e_qB|}{2\pi}V_3, \tag{23}$$

and hence (3) can be rewritten as $\bar{P}_q(B) = \sum_q P_q(B)$,

$$P_q(B) = \sum_{n_\perp,\sigma} 2N_c T\frac{|e_qB|}{2\pi}\frac{1}{2}(\chi(\mu) + \chi(-\mu)), \tag{24}$$

where

$$\chi(\mu) \equiv \int\frac{dp_z}{2\pi}\ln\left(1+\exp\left(\frac{\bar{\mu} - E_{n_\perp}^\sigma(B)}{T}\right)\right). \tag{25}$$

We have introduced in (24) the chemical potential μ with the averaged Polyakov loop factor $\bar{L} = \exp(-\bar{J}/T)$, (see [53] for a corresponding treatment without m.f.),

$$\bar{\mu} = \mu - \bar{J}, \qquad \bar{L}_\mu = \exp\left(\frac{\mu - \bar{J}}{T}\right). \tag{26}$$

Equation (24) can be integrated over dp_z with the result

$$P_q(B) = \frac{N_c|e_qB|T}{\pi^2}\sum_{n_\perp,\sigma}\sum_{n=1}^{\infty}\frac{(-)^{n+1}}{n}\frac{1}{2}(\bar{L}_\mu^n + \bar{L}_{-\mu}^n)\varepsilon_{n_\perp}^\sigma \times K_1\left(\frac{n\varepsilon_{n_\perp}^\sigma}{T}\right), \tag{27}$$

where

$$\varepsilon_{n_\perp}^\sigma = \sqrt{|e_qB|(2n_\perp + 1 - \bar{\sigma}) + m_q^2}. \tag{28}$$

The same result for $\mu = 0$ can be obtained. Extending (20) to the case of nonzero B, using (23) and replacing the exponent in (20) as

$$\left(\frac{m_q^2 + \mathbf{p}^2}{2\omega} + \frac{\omega}{2}\right)n\beta \to \left(\frac{m_q^2 + p_z^2 + (2n_\perp + 1 - \bar{\sigma})|e_qB|}{2\omega} + \frac{\omega}{2}\right)n\beta, \tag{29}$$

$$P_q = 2N_c\frac{|e_qB|}{(2\pi)^2}\sum_{n=1}^{\infty}\frac{(-)^{n+1}}{n\beta}\int_0^\infty d\omega e^{-(\frac{(\varepsilon\sigma)^2}{2\omega}+\frac{\omega}{2})n\beta}, \tag{30}$$

and using the equation

$$\int_0^\infty d\omega e^{-(\frac{\lambda^2}{2\omega}+\frac{\omega}{2})\tau} = 2\lambda K_1(\lambda\tau), \tag{31}$$

we come to Eq. (27).

We turn now to the thermodynamics of hadrons in m.f. The difficulty here is that hadrons are not elementary objects, unlike quarks, and we cannot use for them the energy expressions like (22). Hadrons in m.f. were studied analytically in [60–63] and on the lattice in [41,64,65]. We can use for them an expression such as Eq. (24) or Eq. (27); however, we should write it in a more general way for the charged hadrons,

$$P_H^{(i)}(B) = g_i \frac{|e_H B| T}{2\pi} \int \frac{dP_z}{2\pi} \frac{1}{2} (\chi(\mu_i) + \chi(-\mu_i)), \qquad (32)$$

where

$$\chi(\mu_i) \equiv \sum_{n_\perp, s_i} \ln\left(1 + \exp\left(\frac{\mu_i - E_{N_\perp}^{s_i}(B)}{T}\right)\right), \qquad (33)$$

and we take into account that the total hadron energy $E_{N_\perp}^{s_i}(B)$ depends on the set of two-dimensional oscillator numbers $N_\perp = \{n_\perp(1), n_\perp(2), \ldots n_\perp(\nu)\}$ for each of ν constituents and on the set $s_i = \{\sigma_1, \ldots \sigma_\nu\}$ of spin projections of all constituents. For very large m.f. $eB \gg \sigma$(σ is the string tension) one can approximate $E_{N_\perp}^{s_i}(B)$ as an average of the sum of constituents (2 for mesons and 3 for baryons),

$$E_{N_\perp}^{s_i}(B) = \left\langle \sum_{k=1}^{\nu} \sqrt{m_q^2(k) + |e_k| B \left(2n_\perp(k) + 1 - \frac{e_k \bar{\sigma}_k}{|e_k|}\right) + p_z^2(k)} \right\rangle_{p_z}, \qquad (34)$$

where the average is taken with the functions $\chi(p_z(1), \ldots p_z(\nu))$, satisfying $\sum_{k=1}^{\nu} p_z(k) = P_z$, and taking into account confining dynamics along the z axis (see explicit expressions in the Appendix). In the approximation used in [60–63], when confinement is quadratic, the functions χ are the oscillator eigenfunctions. For neutral hadrons one should use the form (4) instead of (32), where m.f. act on the multiplicity g_i and the mass m_i, which can strongly depend on the m.f., as in the case of ρ_0 and π^0 mesons, see [61,62].

At this point it is useful to compare the systematics of hadrons without m.f. with that of strong m.f. In the first case one classifies a hadron, using, e.g., spin S, partly P isospin I, orbital momentum L, and radial quantum number n_r, or else total angular momentum J. For strong m.f. neither spin S (or J) nor isospin are conserved, and one has, e.g., instead of two states $\rho^0(S_z = 0), \pi^0$ linear combinations $\langle u+, \bar{u} - | \equiv \langle + - |_u, \langle - + |_u, \langle + - |_d, \langle - + |_d$, and similarly $\rho^0(S_z = +1)$ splits into two states: $\langle + + |_u, \langle + + |_d$.

A similar situation occurs in baryons: neutron $S_z = -\frac{1}{2}, (ddu)$ splits into $(- - +), (- + -), (+ - -)$.

A specific role is played here by the so-called "zero states,"for which all constituents have factors in (34) equal to zero,

$$2n_\perp(k) + 1 - \frac{e_k}{|e_k|} \bar{\sigma}_k = 0, \qquad k = 1, 2, \ldots \nu. \qquad (35)$$

Masses of zero states decrease fast with m.f. and for $eB \approx \sigma$ can be $(30 \div 40)\%$ lower than in the absence of m.f. [61,62]. Therefore the role of these states in the forming of $P_h(B)$ grows exponentially, while energies $E_{N_i}^{s_i}(B)$ of all other states according to (34) grow proportionally to $\sqrt{eB}$. Thus in $\rho^0(S_z = 0), \pi^0$ only the states $\langle + - |_u, \langle - + |_d$ are zero states, while for the neutron with $S_z = -\frac{1}{2}$, the only zero state is $(- - +)$. In this way most excited hadron states have energies growing with m.f., and their contribution is strongly suppressed for $eB > \sigma$. However, the same situation occurs for the system of free quarks at large m.f., which can be clearly seen comparing (34) with energies of free quarks; therefore, the main difference occurs for m.f. that are not large, when $eB < \sigma$ and hadron energies change less rapidly than those of free quarks Hence, P_q may grow faster with eB than P_h, which finally results in the decreasing $T_c(B)$, as we show below.

Indeed, for each quark the zero states constitute one half of $n_\perp = 0$ states, namely the states with $\bar{\sigma}_k = 1$, and the corresponding pressure is proportional to $\frac{|e_q| B m_q T}{n} K_1(\frac{m_q n}{T})$, tending to $\frac{|e_q| B T^2}{n}$ at large m.f., thus growing linearly with B.

For hadrons in the same limit, the charged zero states contribute in (32) the amount $\frac{|e_h| B T^2}{n}$, while neutral zero states, like π^0, contribute $\frac{T^4}{n^4}$. Therefore, for $B \geq T^2$ the growth of quark pressure with B is faster than that of hadrons, and one can assert that for $B \geq T^2, \sigma$ the inequality holds,

$$\Delta P_q(B,T) \equiv P_q(B,T) - P_q(0,T) > \Delta P_h(B,T) = P_h(B,T) - P_h(0,T). \qquad (36)$$

In the next section we shall show that (36) leads to the decreasing of the deconfinement temperature $T_c(B)$ with growing B, independent of the character of this transition. In particular, the above arguments were based on the single-line approximation for quarks [48,49], when quarks are treated as independent and vacuum fields create only Polyakov line contributions [$\bar{L}^n$ in (27)]. A more accurate treatment, taking into account the $q\bar{q}$ interaction due to the D_1 correlator [cf. Eq. (12)], shows that the $q\bar{q}$ pairs can form bound states in this interaction [59], and with increasing m.f. the binding energy grows, which lowers the $q\bar{q}$ mass, thus leading to the growth of the quark pressure. Moreover, the introduction of this "intermediate state of deconfinement," existing in the narrow region near T_c and consisting of bound and decaying $q\bar{q}$ pairs, strongly affects the nature of the deconfining transition, making it softer. In addition, the color-electric string tension, which disappears at T_c, decreases gradually in the same region, lifting in this way the hadron pressure and making the transition continuous. This remark, however does not qualitatively change the considerations of the present paper and will be treated in detail elsewhere.

IV. THE QUARK-ANTIQUARK CONTRIBUTION TO THE PRESSURE AT NONZERO M.F.

As was discussed above, the nonperturbative D_1 contribution to a single quark is given by Eqs. (11), (12), where J_n^E can also be written as

$$L_{\rm fund} = \exp(-J_1^E) = \exp\left(-\frac{V_1(\infty)}{2T}\right), \qquad (37)$$

where $V_1(r)$ is the $q\bar{q}$ nonperturbative (np) color-elecric interaction generated by the field correlator $D_1^E(x)$, [59]

$$V_1(r,T) = \int_0^\beta (1-\nu T) d\nu \int_0^r \xi d\xi D_1^E(\sqrt{\nu^2+\xi^2}). \quad (38)$$

As was argued in [66], the asymptotics of $D_1^E(x)$ is expressed via the gluelump mass $M_0 \approx 1$ GeV [67] and can be written as

$$D_1^{(np)}(x) = \frac{A_1}{|x|} e^{-M|x|} + O(\alpha_s^2), \qquad A_1 = 2C_2 \alpha_s \sigma_{\rm adj} M_0, \qquad (39)$$

which leads to

$$V_1(r,T) = V_1(\infty,T) - \frac{A_1}{M_0^2} K_1(M_0 r) M_0 r + O\left(\frac{T}{M_0}\right), \qquad (40)$$

and

$$V_1(\infty,T) = \frac{A_1}{M_0^2}\left[1 - \frac{T}{M_0}(1-e^{-M_0/T})\right],$$
$$\frac{A_1}{M_0^2} \approx \frac{6\alpha_s(M_0)\sigma_f}{M_0} \approx 0.5 \text{ GeV}. \qquad (41)$$

At $r=0, V_1(0,T)=0$.

Above T_c the value of $V_1(\infty,T)$ is decreasing, as seen from (41), (40). This is in agreement with lattice data on Polyakov loops in [68]. Recently, the potential $V_1(r,T)$ was studied on the lattice in [69], yielding a behavior similar for $V_1(\infty,T)$ at $T=1.2T_c$.

We now consider the hadron and quark-gluon pressure in the single-line (the independent particle) approximations with the purpose of defining the deconfinement temperature as a function of m.f.

One starts with the total pressure in the confined phase, phase I, which can be written in the form, generalizing the results of [43–45] for the case of nonzero m.f.,

$$P_I = |\varepsilon_{\rm vac}| + \sum_i P_H^{(i)}(B), \qquad (42)$$

where $\varepsilon_{\rm vac} = \varepsilon_{\rm vac}^{(g)} + \varepsilon_{\rm vac}^{(q)}$ is given in (2), and we assume, that the gluonic condensate does not depend on m.f. in the first approximation, while the quark condensate $|\langle\bar{q}q\rangle|$ grows with m.f., as shown analytically in [70] and on the lattice [41,42]; however, we neglect this contribution in the first approximation and discuss its importance at large eB in the concluding section.

In the deconfined phase (phase II), the pressure can be written in the form (cf. [43–45,48])

$$P_{II} = \frac{1}{2}|\varepsilon_{\rm vac}^{(g)}| + \sum_q P_q(B) + P_g, \qquad (43)$$

where $P_q(B)$ is given in (24)–(26), and we assume that vacuum color-magnetic fields, retained in the deconfined phase at $T \approx T_c$, create one-half of the vacuum condensate G_2 as it happens for $T=0$,

$$\frac{1}{2}|\varepsilon_{\rm vac}^{(g)}| \cong \frac{(11-\frac{2}{3}n_f)}{32}\Delta G_2, \qquad \Delta G_2 \approx \frac{1}{2}G_2. \qquad (44)$$

Taking into account the chemical potential μ, one can rewrite (27) as

$$P_q(B) = \sum_{q=u,d,\ldots} \frac{N_c|e_q B|T}{\pi^2} \times \sum_{n_\perp,\sigma=\pm 1}\sum_{n=1}^{\infty} \frac{(-)^{n+1}}{n} \cosh\frac{\mu n}{T} L_{\rm fund}^n \varepsilon_{n_\perp}^\sigma K_1\left(\frac{n\varepsilon_{n_\perp}^\sigma}{T}\right). \qquad (45)$$

Finally, for gluon pressure we neglect the influence of m.f., which appears in higher $O(\alpha_s)$ orders, and write

$$P_{gl}(B) \cong P_{gl}^{(0)} = \frac{2(N_c^2-1)}{\pi^2}\sum_{n=1}^{\infty}\frac{T^4}{n^4}L_{\rm adj}^n. \qquad (46)$$

As a result we define the deconfinement temperature from the equality

$$P_I(T=T_c) = P_{II}(T=T_c). \qquad (47)$$

The contribution of zero levels of light quarks clearly dominates in (45), when $eB > T^2$, so that keeping for simplicity only the $n=1$, $\sigma=1$ terms for small μ, one has

$$\bar{P}_q(B) \approx P_q^{(0)}(B) = \frac{N_c n_f |\bar{e}_q B| T^2}{\pi^2} L_{\rm fund} \cosh\frac{\mu}{T}, \qquad (48)$$

where$|\bar{e}_q| = \frac{e}{n_f}\sum_{i=1}^{n_f}\frac{|e_i|}{e}$, $\bar{e}_q = \frac{4}{9}e$ for $n_f = 3$.

Neglecting as a first approximation the hadron pressure and $\varepsilon_{\rm vac}^{(q)}$in (42), one obtains an equation for T_c,

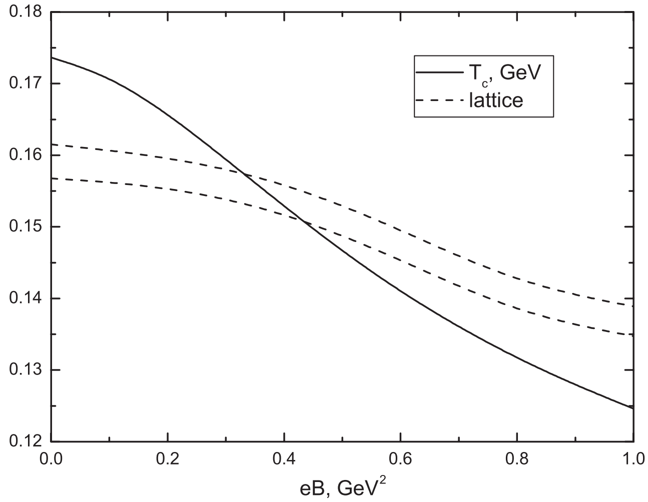

FIG. 1. QCD phase diagram in the $B-T$ plane for $\mu=0$ as given by (49) in comparison with lattice data [40].

$$\frac{1}{2}|\varepsilon_{\rm vac}^{(g)}| = P_{gl}^{(0)} + P_q^{(0)}(B), \tag{49}$$

and finally, neglecting the term $\sum_q m_q|\langle\bar{q}q\rangle|$, and $P_{gl}^{(0)}$ for large $B > T^2$, one obtains the asymptotic expression

$$T_c^2 = \frac{(11-\frac{2}{3}n_f)G_2\pi^2}{64N_c n_f|\bar{e}_q|BL_{\rm fund}\cosh\frac{\mu}{T}}. \tag{50}$$

For $\mu=0$, we take $L_{\rm fund}=\exp(-\frac{V_1}{2T_c}), V_1\approx 0.5$ GeV [59], $n_f=3, \bar{e}_q=\frac{4}{9}e, G_2=0.006$ GeV^4 [71], and we obtain

$$T_c(eB=1\ {\rm GeV}^2)\cong 0.125\ {\rm GeV}. \tag{51}$$

For the same parameters and $B=0$ in [71], one gets $T_c(0)\cong 0.165$ GeV. These values are in a reasonable agreement with the corresponding lattice data in [42], $T_c^{\rm lat}(1\ {\rm GeV}^2)\approx 0.138$ GeV, $T_c^{\rm lat}(0)\approx 0.16$ GeV.

One can now also take into account at large B the contribution of gluons, $P_{gl}=\frac{16}{\pi^2}\exp(-\frac{9V_1}{8T})$, and π^0 mesons, $P_{\pi^0}\approx\frac{\pi^2}{90}T^4$, since the mass of π^0 tends to zero for $eB>\sigma$, while that of π^+,π^- grows as $\sqrt{eB}$ and does not contribute appreciably to the pressure. One can check that solving equation

$$\frac{1}{2}|\varepsilon_{\rm vac}^{(g)}| + P_{\pi^0} = P_{gl}^{(0)} + P_q^{(0)}(B), \tag{52}$$

one obtains $T_c(eB=1\ {\rm GeV}^2)$, which is 2% larger than in (51).

At large $eB\gg T^2$ and $\mu=0, G_2=0.006$ GeV^4, Eq. (50) yields

$$T_c^2 = \frac{0.00205\ {\rm GeV}^2}{eB}\exp\left(\frac{0.25\ {\rm GeV}}{T_c}\right), \tag{53}$$

and one obtains $T_c(eB=6\ {\rm GeV}^2)\approx 0.08$ GeV and a slow decrease for larger eB, $T_c(eB)\sim\frac{1}{\ln(eB)}$.

To investigate the behavior of $T_c(B)$ at all values of B and $\mu=0$, we take into account all Landau levels, as was done in the Appendix, and write the resulting expression for $P_q(B)$ in the case $\mu=0$ [cf., Eq. (A5)],

$$\begin{aligned} P_q(B) = {}& \frac{N_c e_q BT}{\pi^2}\sum_{n=1}^{\infty}\frac{(-)^{n+1}}{n}\bar{L}^n \\ & \times\left\{m_q K_1\left(\frac{nm_q}{T}\right)+\frac{2T}{n}\frac{e_qB+m_q^2}{e_qB}\right. \\ & \times K_2\left(\frac{n}{T}\sqrt{e_qB+m_q^2}\right) \\ & \left.-\frac{ne_qB}{12T}K_0\left(\frac{n}{T}\sqrt{m_q^2+e_qB}\right)\right\}. \end{aligned} \tag{54}$$

In the limiting case of small quark mass, $m_q\ll\{T,e_qB\}$, one can rewrite (54) as

$$\begin{aligned} & P_q(B) \\ & \approx\frac{N_c e_q BT}{\pi^2}\bar{L}\left\{T+2TK_2\left(\frac{\sqrt{e_qB}}{T}\right)-\frac{e_qB}{12T}K_0\left(\frac{\sqrt{e_qB}}{T}\right)\right\}. \end{aligned} \tag{55}$$

Note that Eq. (55) for small eB tends to the limiting B-independent form (13), (18). We shall use the forms (54), (55) at all values of eB, and hence recalculate (49) with $P_q^{(0)}\equiv\bar{P}_q(B)=\sum_{q=u,d,s}P_q(B)$, and $P_q(B)$ from (54), (55).

As a result, from Eq. (49) one obtains the curve $T_c(B)$ shown in Fig. 1 together with the points obtained on the lattice in [40].

V. THE CASE OF NONZERO CHEMICAL POTENTIAL

We first consider the case of very large eB, when one can retain only the lowest Landau levels of quarks.

For nonzero μ and $e_qB\gg 4T^2$, one can keep only the zero Landau level and rewrite (24) in the form

$$P_q(B) = \frac{N_c}{2\pi^2}e_qB(\phi(\mu)+\phi(-\mu)), \tag{56}$$

where $\phi(\mu)$ is

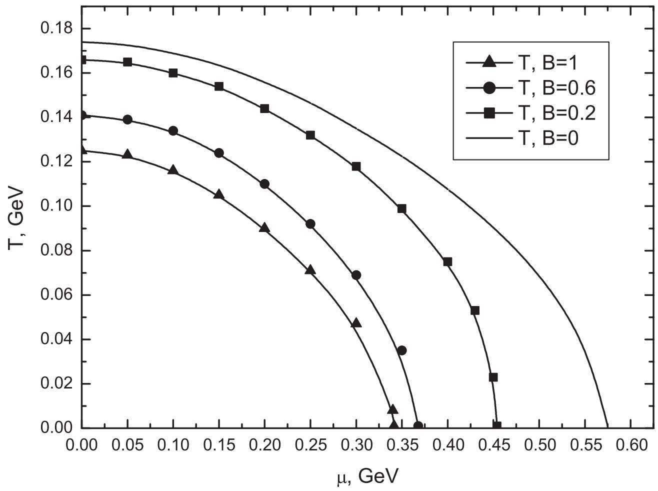

FIG. 2. QCD phase diagram in $\mu - T$ plane for different values of eB.

$$\phi(\mu) = \int_0^\infty \frac{p_z dp_z}{1+e^{\frac{p_z-\bar{\mu}}{T}}}, \qquad \bar{\mu} = \mu - \bar{J} \equiv \mu - \frac{V_1(\infty)}{2}. \tag{57}$$

At large $\frac{\bar{\mu}}{T} \gg 1$ one can use the expansion [55]

$$\phi(\mu) \approx \frac{(\mu-\bar{J})^2}{2} + \frac{\pi^2}{6}T^2, \tag{58}$$

and one obtains in the lowest approximation (neglecting π^0 and the gluon contribution at large eB), which yields ($\frac{1}{2}|\varepsilon_{\rm vac}^{(g)}| = P_q(B)$) the critical value of the chemical potential μ_c,

$$\bar{\mu}_c^2 = \frac{1.386G_2}{\bar{e}_q B}; \qquad \mu_c = \frac{V_1(\infty)}{2} + 1.18\sqrt{\frac{G_2}{eB}}. \tag{59}$$

For $eB \to \infty$ one has $\mu_c(eB\to\infty) = \frac{V_1(\infty)}{2} = 0.25$ GeV, where we have assumed that $V_1(\infty)$ is independent of eB.

Near the critical point the critical curve is easily obtained from (56), (58),

$$\left(\mu_c - \frac{V_1(\infty)}{2}\right)^2 + \frac{\pi^2}{3}T_c^2 = \frac{1.386G_2}{eB}. \tag{60}$$

For small μ the lattice data of [72] reveal that $V_1(\infty)$ depends on eB and may become negative for large T and B.

However, for small T the behavior of L in [72] is compatible with our assumption that $V_1(\infty)$ is weakly dependent on T and is around 0.5 GeV, which supports our form of the phase transition curve (60). Also the μ dependence of the color screening potential $V_1(\infty,\mu)$ was studied in [69] and was found to be rather moderate for $(\mu/T)^2 \le 1$ and $T/T_c = 1.20$ and 1.35. Therefore, we can assume that the behavior in (60) is qualitatively correct for large eB, $eB > \sigma = 0.18$ GeV^2, and should go over for $B\to 0$ into the form found earlier in [71].

Now we turn to the case of arbitrary m.f. As shown in the Appendix, one can sum up in (24) over $n_\perp, \sigma$ in the following way,

$$P_q(B) = \frac{N_c e_q B}{2\pi^2}\left\{\phi(\mu) + \phi(-\mu) + \frac{2}{3}\frac{(\lambda(\mu)+\lambda(-\mu))}{e_q B} - \frac{e_q B}{24}(\tau(\mu)+\tau(-\mu))\right\}, \tag{61}$$

where $\phi(\mu)$ is given in (57), while $\lambda(\mu), \tau(\mu)$ are

$$\lambda(\mu) = \int_0^\infty \frac{p^4 dp}{\sqrt{p^2+\tilde{m}_q^2}} \frac{1}{\exp(\frac{\sqrt{p^2+\tilde{m}_q^2}-\bar{\mu}}{T})+1}, \tag{62}$$

$$\tau(\mu) = \int_0^\infty \frac{dp_z}{\sqrt{p^2+\tilde{m}_q^2}} \frac{1}{\exp(\frac{\sqrt{p_z^2+\tilde{m}_q^2}-\bar{\mu}}{T})+1}. \tag{63}$$

Here $\tilde{\mu}_q^2 = m_q^2 + e_q B$. One can see that $\lambda(\mu), \tau(\mu)$ decay exponentially for $e_q B \to \infty$, and hence one returns to Eq. (56) in this limit. In the opposite case, when $e_q B \to 0$, one recovers the form (61) with only $\lambda(\mu) + \lambda(-\mu)$ present, which exactly coincides with the one studied in [71].

One can now calculate the transition curve $T_c(\mu, B)$ in the (T,μ) plane for different values of eB, using (61) for $\bar{P}_q(B) = \sum_q P_q(B)$ in the equation $\frac{1}{2}|\varepsilon_{\rm vac}^{(g)}| = P_g^{(0)} + \bar{P}_q(B)$. The resulting sequence of curves $T_c(\mu, eB), 0 \le \mu \le \mu_c$, and $eB = (0, 0.2, 0.6, 1)$ GeV^2 is shown in Fig 2. One can see from (59) that asymptotically for large eB, the limiting curve is a cut of the straight line $T_c = 0, 0 \le \mu \le \frac{V_1(\infty)}{2}$. Of special importance is the region of small eB, $eB \lesssim 4T^2$, where the asymptotic (large eB) $P_q(B,\mu)$ from Eq. (56) turns over into the m.f. independent form of Eq. (13).

The details of this transition are given in the Appendix.

VI. DISCUSSION OF RESULTS AND CONCLUSIONS

In this paper we have derived the compact forms of the quark pressure for zero chemical potential and arbitrary T and eB in (54) and for nonzero chemical potential in (61), which take into account higher Landau levels.

Using that, we have calculated $T_c(eB, \mu=0)$ in Eq. (50) and Fig. 1 and $T_c(eB,\mu)$ in Fig. 2 in the lowest approximation, neglecting hadron contributions, except for π^0, and neglecting the possible dependence of gluon vacuum energy $\varepsilon_{\rm vac}^{(g)}$ and Polyakov loop $\bar{L}$ on m.f. This approximation, which can be however a crude one, is supported by available lattice data on the dependence of the interquark potential $V_1(\infty)$ on the magnetic field [69,72].

We have shown that $T_c(eB, \mu = 0)$ is a moderately decreasing function of eB, tending to zero asymptotically as $\frac{1}{\ln eB}$. This fact agrees reasonably well with the realistic lattice data [40]. Qualitatively, the decreasing pattern was obtained in [36] and [37]. However, in both cases T_c dropped much faster, and in [36] T_c passes zero at $eB \approx 0.7\ \mathrm{GeV}^2$.

A common feature of all approaches, resulting in the decreasing $T_c(eB)$, is that they contain a constant piece of pressure in the confinement phase, which is destructed by transition to the quark phase: it is the vacuum condensate in the present paper and [36] and the MIT bag pressure in [37]. However, the treatment of the hadronic pressure in all available papers assumes that hadrons are elementary and their masses are not modified by m.f. In contrast to that, our approach suggests using the real composite hadrons, with masses strongly dependent on m.f., as was found in [61]. In the present paper we have used this notion only marginally, taking massless π^0 (in the limit $B \to \infty$) into account and neglecting π^+, π^-, which have large masses in the limit of strong fields. However, in the next paper we plan to return to this problem and to calculate $T_c(eB, \mu)$ with the lowest mass hadrons.

Concerning the μ dependence of $T_c(\mu, B)$ one can see in Fig. 2 a smooth decreasing behavior for an increasing B, with a limiting piece of straight line $T_c(\mu, B \to \infty) \to 0$, $\mu_c \to \frac{V_1(\infty)}{2}$.

It is important to discuss the accuracy of our approach and possible ways to improve it. The approximations made concern both the case of zero and nonzero magnetic fields. We start with the first case. In the present paper we have used the only parameters of our approach: $|\varepsilon_{\rm vac}^{(g)}|$ and $V_1(\infty)$, with the latter defining the Polyakov loop average $\bar{L} = \exp\left(-\frac{V_1(\infty)}{2T}\right)$. For $|\varepsilon_{\rm vac}^{(g)}|$ we have used $G_2 \equiv \frac{\alpha_s}{\pi}\langle F_{\mu\nu}^a F_{\mu\nu}^a\rangle = 0.006\ \mathrm{GeV}^4$, which was found to give the realistic $T_c(\mu = 0, B = 0) \approx 165$ MeV, in reasonable agreement with most lattice data for $n_f = 3$.

The range of values of G_2, including $G_2 = 0.006\ \mathrm{GeV}^4$, was studied in [50,51] and found to give reasonable values of stable mass configuration for a hybrid star configuration. From the purely theoretical point of view, the values of G_2 are not uniquely defined, and the value $0.006\ \mathrm{GeV}^4$ is within the boundaries of the analysis in [73]. The most explicit estimate of accuracy can be made for the case $n_f = 0$, where our resulting deconfinement temperature is $T_c = 270$ MeV in a 5% agreement with lattice data (see Table 1 of [74] for a full list of comparison for different n_f). Our computation of another parameter—the Polyakov loop—is based on the single quark energy $V_1(\infty, T)$, estimated from the field correlator $D_1(x)$ in [59], and differs from the direct lattice data no more than 10%. Hence one can expect, that the accuracy of our approach for the case $B = \mu = m_q = 0$ is within 10%–15%, as signalled by the values of transition temperatures.

We have neglected the dependence of vacuum parameter $|\varepsilon_{\rm vac}^{(g)}|$ on B, μ, since this dependence can occur only in higher orders of α_s expansion. However, we disregarded the quark component $\varepsilon_{\rm vac}^{(q)} \equiv \sum_q m_q \langle \bar{q} q\rangle$ of the vacuum energy, $|\varepsilon_{\rm vac}| = |\varepsilon_{\rm vac}^{(q)}| + |\varepsilon_{\rm vac}^{(g)}|$, assuming the limit $m_q \to 0$. In reality the contribution of the strange quark with $m_s(2\ \mathrm{GeV}) \cong 0.1$ GeV is significant and grows linearly with eB, which changes the asymptotics of $T_c(eB)$ at large eB from $1/\ln(eB)$ to a constant one. Thus the results of the present paper refer to the case of the $n_f = 3$ massless quarks. In the case of nonzero m_q, one should take into account the quark vacuum energy in (2), as was done in [74], which explains the observed dependence of T_c on m_q for $m_q < \sqrt{\sigma}$.

Consider now the case of $B > 0$ and $\mu > 0$. For large μ we have the high-density region, where both hadron and quark interactions are important. In particular, as shown in [75], confinement is changing due to μ_q, resulting in the 10% decrease of the nucleon mass for $\mu_q \approx 200$ MeV. Qualitatively the same happens for $q\bar{q}$ and $3q$ systems in the quark phase [59], implying the appearance of white correlated $q\bar{q}, 3q$ systems. Therefore, the results for any B and $\mu_q > 200$ MeV are subjected to serious changes, whereas one can retain a 10% accuracy of our results for $\mu_q < 200$ MeV. The density phase transition might be accompanied by the creation of multiquark states, which completely change the structure of the phase transition, and can be important for the physics of the supernovae outbursts [76].

The accuracy of our treatment of nonzero B is relatively less important and can be improved in several directions. First of all, we have neglected the influence of m.f. on the gluon vacuum energy. The structure of the latter, as was shown in [66], is connected to the gluon lumps [67] with the size $r_0 \sim (2\ \mathrm{GeV})^{-1}$, and the effect is proportional to $\alpha_s(r_0)$ and hence can be neglected for $B \lesssim 1\ \mathrm{GeV}^2$. The same holds for the confinement effects in hadrons and for the interaction $V_1(r, T)$ due to correlator $D_1(x)$ in the quark phase. As a result, we expect the same accuracy of ≈10% of our method for nonzero B and $\mu_q < 200$ MeV.

We have not analyzed in this paper the character of the quark-hadron transition, since it needs a careful analysis of the hadronic phase, and a possible change of parameters below T_c, which will be published elsewhere.

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APPENDIX A: QUARK THERMODYNAMICS IN A WEAK MAGNETIC FIELD

We start with the case of $\mu = 0$. Equation (27) can be rewritten as $\bar{P}_q(B) = \sum_q P_q(B), e_q \equiv |e_q|$,

$$P_q(B) = \frac{N_c e_q BT}{\pi^2} \sum_{n=1}^{\infty} \frac{(-)^{n+1}}{n} \bar{L}^n \sum_{n_\perp,\sigma} \varepsilon^\sigma_{n_\perp} K_1\left(\frac{n\varepsilon^\sigma_{n_\perp}}{T}\right), \quad \text{(A1)}$$

where $\varepsilon^\sigma_{n_\perp}$ is given in (28). The sum over $n_\perp, \sigma$ can be transformed as follows,

$$\sum_{n_\perp=0}^{\infty} \sum_{\sigma\pm1} \varepsilon^\sigma_{n_\perp} K_1(n\varepsilon^\sigma_{n_\perp} T) = m_q K_1\left(\frac{nm_q}{T}\right) + 2\sum_{n_\perp=0}^{\infty} \bar{\varepsilon} K_1\left(\frac{n\bar{\varepsilon}}{T}\right). \quad \text{(A2)}$$

Here $\bar{\varepsilon} = \sqrt{m_q^2 + e_q B + 2e_q B(n_\perp + \frac{1}{2})}$.

At this point one can use the well-known approximation for the summation in the limit of weak m.f. (see §59 of [55]),

$$\sum_{k=0}^{\infty} F\left(k + \frac{1}{2}\right) \approx \int_0^\infty F(x)dx + \frac{1}{24} F'(0). \quad \text{(A3)}$$

In the integral in (A3) one can use Eq. (19) and Eq. (31) to write

$$\int_0^\infty dx \sqrt{m_q^2 + e_q B + 2e_q Bx} K_1\left(\frac{n\sqrt{m_q^2 + e_q B + 2e_q Bx}}{T}\right) = \frac{T}{2ne_q B} \int_0^\infty \omega d\omega e^{-\frac{n}{T}(\frac{\omega}{2} + \frac{e_q B + m_q^2}{2\omega})} = \frac{T}{n} \frac{e_q B + m_q^2}{e_q B} K_2\left(\frac{n}{T}\sqrt{e_q B + m_q^2}\right). \quad \text{(A4)}$$

As a result (A1) can be rewritten as

$$P_q(B) = \frac{N_c e_q BT}{\pi^2} \sum_{n=1}^{\infty} \frac{(-)^{n+1}}{n} \bar{L}^n \left\{ m_q K_1\left(\frac{nm_q}{T}\right) + \frac{2T}{n} \frac{e_q B + m_q^2}{e_q B} K_2\left(\frac{n}{T}\sqrt{e_q B + m_q^2}\right) - \frac{ne_q B}{12T} K_0\left(\frac{n}{T}\sqrt{m_q^2 + e_q B}\right) \right\}. \quad \text{(A5)}$$

One can see in (A5) that the sum over n is well saturated by the first term with $n = 1$ (a typical situation for $\mu = 0$), and for $m_q \to 0$ one can write

$$P_q(B) \approx \frac{N_c e_q BT}{\pi^2} \bar{L} \left\{ T + 2TK_2\left(\frac{\sqrt{e_q B}}{T}\right) - \frac{e_q B}{12T} K_0\left(\frac{\sqrt{e_q B}}{T}\right) \right\}. \quad \text{(A6)}$$

For large $e_q B, e_q B \gg T^2$, the first term in the curly brackets dominates, and one returns to Eq. (48) (with $\mu = 0$).

In the opposite case, $e_q B \ll T^2$, one can write, expanding $K_n(z)$ at small z,

$$P_q(B) = \frac{N_c \bar{L}}{\pi^2} \left\{ 4T^4 + (e_q B)^2 \left[\frac{1}{6} \ln\left(\frac{2T}{\sqrt{e_q B}}\right) + \frac{3}{16} - \frac{C}{6} \right] \right\}. \quad \text{(A7)}$$

Here $C = 0.577$ is the Euler constant.

At the point $e_q B = T^2$ one has $K_2(1) = 1.625; K_0(1) = 0.421$ and the coefficient of $\frac{N_c e_q BT^2 \bar{L}}{\pi^2} = \frac{N_c T^4 \bar{L}}{\pi^2}$ is 4.21 instead of 4 in the limiting form for $e_q B \to 0$, which implies that the weak field asymptotics $4T^4$ has the 5% accuracy at $e_q B = T^2$, whereas the form (A7) yields at this point the 0.2% accuracy. Therefore, to treat the whole intermediate region $e_q B \lesssim T^2$ and $e_q B > T^2$ with a few percent accuracy, one can use Eq. (A6).

For $\mu > 0$ one can rewrite $\sum_{n_\perp,\sigma} \chi(\mu)$ in (24) in the same way, as was done in (A2),

$$\sum_{n_\perp,\sigma} \chi(\mu) = \int \frac{dp_z}{2\pi} \left\{ \ln\left(1 + \exp\left(\frac{\bar{\mu} - \sqrt{p_z^2 + m_q^2}}{T}\right)\right) + 2\sum_{n_\perp=0}^{\infty} \ln\left(1 + \exp\left(\frac{\bar{\mu} - \sqrt{p_z^2 + m_q^2 + e_q B + 2e_q B(n_\perp + \frac{1}{2})}}{T}\right)\right) \right. = I_1 + I_2. \quad \text{(A8)}$$

The first term I_1 is easily (integrating by parts) transformed to the m.f. independent term $\phi(\mu)$ in (56),

$$I_1 = \frac{1}{\pi T} \phi(\mu). \quad \text{(A9)}$$

The second term I_2 can be rewritten using (A3) as follows,

$$I_2 = \frac{1}{\pi T}\int_0^\infty \frac{p_z^2 dp_z}{e_q B}\int_0^\infty \frac{d\lambda}{\sqrt{p_z^2+m_q^2+e_qB+\lambda}}\frac{1}{e^{\frac{\sqrt{p_z^2+m_q^2+e_qB+\lambda}-\bar\mu}{T}}+1} - \frac{e_qB}{24\pi T}\int_{-\infty}^\infty \frac{dp_z}{\sqrt{p_z^2+m_q^2+e_qB}}\frac{1}{\exp(\frac{\sqrt{p_z^2+m_q^2+e_qB}-\bar\mu}{T})+1} \equiv I_2' + I_2''. \tag{A10}$$

One can estimate the large eB asymptotics of I_2, $I_2 \sim \exp(-\frac{\sqrt{e_qB+m_q^2}}{T})$, which supports Eq. (56) in this limit. In the opposite limit, $eB \to 0$, the first term I_2' in (A10) behaves as

$$I_2' \approx \frac{2}{3\pi T e_q B}\int_0^\infty \frac{p^4 dp}{\sqrt{p^2+\tilde m_q^2}}\frac{1}{\exp(\frac{\sqrt{p^2+\tilde m_q^2}-\bar\mu}{T})+1}, \tag{A11}$$

where $\tilde m_q^2 = m_q^2 + e_qB$. One can see that $I_2'(e_qB\to 0)$ ensures in (24) the correct limiting form found in [71], namely

$$P_q = \frac{N_c T^4}{3\pi^2}\left[\phi_\nu\left(\frac{\mu-\frac{V_1}{2}}{T}\right)+\phi_\nu\left(-\frac{\mu+\frac{V_1}{2}}{T}\right)\right], \qquad P = \sum_{q=1}^{n_f} P_q, \tag{A12}$$

with

$$\phi_\nu(a) = \int_0^\infty \frac{z^4 dz}{\sqrt{z^2+\nu^2}}\frac{1}{\exp(\sqrt{z^2+\nu^2}-a)+1}, \tag{A13}$$

and $\nu = \frac{\sqrt{m_q^2+e_qB}}{T}$. At the same time the term I_2'' has the order $O(\frac{(e_qB)^2}{T^4})$, as compared to the leading term (A12).

[1] T. Vachaspati, Phys. Lett. B **265**, 258 (1991).
[2] K. Enqvist and P. Olesen, Phys. Lett. B **319**, 178 (1993).
[3] D. E. Kharzeev, L. D. McLerran and H. J. Warringa, Nucl. Phys. **A803**, 227 (2008); V. Skokov, A. Y. Illarionov, and V. Toneev, Int. J. Mod. Phys. A **24**, 5925 (2009).
[4] V. Voronyuk, V. D. Toneev, W. Cassing, E. L. Bratkovskaya, V. P. Konchakovski, and S. A. Voloshin, Phys. Rev. C **83**, 054911 (2011).
[5] A. Bzdak and V. Skokov, Phys. Lett. B **710**, 171 (2012).
[6] W.-T. Deng and X.-G. Huang, Phys. Rev. C **85**, 044907 (2012).
[7] R. C. Duncan and C. Thompson, Astrophys. J. **392**, L9 (1992).
[8] D. E. Kharzeev, K. Landsteiner, A. Schmitt, and H.-U. Yee, Lect. Notes Phys. **871**, 1 (2013).
[9] V. P. Gusynin, V. A. Miransky, and I. A. Shovkovy, Nucl. Phys. **B462**, 249 (1996).
[10] I. Shushpanov and A. V. Smilga, Phys. Lett. B **402**, 351 (1997).
[11] N. O. Agasian and I. Shushpanov, Phys. Lett. B **472**, 143 (2000).
[12] N. O. Agasian, Phys. At. Nucl. **64**, 554 (2001).
[13] T. D. Cohen, D. A. McGady, and E. S. Werbos, Phys. Rev. C **76**, 055201 (2007).
[14] J. O. Andersen, Phys. Rev. D **86**, 025020 (2012).
[15] J. O. Andersen, J. High Energy Phys. 10 (2012) 005.
[16] S. P. Klevansky and R. H. Lemmer, Phys. Rev. D **39**, 3478 (1989).
[17] D. P. Menezes, M. Benghi Pinto, S. S. Avancini, and C. Providencia, Phys. Rev. C **80**, 065805 (2009).
[18] R. Gatto and M. Ruggieri, Phys. Rev. D **83**, 034016 (2011).
[19] R. Gatto and M. Ruggieri, Phys. Rev. D **82**, 054027 (2010).
[20] K. Kashiwa, Phys. Rev. D **83**, 117901 (2011).
[21] J. O. Andersen and R. Khan, Phys. Rev. D **85**, 065026 (2012).
[22] S. S. Avancini, D. P. Menezes, M. B. Pinto, and C. Providencia, Phys. Rev. D **85**, 091901 (2012).
[23] K. Fukushima and J. M. Pawlowski, Phys. Rev. D **86**, 076013 (2012).
[24] A. J. Mizher, M. N. Chernodub, and E. S. Fraga, Phys. Rev. D **82**, 105016 (2010).
[25] J. O. Andersen and A. Tranberg, J. High Energy Phys. 08 (2012) 002.

[26] S. Kanemura, H.-T. Sato, and H. Tochimura, Nucl. Phys. **B517**, 567 (1998).
[27] K. G. Klimenko, Theor. Math. Phys. **90**, 1 (1992).
[28] J. Alexandre, K. Farakos, and G. Koutsoumbas, Phys. Rev. D **63**, 065015 (2001).
[29] D. D. Scherer and H. Gies, Phys. Rev. B **85**, 195417 (2012).
[30] C. V. Johnson and A. Kundu, J. High Energy Phys. 12 (2008) 053.
[31] F. Preis, A. Rebhan, and A. Schmitt, J. High Energy Phys. 03 (2011) 033.
[32] A. J. Mizher, E. S. Fraga, and M. Chernodub, *Proc. Sci.* FACESQCD2010 (**2010**) 020.
[33] J. Gasser and H. Leutwyler, Phys. Lett. B **184**, 83 (1987).
[34] J. Gasser and H. Leutwyler, Phys. Lett. B **188**, 477, (1987).
[35] P. Gerber and H. Leutwyler, Nucl. Phys. **B321**, 387 (1989).
[36] N. Agasian and S. Fedorov, Phys. Lett. B **663**, 445 (2008).
[37] E. S. Fraga and A. J. Mizher, Nucl. Phys. **A820**, 103c (2009); E. S. Fraga and L. F. Palhares, Phys. Rev. D **86**, 016008 (2012).
[38] E. S. Fraga, Lect. Notes Phys. **871**, 121 (2013).
[39] R. Gatto and M. Rugieri, Lect. Notes Phys. **871**, 87 (2013).
[40] G. Bali, F. Bruckmann, G. Endrődi, Z. Fodor, S. D. Katz, S. Krieg, A. Schäfer, and K. K. Szabó, J. High Energy Phys. 02 (2012) 044.
[41] G. S. Bali, F. Bruckmann, G. Endrődi, Z. Fodor, S. D. Katz, and A. Schäfer, Phys. Rev. D **86**, 071502 (2012).
[42] G. Bali, F. Bruckmann, M. Constantinou *et al.*, *Proc. Sci.* ConfinementX2012 (**2012**) 198.
[43] Yu. A. Simonov, JETP Lett. **54**, 249 (1991).
[44] Yu. A. Simonov, JETP Lett. **55**, 627 (1992).
[45] Yu. A. Simonov, Phys. At. Nucl. **58**, 309 (1995).
[46] Yu. A. Simonov, in *Proc. Varenna 1995, Selected Topics in Nonperturbative QCD* (IOS Press, Amsterdam, The Netherlands, 1995), p. 319.
[47] H. G. Dosch, H.-J. Pirner, and Y. A. Simonov, Phys. Lett. B **349**, 335 (1995).
[48] Yu. A. Simonov, Ann. Phys. (Amsterdam) **323**, 783 (2008).
[49] E. V. Komarov and Yu. A. Simonov, Ann. Phys. (Amsterdam) **323**, 1230 (2008).
[50] D. Logoteta and I. Bombaci, Phys. Rev. D **88**, 063001 (2013).
[51] M. Baldo, G. F. Burgio, P. Castorina, S. Plumari, and D. Zappala, Phys. Rev. D **78**, 063009 (2008).
[52] F. I. M. Pereira, Nucl. Phys. **A860**, 102 (2011); **A897**, 151 (2013).
[53] A. V. Nefediev, Yu. A. Simonov, and M. A. Trusov, Int. J. Mod. Phys. E **18**, 549 (2009).
[54] L. D. Landau and E. M. Lifshitz, *Quantum Mechanics: Non-Relativistic Theory* (Pergamon, New York, 1991).
[55] L. D. Landau and E. M. Lifshitz, *Statistical Mechanics* (Pergamon, New York, 1980), Part I, Vol. 5.
[56] Yu. A. Simonov, Nucl. Phys. **B307**, 512 (1988).
[57] Yu. A. Simonov and J. A. Tjon, Ann. Phys. (N.Y.) **228**, 1 (1993); **300**, 54 (2002).
[58] Yu. A. Simonov, Phys. Rev. D **88**, 025028 (2013).
[59] Yu. A. Simonov, Phys. Lett. B **619**, 293 (2005), A. Di Giacomo, E. Meggiolaro, Yu. A. Simonov, and A. I. Veselov, Phys. At. Nucl. **70**, 908 (2007).
[60] M. A. Andreichikov, V. D. Orlovsky, and Yu. A. Simonov, Phys. Rev. Lett. **110**, 162002 (2013).
[61] M. A. Andreichikov, B. O. Kerbikov, V. D. Orlovsky, and Yu. A. Simonov, Phys. Rev. D **87**, 094029 (2013).
[62] V. D. Orlovsky and Yu. A. Simonov, J. High Energy Phys. 09 (2013) 136.
[63] Yu. A. Simonov, Phys. Rev. D **88**, 053004 (2013).
[64] Y. Hidaka and A. Yamamoto, Phys. Rev. D **87**, 094502 (2013).
[65] E. V. Luschevskaya and O. V. Larina, arXiv:1203.5699.
[66] Yu. A. Simonov, Phys. At. Nucl. **69**, 528 (2006); Yu. A. Simonov and V. I. Shevchenko, Adv. High Energy Phys. **2009**, 873051 (2009).
[67] M. Foster and C. Michael, Phys. Rev. D **59**, 094509 (1999); Y. A. Simonov, Nucl. Phys. **B592**, 350 (2000).
[68] S. Gupta, K. Hübner, and O. Kaczmarek, Nucl. Phys. **A785**, 278 (2007).
[69] J. Takahashi, K. Nagata, T. Saito, A. Nakamura, T. Sasaki, H. Kouno, and M. Yahiro, Phys. Rev. D **88**, 114504 (2013).
[70] Yu. A. Simonov, J. High Energy Phys. 01 (2014) 118.
[71] Y. A. Simonov and M. A. Trusov, Phys. Lett. B **650**, 36 (2007); JETP Lett. **85**, 598 (2007).
[72] F. Bruckmann, G. Endródi, and T. G. Kovacs, J. High Energy Phys. 04 (2013) 112.
[73] B. L. Ioffe and K. Zyablyuk, Eur. Phys. J. C **27**, 229 (2003).
[74] V. D. Orlovsky and Yu. A. Simonov, arXiv:1312.4178.
[75] Yu. A. Simonov and M. A. Trusov, arXiv:0908.3276.
[76] M. I. Krivoruchenko, D. K. Nadyozhin, T. L. Rasinkova, Yu. A. Simonov, M. A. Trusov, A. V. Yudin, Phys. At. Nucl. **74**, 371 (2011).