

Constraints to new physics models for the proton charge radius puzzle from the decay $K^+ \rightarrow \mu^+ + \nu + e^- + e^+$

Carl E. Carlson¹ and Benjamin C. Rislow^{1,2}

¹*Department of Physics, College of William and Mary, Williamsburg, Virginia 23187, USA*

²*PRISMA Cluster of Excellence, Institut für Physik, Johannes-Gutenberg-Universität, Mainz D-55099, Germany*

(Received 17 October 2013; published 6 February 2014)

A possible explanation for the discrepancy between electronic and muonic hydrogen measurements of the proton charge radius are new, lepton universality violating interactions. Several new couplings and particles have been suggested that account for this discrepancy. At present, these explanations are poorly constrained. Experiments such as the upcoming kaon decay experiment at the Japan Proton Accelerator Research Complex may constrain or eliminate some explanations by measuring the decay channel $K^+ \rightarrow \mu^+ + \nu + e^- + e^+$. We calculate the predicted contributions of the various explanations to this channel. The predicted signals, if present, should be large enough to be resolved in the experiment.

DOI: [10.1103/PhysRevD.89.035003](https://doi.org/10.1103/PhysRevD.89.035003)

PACS numbers: 12.60.Cn

I. INTRODUCTION

The recent extraction of the charge radius from muonic hydrogen [1,2] is 7σ smaller than the electronically determined Committee on Data for Science and Technology (CODATA) value [3]. This discrepancy has inspired investigations into new physics explanations. To bring the extracted charge radius from muonic measurements into agreement with the electronic determination, any new physics explanation must lower the muonic Lamb shift by $310 \mu\text{eV}$.

Jaeckel and Roy [4] showed that the popular $U(1)'$ dark photon model cannot account for the discrepancy since it would have a larger effect on the Lamb shift of electronic hydrogen. An explanation involving new particles must include larger couplings to muons than electrons (the interactions must violate lepton universality). Explanations must also respect the constraint placed by discrepancy between the measured and calculated values of the muon's anomalous magnetic moment $(g-2)_\mu$. Constructing a model that simultaneously accounts for both discrepancies is challenging because the fractional $(g-2)_\mu$ discrepancy is orders of magnitude smaller than the proton charge radius discrepancy.

Several lepton universality violating explanations have been proposed. Tucker-Smith and Yavin [5] suggested the existence of a low mass particle with either scalar or vector couplings to muons and protons. The coupling strength was set to explain the radius puzzle and the small mass was needed to respect the constraint placed by the $(g-2)_\mu$ discrepancy. Barger *et al.* [6,7] showed that explanations involving scalar, vector, and tensor couplings face stringent constraints placed by meson decays. Batell *et al.* [8] modified the dark photon model by inserting an additional coupling to right-handed muons. Though this explanation receives contributions from a well-motivated,

gauge-invariant model, it also requires the existence of scalar particles to respect the $(g-2)_\mu$ constraint. The right-handed coupling to muons also spoils the gauge invariance.

Our explanations [9] for the proton charge radius introduced new particles with scalar and pseudoscalar (polar and axial vector) couplings to muons and protons. The strength of the scalar (polar vector) coupling was chosen to “explain” the proton charge radius puzzle. The pseudoscalar (axial vector) coupling enters the expression for the anomalous magnetic moment with an opposite sign of the scalar (polar vector) coupling and was fine-tuned to explain the $(g-2)_\mu$ discrepancy. We also use the upper limit to $K \rightarrow \mu + \text{invisible}$ measured by Pang *et al.* [10] (see also [7]) to constrain the masses of these new particles. This experimental limit does not constrain the Batell *et al.* explanation since their modified dark photon decays quickly enough into an e^+e^- pair.

All of these charge radius puzzle explanations should be viewed as proof-of-concepts rather than completed models. More work is needed to make these explanations fully consistent with the gauge structure of the Standard Model. The explanations must also be confronted with more experimental constraints before earning serious consideration as realistic models. Tests of the models will include seeking the new interactions via new radiative decays to processes involving muons.

In this work, after some preliminary remarks, we calculate the predicted signal of the Batell *et al.* explanation in the decay channel $K^+ \rightarrow \mu^+ + \nu + e^- + e^+$. We also calculate the prediction of our model when we modify it to include small couplings to electrons. The latest measurement in this channel [11] probed e^+e^- pair energies ($m_{ee} > 145 \text{ MeV}$) that were too large to detect the new particles considered in the models. The upcoming E36 experiment at the Japan Proton Accelerator Research Complex (JPARC) will measure kaon decays in muonic

channels [12] at lower energies, and its results will constrain or eliminate these explanations.

II. QED CONTRIBUTION TO $K^+ \rightarrow \mu^+ + \nu + e^- + e^+$

The main goal of the E36 experiment is to measure the ratio $\Gamma(K^+ \rightarrow \mu^+ + \nu_\mu)/\Gamma(K^+ \rightarrow e^+ + \nu_e)$. They expect to see a large number of kaon decays, $N(K^+ \rightarrow \mu^+ + \nu_\mu) = 10^{10}$ events [13]. The branching ratio for this decay channel is 0.6355 [14].

New physics explanations for the proton radius puzzle can be tested in the decay channel $K^+ \rightarrow \mu^+ + \nu_\mu + e^+ + e^-$. Figure 1 shows the lowest order diagrams for Standard Model QED contributions to this process. The full Standard Model branching ratio for this decay channel is calculated to be 2.49×10^{-5} [15]. (Independent calculations by us and by the authors of [16] agree with this number; for an early calculation, see [17]). For their expected 50% acceptance of e^+e^- coincidences, the total number of such decays E36 can expect is

$$\begin{aligned} N(K^+ \rightarrow \mu^+ + \nu_\mu + e^+ + e^-) &= \frac{1}{2} \frac{\Gamma(K^+ \rightarrow \mu^+ + \nu_\mu + e^+ + e^-)}{\Gamma(K^+ \rightarrow \mu^+ + \nu_\mu)} N(K^+ \rightarrow \mu^+ + \nu_\mu) \\ &\approx 2 \times 10^5. \end{aligned} \quad (1)$$

If no new particles are present, there will be about 1000 such events per bin, for a bin width of 1 MeV, in the vicinity of $m_{ee} = 30$ MeV. Here, m_{ee} is the energy of the e^+e^- pair. Choosing the normalization $\langle 0 | \bar{u} \gamma_\mu (1 - \gamma_5) s | K \rangle = \sqrt{2} f_K k_\mu$, the amplitude for the QED prediction is

$$i\mathcal{M} = -G_F (-ie)^2 V_{us} \bar{u}(p_1) \gamma_\rho v(p_2) \frac{-i}{q^2} (f_K m_\mu L^\rho - H^{\rho\nu} j_\nu) \quad (2)$$

where

$$\begin{aligned} L^\rho &= \bar{u}(q)(1 + \gamma^5) \left(\frac{(2k - q')^\rho}{2k \cdot q' + q'^2} - \frac{2l^\rho + q' \gamma^\rho}{2l \cdot q' + q'^2} \right) v(l), \\ H^{\rho\nu} &= -iV_1 \epsilon^{\rho\nu\alpha\beta} q'_\alpha k_\beta - A_1(q' \cdot (k - q')) g^{\rho\nu} \\ &\quad - (k - q')^\rho q'^\nu - A_2(q'^2 g^{\rho\nu} - q'^\rho q'^\nu), \end{aligned} \quad (3)$$

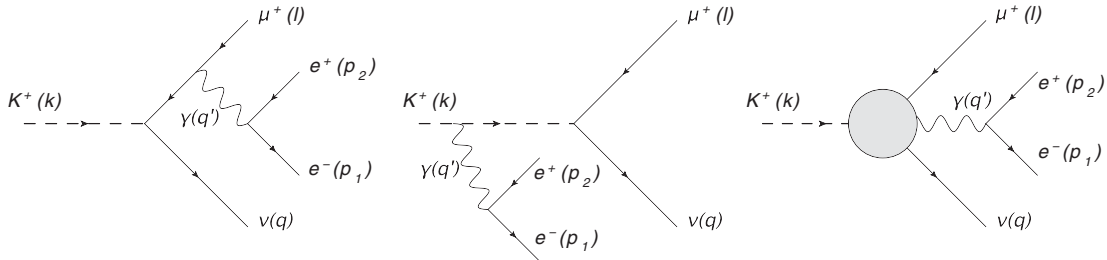


FIG. 1. QED contribution to $K^+ \rightarrow \mu^+ + \nu_\mu + e^+ + e^-$.

and

$$j_\nu = \bar{u}(q) \gamma_\nu (1 + \gamma^5) v(l). \quad (4)$$

If the kaon were pointlike, $H^{\rho\nu}$ would be zero. In the energy regions examined in this work, the inner Bremsstrahlung (IB) term L^ρ dominates while $H^{\rho\nu}$ contributes less than 1% to the plots. Actual values for V_1 , A_1 , A_2 , and f_K can be found in Poblaguev *et al.* [11]. V_1 , A_1 , and A_2 are given by

$$-\sqrt{2} m_K (A_1, A_2, V_1) = (F_A, R, F_V), \quad (5)$$

where $F_A = 0.031$, $R = 0.235$, and $F_V = 0.124$. Their value for the decay constant maps to $f_K = 113$ MeV.

III. DARK PHOTON CONTRIBUTION TO $K^+ \rightarrow \mu^+ + \nu + e^- + e^+$

Before discussing the predictions of the Batell *et al.* model, we consider the original dark photon model it modifies. A simple extension to the Standard Model, dark photons are the quanta of a hypothetical $U(1)'$ gauge field that interacts with fermions through kinetic mixing with the $U(1)$ field associated with weak hypercharge. Determining constraints on the dark photon mass and its effective coupling to fermions is an active area of research, and it is worth exploring whether JPARC can help this cause.

As the name suggests, the dark photon shares many of the properties of the QED photon. If it exists, the dark photon would contribute to the decay $K^+ \rightarrow \mu^+ + \nu + e^- + e^+$. The dark photon's contribution to this decay is found by modifying the photon's propagator and coupling:

$$\frac{-i}{q'^2} \rightarrow \frac{-i}{q'^2 - m_{A'}^2 + im_{A'} \Gamma}, \quad (6)$$

$$-ie \rightarrow -i\epsilon e. \quad (7)$$

In the above expressions, $m_{A'}$ is the mass of the dark photon, Γ is its decay rate into $e^+ + e^-$ (assumed to be the dominant decay), and ϵe is its effective coupling to fermions. If a dark photon exists, a bump in the data will

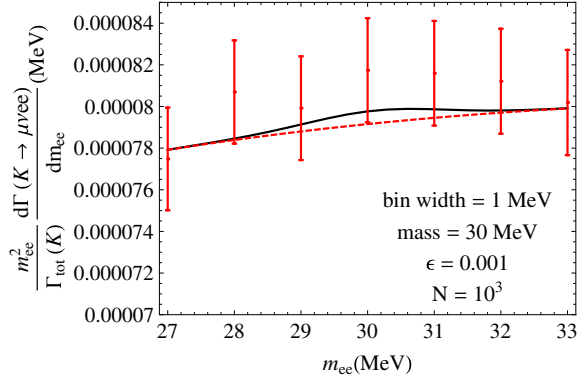


FIG. 2 (color online). QED prediction for the normalized differential decay rate of $K^+ \rightarrow \mu^+ + \nu_\mu + e^+ + e^-$ (red dashed curve) and the prediction with an additional dark photon (black curve). Data points are simulated and possess fluctuations and error bars accounting for the statistical uncertainty given the number $N = 1000$ anticipated events per bin.

be centered around the propagator's pole and will determine the dark photon's mass. The size of the deviation will indicate the value of ϵ . Of course, a lack of observed deviation from the Standard Model will place constraints on $m_{A'}$ and ϵe .

The square of the propagator produces a sharp peak. When the experimental resolution is larger than the natural width of the state, as we expect here, one will observe a peak that is smeared into a Gaussian in $q'^2 = m_{ee}^2$, with a width given by the experimental resolution. To simulate the experimental spread in our decay rate calculation, we replaced the Lorentzian with a Gaussian. The normalization for the Gaussian was chosen to produce the same total decay rate when integrated over q'^2 :

$$\int dq'^2 \frac{1}{(q'^2 - m_{A'}^2)^2 + m_{A'}^2 \Gamma^2} = \frac{\pi}{m_{A'} \Gamma} \int dq'^2 \frac{1}{2q'} \frac{1}{\sigma \sqrt{2\pi}} e^{-\frac{(q' - m_{A'})^2}{2\sigma^2}}, \quad (8)$$

where $\sigma = 1$ MeV is the bin width.

As an example using JPARC's rate and an experimental resolution of 1 MeV, we calculated the signal of a dark photon with the currently allowed [16] parameters $m_{A'} = 30$ MeV and $\epsilon = 10^{-3}$. The overall branching ratio for $K \rightarrow \mu \nu A' \rightarrow \mu \nu e^+ e^-$ is of order 10^{-9} for these parameters. The result is shown in Fig. 2. The dotted red curve is the expected signal from QED while the black curve is the signal due to QED and an additional dark photon. The simulated data points possess error bars accounting for the statistical uncertainty of 1000 events per bin. Given the relative size of the bump and error bars, it will be very hard for JPARC to detect the presence of low mass dark photons, at least with present statistics [16]. However, we continue to more compelling possibilities.

IV. BATELL *et al.* MODIFIED DARK PHOTON CONTRIBUTION TO $K^+ \rightarrow \mu^+ + \nu + e^- + e^+$

JPARC could be quite sensitive to the lepton universality violating particles present in proton charge radius puzzle explanations. The model of Batell *et al.* [8] contains dark photonlike particles that possess an additional, direct coupling to right-handed muons. To account for this particle's effect on the amplitude $K^+ \rightarrow \mu^+ + \nu_\mu + e^+ + e^-$, one modifies the electromagnetic coupling to particles other than the muon by $e \rightarrow \kappa e$ and modifies the photon propagator and muon coupling to

$$\frac{-i}{q^2} \rightarrow \frac{-i}{q^2 - m_{A'}^2 + im_{A'} \Gamma}, \quad (9)$$

$$-ie\gamma^\mu \rightarrow -i\kappa e\gamma^\mu - i\frac{g_R}{2}\gamma^\mu(1 + \gamma^5), \quad (10)$$

where g_R is its additional right-handed coupling to muons. Batell *et al.* give three examples for values of $m_{A'}$, κ , and g_R that satisfy all present constraints. The overall branching ratios for $K \rightarrow \mu \nu A' \rightarrow \mu \nu e^+ e^-$ range from a few tenths to about 10^{-4} for the parameter values that they give. Figure 3 displays the predictions for their parameter values. The figure shows the $e^+ e^-$ signal and the QED background for an experimental resolution of 1 MeV, plotted vs the $e^+ e^-$ mass. This signal is several orders of magnitude greater than the one due to “standard” dark photons and considerably larger than the QED background in the mass range of the new particle. As can be seen by the expected experimental error bars, a dark photon with the Batell *et al.* enhanced muon coupling should be detectable at JPARC.

V. OUR MODEL CONTRIBUTION TO $K^+ \rightarrow \mu^+ + \nu + e^- + e^+$

In our model [9] we considered new particles with polar and axial vector couplings to muons and protons. The new particle's propagator is the same as in the Batell *et al.* model. Our original model omitted a coupling to electrons and would not contribute to $K^+ \rightarrow \mu^+ + \nu + e^- + e^+$. We now modify our model to include small couplings to electrons, being careful that the coupling strength respects the constraint placed by the uncertainty of the electron's anomalous magnetic moment. According to the Particle Data Group value of $\delta(g-2)_e$ [14] and the latest $\delta(g-2)_\mu$ calculation [18], the new particle's coupling to the electron must be at least

$$\epsilon^2 = \frac{\delta(g-2)_e}{\delta(g-2)_\mu} = 1.1 \times 10^{-4} \quad (11)$$

smaller than that of the muon.

It is important to note that as long as the electron coupling is large enough that the new particle decays in

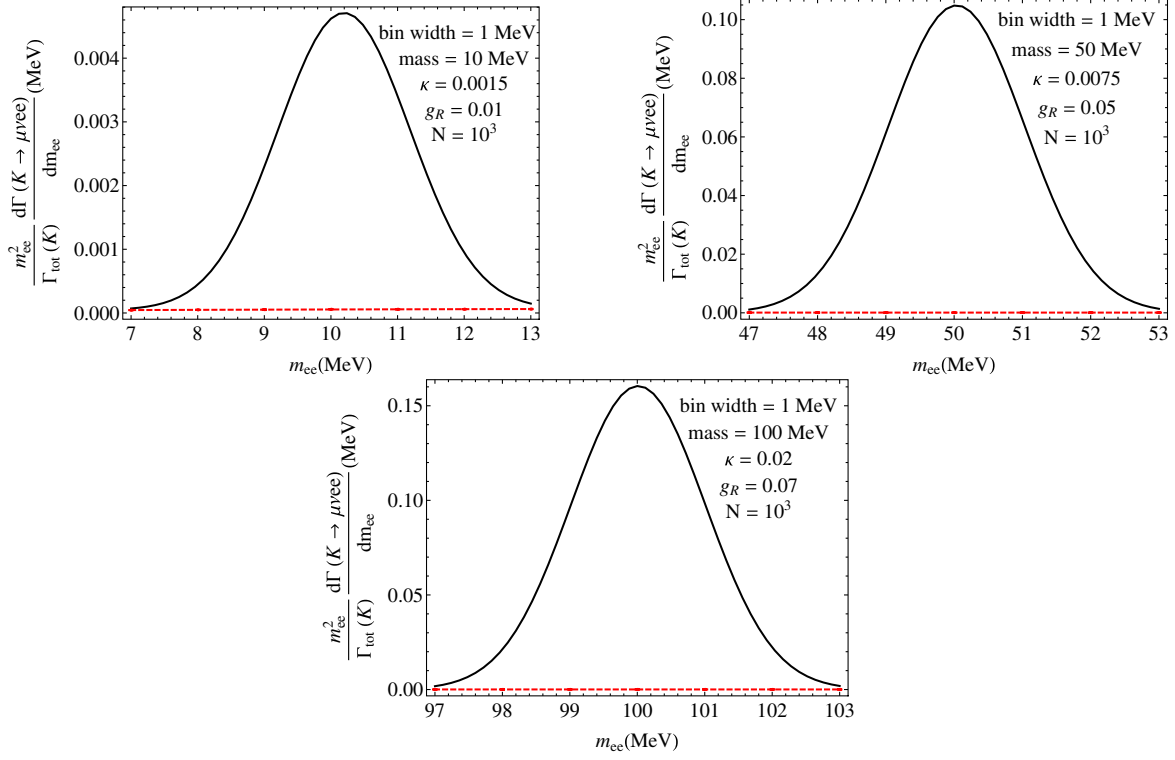


FIG. 3 (color online). QED prediction for $K^+ \rightarrow \mu^+ + \nu_\mu + e^+ + e^-$ (red dashed curve) and the prediction with the additional lepton-universality violating particle of Batell *et al.* (black curve). Simulated data points with error bars accounting for the statistical uncertainty, anticipating $N = 1000$ events per bin, are smaller than the width of the red line; κ and g_R are couplings in the Batell *et al.* model [8].

the detector and is small enough that the decay width is narrower than the experimental resolution, the size of the bump that may be seen in the e^+e^- spectrum is independent of its value. The allowed range of couplings is rather wide.

To account for this particle's contribution to the amplitude $K^+ \rightarrow \mu^+ + \nu_\mu + e^+ + e^-$, the photon's propagator and charged fermion couplings are modified to

$$\frac{-i}{q^2} \rightarrow \frac{-i}{q^2 - m_{A'}^2 + im_{A'}\Gamma}, \quad (12)$$

$$-ie\gamma^\mu \rightarrow -i\gamma^\mu \epsilon(C_V(m_{A'}) + C_A(m_{A'})\gamma^5), \quad (13)$$

where $C_V(m_{A'})$ and $C_A(m_{A'})$ are the polar and axial vector couplings to muons calculated in [9]. The $C_{V,A}$ are chosen, as described in [9], so that the muonic Lamb shift is lowered by the correct amount, and so that the $(g-2)_\mu$ discrepancy is at its known value. The values depend on the mass $m_{A'}$. The parameter $\epsilon = 1$ for the muon and is the square root of Eq. (11) for the electron. We assume the electron's value of ϵ can also be applied to the kaon. See Fig. 4 for the prediction of a 30 MeV particle. As mentioned in Sec. II, $H^{\rho\nu}$ is very small and we only consider the IB contributions. The peak is so large that omitting $H^{\rho\nu}$ will not affect the conclusion that this new particle will be detectable at JPARC. The overall

$K \rightarrow \mu\nu A' \rightarrow \mu\nu e^+e^-$ branching ratio is a few times 10^{-5} for polar/axial vector A' and this mass.

The new particle may instead have scalar and pseudo-scalar couplings to the muon and proton, and we also model

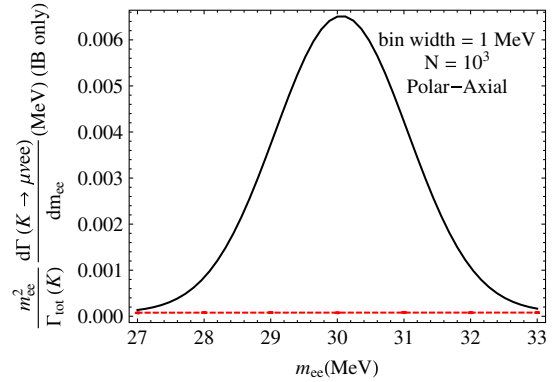


FIG. 4 (color online). QED prediction for $K^+ \rightarrow \mu^+ + \nu_\mu + e^+ + e^-$ (red dashed curve) and the prediction with the additional 30 MeV lepton-universality violating polar/axial vector particle of our model [9] (black curve). Simulated data points with error bars accounting for the statistical uncertainty are smaller than the width of the red line. The couplings to the new exchanged particle are chosen to fit the muonic Lamb shift and $\delta(g-2)_\mu$; for this mass, $C_V = 4.6 \times 10^{-3}$ and $C_A = 6 \times 10^{-4}$ are the geometric means of the proton and muon couplings [with the electron coupling limited by the square root of Eq. (11)].

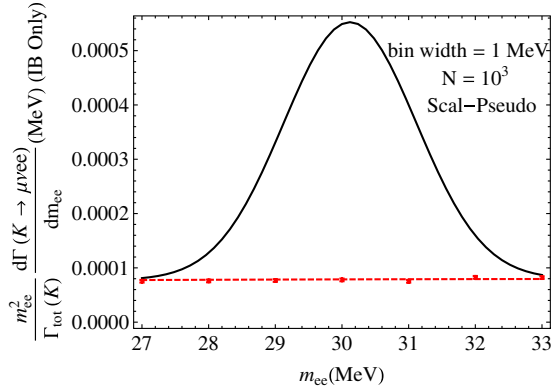


FIG. 5 (color online). QED prediction for $K^+ \rightarrow \mu^+ + \nu_\mu + e^+ + e^-$ (red dashed curve) and the prediction with the additional 30 MeV lepton-universality violating scalar/pseudoscalar particle of our model (black curve). Data points are simulated and possess error bars accounting for the statistical uncertainty. This plot has $C_S = 4.6 \times 10^{-3}$ and $C_P = 6.8 \times 10^{-3}$ as the geometric means of the proton and muon couplings.

this. Its contribution to the amplitude $K^+ \rightarrow \mu^+ + \nu_\mu + e^+ + e^-$ is found by modifying the photon's propagator as before and modifying the charged fermion couplings as

$$-ie\gamma^\mu \rightarrow -ie(C_S(m_{A'}) + iC_P(m_{A'})\gamma^5). \quad (14)$$

Here, $C_S(m_{A'})$ and $C_P(m_{A'})$ are the scalar and pseudoscalar couplings to muons calculated in [9]. For muons, $\varepsilon = 1$ and it is smaller for other fermions. See Fig. 5 for the prediction of a 30 MeV particle with scalar and pseudoscalar couplings. Here, we also only consider the IB contribution since $H^{\rho\nu}$ is so small. The overall $K \rightarrow \mu\nu A' \rightarrow \mu\nu e^+ e^-$ branching ratio is a few times 10^{-6} for this spin and mass.

VI. CONCLUSIONS

Analyses of the proton radius conundrum have provoked suggestions of muon-electron nonuniversality that may also explain the $(g-2)_\mu$ puzzle but whose mass and couplings can otherwise be arranged to not contradict known constraints (see [5–9]). Further tests should be made. Low mass new particles that couple to muons must show themselves throughout corrections to particle decays involving muons.

Here we have focused on what might be called leptonic Dalitz decays of the charged kaon $K \rightarrow \mu\nu e^+ e^-$. There is a calculated [15] and partly measured (in the higher m_{ee} mass region) QED background [11]. New light massive particles that couple well to muons, and to electrons at least enough to decay before leaving the detector apparatus, should stand out as bumps in the e^+e^- spectrum at the mass of the new particles.

An experiment like E36 at JPARC that anticipates 10^{10} kaon decays will see, by calculation, about 200,000 QED produced $K \rightarrow \mu\nu e^+ e^-$ decays with a smooth and predictable background. With these statistics and the masses and couplings of the new particles tuned to explain the proton radius discrepancy and to accommodate the $(g-2)_\mu$ discrepancy, the bumps in the e^+e^- spectrum will be very striking and very observable if the particles exist.

ACKNOWLEDGMENTS

We thank Michael Kohl and Marc Vanderhaeghen for useful conversations. We thank the National Science Foundation for support under Grant No. PHY-1205905. B. R. also thanks the DFG in the Grant No. SFB 1044 for support.

-
- [1] R. Pohl, A. Antognini, F. Nez, F. D. Amaro, F. Biraben *et al.*, *Nature (London)* **466**, 213 (2010).
 - [2] A. Antognini, F. Nez, K. Schuhmann, F. D. Amaro, F. Biraben *et al.*, *Science* **339**, 417 (2013).
 - [3] P. J. Mohr, B. N. Taylor, and D. B. Newell, *Rev. Mod. Phys.* **84** 1527 (2012).
 - [4] J. Jaeckel and S. Roy, *Phys. Rev. D* **82**, 125020 (2010).
 - [5] D. Tucker-Smith and I. Yavin, *Phys. Rev. D* **83**, 101702 (2011).
 - [6] V. Barger, C.-W. Chiang, W.-Y. Keung, and D. Marfatia, *Phys. Rev. Lett.* **106**, 153001 (2011).
 - [7] V. Barger, C.-W. Chiang, W.-Y. Keung, and D. Marfatia, *Phys. Rev. Lett.* **108**, 081802 (2012).
 - [8] B. Batell, D. McKeen, and M. Pospelov, *Phys. Rev. Lett.* **107**, 011803 (2011).
 - [9] C. E. Carlson and B. C. Rislow, *Phys. Rev. D* **86**, 035013 (2012).
 - [10] C. Pang, R. Hildebrand, G. Cable, and R. Stiening, *Phys. Rev. D* **8**, 1989 (1973).
 - [11] A. Poblaguev, R. Appel, G. Atoyan, B. Bassalleck, D. Bergman *et al.*, *Phys. Rev. Lett.* **89**, 061803 (2002).
 - [12] C. Djalali (TREK Collaboration), AIP Conf. Proc. **1423**, 297 (2012); M. Kohl (private communication).
 - [13] M. Kohl (private communication).
 - [14] J. Beringer *et al.* (Particle Data Group), *Phys. Rev. D* **86**, 010001 (2012).
 - [15] J. Bijnens, G. Ecker, and J. Gasser, *Nucl. Phys.* **B396**, 81 (1993).
 - [16] T. Beranek, H. Merkel, and M. Vanderhaeghen, *Phys. Rev. D* **88**, 015032 (2013).
 - [17] S. Krishna and H. Mani, *Phys. Rev. D* **5**, 678 (1972).
 - [18] T. Aoyama, M. Hayakawa, T. Kinoshita, and M. Nio, *Phys. Rev. Lett.* **109**, 111808 (2012).