PHYSICAL REVIEW D **89**, 034017 (2014)

Analysis of hadronic coupling constants $G_{B_c^*B_c\Upsilon}$, $G_{B_c^*B_cJ/\psi}$, $G_{B_cB_c\Upsilon}$ and $G_{B_cB_cJ/\psi}$ with QCD sum rules

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(Received 26 August 2013; published 14 February 2014)

In this article, we study the momentum dependence of the hadronic coupling constants $G_{B_c^*B_c\Upsilon}$, $G_{B_c^*B_cJ/\psi}$, $G_{B_cB_c\Upsilon}$, and $G_{B_cB_cJ/\psi}$ with the off-shell Υ and J/ψ using the three-point QCD sum rules. Then we fit the hadronic coupling constants into analytical functions and extrapolate them into deep time-like regions to obtain the on-shell values $G_{B_c^*B_c\Upsilon}(q^2 = M_\Upsilon^2)$, $G_{B_c^*B_cJ/\psi}(q^2 = M_{J/\psi}^2)$, $G_{B_cB_c\Upsilon}(q^2 = M_\Upsilon^2)$, and $G_{B_cB_cJ/\psi}(q^2 = M_{J/\psi}^2)$ for the first time. These hadronic coupling constants can be taken as basic input parameters in phenomenological analyses.

DOI: 10.1103/PhysRevD.89.034017 PACS numbers: 12.38.Lg, 14.40.Pq

I. INTRODUCTION

The suppression of J/ψ production in relativistic heavy ion collisions is considered an important signature to identify the quark-gluon plasma [1]. The dissociation of J/ψ in the quark-gluon plasma due to color screening can lead to a reduction of its production. The bottomonium states are also sensitive to the color screening: the Υ suppression in high-energy heavy ion collisions can also be taken as a signature to identify the quark-gluon plasma [2]. The suppressions of Υ production in ultrarelativistic heavy ion collisions will be studied in detail at the Relativistic Heavy Ion Collider (RHIC) and LHC. Before drawing a definite conclusion on the appearance of the quark-gluon plasma, we have to disentangle the color screening versus recombination of the off-diagonal $\bar{c}c$ (or $\bar{b}b$) pairs in the hot dense medium versus cold nuclear matter effects, such as nuclear absorption, shadowing, and antishadowing [3,4].

We can study the heavy quarkonium absorptions with the effective Lagrangians in meson-exchange models [5], and calculate the absorption cross sections based on the interactions among the heavy quarkonia and heavy mesons, where the hadronic coupling constants are basic input parameters. A detailed knowledge of the hadronic coupling constants is of great importance in understanding the effects of heavy quarkonium absorptions in hadronic matter. Furthermore, the hadronic coupling constants among the heavy quarkonia and heavy mesons play an important role in understanding final-state interactions in the heavy quarkonium decays [6].

The hadronic coupling constants in the $D^*D\pi$, D^*D_sK, D_s^*DK, $B^*B\pi$, B_s^*BK, $DD\rho$, D_sDK^*, B_sBK^*, $D^*D\rho$, $D_s^*DK^*$, $B_s^*BK^*$, $D^*D^*\rho$, $B^*B^*\rho$, $B_{s0}BK$, $B_{s1}B^*K$, $D_s^*DK_1$, $B_s^*BK_1$, $J/\psi DD$, $J/\psi DD^*$, $J/\psi D^*D^*$ vertices have been studied with the three-point QCD sum rules (QCDSR) [7,8], while the hadronic coupling constants in the $D^*D\pi$, D^*D_sK, D_s^*DK, $B^*B\pi$, $DD\rho$, DD_sK^*, $D_sD_s\phi$, $BB\rho$, $D^*D\rho$, $D^*D_sK^*$, $D_s^*D_s\phi$, $B^*B\rho$, $D^*D^*\pi$, $D^*D_s^*K$, $B^*B^*\pi$, $D^*D^*\rho$, $D_0D\pi$, $B_0B\pi$, D_0D_sK, $D_{s0}DK$, $B_{s0}BK$, $D_1D^*\pi$, $B_1B^*\pi$, $D_{s1}D^*K$, $B_{s1}B^*K$, $B_1B_0\pi$, $B_2B_1\pi$, $B_2B^*\pi$, $B_1B^*\rho$, $B_1B\rho$, $B_2B^*\rho$, $B_2B_1\rho$ vertices have been studied with the light-cone QCDSR [9].

To our knowledge, the hadronic coupling constants among the heavy quarkonium states have not been studied with the three-point QCDSR or light-cone QCDSR. In this article, we study the vertices $B_c^*B_c\Upsilon$, $B_c^*B_cJ/\psi$, $B_cB_c\Upsilon$, and B_cB_cJ/ψ with the three-point QCDSR. The QCD sum rules are a powerful nonperturbative approach in studying the heavy quarkonium states, and they have given many successful descriptions of the masses, decay constants, form factors, and hadronic coupling constants [10–12].

The $B_c^{*\pm}$ mesons have not been observed yet, but they are expected to be observed at the LHC through the radiative transitions. In previous works, we studied the vector and axial-vector B_c mesons with the QCDSR and made reasonable predictions of the masses and decay constants, then calculated the $B_c^* \to B_c$ electromagnetic form factor with the three-point QCDSR, and finally obtained the decay width of the radiative transitions $B_c^{*\pm} \to B_c^\pm\gamma$ [13,14].

This article is arranged as follows. We study the $B_c^*B_c\Upsilon$, $B_c^*B_cJ/\psi$, $B_cB_c\Upsilon$, and B_cB_cJ/ψ vertices using the three-point QCDSR in Sec. II. In Sec. III, we present the numerical results and discussions, and Sec. IV is reserved for our conclusions.

II. THE $B_c^*B_c\Upsilon$ AND $B_cB_c\Upsilon$ (ALSO $B_c^*B_cJ/\psi$ AND B_cB_cJ/ψ) VERTICES WITH QCD SUM RULES

We study the $B_c^*B_c\Upsilon$ and $B_cB_c\Upsilon$ vertices with the three-point correlation functions $\Pi_{\mu\nu}(p,p')$ and $\Pi_\mu(p,p')$, respectively,

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$$\Pi_{\mu\nu}(p,p') = i^2 \int d^4x d^4y e^{ip'\cdot x + i(p-p')\cdot y} \langle 0|T\{J_5(x) j_\mu(y) J_\nu^\dagger(0)\}|0\rangle,$$
$$\Pi_{\mu}(p,p') = i^2 \int d^4x d^4y e^{ip'\cdot x + i(p-p')\cdot y} \langle 0|T\{J_5(x) j_\mu(y) J_5^\dagger(0)\}|0\rangle, \tag{1}$$

where

$$j_\mu(x) = \bar{b}(x)\gamma_\mu b(x),$$
$$J_5(x) = \bar{c}(x) i\gamma_5 b(x),$$
$$J_\nu^\dagger(x) = \bar{b}(x)\gamma_\nu c(x), \tag{2}$$

and the currents $j_\mu(x)$, $J_5(x)$, and $J_\nu^\dagger(x)$ interpolate the heavy quarkonia Υ, B_c, and B_c^*, respectively.

We can insert a complete set of intermediate hadronic states with the same quantum numbers as the current operators $j_\mu(y)$, $J_5(x)$, $J_\nu^\dagger(0)$, and $J_5^\dagger(0)$ into the correlation functions $\Pi_{\mu\nu}(p,p')$ and $\Pi_\mu(p,p')$ to obtain the hadronic representation [10,11]. After isolating the ground-state contributions coming from the heavy quarkonia Υ, B_c, and B_c^*, we get

$$\Pi_{\mu\nu}(p,p') = \frac{\langle 0|J_5(0)|B_c(p')\rangle\langle 0|j_\mu(0)|\Upsilon(q)\rangle\langle B_c^*(p)|J_\nu^\dagger(0)|0\rangle\langle B_c(p')\Upsilon(q)|\mathcal{L}(0)|B_c^*(p)\rangle}{(M_{B_c}^2 - p'^2)(M_\Upsilon^2 - q^2)(M_{B_c^*}^2 - p^2)} + \cdots$$
$$= -\frac{f_{B_c} M_{B_c}^2 f_{B_c^*} M_{B_c^*} f_\Upsilon M_\Upsilon}{(m_b + m_c)(M_{B_c}^2 - p'^2)(M_{B_c^*}^2 - p^2)(M_\Upsilon^2 - q^2)} G_{B_c^* B_c \Upsilon}(q^2) \epsilon_{\mu\nu\alpha\beta} p^\alpha p'^\beta + \cdots, \tag{3}$$

$$\Pi_{\mu}(p,p') = \frac{\langle 0|J_5(0)|B_c(p')\rangle\langle 0|j_\mu(0)|\Upsilon(q)\rangle\langle B_c(p)|J_5^\dagger(0)|0\rangle\langle B_c(p')\Upsilon(q)|\mathcal{L}(0)|B_c(p)\rangle}{(M_{B_c}^2 - p'^2)(M_\Upsilon^2 - q^2)(M_{B_c}^2 - p^2)} + \cdots$$
$$= \frac{f_{B_c}^2 M_{B_c}^4 f_\Upsilon M_\Upsilon}{(m_b + m_c)^2(M_{B_c}^2 - p'^2)(M_{B_c}^2 - p^2)(M_\Upsilon^2 - q^2)} G_{B_c B_c \Upsilon}(q^2)(p+p')_\mu + \cdots$$
$$= \Gamma_p(p,p') p_\mu + \Gamma_{p'}(p,p') p'_\mu + \cdots, \tag{4}$$

where we have used the following effective Lagrangian $\mathcal{L}$ and definitions for the decay constants $f_{B_c^*}$, f_{B_c}, f_Υ:

$$\mathcal{L} = G_{B_c^* B_c \Upsilon} \epsilon_{\lambda\tau\rho\sigma} B_c^\dagger \partial^\lambda \Upsilon^\tau \partial^\rho B_c^{*\sigma} + i G_{B_c B_c J/\psi} \Upsilon^\mu (B_c^\dagger \partial_\mu B_c - \partial_\mu B_c^\dagger B_c), \tag{5}$$

$$\langle 0|J_\mu(0)|B_c^*(p)\rangle = f_{B_c^*} M_{B_c^*} \zeta_\mu,$$
$$\langle 0|J_5(0)|B_c(p')\rangle = \frac{f_{B_c} M_{B_c}^2}{m_b + m_c},$$
$$\langle 0|j_\mu(0)|\Upsilon(q)\rangle = f_\Upsilon M_\Upsilon \xi_\mu, \tag{6}$$

where $q_\mu = (p - p')_\mu$, and ζ_μ and ξ_μ are the polarization vectors. The tensor structures p_μ and p'_μ are associated with the correlation functions $\Gamma_p(p,p')$ and $\Gamma_{p'}(p,p')$, respectively, and we obtain the QCDSR by considering the combination $\Gamma_p(p,p') + \Gamma_{p'}(p,p')$.

The effective fields describe point-like particles only in the case that all the interacting particles are on the mass shell. When at least one particle in the vertex is off-shell, the finite-size effects of the hadrons become important. We should introduce form factors in the hadronic coupling constants to parametrize the off-shell effects, which are of great importance in calculating scattering amplitudes at the hadronic level. In this article, we parametrize the q^2 dependence of the hadronic coupling constants $G(q^2)$ with suitable functions, then obtain the on-shell values $G(q^2 = -Q^2 = M_\Upsilon^2)$ by analytically continuing the q^2 to the physical region.

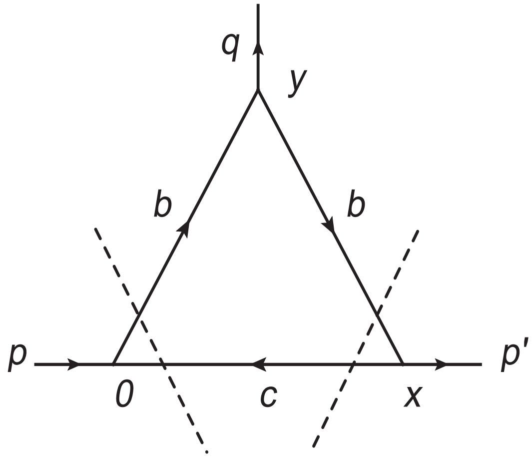

FIG. 1. The leading-order contributions. The dashed lines denote the Cutkosky cuts.

Now we briefly outline the operator product expansion for the correlation functions $\Pi_{\mu\nu}(p,p')$ and $\Pi_{\mu}(p,p')$. First, we contract the quark fields in the correlation functions $\Pi_{\mu\nu}(p,p')$ and $\Pi_{\mu}(p,p')$ with Wick's theorem,

$$\begin{aligned}\Pi_{\mu\nu}(p,p') &= \int d^4x d^4y e^{ip'\cdot x+i(p-p')\cdot y} \\ &\quad\times \mathrm{Tr}\{i\gamma_5 B^{mn}(x-y)\gamma_\mu B^{nk}(y)\gamma_\nu C^{km}(-x)\}, \\ \Pi_{\mu}(p,p') &= \int d^4x d^4y e^{ip'\cdot x+i(p-p')\cdot y} \\ &\quad\times \mathrm{Tr}\{i\gamma_5 B^{mn}(x-y)\gamma_\mu B^{nk}(y) i\gamma_5 C^{km}(-x)\}.\end{aligned} \tag{7}$$

Then, we replace the b and c quark propagators $B^{ij}(x)$ and $C^{ij}(x)$ with the corresponding full propagators $S_{ij}(x)$,

$$S_{ij}(x) = \frac{i}{(2\pi)^4}\int d^4k e^{-ik\cdot x}\left\{\frac{\delta_{ij}}{\not{k}-m_Q} - \frac{g_s G^n_{\alpha\beta} t^n_{ij}}{4}\frac{\sigma^{\alpha\beta}(\not{k}+m_Q)+(\not{k}+m_Q)\sigma^{\alpha\beta}}{(k^2-m_Q^2)^2} + \frac{\delta_{ij}\langle g_s^2 GG\rangle}{12}\frac{m_Q k^2+m_Q^2\not{k}}{(k^2-m_Q^2)^4}+\cdots\right\}, \tag{8}$$

where $Q=c$, b, $\langle g_s^2 GG\rangle = \langle g_s^2 G^n_{\alpha\beta}G^{n\alpha\beta}\rangle$, $t^n=\frac{\lambda^n}{2}$, the λ^n are the Gell-Mann matrixes, and the i, j are color indices [11]. Finally, we then compute the integrals. In this article, we take into account the leading-order contributions $\Pi^0_{\mu\nu}(p,p')$, $\Pi^0_{\mu}(p,p')$ and gluon condensate contributions $\Pi^{GG}_{\mu\nu}(p,p')$, $\Pi^{GG}_{\mu}(p,p')$ in the operator product expansion, and show them explicitly using the Feynman diagrams in Figs. 1 and 2.

The leading-order contributions $\Pi^0_{\mu\nu}(p,p')$, $\Pi^0_{\mu}(p,p')$ can be written as

$$\Pi^0_{\mu\nu}(p,p') = \frac{3}{(2\pi)^4}\int d^4k \frac{\mathrm{Tr}\{\gamma_5[\not{k}+m_b]\gamma_\mu[\not{k}+\not{p}-\not{p}'+m_b]\gamma_\nu[\not{k}-\not{p}'+m_c]\}}{[k^2-m_b^2][(k+p-p')^2-m_b^2][(k-p')^2-m_c^2]} = \int dsdu \frac{\rho_{\mu\nu}(s,u,q^2)}{(s-p^2)(u-p'^2)}, \tag{9}$$

$$\Pi^0_{\mu}(p,p') = \frac{3i}{(2\pi)^4}\int d^4k \frac{\mathrm{Tr}\{\gamma_5[\not{k}+m_b]\gamma_\mu[\not{k}+\not{p}-\not{p}'+m_b]\gamma_5[\not{k}-\not{p}'+m_c]\}}{[k^2-m_b^2][(k+p-p')^2-m_b^2][(k-p')^2-m_c^2]} = \int dsdu \frac{\rho_{\mu}(s,u,q^2)}{(s-p^2)(u-p'^2)}. \tag{10}$$

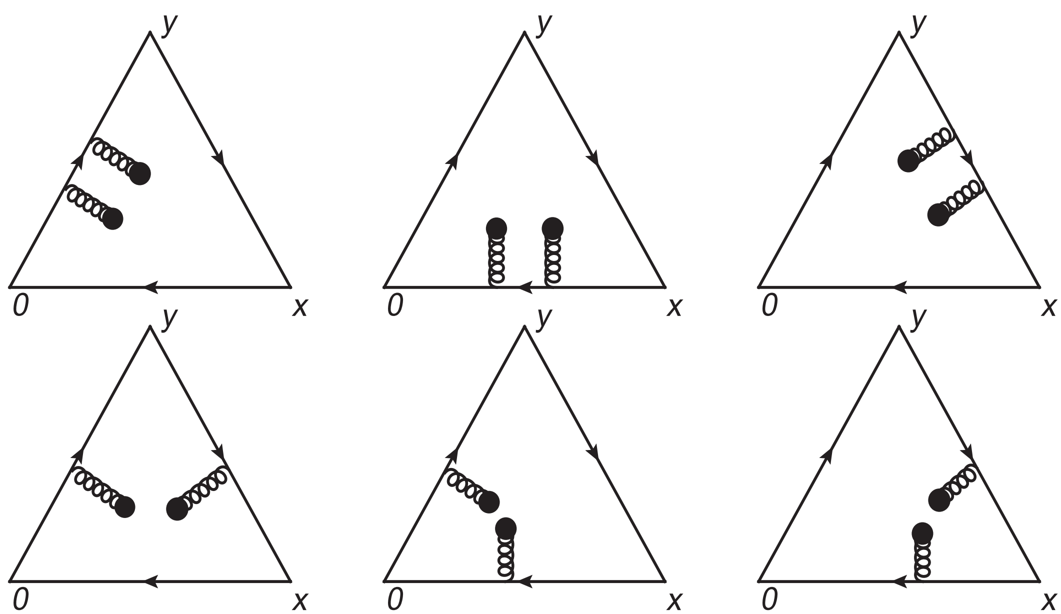

FIG. 2. The gluon condensate contributions.

We put all the quark lines on mass shell using Cutkosky's rules (see Fig. 1) and obtain the leading-order spectral densities $\rho_{\mu\nu}(s,u,q^2)$ and $\rho_\mu(s,u,q^2)$,

$$\begin{aligned}\rho_{\mu\nu}(s,u,q^2) &= -\frac{3i}{(2\pi)^3}\int d^4k\delta[k^2-m_b^2]\delta[(k+p-p')^2-m_b^2]\delta[(k-p')^2-m_c^2]\\ &\quad\times \mathrm{Tr}\{\gamma_5[\not{k}+m_b]\gamma_\mu[\not{k}+\not{p}-\not{p}'+m_b]\gamma_\nu[\not{k}-\not{p}'+m_c]\}\\ &= -\frac{3\epsilon_{\mu\nu\alpha\beta}p^\alpha p'^\beta}{4\pi^2\sqrt{\lambda(s,u,q^2)}}\left\{m_b+\frac{(m_b-m_c)(s+u-q^2+2m_b^2-2m_c^2)q^2}{\lambda(s,u,q^2)}\right\},\end{aligned} \tag{11}$$

$$\begin{aligned}\rho_\mu(s,u,q^2) &= \frac{3}{(2\pi)^3}\int d^4k\delta[k^2-m_b^2]\delta[(k+p-p')^2-m_b^2]\delta[(k-p')^2-m_c^2]\\ &\quad\times \mathrm{Tr}\{\gamma_5[\not{k}+m_b]\gamma_\mu[\not{k}+\not{p}-\not{p}'+m_b]\gamma_5[\not{k}-\not{p}'+m_c]\}\\ &= \frac{3}{8\pi^2\sqrt{\lambda(s,u,q^2)}}\{[s+u-q^2-2(m_b-m_c)^2](C_p p_\mu+C_{p'}p'_\mu)+[u-(m_b-m_c)^2]q_\mu+q^2p'_\mu\},\end{aligned} \tag{12}$$

where

$$\begin{aligned}C_p &= \frac{(s+u-q^2)(u+m_b^2-m_c^2)-2u(u-q^2+m_b^2-m_c^2)}{\lambda(s,u,q^2)},\\ C_{p'} &= \frac{(s+u-q^2)(u-q^2+m_b^2-m_c^2)-2s(u+m_b^2-m_c^2)}{\lambda(s,u,q^2)},\end{aligned} \tag{13}$$

and $\lambda(a,b,c)=a^2+b^2+c^2-2ab-2bc-2ca$. (We have used the formulas presented in Refs. [15,16] to compute the integrals.)

We calculate the gluon condensate contributions directly (see Fig. 2) and obtain the following formulas:

$$\begin{aligned}\Pi_{\mu\nu}^{GG}(p,p') &= \frac{i\epsilon_{\mu\nu\alpha\beta}}{4\pi^2}\left\langle\frac{\alpha_sGG}{\pi}\right\rangle\{-m_b^3(\bar{I}_{411}+\bar{I}_{141})p^\alpha p'^\beta-m_b^2(m_b-m_c)(\bar{I}^\alpha_{411}+\bar{I}^\alpha_{141})q^\beta\\ &\quad-m_bm_c^2\bar{I}_{114}p^\alpha p'^\beta+m_c^2(m_c-m_b)\bar{I}^\alpha_{114}q^\beta-m_b\bar{I}_{311}p^\alpha p'^\beta-m_b\bar{I}^\alpha_{311}p^\beta+m_b\bar{I}^\alpha_{131}p'^\beta+m_c\bar{I}^\alpha_{113}q^\beta\}\\ &\quad+\frac{i\epsilon_{\mu\nu\alpha\beta}}{24\pi^2}\left\langle\frac{\alpha_sGG}{\pi}\right\rangle\{(m_b-m_c)(\bar{I}^\alpha_{221}+\bar{I}^\alpha_{122})q^\beta+m_b(\bar{I}_{221}+\bar{I}_{122})p^\alpha p'^\beta\\ &\quad-2m_b\bar{I}^\alpha_{212}p'^\beta-3(m_b-m_c)\bar{I}^\alpha_{212}q^\beta-3m_b\bar{I}_{212}p^\alpha p'^\beta\},\end{aligned} \tag{14}$$

$$\begin{aligned}\Pi^\mu_{GG}(p,p') &= \frac{i}{8\pi^2}\left\langle\frac{\alpha_sGG}{\pi}\right\rangle\{m_b^2[-(2(m_b-m_c)^2+q^2)(\bar{I}^\mu_{411}+\bar{I}^\mu_{141})+\bar{K}^\mu_{411}+\bar{K}^\mu_{141}+\bar{N}^\mu_{411}\\ &\quad+\bar{N}^\mu_{141}-(m_b-m_c)^2(\bar{I}_{411}+\bar{I}_{141})q^\mu+(\bar{N}_{411}+\bar{N}_{141})q^\mu+q^2(\bar{I}_{411}+\bar{I}_{141})p'^\mu]\\ &\quad+m_c^2[-(2(m_b-m_c)^2+q^2)\bar{I}^\mu_{114}+\bar{K}^\mu_{114}+\bar{N}^\mu_{114}-(m_b-m_c)^2\bar{I}_{114}q^\mu+\bar{N}_{114}q^\mu\\ &\quad+q^2\bar{I}_{114}p'^\mu]+m_b[2(m_c-m_b)(\bar{I}^\mu_{311}+\bar{I}^\mu_{131})+m_b(\bar{I}_{311}+\bar{I}_{131})p'^\mu\\ &\quad+(2m_c-m_b)\bar{I}_{311}q^\mu]+m_c[(2m_b-m_c)(2\bar{I}^\mu_{113}+\bar{I}_{113}q^\mu)]\}\\ &\quad+\frac{i}{48\pi^2}\left\langle\frac{\alpha_sGG}{\pi}\right\rangle\{(12m_bm_c-4m_b^2-2m_c^2+q^2)\bar{I}^\mu_{221}+\bar{K}^\mu_{221}+\bar{N}^\mu_{221}\\ &\quad+[(6m_bm_c-m_b^2-m_c^2)(\bar{I}_{221}+\bar{I}_{122})+\bar{N}_{221}+\bar{N}_{122}]q^\mu+(2m_b^2-q^2)\bar{I}_{221}p'^\mu\\ &\quad+(18m_bm_c-8m_b^2-4m_c^2-q^2)(\bar{I}^\mu_{122}+\bar{I}^\mu_{212})+3\bar{K}^\mu_{122}+\bar{N}^\mu_{122}+\bar{K}^\mu_{212}+3\bar{N}^\mu_{212}\\ &\quad+(4m_b^2+q^2)(\bar{I}_{122}+\bar{I}_{212})p'^\mu+3(4m_bm_c-m_b^2-m_c^2)\bar{I}_{212}q^\mu+3\bar{N}_{212}q^\mu\\ &\quad-(2\bar{I}_{121}+3\bar{I}_{112}+3\bar{I}_{211})q^\mu+(\bar{I}_{121}-2\bar{I}_{112}+\bar{I}_{211})p'^\mu-(6\bar{I}^\mu_{121}+4\bar{I}^\mu_{112}+6\bar{I}^\mu_{211})\},\end{aligned} \tag{15}$$

where

$$\begin{aligned}
\bar{I}_{ijn} &= \int d^4k \frac{1}{[k^2-m_b^2]^i[(k+p-p')^2-m_b^2]^j[(k-p')^2-m_c^2]^n},\\
\bar{K}_{ijn} &= \int d^4k \frac{p^2}{[k^2-m_b^2]^i[(k+p-p')^2-m_b^2]^j[(k-p')^2-m_c^2]^n},\\
\bar{N}_{ijn} &= \int d^4k \frac{p'^2}{[k^2-m_b^2]^i[(k+p-p')^2-m_b^2]^j[(k-p')^2-m_c^2]^n},\\
\bar{I}^{\alpha}_{ijn} &= \int d^4k \frac{k^\alpha}{[k^2-m_b^2]^i[(k+p-p')^2-m_b^2]^j[(k-p')^2-m_c^2]^n},\\
\bar{K}^{\alpha}_{ijn} &= \int d^4k \frac{p^2k^\alpha}{[k^2-m_b^2]^i[(k+p-p')^2-m_b^2]^j[(k-p')^2-m_c^2]^n},\\
\bar{N}^{\alpha}_{ijn} &= \int d^4k \frac{p'^2k^\alpha}{[k^2-m_b^2]^i[(k+p-p')^2-m_b^2]^j[(k-p')^2-m_c^2]^n}.
\end{aligned} \tag{16}$$

We take quark-hadron duality below the thresholds s_0 and u_0 for the mesons B_c^* (or B_c) and B_c, respectively, perform a double Borel transform with respect to the variables $P^2=-p^2$ and $P'^2=-p'^2$, respectively, and obtain the QCDSR for the coupling constants $G_{B_c^*B_c\Upsilon}(q^2)$ and $G_{B_cB_c\Upsilon}(q^2)$,

$$\begin{aligned}
G_{B_c^*B_c\Upsilon}(q^2) =& \frac{(m_b+m_c)(M_\Upsilon^2-q^2)}{f_\Upsilon f_{B_c^*}f_{B_c}M_\Upsilon M_{B_c^*}M_{B_c}^2}\exp\left(\frac{M_{B_c^*}^2}{M_1^2}+\frac{M_{B_c}^2}{M_2^2}\right)\frac{3}{4\pi^2}\int_{(m_b+m_c)^2}^{s_0}ds\int_{(m_b+m_c)^2}^{u_0}du\frac{\mathcal{C}}{\sqrt{\lambda(s,u,q^2)}}\exp\left(-\frac{s}{M_1^2}-\frac{u}{M_2^2}\right)\\
&\times\left\{m_b+\frac{(m_b-m_c)(s+u-q^2+2m_b^2-2m_c^2)q^2}{\lambda(s,u,q^2)}\right\}\Bigg|_{|f(s,u,q^2)|\le 1} - \frac{(m_b+m_c)(M_\Upsilon^2-q^2)M_1^2M_2^2}{f_\Upsilon f_{B_c^*}f_{B_c}M_\Upsilon M_{B_c^*}M_{B_c}^2}\left\langle\frac{\alpha_sGG}{\pi}\right\rangle\\
&\times\exp\left(\frac{M_{B_c^*}^2}{M_1^2}+\frac{M_{B_c}^2}{M_2^2}\right)\left\{\frac{m_b^3}{4\pi^2}(I_0^{411}+I_0^{141})-\frac{m_b^2(m_b-m_c)}{4\pi^2}(I_{01}^{411}+I_{01}^{141})+\frac{m_bm_c^2}{4\pi^2}I_0^{114}+\frac{m_c^2(m_c-m_b)}{4\pi^2}I_{01}^{114}\right.\\
&+\frac{m_b}{4\pi^2}(I_0^{311}-I_{01}^{311}+I_{10}^{311})-\frac{m_b}{4\pi^2}I_{10}^{131}+\frac{m_c}{4\pi^2}I_{01}^{113}+\frac{m_b}{8\pi^2}I_0^{212}+\frac{m_b-m_c}{24\pi^2}(I_{01}^{221}+I_{01}^{122})-\frac{m_b}{24\pi^2}(I_0^{221}+I_0^{122})\\
&\left.+\frac{m_b}{12\pi^2}I_{10}^{212}-\frac{m_b-m_c}{8\pi^2}I_{01}^{212}\right\},
\end{aligned} \tag{17}$$

$$\begin{aligned}
G_{B_cB_c\Upsilon}(q^2) =& \frac{(m_b+m_c)^2(M_\Upsilon^2-q^2)}{2f_\Upsilon f_{B_c}^2M_\Upsilon M_{B_c}^4}\exp\left(\frac{M_{B_c}^2}{M_1^2}+\frac{M_{B_c}^2}{M_2^2}\right)\frac{3}{8\pi^2}\int_{(m_b+m_c)^2}^{s_0}ds\int_{(m_b+m_c)^2}^{u_0}du\frac{\mathcal{C}}{\sqrt{\lambda(s,u,q^2)}}\exp\left(-\frac{s}{M_1^2}-\frac{u}{M_2^2}\right)\\
&\times\left\{[s+u-q^2-2(m_b-m_c)^2]\left[\frac{(u-s-q^2)(u+m_b^2-m_c^2)}{\lambda(s,u,q^2)}+\frac{(s-u-q^2)(u-q^2+m_b^2-m_c^2)}{\lambda(s,u,q^2)}\right]+q^2\right\}\Bigg|_{|f(s,u,q^2)|\le 1}\\
&-\frac{(m_b+m_c)^2(M_\Upsilon^2-q^2)M_1^2M_2^2}{2f_\Upsilon f_{B_c}^2M_\Upsilon M_{B_c}^4}\left\langle\frac{\alpha_sGG}{\pi}\right\rangle\exp\left(\frac{M_{B_c}^2}{M_1^2}+\frac{M_{B_c}^2}{M_2^2}\right)\left\{\frac{m_b^2}{8\pi^2}[-(2(m_b-m_c)^2+q^2)(I_{01}^{411}+I_{01}^{141})\right.\\
&+K_{01}^{411}+K_{01}^{141}+N_{01}^{411}+N_{01}^{141}+q^2(I_0^{411}+I_0^{141})]+\frac{m_c^2}{8\pi^2}[-(2(m_b-m_c)^2+q^2)I_{01}^{114}+K_{01}^{114}+N_{01}^{114}+q^2I_0^{114}]\\
&+\frac{2m_b(m_c-m_b)(I_{01}^{311}+I_{01}^{131})+m_b^2(I_0^{311}+I_0^{131})+2m_c(2m_b-m_c)I_{01}^{113}}{8\pi^2}\\
&+\frac{12m_bm_c-4m_b^2-2m_c^2+q^2}{48\pi^2}I_{01}^{221}+\frac{18m_bm_c-8m_b^2-4m_c^2-q^2}{48\pi^2}(I_{01}^{122}+I_{01}^{212})\\
&+\frac{K_{01}^{221}+N_{01}^{221}+3K_{01}^{122}+N_{01}^{122}+K_{01}^{212}+3N_{01}^{212}+(2m_b^2-q^2)I_0^{221}}{48\pi^2}\\
&\left.+\frac{(4m_b^2+q^2)(I_0^{122}+I_0^{212})}{48\pi^2}-\frac{3I_{01}^{121}+2I_{01}^{112}+3I_{01}^{211}}{24\pi^2}+\frac{I_0^{121}-2I_0^{112}+I_0^{211}}{48\pi^2}\right\},
\end{aligned} \tag{18}$$

where

$$f(s,u,q^2)=\frac{(s+u-q^2)(u-q^2+m_b^2-m_c^2)-2s(u+m_b^2-m_c^2)}{\sqrt{\lambda(s,u,q^2)[(u-q^2+m_b^2-m_c^2)^2-4sm_b^2]}},$$
$$\mathcal{C}=\sqrt{\frac{4\pi\alpha_s^{\mathcal{C}}}{3v_s}\left[1-\exp\left(-\frac{4\pi\alpha_s^{\mathcal{C}}}{3v_s}\right)\right]^{-1}}\sqrt{\frac{4\pi\alpha_s^{\mathcal{C}}}{3v_u}\left[1-\exp\left(-\frac{4\pi\alpha_s^{\mathcal{C}}}{3v_u}\right)\right]^{-1}},$$
$$v_s=\sqrt{1-\frac{4m_bm_c}{s-(m_b-m_c)^2}},$$
$$v_u=\sqrt{1-\frac{4m_bm_c}{u-(m_b-m_c)^2}}. \tag{19}$$

explicit expressions for I_0^{ijn}, K_0^{ijn}, N_0^{ijn}, I_{10}^{ijn}, I_{01}^{ijn}, K_{10}^{ijn}, K_{01}^{ijn}, N_{10}^{ijn}, and N_{01}^{ijn} are presented in the Appendix. For the heavy quarkonium states B_c^* and B_c, the relative velocities of quark movements are small; we should account for the Coulomb-like corrections $\frac{\alpha_s^{\mathcal{C}}}{v_s}$ and $\frac{\alpha_s^{\mathcal{C}}}{v_u}$ corresponding to the currents $J_\nu^\dagger(0)$ [or $J_5^\dagger(0)$] and $J_5(x)$, respectively. After taking into account all the Coulomb-like contributions shown in Fig. 3, we obtain the coefficient $\mathcal{C}$ to dress the quark-meson vertices [17,18], and take the approximation $\alpha_s^{\mathcal{C}}=\alpha_s(\mu)$ in numerical calculations [13].

We can obtain the hadronic coupling constants $G_{B_c^*B_cJ/\psi}(q^2)$ and $G_{B_cB_cJ/\psi}(q^2)$ with the following simple replacements:

$$G_{B_c^*B_cJ/\psi}(q^2)=G_{B_c^*B_c\Upsilon}(q^2)\Big|_{m_b\leftrightarrow m_c,f_\Upsilon\to f_{J/\psi},M_\Upsilon\to M_{J/\psi}},$$
$$G_{B_cB_cJ/\psi}(q^2)=G_{B_cB_c\Upsilon}(q^2)\Big|_{m_b\leftrightarrow m_c,f_\Upsilon\to f_{J/\psi},M_\Upsilon\to M_{J/\psi}}. \tag{20}$$

In this article, we calculate the hadronic coupling constants $G_{B_c^*B_c\Upsilon}$, $G_{B_cB_c\Upsilon}$, $G_{B_c^*B_cJ/\psi}$, and $G_{B_cB_cJ/\psi}$ in the space-like region $Q^2=-q^2\geq 1\times$ GeV^2, then fit the

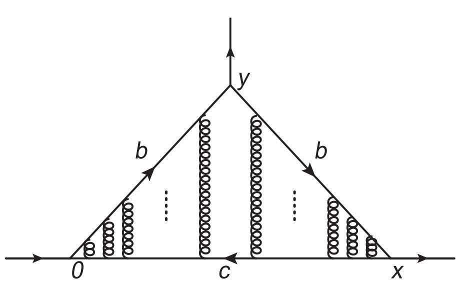

FIG. 3. The ladder Feynman diagram for the Coulomb-like interactions.

$G_{B_c^*B_c\Upsilon}$, $G_{B_cB_c\Upsilon}$, $G_{B_c^*B_cJ/\psi}$, and $G_{B_cB_cJ/\psi}$ into suitable analytical functions, and finally obtain the values $G_{B_c^*B_c\Upsilon}(q^2{=}M_\Upsilon^2)$, $G_{B_cB_c\Upsilon}(q^2{=}M_\Upsilon^2)$, $G_{B_c^*B_cJ/\psi}(q^2{=}M_{J/\psi}^2)$, and $G_{B_cB_cJ/\psi}(q^2=M_{J/\psi}^2)$ by analytically continuing the variable q^2 to the physical regions.

III. NUMERICAL RESULTS AND DISCUSSIONS

The hadronic input parameters are taken as $f_{B_c^*}=0.384$ GeV, $M_{B_c^*}=6.337$ GeV from the QCDSR [13], $f_{B_c}=395$ MeV from the QCD-motivated potential model [19], and $M_{B_c}=6.277$ GeV, $M_\Upsilon=9.4603$ GeV, $M_{J/\psi}=3.096916$ GeV from the Particle Data Group [20]. We extract the values of the decay constants $f_\Upsilon=0.700$ GeV and $f_{J/\psi}=0.415$ GeV from the decays $\Upsilon\to e^+e^-$ and $J/\psi\to e^+e^-$, respectively [20]. The decay constants have the relation $f_{B_c^*}\approx f_{B_c}$, and the masses have the splitting $M_{B_c^*}-M_{B_c}=60$ MeV. The calculations based on the nonrelativistic renormalization group indicate that $M_{B_c(1^-)}-M_{B_c(0^-)}=(50\pm17^{+15}_{-12})$ MeV [21]; the mass $M_{B_c^*}=6.337$ GeV from the QCDSR is satisfactory. Accordingly, we take the threshold parameters and Borel parameters as $s_0=u_0=(45\pm1)$ GeV^2, $M_1^2=M_2^2=(5-7)$ GeV^2 from the QCDSR [13]. The uncertainties of the hadronic coupling constants $\delta G_{B_c^*B_c\Upsilon}(Q^2)$, $\delta G_{B_cB_c\Upsilon}(Q^2)$, $\delta G_{B_c^*B_cJ/\psi}(Q^2)$, and $\delta G_{B_cB_cJ/\psi}(Q^2)$ originating from the decay constants f_i can be estimated as $\frac{\delta f_i}{f_i}$, where $i=\Upsilon,J/\psi,B_c^*,B_c$. For more references on the decay constants $f_{B_c^*}$ and f_{B_c}, one can consult Ref. [14].

The value of the gluon condensate $\langle\frac{\alpha_sGG}{\pi}\rangle$ has been updated from time to time and changes greatly; we use the recently updated value $\langle\frac{\alpha_sGG}{\pi}\rangle=(0.022\pm0.004)$ GeV^4 [22]. For the heavy-quark masses, we take the $\bar{M}S$ masses $m_c(m_c^2)=(1.275\pm0.025)$ GeV and $m_b(m_b^2)=(4.18\pm0.03)$ GeV from the Particle Data Group [20],

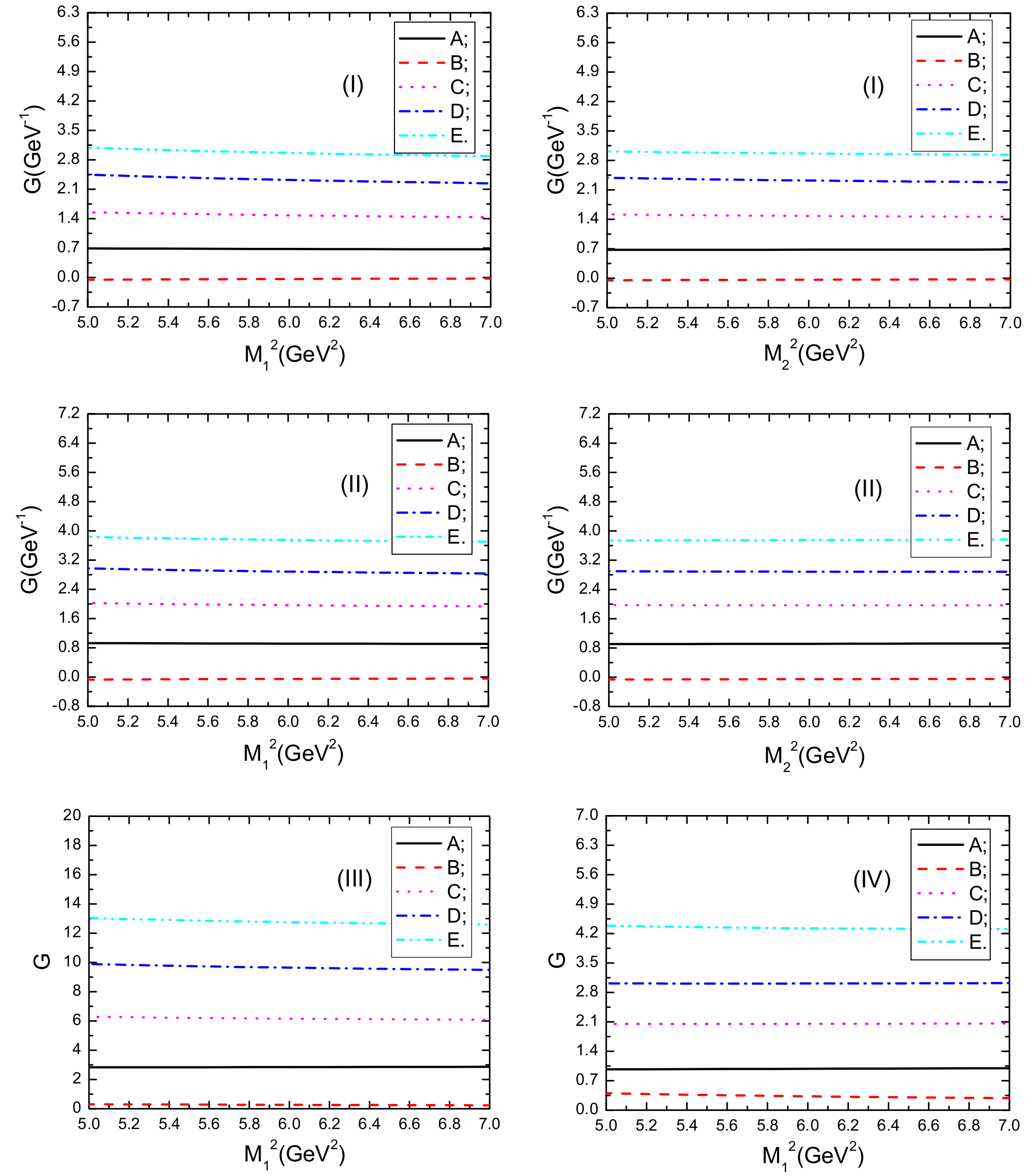

FIG. 4 (color online). The hadronic coupling constants $G_{B_c^*B_c\Upsilon}(Q^2)$ (I), $G_{B_c^*B_cJ/\psi}(Q^2)$ (II), $G_{B_cB_c\Upsilon}(Q^2)$ (III), and $G_{B_cB_cJ/\psi}(Q^2)$ (IV) with variations of the Borel parameters M_1^2 or M_2^2 at the value $Q^2 = 1$ GeV^2. A, B, C, D, and E denote the perturbative contributions, gluon condensate contributions, leading-order Coulomb-like corrections $[\mathcal{O}(\alpha_s^{\mathcal{C}}/v_s, \alpha_s^{\mathcal{C}}/v_u)]$, total Coulomb-like corrections, and total contributions, respectively. The values of unplotted parameters are tacitly taken as $M_1^2 = 6$ GeV^2 or $M_2^2 = 6$ GeV^2.

and account for the energy-scale dependence of the $\bar{M}S$ masses,

$$m_c(\mu^2) = m_c(m_c^2)\left[\frac{\alpha_s(\mu)}{\alpha_s(m_c)}\right]^{\frac{12}{25}},$$

$$m_b(\mu^2) = m_b(m_b^2)\left[\frac{\alpha_s(\mu)}{\alpha_s(m_b)}\right]^{\frac{12}{23}},$$

$$\alpha_s(\mu) = \frac{1}{b_0 t}\left[1 - \frac{b_1}{b_0^2}\frac{\log t}{t} + \frac{b_1^2(\log^2 t - \log t - 1) + b_0 b_2}{b_0^4 t^2}\right], \tag{21}$$

where $t = \log\frac{\mu^2}{\Lambda^2}$, $b_0 = \frac{33-2n_f}{12\pi}$, $b_1 = \frac{153-19n_f}{24\pi^2}$, $b_2 = \frac{2857-\frac{5033}{9}n_f+\frac{325}{27}n_f^2}{128\pi^3}$, and $\Lambda = 213$ MeV, 296 MeV, and 339 MeV for the flavors $n_f = 5, 4$, and 4, respectively [20]. In this article, we take the typical energy scale $\mu = 2$ GeV, as in Refs. [13,14].

In Fig. 4, we plot the contributions to the hadronic coupling constants $G_{B_c^*B_c\Upsilon}(Q^2)$, $G_{B_c^*B_cJ/\psi}(Q^2)$, $G_{B_cB_c\Upsilon}(Q^2)$, and $G_{B_cB_cJ/\psi}(Q^2)$ from different terms in the operator product expansion at the value $Q^2 = 1$ GeV^2 with variations of the Borel parameters M_1^2 and M_2^2. From the figure, we can see that the values are rather stable with

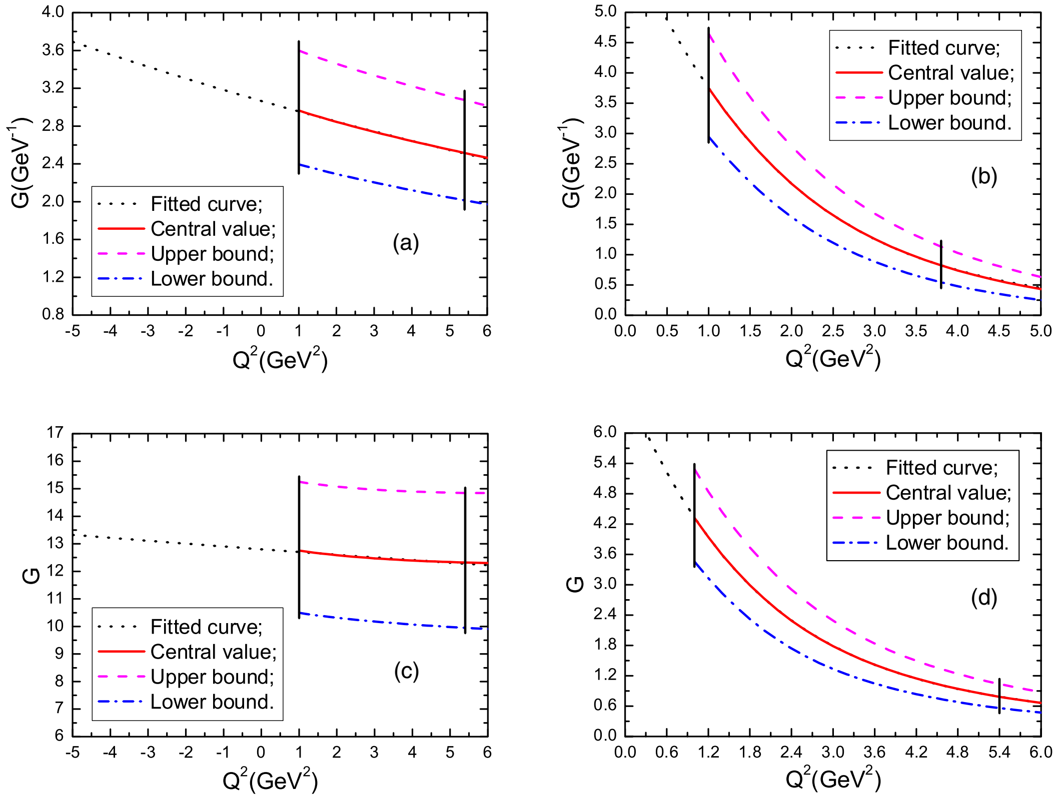

FIG. 5 (color online). The hadronic coupling constants $G_{B_c^*B_c\Upsilon}(Q^2)$ (a), $G_{B_c^*B_cJ/\psi}(Q^2)$ (b), $G_{B_cB_c\Upsilon}(Q^2)$ (c), and $G_{B_cB_cJ/\psi}(Q^2)$ (d) with variations of $Q^2 = -q^2$, where the fitted curve denotes the central values of the fitted functions. The data between the two perpendicular lines are used to fit the parameters of the hadronic coupling constants.

variations of the Borel parameters, and Borel platforms appear. The ratios among the perturbative contributions, gluon condensate contributions, leading-order Coulomb-like corrections $[\mathcal{O}(\alpha_s^{\mathcal{C}}/v_s, \alpha_s^{\mathcal{C}}/v_u)]$, total Coulomb-like corrections, and total contributions are about $(20-25)\%:(1-8)\%:50\%:(70-80)\%:1$. Although the contributions of the leading-order Coulomb-like corrections are twice as large as those of the perturbative terms, the Coulomb-like corrections decrease quickly with an increase of the orders of $\alpha_s^{\mathcal{C}}/v_s, \alpha_s^{\mathcal{C}}/v_u$,

$$\frac{4\pi\alpha_s^{\mathcal{C}}}{3v}\frac{1}{1-\exp\left(-\frac{4\pi\alpha_s^{\mathcal{C}}}{3v}\right)} = 1 + \frac{2\pi\alpha_s^{\mathcal{C}}}{3v} + \frac{1}{12}\left(\frac{4\pi\alpha_s^{\mathcal{C}}}{3v}\right)^2 - \frac{1}{720}\left(\frac{4\pi\alpha_s^{\mathcal{C}}}{3v}\right)^4 + \cdots, \qquad (22)$$

where v denotes v_s and v_u; hence, the operator product expansion is well convergent.

In calculations, we observe that $0.0001 \le \exp(-\frac{s_0}{M_1^2}) \le 0.00186$ and $0.0001 \le \exp(-\frac{u_0}{M_2^2}) \le 0.00186$. The contributions of high resonances and continuum states are greatly suppressed, and the hadronic coupling constants $G_{B_c^*B_c\Upsilon}(Q^2)$, $G_{B_c^*B_cJ/\psi}(Q^2)$, $G_{B_cB_c\Upsilon}(Q^2)$, and $G_{B_cB_cJ/\psi}(Q^2)$ are not sensitive to the threshold parameters. The two criteria (pole dominance and convergence of the operator product expansion) of the QCDSR are fully satisfied. Furthermore, there exist Borel platforms to extract the numerical values of the hadronic coupling constants $G_{B_c^*B_c\Upsilon}(Q^2)$, $G_{B_c^*B_cJ/\psi}(Q^2)$, $G_{B_cB_c\Upsilon}(Q^2)$, and $G_{B_cB_cJ/\psi}(Q^2)$.

The numerical values of the hadronic coupling constants $G_{B_c^*B_c\Upsilon}(Q^2)$, $G_{B_c^*B_cJ/\psi}(Q^2)$, $G_{B_cB_c\Upsilon}(Q^2)$, and $G_{B_cB_cJ/\psi}(Q^2)$ are shown explicitly in Fig. 5, and are fitted into the following analytical functions using MINUIT:

$$\begin{aligned}
G_{B_c^*B_c\Upsilon}(Q^2) &= A\ \exp\,(-BQ^2),\\
G_{B_c^*B_cJ/\psi}(Q^2) &= \frac{C}{1+DQ^2+EQ^4}\ \exp\,(-FQ^2)+H,\\
G_{B_cB_c\Upsilon}(Q^2) &= \frac{A'}{1+B'Q^2},\\
G_{B_cB_cJ/\psi}(Q^2) &= \frac{C'}{1+D'Q^2+E'Q^4}\ \exp\,(-F'Q^2)+H',
\end{aligned} \qquad (23)$$

where

$$\begin{aligned}
A &= 3.0667 \pm 0.43429\ \mathrm{GeV}^{-1},\\
B &= 0.037120 \pm 0.040971\ \mathrm{GeV}^{-2},\\
C &= 6.1944 \pm 11.883\ \mathrm{GeV}^{-1},\\
D &= 0.18488 \pm 3.1480\ \mathrm{GeV}^{-2},\\
E &= 0.063663 \pm 1.2289\ \mathrm{GeV}^{-4},\\
F &= 0.29199 \pm 2.9767\ \mathrm{GeV}^{-2},\\
H &= 0.044515 \pm 3.6666\ \mathrm{GeV}^{-1},\\
A' &= 12.802 \pm 0.68184,\\
B' &= 0.0078868 \pm 0.016091\ \mathrm{GeV}^{-2},\\
C' &= 6.6854 \pm 9.7404,\\
D' &= 0.31276 \pm 1.9985\ \mathrm{GeV}^{-2},\\
E' &= 0.086261 \pm 1.0481\ \mathrm{GeV}^{-4},\\
F' &= 0.14764 \pm 1.7194\ \mathrm{GeV}^{-2},\\
H' &= 0.20721 \pm 2.2281. \qquad (24)
\end{aligned}$$

Although the uncertainties of the parameters in $G_{B_c^*B_cJ/\psi}(Q^2)$, $G_{B_cB_cJ/\psi}(Q^2)$ are very large, the central values of the fitted functions $G_{B_c^*B_c\Upsilon}(Q^2)$, $G_{B_c^*B_cJ/\psi}(Q^2)$, $G_{B_cB_c\Upsilon}(Q^2)$, and $G_{B_cB_cJ/\psi}(Q^2)$ coincide with the central values from the QCDSR.

From the numerical values of $G_{B_c^*B_c\Upsilon}(Q^2)$, $G_{B_c^*B_cJ/\psi}(Q^2)$, $G_{B_cB_c\Upsilon}(Q^2)$, and $G_{B_cB_cJ/\psi}(Q^2)$ at $Q^2 = 1\ \mathrm{GeV}^2$,

$$\begin{aligned}
G_{B_c^*B_c\Upsilon}(Q^2 = 1\ \mathrm{GeV}^2) &= 3.0 \pm 0.6\ \mathrm{GeV}^{-1},\\
G_{B_c^*B_cJ/\psi}(Q^2 = 1\ \mathrm{GeV}^2) &= 3.7 \pm 0.8\ \mathrm{GeV}^{-1},\\
G_{B_cB_c\Upsilon}(Q^2 = 1\ \mathrm{GeV}^2) &= 12.8 \pm 2.3,\\
G_{B_cB_cJ/\psi}(Q^2 = 1\ \mathrm{GeV}^2) &= 4.3 \pm 0.9, \qquad (25)
\end{aligned}$$

we can obtain the ratios

$$\begin{aligned}
\frac{G_{B_cB_c\Upsilon}(Q^2 = 1\ \mathrm{GeV}^2)}{G_{B_c^*B_c\Upsilon}(Q^2 = 1\ \mathrm{GeV}^2)} &= 4.3 \pm 1.2\ \mathrm{GeV} \approx m_b(m_b^2),\\
\frac{G_{B_cB_cJ/\psi}(Q^2 = 1\ \mathrm{GeV}^2)}{G_{B_c^*B_cJ\psi}(Q^2 = 1\ \mathrm{GeV}^2)} &= 1.2 \pm 0.4\ \mathrm{GeV} \approx m_c(m_c^2) \qquad (26)
\end{aligned}$$

if the uncertainties are neglected. The ratio $\frac{G_{B_cB_c\Upsilon}(Q^2)}{G_{B_c^*B_c\Upsilon}(Q^2)}$ increases slowly with an increase of Q^2, while the ratio $\frac{G_{B_cB_cJ/\psi}(Q^2)}{G_{B_c^*B_cJ\psi}(Q^2)}$ increases quickly with an increase of Q^2 in the range $Q^2 = (1-6)\ \mathrm{GeV}^2$.

In Fig. 6, we analytically extrapolate the hadronic coupling constants $G_{B_c^*B_c\Upsilon}(Q^2)$, $G_{B_c^*B_cJ/\psi}(Q^2)$, $G_{B_cB_c\Upsilon}(Q^2)$, and $G_{B_cB_cJ/\psi}(Q^2)$ into the deep time-like

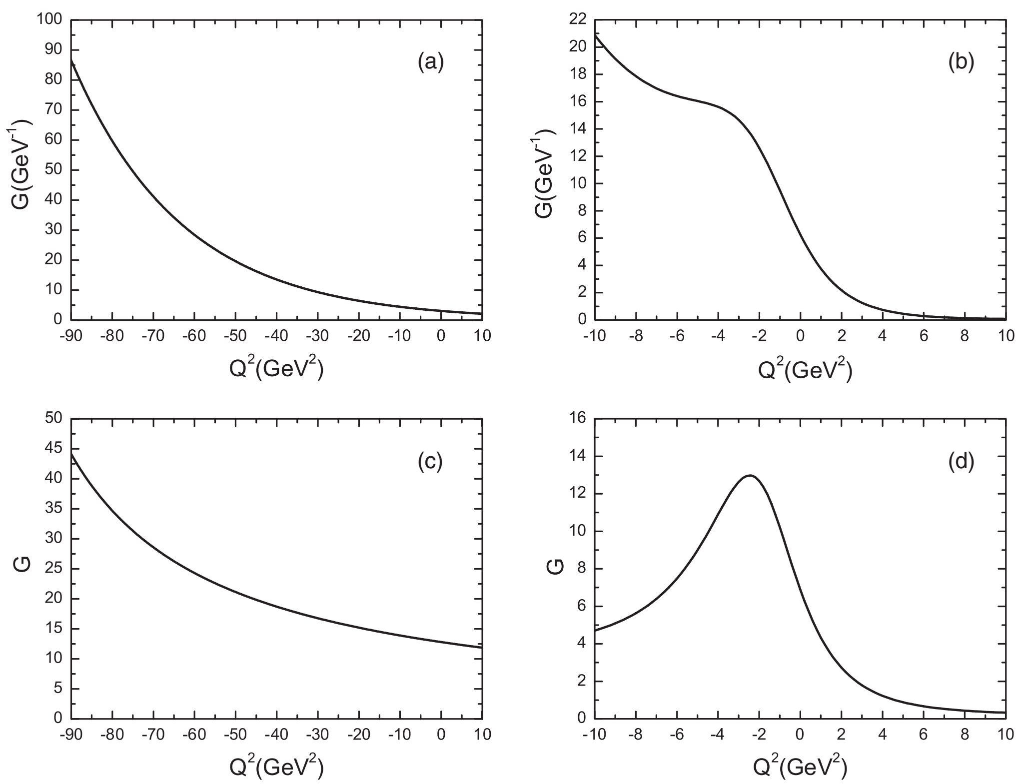

FIG. 6. The central values of the hadronic coupling constants $G_{B_c^*B_c\Upsilon}(Q^2)$ (a), $G_{B_c^*B_cJ/\psi}(Q^2)$ (b), $G_{B_cB_c\Upsilon}(Q^2)$ (c), and $G_{B_cB_cJ/\psi}(Q^2)$ (d) extrapolated into the time-like regions.

regions. From the figure, we can see that $G_{B_c^*B_c\Upsilon}(Q^2)$ and $G_{B_cB_c\Upsilon}(Q^2)$ increase monotonously with an increase of the squared momentum $q^2=-Q^2$. $G_{B_c^*B_cJ/\psi}(Q^2)$ also increases steadily with an increase of the squared momentum $q^2=-Q^2$ and develops a shoulder at about $q^2=3\ \mathrm{GeV}^2$, while $G_{B_cB_cJ/\psi}(q^2)$ develops a broad peak at about $q^2=2.4\ \mathrm{GeV}^2$. The fitted functions $G_{B_c^*B_cJ/\psi}(q^2)$ and $G_{B_cB_cJ/\psi}(q^2)$ in the time-like region $q^2=-Q^2>0$ can be reexpressed in the following forms:

$$\begin{aligned}
G_{B_c^*B_cJ/\psi}(q^2) &= \frac{C\,\exp\,(Fq^2)}{1-Dq^2+Eq^4}+H\\
&= \frac{C\,\exp\,(Fq^2)}{(1-\frac{D}{2}q^2)^2+(E-\frac{D^2}{4})q^4}+H,\\
G_{B_cB_cJ/\psi}(q^2) &= \frac{C'\,\exp\,(F'q^2)}{1-D'q^2+E'q^4}+H'\\
&= \frac{C'\,\exp\,(F'q^2)}{(1-\frac{D'}{2}q^2)^2+(E'-\frac{D'^2}{4})q^4}+H', \qquad (27)
\end{aligned}$$

with $E-\frac{D^2}{4}>0$ and $E'-\frac{D'^2}{4}>0$ for the central values of the parameters. Peaks may appear in the neighborhood of the values $q^2=\frac{1}{D}$, $\frac{2}{D}$, $\frac{1}{D'}$ and $\frac{2}{D'}$.

The extrapolation to deep time-like regions is highly mode dependent and leads to systematic uncertainties for the hadronic coupling constants. In order to minimize the systematic uncertainties, we can study the vertices simultaneously by putting B_c^*, B_c, and Υ (or J/ψ) off shell sequentially, and then fitting the hadronic coupling constants to suitable analytical functions and extrapolating them to the physical regions by requiring that the on-shell values of the hadronic coupling constants coincide [8]. We postpone these tedious calculations to our next work.

Finally, we obtain the on-shell values of $G_{B_c^*B_c\Upsilon}(q^2)$, $G_{B_cB_c\Upsilon}(q^2)$, $G_{B_c^*B_cJ/\psi}(q^2)$, and $G_{B_cB_cJ/\psi}(q^2)$ from the fitted functions,

$$\begin{aligned}
G_{B_c^*B_c\Upsilon}(q^2=M_\Upsilon^2) &= 85\ \mathrm{GeV}^{-1},\\
G_{B_c^*B_cJ/\psi}(q^2=M_{J/\psi}^2) &= 20\ \mathrm{GeV}^{-1},\\
G_{B_cB_c\Upsilon}(q^2=M_\Upsilon^2) &= 44,\\
G_{B_cB_cJ/\psi}(q^2=M_{J/\psi}^2) &= 5, \qquad (28)
\end{aligned}$$

where we retain the central values only, as the uncertainties are too large to make sense. The uncertainties originating from the uncertainties δB, δC, δD, δE, δF, $\delta B'$, $\delta C'$, $\delta D'$, $\delta E'$, and $\delta F'$ are greatly amplified in the deep time-like regions and are much larger than the central values, while the uncertainties originating from the uncertainties δA and $\delta A'$ are moderate. For example, the uncertainties $\delta A=\pm 0.43429\ \mathrm{GeV}^{-1}$, $\delta B=\pm 0.040971\ \mathrm{GeV}^{-2}$, $\delta A'=\pm 0.68184$, and $\delta B'=\pm 0.016091\ \mathrm{GeV}^{-2}$ lead to $\delta G_{B_c^*B_c\Upsilon}(q^2=M_\Upsilon^2)=\pm 12\ \mathrm{GeV}^{-1}$, $\delta G_{B_c^*B_c\Upsilon}(q^2=M_\Upsilon^2)=\pm 312\ \mathrm{GeV}^{-1}$, $\delta G_{B_cB_c\Upsilon}(q^2=M_\Upsilon^2)=\pm 2$, and $\delta G_{B_cB_c\Upsilon}(q^2=M_\Upsilon^2)=\pm 213$, respectively. It is obvious that the uncertainties $\delta G_{B_c^*B_c\Upsilon}(q^2=M_\Upsilon^2)=\pm 312\ \mathrm{GeV}^{-1}$ and $\delta G_{B_cB_c\Upsilon}(q^2=M_\Upsilon^2)=\pm 213$ are too large to make sense. On the other hand, although the uncertainties δH and $\delta H'$ do not vary with q^2, they are about 10 times as large as the corresponding central values. So we only retain the central values, which are more reasonable than the uncertainties. We can take these hadronic coupling constants as basic input parameters to study final-state interactions in the heavy quarkonium decays, or calculate the absorption cross sections at the hadronic level to understand the heavy quarkonium absorptions in hadronic matter.

IV. CONCLUSION

In this article we studied the momentum dependence of the hadronic coupling constants $G_{B_c^*B_c\Upsilon}$, $G_{B_c^*B_cJ/\psi}$, $G_{B_cB_c\Upsilon}$, and $G_{B_cB_cJ/\psi}$ with the off-shell Υ and J/ψ using the three-point QCDSR. Then we fit the hadronic coupling constants $G_{B_c^*B_c\Upsilon}(Q^2)$, $G_{B_c^*B_cJ/\psi}(Q^2)$, $G_{B_cB_c\Upsilon}(Q^2)$, and $G_{B_cB_cJ/\psi}(Q^2)$ into analytical functions, extrapolated them into the deep time-like regions, and obtained the on-shell values $G_{B_c^*B_c\Upsilon}(Q^2=-M_\Upsilon^2)$, $G_{B_c^*B_cJ/\psi}(Q^2=-M_{J/\psi}^2)$, $G_{B_cB_c\Upsilon}(Q^2=-M_\Upsilon^2)$, and $G_{B_cB_cJ/\psi}(Q^2=-M_{J/\psi}^2)$ for the first time. No other theoretical work on this subject exists. The hadronic coupling constants can be taken as basic input parameters in studying the heavy quarkonium absorptions in hadronic matter and final-state interactions in the heavy quarkonium hadronic decays.

ACKNOWLEDGMENTS

This work is supported by the National Natural Science Foundation, Grant No. 11375063, and the Fundamental Research Funds for the Central Universities.

APPENDIX

The explicit expressions of I_0^{ijn}, K_0^{ijn}, N_0^{ijn}, I_{10}^{ijn}, I_{01}^{ijn}, K_{10}^{ijn}, K_{01}^{ijn}, N_{10}^{ijn}, and N_{01}^{ijn} are as follows:

$$\begin{aligned}
iI_0^{ijn} &= B_{-p^2\to M_1^2}B_{-p'^2\to M_2^2}\bar{I}_{ijn}\\
&= \frac{(-1)^{i+j+n}i\pi^2}{\Gamma(i)\Gamma(j)\Gamma(n)(M_2^2)^i(M_1^2)^j(M^2)^{n-2}}\int_0^1 d\lambda\frac{\lambda^{1-i-j}}{(1-\lambda)^{n-1}}\ \exp\left\{-\frac{(1-\lambda)Q^2}{\lambda(M_1^2+M_2^2)}-\frac{m_b^2}{\lambda M^2}-\frac{m_c^2}{(1-\lambda)M^2}\right\}, \qquad (A1)
\end{aligned}$$

$$
\begin{aligned}
iK_0^{ijn} &= B_{-p^2\to M_1^2}B_{-p'^2\to M_2^2}\bar{K}_{ijn}\\
&= \frac{d}{dt}\frac{(-1)^{i+j+n}i\pi^2}{\Gamma(i)\Gamma(j)\Gamma(n)(\bar{M}_2^2)^i(\bar{M}_1^2)^{j-1}(\bar{M}^2)^{n-2}}\int_0^1 d\lambda\frac{\lambda^{1-i-j}}{(1-\lambda)^{n-1}}\exp\left\{-\frac{(1-\lambda)Q^2}{\lambda(\bar{M}_1^2+\bar{M}_2^2)}-\frac{m_b^2}{\lambda\bar{M}^2}-\frac{m_c^2}{(1-\lambda)\bar{M}^2}\right\}\bigg|_{t=1,r=1}, \quad \text{(A2)}
\end{aligned}
$$

$$
\begin{aligned}
iN_0^{ijn} &= B_{-p^2\to M_1^2}B_{-p'^2\to M_2^2}\bar{N}_{ijn}\\
&= \frac{d}{dr}\frac{(-1)^{i+j+n}i\pi^2}{\Gamma(i)\Gamma(j)\Gamma(n)(\bar{M}_2^2)^{i-1}(\bar{M}_1^2)^{j}(\bar{M}^2)^{n-2}}\int_0^1 d\lambda\frac{\lambda^{1-i-j}}{(1-\lambda)^{n-1}}\exp\left\{-\frac{(1-\lambda)Q^2}{\lambda(\bar{M}_1^2+\bar{M}_2^2)}-\frac{m_b^2}{\lambda\bar{M}^2}-\frac{m_c^2}{(1-\lambda)\bar{M}^2}\right\}\bigg|_{t=1,r=1}, \quad \text{(A3)}
\end{aligned}
$$

$$
\begin{aligned}
iI_{ijn}^{\mu} &= B_{-p^2\to M_1^2}B_{-p'^2\to M_2^2}\bar{I}_{ijn}^{\mu}\\
&= \frac{(-1)^{i+j+n+1}i\pi^2}{\Gamma(i)\Gamma(j)\Gamma(n)(M_2^2)^i(M_1^2)^{j+1}(M^2)^{n-3}}\int_0^1 d\lambda\frac{\lambda^{1-i-j}}{(1-\lambda)^{n-2}}\exp\left\{-\frac{(1-\lambda)Q^2}{\lambda(M_1^2+M_2^2)}-\frac{m_b^2}{\lambda M^2}-\frac{m_c^2}{(1-\lambda)M^2}\right\}(p-p')^{\mu}\\
&\quad+\frac{(-1)^{i+j+n}i\pi^2}{\Gamma(i)\Gamma(j)\Gamma(n)(M_2^2)^i(M_1^2)^{j}(M^2)^{n-2}}\int_0^1 d\lambda\frac{\lambda^{2-i-j}}{(1-\lambda)^{n-1}}\exp\left\{-\frac{(1-\lambda)Q^2}{\lambda(M_1^2+M_2^2)}-\frac{m_b^2}{\lambda M^2}-\frac{m_c^2}{(1-\lambda)M^2}\right\}p'^{\mu}\\
&= iI_{10}^{ijn}(p-p')^{\mu}+iI_{01}^{ijn}p'^{\mu}, \qquad \text{(A4)}
\end{aligned}
$$

$$
\begin{aligned}
iK_{ijn}^{\mu} &= B_{-p^2\to M_1^2}B_{-p'^2\to M_2^2}\bar{K}_{ijn}^{\mu}\\
&= \frac{d}{dt}\frac{(-1)^{i+j+n+1}i\pi^2}{\Gamma(i)\Gamma(j)\Gamma(n)(\bar{M}_2^2)^i(\bar{M}_1^2)^{j}(\bar{M}^2)^{n-3}}\int_0^1 d\lambda\frac{\lambda^{1-i-j}}{(1-\lambda)^{n-2}}\exp\left\{-\frac{(1-\lambda)Q^2}{\lambda(\bar{M}_1^2+\bar{M}_2^2)}-\frac{m_b^2}{\lambda\bar{M}^2}-\frac{m_c^2}{(1-\lambda)\bar{M}^2}\right\}\bigg|_{t=1,r=1}(p-p')^{\mu}\\
&\quad+\frac{d}{dt}\frac{(-1)^{i+j+n}i\pi^2}{\Gamma(i)\Gamma(j)\Gamma(n)(\bar{M}_2^2)^i(\bar{M}_1^2)^{j-1}(\bar{M}^2)^{n-2}}\int_0^1 d\lambda\frac{\lambda^{2-i-j}}{(1-\lambda)^{n-1}}\exp\left\{-\frac{(1-\lambda)Q^2}{\lambda(\bar{M}_1^2+\bar{M}_2^2)}-\frac{m_b^2}{\lambda\bar{M}^2}-\frac{m_c^2}{(1-\lambda)\bar{M}^2}\right\}\bigg|_{t=1,r=1}p'^{\mu}\\
&= iK_{10}^{ijn}(p-p')^{\mu}+iK_{01}^{ijn}p'^{\mu}, \qquad \text{(A5)}
\end{aligned}
$$

$$
\begin{aligned}
iN_{ijn}^{\mu} &= B_{-p^2\to M_1^2}B_{-p'^2\to M_2^2}\bar{N}_{ijn}^{\mu}\\
&= \frac{d}{dr}\frac{(-1)^{i+j+n+1}i\pi^2}{\Gamma(i)\Gamma(j)\Gamma(n)(\bar{M}_2^2)^{i-1}(\bar{M}_1^2)^{j+1}(\bar{M}^2)^{n-3}}\int_0^1 d\lambda\frac{\lambda^{1-i-j}}{(1-\lambda)^{n-2}}\exp\left\{-\frac{(1-\lambda)Q^2}{\lambda(\bar{M}_1^2+\bar{M}_2^2)}-\frac{m_b^2}{\lambda\bar{M}^2}-\frac{m_c^2}{(1-\lambda)\bar{M}^2}\right\}\bigg|_{t=1,r=1}(p-p')^{\mu}\\
&\quad+\frac{d}{dr}\frac{(-1)^{i+j+n}i\pi^2}{\Gamma(i)\Gamma(j)\Gamma(n)(\bar{M}_2^2)^{i-1}(\bar{M}_1^2)^{j}(\bar{M}^2)^{n-2}}\int_0^1 d\lambda\frac{\lambda^{2-i-j}}{(1-\lambda)^{n-1}}\exp\left\{-\frac{(1-\lambda)Q^2}{\lambda(\bar{M}_1^2+\bar{M}_2^2)}-\frac{m_b^2}{\lambda\bar{M}^2}-\frac{m_c^2}{(1-\lambda)\bar{M}^2}\right\}\bigg|_{t=1,r=1}p'^{\mu}\\
&= iN_{10}^{ijn}(p-p')^{\mu}+iN_{01}^{ijn}p'^{\mu}, \qquad \text{(A6)}
\end{aligned}
$$

where

$$
M^2=\frac{M_1^2M_2^2}{M_1^2+M_2^2},\qquad \bar{M}^2=\frac{\bar{M}_1^2\bar{M}_2^2}{\bar{M}_1^2+\bar{M}_2^2},\qquad \bar{M}_1^2=tM_1^2,\qquad \bar{M}_2^2=rM_2^2, \qquad \text{(A7)}
$$

and $B_{-p^2\to M_1^2}B_{-p'^2\to M_2^2}$ denotes the double Borel transform.

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