

Universal suppression of heavy quarkonium production in pA collisions at low transverse momentum

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The nuclear suppression of heavy quarkonium production at low transverse momentum in pA collisions in high energy scatterings is investigated in the small- x factorization formalism. A universal suppression is found in the large N_c limit between the two formalisms to describe the heavy quarkonium production: the nonrelativistic QCD and the color-evaporation model. This provides an important probe to the saturation momentum at small x in the big nucleus. We also comment on the phenomenological applications of our results.

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I. INTRODUCTION

Gluon saturation at small x in a large nucleus has attracted great attention in the past few decades, and progress has been made theoretically [1–5]. The emergence of this phenomenon changes the landscape of parton distributions inside the nucleon/nucleus. In the dense region, the characteristics of QCD dynamics is different from that in the dilute region, for example, the evolution equation has to be modified to account for coherent interactions among high density gluons. To better explore the strong interaction dynamics in the dense region, it is important to have physical processes with two distinct scales, one hard scale Q to localize the probe and a relatively soft scale to be sensitive to the saturation physics [6]. It was realized that two-scale observables make it possible to perform the small- x calculations in terms of the factorization approach [7], and to provide directly access to the unintegrated gluon distributions (UGDs), which are important ingredients in the saturation physics, and unveil the importance of the coherent multiple interaction effects in the small- x calculations [4]. Several processes have been proposed in the literature, including semi-inclusive deep inelastic scattering [8], low $P_\perp$ Drell-Yan [9], and back-to-back dihadron correlations in forward pA processes [7,10–19].

In this paper, we will demonstrate that the heavy quarkonium production in the forward region of pA collisions is an excellent probe of the saturation physics at small x . Earlier works to calculate the heavy quarkonium production using the saturation formalism have been done in Refs. [20–24], where the inclusive productions are the main focus and the calculations are done in the simple color-singlet model. In our calculations below, we concentrate on rapidity and transverse momentum distribution, and follow an effective factorization approach [7] with two models dealing with the hadronization from heavy quark

pairs to physical quarkonia. One model is based on non-relativistic QCD (NRQCD) [25], an effective theory of QCD, and the other is the so-called color-evaporation model (CEM) [26]. Both the NRQCD model and CEM share a similar idea for heavy quarkonium production: heavy quark pairs are produced at short distance, and all pairs, at various spin states with/without color, could hadronize into physical heavy quarkonia. In NRQCD, the hadronization probability from different states of heavy quark pairs is organized by the power of α_s and the power of heavy quark velocity v in the pair's rest frame. In CEM, on the other hand, every produced heavy quark pair has the same probability to transform into the same quarkonium state, so long as the pairs' invariant masses are within the open charm threshold. Both approaches give a reasonable description of unpolarized J/ψ and Υ production at a large transverse momentum at collider energies [27].

We focus our calculations of heavy quarkonium production in pA collisions in the forward region (large rapidity region along the direction of proton), as sketched in Fig. 1, where we believe that a factorization approach might be justified. In addition, we concentrate on the production of heavy quarkonium with relatively low transverse momentum $P_\perp$, which is mainly generated by coherent multiple scattering of soft gluons from the nuclei. More specifically, we present our calculations for the

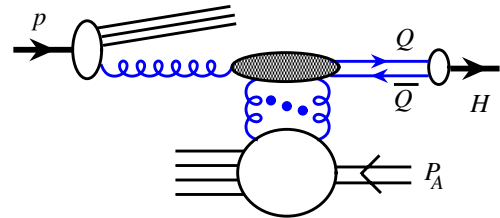


FIG. 1 (color online). Sketch for heavy quarkonium production at a forward rapidity in pA collisions.

region, $\Lambda_{\text{QCD}} \ll P_{\perp} \sim Q_s(A) \ll M$, where M is quarkonium mass and $Q_s(A)$ is the saturation scale of the nuclei with atomic weight A . In general, the factorization between the production of heavy quark pair and the pair's hadronization to a physical quarkonium, assumed to be true in both NRQCD and CEM, is not necessarily valid in this kinematic regime. When $Q_s(A) \sim mv$ with heavy quark mass $m \sim M/2$, multiple scattering of the heavy quark pair with gluons from nuclei could interfere with the pair's hadronization process, and invalidate the velocity expansion of the NRQCD approach. However, in the very forward region, the system of heavy quark pair moves through nuclear matter with a very large longitudinal momentum $P_{\parallel}$, the dynamics of coherent interaction between the pair, which leads to the formation of a bound quarkonium, is strongly time dilated. With a time dilation factor $P_{\parallel}/M$, we conclude that if

$$\frac{1}{mv} \left(\frac{P_{\parallel}}{M} \right) \gg \frac{1}{P_{\perp}} \sim \frac{1}{Q_s(A)} \\ \Leftrightarrow y \gg \ln \left(\frac{2mv}{P_T} \right) \sim \ln \left(\frac{Mv}{Q_s(A)} \right), \quad (1)$$

the pair's hadronization is effectively frozen (or not even started) when the pair passes through and interacts with the nuclei, so that the pair's hadronization is effectively factorized from the production of the pair.

The condition, $\Lambda_{\text{QCD}} \ll P_{\perp}$, effectively ensures that the interaction between the heavy quark pair and the spectators of incoming proton is suppressed by powers of $Q_s(p)/P_{\perp}$ with proton saturation scale $Q_s(p) \sim \Lambda_{\text{QCD}}$ in this region, and we could use collinear factorization to factorize the active gluon from the colliding proton. The condition in Eq. (1) allows us to calculate the multiple interactions of the pair with the nuclear target. It was shown [7] that for hard processes in pA collisions, these multiple interactions could be reduced to the unintegrated gluon distributions which can be easily formulated in small- x saturation formalism. We show that such an effective $k_{\perp}$ factorization from the nucleus is also valid for heavy quarkonium production in pA collisions, where a very important feature emerges that the unintegrated gluon distribution is the same in both NRQCD and CEM approaches in the large N_c limit. Therefore, these two approaches will give the same predictions for the nuclear suppression factor in our calculations.

When $P_{\perp} \ll M$, the transverse momentum distribution of the pair is strongly influenced by the initial-state gluon shower, which leads to $\alpha_s^m \ln^{2m-1}(M^2/P_{\perp}^2)$ -type Sudakov large logarithms in perturbative calculation of the production rate [28]. Such large logarithmic contributions have to be resummed. We use the Collins-Soper-Sterman (CSS) formalism for the resummation [29]. For heavy quarkonium production in hadronic collisions, resummation of Sudakov large logarithms was first addressed in Ref. [30]

by using CSS formalism and CEM for the formation of quarkonia. Recently, the same resummation was carried out in the NRQCD formalism as well [31].

It was demonstrated recently that the soft gluon resummation and small- x evolution can be performed consistently for the hard processes in pA collisions [32]. We apply the same technique to heavy quarkonium production in pA collisions. The combination of the QCD resummation and the effective $k_{\perp}$ factorization from the nucleus in pA collisions at small x makes the heavy quarkonium production a powerful tool to investigate the saturation physics. Most importantly, we show that the hadronization mechanism does not affect the sensitivity of the nuclear suppression of heavy quarkonium production in the forward pA collisions.

The rest of the paper is organized as the following. In Sec. II, we formulate the heavy quarkonium production at low transverse momentum in the small- x formalism and NRQCD. An effective $k_{\perp}$ factorization emerges for heavy quarkonium production with two different unintegrated gluon distributions in the NRQCD formalism. We extend the Sudakov resummation to heavy quarkonium production in Sec. III, and discuss heavy quarkonium suppression in pA collisions in Sec. IV. Finally, we summarize our paper and comment on the phenomenological applications in Sec. V.

II. UNINTEGRATED GLUON DISTRIBUTIONS IN HEAVY QUARKONIUM PRODUCTION IN pA COLLISIONS

In this section, we calculate the heavy quarkonium production at low transverse momentum in pA collisions in the small- x factorization formalism, and demonstrate that an effective $k_{\perp}$ factorization applies for both NRQCD and CEM models. We take the $P_{\perp} \ll M_Q$ limit, and neglect all higher order corrections. Since the details in the production mechanism are different in NRQCD and CEM, we present the calculations separately.

Let us recall the two basic unintegrated gluon distributions in the small- x formalism. The first gluon distribution, the so-called Weizsäcker-Williams (WW) gluon distribution, can be defined following the conventional gluon distribution [33,34]

$$xG^{(1)}(x, k_{\perp}) = \int \frac{d\xi^- d^2\xi_{\perp}}{(2\pi)^3 P^+} e^{ixP^+ \xi^- - ik_{\perp} \cdot \xi_{\perp}} \\ \times \langle P | F^{+i}(\xi^-, \xi_{\perp}) \mathcal{L}_{\xi}^{\dagger} \mathcal{L}_0 F^{+i}(0) | P \rangle, \quad (2)$$

where $F^{\mu\nu}$ is the gauge field strength tensor $F_a^{\mu\nu} = \partial^{\mu} A_a^{\nu} - \partial^{\nu} A_a^{\mu} - gf_{abc} A_b^{\mu} A_c^{\nu}$ with f_{abc} the antisymmetric structure constants for $SU(3)$ with a, b, c , the color indices, and $\mathcal{L}_{\xi}$ is the gauge link in the adjoint representation $A^{\mu} = A_a^{\mu} t_a$ with $t_a = -if_{abc}$. In the small- x factorization calculations, this distribution can be written in terms of the correlator of four Wilson lines as [7]

$$xG^{(1)}(x, k_\perp) = -\frac{2}{\alpha_s} \int \frac{d^2v}{(2\pi)^2} \frac{d^2v'}{(2\pi)^2} e^{-ik_\perp \cdot (v-v')} \times \langle \text{Tr}[\partial_i U(v)] U^\dagger(v') [\partial_i U(v')] U^\dagger(v) \rangle_{x_g}, \quad (3)$$

where the Wilson line $U(x_\perp)$ is defined as $U^n[-\infty, +\infty; x_\perp]$, and the notation $\langle \dots \rangle_{x_g}$ is used for the color glass condensate average of the color charges over the nuclear wave function where x_g is the smallest fraction of longitudinal momentum probed, and is determined by the kinematics. For a large nucleus, this distribution at a small x can be evaluated using the McLerran-Venugopalan model [3]

$$xG^{(1)}(x, k_\perp) = \frac{S_\perp}{\pi^2 \alpha_s} \frac{N_c^2 - 1}{N_c} \int \frac{d^2r_\perp}{(2\pi)^2} \frac{e^{-ik_\perp \cdot r_\perp}}{r_\perp^2} (1 - e^{-\frac{r_\perp^2 Q_s^2}{4}}), \quad (4)$$

where $N_c = 3$ is the number of colors, $S_\perp$ is the transverse area of the target nucleus, and $Q_s^2 = \frac{q_\perp^2 N_c}{4\pi} \ln \frac{1}{r_\perp^2 \lambda^2} \int dx^- \mu^2(x^-)$ is the gluon saturation scale with μ^2 the color charge density in a large nuclei.

The second gluon distribution, the Fourier transform of the dipole cross section, is defined in the fundamental representation,

$$xG^{(2)}(x, k_\perp) = 2 \int \frac{d\xi^- d\xi_\perp}{(2\pi)^3 P^+} e^{ixP^+ \xi^- - ik_\perp \cdot \xi_\perp} \times \langle P | \text{Tr}[F^{+i}(\xi^-, \xi_\perp) \mathcal{U}^{[-]\dagger} F^{+i}(0) \mathcal{U}^{[+]}] | P \rangle, \quad (5)$$

where the gauge link $\mathcal{U}_\xi^{[-]} = U^n[0, -\infty; 0] U^n[-\infty, \xi^-; \xi_\perp]$ stands for initial state interactions. Thus, the dipole gluon distribution contains both initial and final state interactions in the definition, and can be calculated in the MV model,

$$\begin{aligned} xG^{(2)}(x, q_\perp) &= \frac{q_\perp^2 N_c}{2\pi^2 \alpha_s} S_\perp \int \frac{d^2r_\perp}{(2\pi)^2} e^{-iq_\perp \cdot r_\perp} \\ &\times \frac{1}{N_c} \langle \text{Tr} U(0) U^\dagger(r_\perp) \rangle_{x_g} \\ &\equiv \frac{q_\perp^2 N_c}{2\pi^2 \alpha_s} S_\perp F(q_\perp). \end{aligned} \quad (6)$$

A. NRQCD calculations

In the NRQCD formalism, we can write down the production of heavy quarkonium into the following form:

$$d\sigma = \sum_n d\hat{\sigma}(Q\bar{Q}[n] + X) \langle \mathcal{O}^H[n] \rangle. \quad (7)$$

Here $\hat{\sigma}(Q\bar{Q}[n] + X)$ is the cross section at the parton level, which represents the production of a pair of heavy quark in a fixed color, spin, and orbital angular momentum state n and can be calculated perturbatively. The long distant matrix element (LDME) $\langle \mathcal{O}^H[n] \rangle$ describes the transition of the heavy quark pair in the configuration of $Q\bar{Q}[n]$ into the final state heavy quarkonium. LDMEs are organized in terms of the velocity v expansion in the NRQCD framework. For J/ψ production in pp collisions, the differential cross sections depend on the following three color-octet matrix elements:

$$\langle \mathcal{O}^{J/\psi}[^3S_1^8] \rangle, \quad \langle \mathcal{O}^{J/\psi}[^1S_0^8] \rangle, \quad \langle \mathcal{O}^{J/\psi}[^3P_J^8] \rangle, \quad (8)$$

which are at the same order in the velocity expansion.

In the following, we will take the example for the 1S_0 channel in both color-singlet and color-octet configurations to show how the unintegrated gluon distributions enter into the factorization formalism at low transverse momentum. The typical diagrams are plotted in Fig. 2, where the final state $Q\bar{Q}$ can be either color singlet or color octet. According to the NRQCD factorization, we calculate the perturbative part of heavy quark pair production, and project out different spin and color configuration. Using the spin projection we listed in Appendix A, we find that the amplitude for 1S_0 can be written as

$$\begin{aligned} \mathcal{M}^\mu(^1S_0^8) &= - \int \frac{d^2k_{g1\perp}}{(2\pi)^2} d^2x_{1\perp} d^2x_{2\perp} e^{ik_{g1\perp} \cdot x_{1\perp} + ik_{g2\perp} \cdot x_{2\perp}} \\ &\times \frac{16m_c \epsilon^{\mu(k_{g1} - k_{g2})np}}{4m_c^2 - (k_{g1\perp} - k_{g2\perp})^2} \frac{1}{\sqrt{4m_c^3}} \\ &\times 2\pi\delta(k_g^-) (\text{Tr}[T^b U_{k_{g1}}(x_{1\perp}) T^a U_{k_{g2}}^\dagger(x_{2\perp})]), \end{aligned} \quad (9)$$

$$\begin{aligned} \mathcal{M}^\mu(^1S_0^1) &= - \int \frac{d^2k_{g1\perp}}{(2\pi)^2} d^2x_{1\perp} d^2x_{2\perp} e^{ik_{g1\perp} \cdot x_{1\perp} + ik_{g2\perp} \cdot x_{2\perp}} \\ &\times \frac{16m_c \epsilon^{\mu(k_{g1} - k_{g2})np}}{4m_c^2 - (k_{g1\perp} - k_{g2\perp})^2} \frac{1}{\sqrt{4m_c^3}} \\ &\times 2\pi\delta(k_g^-) (\text{Tr}[U_{k_{g1}}(x_{1\perp}) T^a U_{k_{g2}}^\dagger(x_{2\perp})]), \end{aligned} \quad (10)$$

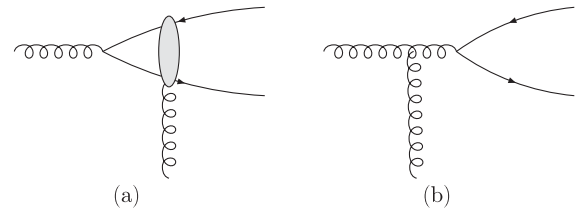


FIG. 2. Multiple interaction in heavy quarkonium production in pA collisions: (a) multiple interactions between the heavy quark pair and the nucleus target; (b) multiple interactions between the incoming gluon and the nucleus target.

where a is the color index for the incoming gluon, and b for that in the final state if it is a color-octet channel. In the above equations, U represents the multiple interaction with the nucleus target $U(x_\perp) = \mathcal{P} \exp \{ig_S \int_{-\infty}^{+\infty} dx^+ T^c A_c^-(x^+, x_\perp)\}$.

First, let us examine the color-singlet channel. In the small transverse momentum limit, $P_\perp \ll M_Q$, we have $k_{g1\perp} \sim P_\perp \ll M_Q$, and we can expand the amplitude in terms of $k_{g1\perp}/M_Q$. In the leading order expansion, there is no dependence on $k_{g1\perp}$ except the Fourier transformation factor, which leads to $\delta^2(x_{1\perp} - x_{2\perp})$. The latter equation results into vanishing contribution, because $\text{Tr}(T^a) = 0$. Therefore, nonzero leading order expansion is proportional to $k_{g1\perp}^\alpha - k_{g2\perp}^\alpha$, and its contribution leads to $\partial_\perp^\alpha U^\dagger(x_\perp)U(x_\perp)$. This is exactly the WW gluon distribution. It is consistent to the color single, because there are only the initial state interaction effects, and the gluon distribution is the same as that in the Higgs boson production process.

For the color-octet channel, however, it has final state interaction as well. The UGD will be different from WW gluon distribution. Again, if we work at the leading nonzero order, we find that, in the large N_c limit, the relevant gluon distribution $\mathcal{F}^{(c\bar{c})[8]}$ is

$$\frac{S_\perp}{\pi^2 \alpha_s} \frac{N_c^2 - 1}{4N_c} \int d^2 q_1 d^2 q_2 \delta^{(2)}(P_\perp - q_1 - q_2) \times F(q_1)F(q_2)(q_1 - q_2)^2. \quad (11)$$

In terms of the notation of Ref. [7],

$$\mathcal{F}^{(c\bar{c})[8]} = \mathcal{F}_{gg}^{(1)} + \mathcal{F}_{gg}^{(2)}, \quad (12)$$

where $\mathcal{F}_{gg}^{(1,2)}$ are defined in Ref. [7].

Now, we turn to the 3S_1 channel. It is different from the 1S_0 channel; there will be an additional diagram in Fig. 2(b) contributing to the color-octet 3S_1 channel. In the following, we will show, however, that the 3S_1 channel is power suppressed both for the color-singlet and color-octet contributions. This is consistent with the power counting in the collinear factorization calculation in the NRQCD of Ref. [31].

First, let us write down the contribution from Fig. 2(a), which reads

$$\begin{aligned} \mathcal{M}^{\rho\mu}(^3S_1^1) &= \int \frac{d^2 k_{g1\perp}}{(2\pi)^2} d^2 x_{1\perp} d^2 x_{2\perp} \\ &\times e^{ik_{g1\perp} \cdot x_{1\perp} + ik_{g2\perp} \cdot x_{2\perp}} \frac{1}{\sqrt{4m_c^3}} 2\pi\delta(k_g^-) \\ &\times \frac{16im_c^2(k_{g1}^\mu n^\rho + k_{g2}^\mu n^\rho - 2g^{\mu\rho} n \cdot p)}{(k_{g1\perp} - k_{g2\perp})^2 - 4m_c^2} \\ &\times (\text{Tr}[U(x_{1\perp})T^a U^\dagger(x_{2\perp})]), \end{aligned} \quad (13)$$

for the color-singlet channel. In the low transverse momentum region, the power counting analysis tells us that the above vanishes in the leading power of $P_\perp/M_Q$. The first term cancels out that of the second term because of $\partial_\perp^\alpha U^\dagger(x_\perp)U(x_\perp) = -U^\dagger \partial_\perp^\alpha U(x_\perp)$. The third term is power suppressed because of $\text{Tr}(T^a) = 0$.

The color-octet 3S_1 can be analyzed in a similar manner. The contribution from Fig. 2(a) can be written as

$$\begin{aligned} \mathcal{M}^{\rho\mu}(^3S_1^8)_{\text{fig.2(a)}} &= \int \frac{d^2 k_{g1\perp}}{(2\pi)^2} d^2 x_{1\perp} d^2 x_{2\perp} \\ &\times e^{ik_{g1\perp} \cdot x_{1\perp} + ik_{g2\perp} \cdot x_{2\perp}} \frac{1}{\sqrt{4m_c^3}} 2\pi\delta(k_g^-) \\ &\times \frac{16im_c^2(k_{g1}^\mu n^\rho + k_{g2}^\mu n^\rho - 2g^{\mu\rho} n \cdot p)}{(k_{g1\perp} - k_{g2\perp})^2 - 4m_c^2} \\ &\times (\text{Tr}[T^b U(x_{1\perp})T^a U^\dagger(x_{2\perp}) - T^b T^a]), \end{aligned} \quad (14)$$

while the contribution from Fig. 2(b) is

$$\begin{aligned} \mathcal{M}^{\rho\mu}(^3S_1^8)_{\text{fig.2(b)}} &= \int d^2 x_\perp e^{iP_\perp \cdot x_\perp} \frac{1}{\sqrt{4m_c^3}} 2\pi\delta(k_g^-) \\ &\times \frac{16im_c^2(k_g^\mu n^\rho - 2g^{\mu\rho} n \cdot p)}{-4m_c^2} \frac{1}{\sqrt{4m_c^3}} \\ &\times (\text{Tr}[T^b U(x_\perp)T^a U^\dagger(x_\perp) - T^b T^a]). \end{aligned} \quad (15)$$

Clearly, the leading order expansions cancel out each other between these two diagrams. Therefore, in the leading power, there is no contribution from the 3S_1 channel in either color-singlet or color-octet channel. This again, is consistent with the collinear factorization calculation in NRQCD.

We have also calculated the contributions from 3P_J states in both color-singlet and color-octet channels (see the detailed derivations in Appendix B), and we find that the nonvanishing contribution in the color-singlet channel is proportional to the WW gluon distribution, whereas that from the color-octet channel is proportional to the same UGD as that for the $^1S_0^8$ channel. Therefore, in summary, we can write down an effective $k_\perp$ factorization for heavy quarkonium production in the low $P_\perp$ region as

$$\frac{d\sigma^{(1)}}{dyd^2P_\perp} = \sigma_0^{(1)} xg_p(x)x_A G^{(1)}(x_A, P_\perp), \quad (16)$$

$$\frac{d\sigma^{(8)}}{dyd^2P_\perp} = \sigma_0^{(8)} xg_p(x)\mathcal{F}^{(c\bar{c})[8]}(x_A, P_\perp), \quad (17)$$

for the color-singlet and octet channels, respectively. The leading Born cross sections $\sigma_0^{(1,8)}$ are the same as those calculated in Ref. [31].

B. CEM calculations

Similar calculations can be done in the color-evaporation model. The only difference is that the hard partonic cross sections are calculated in terms of open heavy flavor pair production with certain invariant mass range,

$$d\sigma = \int dM_{Q\bar{Q}} d\hat{\sigma}(Q\bar{Q}(M_{Q\bar{Q}}) + X) f_{M_{Q\bar{Q}}}, \quad (18)$$

where the invariant mass of the heavy quark pair is restricted in the range of

$$M_\Psi \leq M_{Q\bar{Q}} \leq M_{D\bar{D}}, \quad (19)$$

where M_Ψ represents the bound state mass and $M_{D\bar{D}}$ for the threshold for the open heavy meson pair. In each case of charm and bottom quarks, we notice that the invariant mass difference between the open heavy meson and the bound state is very small. We argue that in this model, the heavy quarkonium production is dominated by the pair production close to the bound state mass range. In the partonic scattering process, this argument tells us that the momentum fraction of the heavy quark and antiquark is restricted to the $z \approx 1/2$ region. That will simplify the derivation of the $k_\perp$ factorization formalism.

In the large N_c limit, the heavy quark pair production in pA collisions can be formulated following the results of Refs. [7,11], and we find the leading power contribution,

$$\begin{aligned} \frac{d\sigma^{CEM}}{dy d^2 P_\perp} &\propto \int dx_p g_f(x_p) \alpha_S [z^2 + (1-z)^2] z(1-z) \frac{S_\perp}{(2\pi)^2} \\ &\times \int d^2 q_1 d^2 q_2 \delta^{(2)}(P_\perp - q_1 - q_2) F_{x_g}(q_1) \\ &\times F_{x_g}(q_2) \frac{(zq_1 - (1-z)q_2)^2}{[M_Q^2 + (zq_1 - (1-z)q_2)^2]^2}. \end{aligned} \quad (20)$$

If we take the limit of small transverse momentum $P_\perp \ll M_Q$, the above expression can be simplified as

$$\begin{aligned} &\int d^2 q_1 d^2 q_2 \delta^{(2)}(P_\perp - q_1 - q_2) F_{x_g}(q_1) \\ &\times F_{x_g}(q_2) (zq_1 - (1-z)q_2)^2. \end{aligned} \quad (21)$$

Now, we apply the approximation $z \sim (1-z) \sim 1/2$, and we will find that the differential cross section of the heavy quark pair in the invariant mass range close to the bound state can be written as

$$\begin{aligned} &\int d^2 q_1 d^2 q_2 \delta^{(2)}(P_\perp - q_1 - q_2) F_{x_g}(q_1) \\ &\times F_{x_g}(q_2) (q_1 - q_2)^2 \propto \mathcal{F}^{(c\bar{c})[8]}. \end{aligned} \quad (22)$$

Clearly, this tells us that the UGD from the nucleus is the same as that in the color-octet contribution in the NRQCD.

The above derivation is consistent with the argument that the CEM and NRQCD share a similar factorization argument for heavy quarkonium production. Although they differ in the hadronization of the heavy quark pair, the nuclear dependence at small x is the same. This is an important message for us to use the heavy quarkonium production as a probe to the nuclear effects at small x .

III. QCD SOFT GLUON RESUMMATION

An important physics has to be taken care of in the low transverse momentum region of the QCD hard processes. That is the Sudakov double logarithms in terms of $\ln M^2/P_\perp^2$ from the initial-state gluon shower [30,31], which has to be resummed to have reliable predictions. Recently, there have been studies to show that the Sudakov resummation can be consistently carried out with the small- x resummation [32]. In the following, we conjecture that it is the same for heavy quarkonium production. It will be interesting to have a rigorous calculation to demonstrate this. We hope to come back to this issue in a later publication.

First, we recall the CSS resummation formalism [29]. The resummation part of total cross section can be written as

$$\left. \frac{d\sigma}{d^2 P_\perp dy} \right|_{P_\perp \ll M} = \frac{1}{(2\pi)^2} \int d^2 b e^{i\vec{P}_\perp \cdot \vec{b}} W(b, M, x_1, x_2), \quad (23)$$

where

$$W(b, M^2) = e^{-\mathcal{S}_{\text{Sud}}(M^2, b)} W(b), \quad (24)$$

with the Sudakov form factor is

$$\mathcal{S}_{\text{Sud}} = \int_{c_0^2/b^2}^{M^2} \frac{d\mu^2}{\mu^2} \left[\ln \left(\frac{M^2}{\mu^2} \right) A + B \right]. \quad (25)$$

$W(b, M, x_1, x_2)$ can be written as

$$W(b) = \sigma_0 x f(x, c_0/b) x' f(x', c_0/b) \left(1 + \frac{\alpha_s}{\pi} \right), \quad (26)$$

where $x = Me^y/\sqrt{S}$, $x' = Me^{-y}/\sqrt{S}$, and the scale $\mu = C_3/b$.

In Ref. [32], the Sudakov resummation has been extended to the small- x saturation formalism, taking the example of massive color-neutral scalar particle production

in pA collisions. We assume that the general arguments for separating Sudakov large logarithms and small- x resummation hold for any hard processes, in particular in the leading double logarithmic approximation [32]. Therefore, in this paper, we follow the similar formalism, and conjecture that the Sudakov resummation for heavy quarkonium production can be included in the small- x factorization formalism. We can write

$$\begin{aligned} \left. \frac{d\sigma^{(\text{resum})}}{dy d^2 P_\perp} \right|_{P_\perp \ll M} &= \sigma_0 \int \frac{d^2 x_\perp d^2 x'_\perp}{(2\pi)^2} e^{iP_\perp \cdot r_\perp} \\ &\times e^{-\mathcal{S}_{\text{sud}}(M^2, r_\perp^2)} \mathcal{F}_{Y=\ln 1/x_a}^{(c\bar{c})[8]}(x_\perp, x'_\perp) \\ &\times xg_p(x, \mu^2 = c_0^2/r_\perp^2), \end{aligned} \quad (27)$$

for the color-octet contribution in NRQCD and CEM model calculations in the large N_c limit. Here, σ_0 represents the leading order normalization, $r_\perp = x_\perp - x'_\perp$, $\mathcal{F}^{(c\bar{c})[8]}$ denotes the gluon distribution from the nucleus defined in Eq. (12), $g_p(x)$ the integrated gluon distribution from the nucleon. The Sudakov form factor $\mathcal{S}_{\text{sud}}$ contains all order resummation, and can be calculated perturbatively [32]. For the color-singlet model, we have to replace $\mathcal{F}^{(c\bar{c})[8]}$ by the WW gluon distribution $xG_A^{(1)}(x_\perp, x'_\perp)$.

The above result is valid in the limit of $P_\perp \ll M$, and we have applied the small- x factorization where higher order in $1/\ln(1/x_a)$ has been neglected as well. Therefore, the only nuclear effects can be formulated in terms of the initial state input of the UGDs. When $P_\perp \sim Q_s(A) \sim M$ and the condition in Eq. (1) is satisfied, the nuclear multiple scattering based effective $k_\perp$ factorization for heavy quarkonium production in pA collisions in the forward region could be still valid, and a matching Y term could be added to the resummed contribution [29] to extend the validity of Eq. (27) to the region where $M \sim Q_s(A)$.

IV. NUCLEAR SUPPRESSION

When applying Eq. (27) to calculate the nuclear suppressions for heavy quarkonium production in pA collisions, we need input of the UGDs. First, we find that the UGD involved in heavy quarkonium production can be calculated in the MV model,

$$xG_A^{(1)}(r_\perp) = \frac{S_\perp}{\pi^2 \alpha_s} \frac{N_c^2 - 1}{N_c} \frac{1}{r_\perp^2} (1 - e^{-\frac{r_\perp^2 Q_s^2(A)}{4}}), \quad (28)$$

$$\mathcal{F}^{(c\bar{c})[8]}(r_\perp) = \frac{S_\perp}{\pi^2 \alpha_s} \frac{N_c^2 - 1}{N_c} \frac{Q_s^2(A)}{4} e^{-\frac{r_\perp^2 Q_s^2(A)}{4}}, \quad (29)$$

where $Q_s(A)$ is the saturation scale in the adjoint representation. At small dipole size, the above UGDs are the same, and both proportional to the integrated gluon distributions,

$$Q_s^2(A) = \frac{2N_c^2}{N_c^2 - 1} A^{1/3} Q_{sp}^2 \approx \frac{2N_c^2}{N_c^2 - 1} \frac{2\pi^2 \alpha_s}{N_c S_\perp} xG_A(x, 1/r_\perp), \quad (30)$$

where the last quantity is the integrated gluon distribution. It is easy to find that this gives the correct normalization after integrating over the transverse momentum $P_\perp$. From the calculations in Sec. II, we know that the heavy quarkonium production at low $P_\perp$ is dominated $\mathcal{F}^{(c\bar{c})[8]}$ gluon distribution (coming from the color-octet channel in NRQCD factorization or quark pair in large N_c in the CEM model). Therefore, our following discussions will focus on this part of the contribution.

With the above initial parametrization for the UGDs, we will be able to calculate the transverse momentum distribution of heavy quarkonium in pA collisions, and study the nuclear suppression factor. Although the precise predictions of the nuclear suppression depend on the numeric calculations for heavy quarkonium production in both pp and pA collisions, a number of interesting features can be derived from the above formulas. First, the suppressions when $P_\perp \rightarrow 0$ is less sensitive to the detailed shape of the UGDs. This is because, without $e^{iP_\perp \cdot r_\perp}$ when $P_\perp = 0$, the differential cross section is sensitive to the integral over full range of $r_\perp$, rather than the limited region, $r_\perp \lesssim 1/P_\perp$, as shown in Eq. (27). Second, since higher saturation scale broadens the $P_\perp$ -distribution [as can be seen from Eq. (28)], we will have less suppression at higher $P_\perp$. This phenomena is generic [35] and has been seen from experiments. That is, the $P_\perp$ distribution of heavy quarkonium production in the forward region of pA collisions is very sensitive to the shape of the UGDs.

Therefore, heavy quarkonium production at low transverse momentum in the forward region of pA collisions has a strong advantage to probe the saturation physics at small x in a large nucleus. We find that the nuclear suppression in this region is not very sensitive to the details on how a produced heavy quark pair transforms into a physical quarkonium since the NRQCD and CEM yield the same suppression factor in the low $P_\perp$ region.

V. SUMMARY

In this paper, we have investigated the heavy quarkonium production in the forward region of pA collisions in the low transverse momentum region in the small- x factorization formalism. Following the previous example, we developed an effective $k_\perp$ factorization, where the new combination of the unintegrated gluon distributions dominate the production of heavy quarkonium in pA collisions. More importantly, predictions calculated using both NRQCD and CEM depend on the same UGD in the large N_c limit, which will yield a universal nuclear suppression factor for heavy quarkonium production. In our calculations, we only focus on heavy quarkonium productions in

pA collisions. The factorization we argued in this paper does not apply to the case of AA collisions.

In Ref. [31], heavy quarkonium production at low $P_\perp$ has been applied to a wide energy range of the pp collisions, from fixed target to colliders such as the LHC and Tevatron. It will be interesting to extend the numerical calculations to the pA collisions, by applying the results derived in this paper. We will leave this to a future publication. We notice that the small- x calculation of heavy quarkonium production in the CEM has recently been published [36]. Our calculations focus on the low transverse momentum region where the factorization is different from that for the total cross section which was the main object of Ref. [36]. A straightforward application of the calculations of Ref. [36] to the low transverse momentum region will suffer large Sudakov effects.

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Note added—When this paper was finished, we noticed a preprint [37] about the same topic. In terms of the leading order formulation of heavy quarkonium production in NRQCD and small- x formalism, our calculations agree with those in Ref. [37]. However, we further discussed the low transverse momentum region where additional Sudakov effects have to be considered.

APPENDIX A: BASICS IN NRQCD CALCULATIONS

The spin projectors for outgoing heavy quark pairs with momentum $Q = P/2 + q$ and $\bar{Q} = P/2 - q$, can be written as

$$\Pi_0 = \frac{1}{\sqrt{8m^3}} \left(\frac{P}{2} - \not{q} - m \right) \gamma_5 \left(\frac{P}{2} + \not{q} + m \right), \quad (\text{A1})$$

$$\Pi_1^\rho = \frac{1}{\sqrt{8m^3}} \left(\frac{P}{2} - \not{q} - m \right) \gamma^\rho \left(\frac{P}{2} + \not{q} + m \right). \quad (\text{A2})$$

After projecting these spin projector on parton level amplitude, we can get the amplitudes for outgoing heavy quark pairs with different spin structure:

$$\mathcal{M}_{S=0,L=0} = \text{Tr}[\Pi_0 \mathcal{M}]|_{q=0}, \quad (\text{A3})$$

$$\mathcal{M}_{S=1,L=0} = \epsilon_\rho \text{Tr}[\Pi_1^\rho \mathcal{M}]|_{q=0}, \quad (\text{A4})$$

$$\mathcal{M}_{S=0,L=1} = \epsilon_\nu \frac{d}{q_\nu} \text{Tr}[\Pi_0 \mathcal{M}]|_{q=0}, \quad (\text{A5})$$

$$\mathcal{M}_{S=1,L=1} = \epsilon_{\rho\nu} \frac{d}{q_\nu} \text{Tr}[\Pi_1^\rho \mathcal{A}]|_{q=0}. \quad (\text{A6})$$

APPENDIX B: COLOR-OCTET $^3P_J^{(1,8)}$ CHANNEL

Different from the S -wave state, for the P -wave state, we have to do the expansion of amplitude to order q^ν as shown in Eqs. (A5) and (A6). After a lengthy evaluation, we find that the Fig. 1(a) contribution can be written as

$$\begin{aligned} \mathcal{M}^{\mu\rho\nu} (^3P_J^8)_{\text{fig.1(a)}} &= \Xi^{\rho\nu\mu} [k_{g1\perp}, k_{g2\perp}] \frac{1}{\sqrt{4m_c^3}} \\ &\times 2\pi\delta(k_g^-) (\text{Tr}[T^b U(x_\perp) T^a U^\dagger(x_\perp)]), \end{aligned} \quad (\text{B1})$$

where

$$\begin{aligned} \Xi^{\rho\nu\mu} [k_{g1\perp}, k_{g2\perp}] &= \frac{d}{dq^\nu} \text{Tr} \left[\not{n} \frac{\not{P} + \not{q} - \not{k}_{g1} - m_c}{d} \right. \\ &\times \gamma^\mu \frac{\not{P} + \not{q} + \not{k}_{g2} - m_c}{-P + \not{q} + \not{k}_{g2} - m_c} \\ &\left. \times \not{n} (P - \not{q} - m_c) \gamma^\rho (P + \not{q} + m_c) \right] \end{aligned}$$

A similar expression is found for the color-singlet channel,

$$\begin{aligned} \mathcal{M}^{\mu\rho\nu} (^3P_J^1)_{\text{fig.1(a)}} &= \Xi^{\rho\nu\mu} [k_{g1\perp}, k_{g2\perp}] \frac{1}{\sqrt{4m_c^3}} \\ &\times 2\pi\delta(k_g^-) (\text{Tr}[U(x_\perp) T^a U^\dagger(x_\perp)]). \end{aligned} \quad (\text{B2})$$

$$\mathcal{M}_{S=1,L=1} = \mathcal{E}_{\rho\nu} \frac{d}{q_\nu} \text{Tr}[\mathcal{C}\Pi_1^\rho \mathcal{M}]|_{q=0} \quad \text{spin triplet } P \text{ states.} \quad (\text{B3})$$

We define

$$\Pi^{\rho\nu} = -g^{\rho\nu} + \frac{p^\rho p^\nu}{m_c^2} \quad (\text{B4})$$

and the sum of polarization for a 3P_J state, which own the forms

$$\epsilon_{\rho\nu}^{(0)} \epsilon_{\rho'\nu'}^{(0)} = \frac{1}{D-1} \Pi^{\rho\nu} \Pi^{\rho'\nu'}, \quad (\text{B5})$$

$$\sum \epsilon_{\rho\nu}^{(1)} \epsilon_{\rho'\nu'}^{(1)} = \frac{1}{2} [\Pi^{\rho\rho'} \Pi^{\nu\nu'} - \Pi^{\rho\nu'} \Pi^{\rho'\nu}], \quad (\text{B6})$$

$$\sum \epsilon_{\rho\nu}^{(2)} \epsilon_{\rho'\nu'}^{(2)} = \frac{1}{2} [\Pi^{\rho\rho'} \Pi^{\nu\nu'} + \Pi^{\rho\nu'} \Pi^{\rho'\nu}] - \frac{1}{D-1} \Pi^{\rho\nu} \Pi^{\rho'\nu'}, \quad (\text{B7})$$

$$\begin{aligned} \epsilon_{\rho\nu}^{(0)} \mathcal{M}(^3P_0^8)^{\mu\rho\nu} &= \frac{16i(k_{g2}^\mu - k_{g1}^\mu)(-12m_c^2 + (k_{g2} - k_{g1})^2)n \cdot p}{(-4m_c^2 - (k_{g2} - k_{g1})^2)^2} \\ &\times \frac{1}{\sqrt{4m_c^3}} 2\pi\delta(k_g^-) \\ &\times (\text{Tr}[T^b U(x_\perp) T^a U^\dagger(x_\perp)]). \end{aligned} \quad (\text{B8})$$

At the leading order of $k_\perp$,

$$\begin{aligned} \epsilon_{\rho\nu}^{(0)} \mathcal{M}(^3P_0^8)^{\mu\rho\nu} &= \frac{i(k_{g2}^\mu - k_{g1}^\mu)(-12)n \cdot p}{m_c^2} \frac{1}{\sqrt{4m_c^3}} \\ &\times 2\pi\delta(k_g^-) (\text{Tr}[T^b U(x_\perp) T^a U^\dagger(x_\perp)]), \end{aligned} \quad (\text{B9})$$

similarly

$$\begin{aligned} \epsilon_{\rho\nu}^{(0)} \mathcal{M}(^3P_0^1)^{\mu\rho\nu} &= \frac{16i(k_{g2}^\mu - k_{g1}^\mu)(-12m_c^2 + (k_{g2} - k_{g1})^2)n \cdot p}{(-4m_c^2 - (k_{g2} - k_{g1})^2)^2} \\ &\times \frac{1}{\sqrt{4m_c^3}} 2\pi\delta(k_g^-) (\text{Tr}[U(x_\perp) T^a U^\dagger(x_\perp)]). \end{aligned} \quad (\text{B10})$$

At the leading order of $k_\perp$,

$$\begin{aligned} \epsilon_{\rho\nu}^{(0)} \mathcal{M}(^3P_0^1)^{\mu\rho\nu} &= \frac{i(k_{g2}^\mu - k_{g1}^\mu)(-12)n \cdot p}{m_c^2} \frac{1}{\sqrt{4m_c^3}} \\ &\times 2\pi\delta(k_g^-) (\text{Tr}[U(x_\perp) T^a U^\dagger(x_\perp)]), \end{aligned} \quad (\text{B11})$$

and for the color-octet P wave state,

$$\begin{aligned} \mathcal{M}(^3P_2^8)^{\mu\rho\nu} \mathcal{M}(^3P_2^8)^{\mu'\rho'\nu'} \epsilon_{\rho\nu}^{(2)} \epsilon_{\rho'\nu'}^{(2)} &= \left(\frac{32g^{\mu\mu'}(k_{g1} - k_{g2}) \cdot (k'_{g1} - k'_{g2})(n \cdot p)^2}{m_c^4} \right) \\ &\times \left(\frac{1}{\sqrt{4m_c^3}} 2\pi\delta(k_g^-) (\text{Tr}[T^b U_{k_{g1}}(x_{1\perp}) T^a U_{k_{g2}}^\dagger(x_{2\perp})]) \right) \\ &\times \left(\frac{1}{\sqrt{4m_c^3}} 2\pi\delta(k'_g^-) (\text{Tr}[T^b U_{k'_{g1}}(x'_{1\perp}) T^a U_{k'_{g2}}^\dagger(x'_{2\perp})]) \right)^\dagger. \end{aligned} \quad (\text{B12})$$

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