

Self-consistent covariant description of vector meson decay constants and chirality-even quark-antiquark distribution amplitudes up to twist 3 in the light-front quark model

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Although the meson decay amplitude described by a two-point function may be regarded as one of the simplest possible physical observables, it is interesting that this apparently simple amplitude bears abundant fundamental information on QCD vacuum dynamics and chiral symmetry. The light-front zero-mode issue of the vector meson decay constant f_V is in this respect highly nontrivial and deserves careful analysis. We discuss the zero-mode issue in the light-front quark model (LFQM) prediction of f_V from the perspective of the vacuum fluctuation consistent with the chiral symmetry of QCD. We extend the exactly solvable, manifestly covariant Bethe-Salpeter model calculation to the more phenomenologically accessible, realistic light-front quark model and present a self-consistent covariant description of f_V , analyzing the twist-2 and twist-3 quark-antiquark distribution amplitudes with even chirality.

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I. INTRODUCTION

Meson decay constants provide essential information on the QCD interaction between a quark and an antiquark. They are the lowest moments of the light-cone distribution amplitudes (DAs) for a quark and an antiquark inside a meson. They are also important ingredients in studying the CP violation in leptonic or nonleptonic weak decays of mesons.

Many theoretical works have been devoted to predicting these fundamental constants of mesons, e.g. the lattice QCD [1], the QCD sum rules [2], and the light-front quark model (LFQM) [3–15] based on the LF quantization [13] of QCD. Among various theoretical approaches, the LFQM has been successful in computing not only the meson mass spectra [10,14] but also the decay constants and weak transition form factors of mesons [3–15]. In particular, the light-front dynamics (LFD) carries the maximum number (seven) of the kinetic (or interaction-independent) generators, and thus less effort in dynamics is necessary in order to get the QCD solutions that reflect the full Poincaré symmetries. Also, the rational energy-momentum dispersion relation of LFD yields the sign correlation between the LF energy $k^- (= k^0 - k^3)$ and the LF longitudinal momentum $k^+ (= k^0 + k^3)$ and leads to the suppression of vacuum fluctuations.

Despite these advantages in LFD, the zero-mode ($k^+ = 0$) [16] complication in the matrix element has been noticed for the vector meson decay constant f_V [4–7,17], as well as some electroweak form factors involving a spin-1 particle [4–9]. For the case of f_V , there has been a debate about the zero-mode contribution to the matrix element of

the plus component of the weak current J_W^μ . Unlike the electroweak form factor described by a three-point function involving an external probe, the meson decay amplitude is described by a two-point function and may be regarded as one of the simplest possible physical observables. It is interesting that this apparently simple amplitude bears abundant fundamental information on QCD vacuum dynamics and chiral symmetry. In this respect, the zero-mode issue of f_V in LFD is highly nontrivial and deserves careful analysis. Indeed, we found [7,17] that the existence or absence of the zero mode may depend on the model, especially on the form of vector meson vertex operator Γ^μ , while Jaus [4,5] claimed that there exists a zero-mode contribution to f_V even for the case of the good current J_W^+ . The purpose of this work is not just to clarify this zero-mode issue in the f_V prediction from LFQM but also to discuss this topic in relation to the vacuum fluctuation consistent with the chiral symmetry of QCD. With this aim, we attempt to extend our previous analysis [17] from the exactly solvable, manifestly covariant Bethe-Salpeter (BS) model to the more phenomenologically accessible, realistic LFQM [3,10–12,18–20] and discuss a self-consistent covariant description of the vector meson decay constant in view of the link between QCD and LFQM.

A systematic study of twist-3 light-cone DAs of vector mesons in QCD was presented in Refs. [21,22]. It was based on the conformal expansion, taking into account meson and quark mass corrections. Two-particle DAs of vector mesons were classified in the same way as the more familiar nucleon structure functions, i.e. parton distribution functions (PDFs), which correspond to the independent

tensor structures in nonlocal matrix elements. Nowadays, we know that there are 12 independent generalized parton distributions in deeply virtual Compton scattering [23]. For the forward case with zero skewness, however, nine independent PDFs are found and classified by twist, spin, and chirality [24]. Similarly, the analysis of vector meson DAs revealed an analogous pattern, as the operator structures are the same with the case of PDFs, and the ρ meson polarization vector formally substitutes the nucleon spin vector in the Lorentz structures [22]. Eight independent two-particle DAs were found with the classification due to twist, spin, and chirality [22]. In this work, we focus on the chirality-even distributions up to twist 3.

The light-cone DAs of a vector meson are defined in terms of the following matrix elements of quark-antiquark nonlocal gauge invariant operators at lightlike separation [21,22,25,26]:

$$\begin{aligned} \langle 0 | \bar{q}(0) [0, z] \gamma^\mu q(z) | V(P, h) \rangle \\ = f_V M \int_0^1 dx e^{-ixP \cdot z} \left\{ P^\mu \frac{\epsilon_h \cdot z}{P \cdot z} \phi_{2,V}^\parallel(x) \right. \\ \left. + \left(\epsilon_h^\mu - P^\mu \frac{\epsilon_h \cdot z}{P \cdot z} \right) \phi_{3,V}^\perp(x) + (\cdots) z^\mu \right\}, \end{aligned} \quad (1)$$

where $z^2 = 0$, and the path-ordered gauge factor

$$[0, z] = P \exp \left[ig \int_0^1 dt (-z)_\mu A^\mu((1-t)z) \right] \quad (2)$$

ensures the gauge invariance of the matrix elements and is equal to unity in the LF gauge $A^+ = 0$. According to the classification of Ball and Braun [21,22], $\phi_{2,V}^\parallel(x)$ and $\phi_{3,V}^\perp(x)$ correspond to the twist-2 and twist-3 two-particle DAs, respectively. The ellipses in Eq. (1) represent the higher twist contribution [21,22,25,26], which we do not consider in this work. The normalization of the two DAs $\Phi = \{\phi_{2,V}^\parallel, \phi_{3,V}^\perp\}$ is given by

$$\int_0^1 dx \Phi(x) = 1. \quad (3)$$

In order to establish a connection between these DAs and the LF wave functions of vector mesons, we need to apply the equal LF time condition, $z^+ = 0$, and choose the LF gauge $A^+ = 0$. Then, neglecting the higher-twist DAs, Eq. (1) can be rewritten as

$$\begin{aligned} \langle 0 | \bar{q}(0) \gamma^\mu q(z) | V(P, h) \rangle |_{z^+ = z_\perp = 0} \\ = f_V M \int_0^1 dx e^{-ixP \cdot z} \left\{ P^\mu \frac{\epsilon_h^+}{P^+} \phi_{2,V}^\parallel(x) \right. \\ \left. + \left(\epsilon_h^\mu - P^\mu \frac{\epsilon_h^+}{P^+} \right) \phi_{3,V}^\perp(x) \right\}. \end{aligned} \quad (4)$$

To isolate the twist-2 DA, $\phi_{2,V}^\parallel$, we may take the plus component ($\mu = +$) of the current with the longitudinal polarization ($h = 0$) and obtain

$$\langle 0 | \bar{q}(0) \gamma^+ q(z^-) | V(P, h) \rangle = f_V M \epsilon_0^+ \int_0^1 dx e^{-ixP \cdot z} \phi_{2,V}^\parallel(x). \quad (5)$$

On the other hand, to isolate the twist-3 DA, $\phi_{3,V}^\perp$, we take the perpendicular component ($\mu = \perp$) of the current with the transverse polarization ($h = +$) and obtain

$$\langle 0 | \bar{q}(0) \gamma^\perp q(z^-) | V(P, h) \rangle = f_V M \epsilon_\perp^\pm \int_0^1 dx e^{-ixP \cdot z} \phi_{3,V}^\perp(x). \quad (6)$$

It is a common practice to utilize an exactly solvable, manifestly covariant model to check the existence (or absence) of the zero mode and substitute the radial and spin-orbit wave functions of the exactly solvable model with the more phenomenologically accessible model wave functions that can be provided by LFQM. To discuss the nature of the LF zero mode in meson decay amplitude, we may denote the total LF longitudinal momentum of the meson, $P^+ = k_Q^+ + k_{\bar{Q}}^+$, where k_Q^+ and $k_{\bar{Q}}^+$ are the individual quark and antiquark LF longitudinal momenta, respectively. Similarly, the LF energy P^- is shared by k_Q^- and $k_{\bar{Q}}^-$, i.e. $P^- = k_Q^- + k_{\bar{Q}}^-$. The LF energy integration is done typically by using Cauchy's theorem for a contour integration. For the LF energy integration of the two-point function to compute the meson decay amplitude, one may pick up a LF energy pole, e.g. either $[k_Q^-]_{\text{on}}$ (i.e. the on shell value of k_Q^-) from the quark propagator or $[k_{\bar{Q}}^-]_{\text{on}}$ from the antiquark propagator. However, it is crucial to note that the poles move to infinity (or fly away in the complex plane) as the LF longitudinal momentum, either k_Q^+ or $k_{\bar{Q}}^+$, goes to zero [27]. Unless the contribution from the pole flown into infinity vanishes, it must be kept in computing the physical observable. Since such a contribution, if it exists, appears either from $k_Q^+ = 0$ and $k_{\bar{Q}}^+ = P^+$ or from $k_{\bar{Q}}^+ = 0$ and $k_Q^+ = P^+$, we call it as the zero-mode contribution. In our previous work [8,14,17], we discussed the power-counting method, which can reveal the existence or absence of the zero-mode contribution by analyzing the power behavior of the integration variable such as k_Q^+ or $k_{\bar{Q}}^+$ and presented an effective method of identifying the corresponding zero-mode operators. If the zero mode exists, it is critical to take into account its contribution in order to get the identical result to the one obtained by manifestly covariant calculation.

As discussed above, in the case of the two-point function for the computation of the meson decay constant, the zero-mode contribution is locked into a single point of the LF

longitudinal momentum, i.e. either $k_Q^+ = 0$, where $k_Q^+ = P^+$, or $k_Q^+ = 0$, where $k_Q^+ = P^+$. Since one of the constituents of the meson carries the entire momentum P^+ of the meson, the other constituent carries the zero LF longitudinal momentum and thus can be regarded as the zero-mode quantum fluctuation linked to the vacuum. This link is due to a pair creation of particles with zero LF longitudinal momenta from the vacuum. It is important to capture the vacuum effect for consistency with the chiral symmetry properties of the strong interactions [28]. With this link, the zero-mode contribution in the meson decay process can be considered effectively as the effect of vacuum fluctuation consistent with the chiral symmetry of the strong interactions. In this respect, the LFQM with effective degrees of freedom represented by the constituent quark and antiquark may be linked to the QCD. The zero-mode link to the QCD vacuum may provide the view of an effective zero-mode cloud around the quark and antiquark inside the meson, and the constituents dressed by the zero-mode cloud may satisfy the chiral symmetry consistent with the QCD. Since the constituent quark and antiquark used in the LFQM have already absorbed the zero-mode cloud, the zero-mode contribution in the LFQM may not be as explicit as in the manifestly covariant model calculation, but it effectively provides consistency with the chiral symmetry.

In the LFQM presented in Refs. [3,10–12,18–20], the constituent quark and antiquark in a bound state are required to be on mass shell, which is different from the covariant formalism, in which the constituents are off mass shell. The spin-orbit wave function in the LFQM is obtained by the interaction-independent Melosh transformation [29] from the ordinary equal-time static spin-orbit wave function assigned by the quantum number J^{PC} . The common feature of the LFQM presented in Refs. [3,10–12,18–20] is to use the sum of the light-front energy of the constituent quark and antiquark for the meson mass in the spin-orbit wave function. In the standard light-front (SLF) approach used in the LFQM [3,10–12,18–20], the vector meson decay constant f_V^{SLF} is obtained by the matrix element of the plus component of the currents in three-dimensional LF momentum space. As the constituent quark and antiquark in LFQM are the dressed constituents including the zero-mode effect, the SLF approach within the phenomenological LFQM [3,10–12,18–20] is not amenable to determine the zero-mode contribution by itself. We thus utilize a manifestly covariant model to check the existence (or absence) of the zero mode and substitute the radial and spin-orbit wave functions with the phenomenologically accessible model wave functions provided by the LFQM analysis of meson mass spectra. If the on-mass-shell spin structure of the matrix element in the SLF approach is not exactly reproducible from the manifestly covariant model, the SLF result is identified to be necessary to take into account the zero mode. We discuss

the zero-mode operator necessary for the SLF analysis of the meson decay process.

To further clarify the zero-mode issue regarding f_V , we analyze the twist-2 and twist-3 two-particle DAs and examine a fundamental constraint anticipated from the LFQM [3,10–12,18–20]: i.e., symmetric quark-antiquark DAs for the equal quark and antiquark bound-state mesons such as ρ . As we shall show in this work, the existence of the zero-mode contribution to f_V claimed by Jaus [4,5] and subsequently advocated by other authors [6] contradicts with this anticipated constraint. We also note that the two equivalent decay constants obtained from $(J_W^\mu, \epsilon_h) = (J_W^+, \epsilon_0)$ and (J_W^μ, ϵ_+) are related to the twist-2 and twist-3 two-particle DAs of a vector meson [21,22,25,26], respectively.

The paper is organized as follows: In Sec. II, we briefly discuss the vector meson decay constant in an exactly solvable model based on the covariant BS model of $(3+1)$ -dimensional fermion field theory. We then present our LF calculation of the vector meson decay constant using two different combinations of LF weak currents J_W^μ and polarization vectors ϵ_h , i.e. (J_W^+, ϵ_0) and (J_W^μ, ϵ_+) , and check the LF covariance of the decay constant within the covariant BS model. Especially, we identify the zero-mode contributions to the decay constant and find the corresponding zero-mode operators. In Sec. III, we present the SLF calculation of the decay constant in a phenomenologically more realistic LFQM with the Gaussian wave function. In Sec. IV, we discuss the correct relation linking the manifestly covariant model to the standard LFQM. We present self-consistent covariant descriptions of vector meson decay constants, as well as twist-2 and twist-3 two-particle DAs in the standard LFQM. In Sec. V, we present our numerical results for the explicit demonstration of our findings. Summary and discussion follow in Sec. VI. The details of the spin structure in standard LFQM are summarized in Appendix A, and the analyses of the pseudoscalar meson decay constant are presented in Appendix B.

II. MANIFESTLY COVARIANT MODEL

The decay constant f_V of a vector meson with the four-momentum P and the mass M as a $q\bar{q}$ bound state is defined by the matrix element of the vector current

$$\langle 0 | \bar{q} \gamma^\mu q | V(P, h) \rangle = f_V M \epsilon_h^\mu, \quad (7)$$

where the polarization vector ϵ_h of a vector meson satisfies the Lorentz condition $\epsilon_h \cdot P = 0$.

The matrix element $A_h^\mu \equiv \langle 0 | \bar{q} \gamma^\mu q | V(P, h) \rangle$ is given in the one-loop approximation (see Fig. 1) as a momentum integral

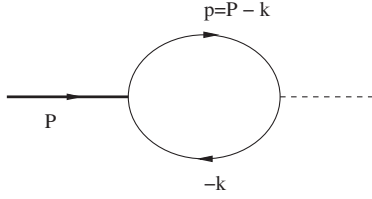


FIG. 1. Feynman diagram for a decay constant.

$$A_h^\mu = N_c \int \frac{d^4 k}{(2\pi)^4} \frac{H_V}{N_p N_k} S_h^\mu, \quad (8)$$

where N_c denotes the number of colors. The denominators N_p and N_k come from the fermion propagators of mass m_1 and m_2 carrying the internal four-momenta $p = P - k$ and k , respectively, and they are given by $N_p = p^2 - m_1^2 + i\epsilon$ and $N_k = k^2 - m_2^2 + i\epsilon$. In order to regularize the covariant loop in $(3 + 1)$ dimensions, we use the usual multipole ansatz [4] for the $q\bar{q}$ bound-state vertex function H_V of a vector meson:

$$H_V = \frac{g}{N_\Lambda^n}, \quad (9)$$

where $N_\Lambda = p^2 - \Lambda^2 + i\epsilon$, and g and Λ are constant parameters, and the power n for the multipole ansatz should be $n \geq 2$ for the regularization. For our purpose, we take $n = 2$, since our qualitative results in conjunction with the zero-mode issue do not depend on the value of n .

The trace term S_h^μ in Eq. (8) is given by

$$S_h^\mu = \text{Tr}[\gamma^\mu (\not{p} + m_1) \Gamma \cdot \epsilon_h (-\not{k} + m_2)], \quad (10)$$

where the vector meson vertex operator Γ^μ is given by

$$\Gamma^\mu = \gamma^\mu - \frac{(p - k)^\mu}{D}. \quad (11)$$

While the Dirac coupling γ^μ is intrinsic to the vector meson vertex, the model dependence of a vector meson is implemented through the factor D in Eq. (11). For the explicit comparison between the manifestly covariant calculation and the LF calculation, checking the existence (or absence) of the zero-mode contribution to the vector meson decay constant, we analyze Γ^μ in this section with a constant D factor, i.e. $D = D_{\text{con}} = M + m_1 + m_2$. We will discuss a more realistic (but not manifestly covariant) D factor such as $D_{\text{LF}} = M_0 + m_1 + m_2$ with the invariant mass M_0 of the vector meson in Sec. III, devoted to phenomenologically more accessible LFQM [3,10–12,18–20].

The manifestly covariant result for $n = 2$ case is given by [17]

$$f_V^{\text{cov}} = \frac{N_c g}{4\pi^2 M} \int_0^1 dx \int_0^{1-x} dy (1 - x - y) \times \left\{ \frac{y(1-y)M^2 + m_1 m_2}{C_{\text{cov}}^2} - \frac{1 + \frac{m_1 + m_2}{D_{\text{con}}}}{C_{\text{cov}}} \right\}, \quad (12)$$

where

$$C_{\text{cov}} = y(1-y)M^2 - x m_1^2 - y m_2^2 - (1-x-y)\Lambda^2. \quad (13)$$

Performing the LF calculation in parallel with the manifestly covariant one, we use two different combinations of the currents and the polarization vectors to obtain the decay constant: (1) the plus component ($\mu = +$) of the currents with the longitudinal polarization ϵ_0^μ and (2) the perpendicular components ($\mu = \perp$) of the currents with the transverse polarization $\epsilon_\pm^\mu$. The polarization vectors used in this manifestly covariant analysis are given by

$$\epsilon_0^\mu = [\epsilon^+, \epsilon^-, \epsilon_\perp] = \frac{1}{M} \left[P^+, \frac{\mathbf{P}_\perp^2 - M^2}{P^+}, \mathbf{P}_\perp \right], \quad \epsilon_\pm^\mu = \left[0, \frac{2}{P^+} \epsilon_\pm^\perp \cdot \mathbf{P}_\perp, \epsilon_\pm^\perp \right], \quad \epsilon_\pm^\perp = \mp \frac{(1, \pm i)}{\sqrt{2}}, \quad (14)$$

where we use the metric convention $a \cdot b = \frac{1}{2}(a^+ b^- + a^- b^+) - \mathbf{a}_\perp \cdot \mathbf{b}_\perp$.

The trace term S_h^μ in Eq. (10) can be separated into the on-mass-shell propagating part $[S_h^\mu]_{\text{on}}$ and the off-mass-shell instantaneous part $[S_h^\mu]_{\text{inst}}$ via $\not{q} = \not{q}_{\text{on}} + \frac{1}{2}\gamma^+(q^- - q_{\text{on}}^-)$ as

$$S_h^\mu = [S_h^\mu]_{\text{on}} + [S_h^\mu]_{\text{inst}}, \quad (15)$$

where

$$[S_h^\mu]_{\text{on}} = 4[\epsilon_h^\mu (p_{\text{on}} \cdot k_{\text{on}} + m_1 m_2) - p_{\text{on}}^\mu (\epsilon_h \cdot k_{\text{on}}) - k_{\text{on}}^\mu (\epsilon_h \cdot p_{\text{on}})] + 8 \frac{\epsilon_h \cdot k_{\text{on}}}{D} (m_2 p_{\text{on}}^\mu - m_1 k_{\text{on}}^\mu) \quad (16)$$

and

$$[S_h^\mu]_{\text{inst}} = 2\Delta_k^-(\epsilon_h^\mu p^+ - \epsilon_h^+ p_{\text{on}}^\mu) + 2\Delta_p^-(\epsilon_h^\mu k^+ - \epsilon_h^+ k_{\text{on}}^\mu) + 4 \frac{\epsilon_h^+ \Delta_k^-}{D} (m_2 p_{\text{on}}^\mu - m_1 k_{\text{on}}^\mu) + (\cdots) g^{\mu+}, \quad (17)$$

with $\Delta_q^\mu = q^\mu - q_{\text{on}}^\mu$. Since the $g^{\mu+}$ term in Eq. (17) vanishes when $\mu = +$ or $\perp$ is taken, we do not list the explicit forms of the ellipses in front of $g^{\mu+}$. Furthermore, we take the reference frame where $\mathbf{P}_\perp = 0$, i.e., $P = (P^+, M^2/P^+, 0)$. In this case, the four-momenta of the on-mass-shell constituents are given by

$$p_{\text{on}} = \left[xP^+, \frac{\mathbf{k}_\perp^2 + m_1^2}{xP^+}, -\mathbf{k}_\perp \right],$$

$$k_{\text{on}} = \left[(1-x)P^+, \frac{\mathbf{k}_\perp^2 + m_2^2}{(1-x)P^+}, \mathbf{k}_\perp \right], \quad (18)$$

where $x = p^+/P^+$ is the LF longitudinal momentum fraction of the quark.

By the integration over k^- in Eq. (8) and closing the contour in the lower half of the complex k^- plane, one picks up the residue at $k^- = k_{\text{on}}^-$ in the region $0 < k^+ < P^+$ (or $0 < x < 1$). Thus, the Cauchy integration formula for the k^- integral in Eq. (8) yields

$$A_h^\mu = \frac{N_c}{16\pi^3} \int_0^1 \frac{dx}{(1-x)} \int d^2\mathbf{k}_\perp \chi(x, \mathbf{k}_\perp) S_h^\mu(k^- = k_{\text{on}}^-), \quad (19)$$

where

$$\chi(x, \mathbf{k}_\perp) = \frac{g}{[x(M^2 - M_0^2)][x(M^2 - M_\Lambda^2)]^n} \quad (20)$$

and

$$M_{0(\Lambda)}^2 = \frac{\mathbf{k}_\perp^2 + m_1^2(\Lambda^2)}{x} + \frac{\mathbf{k}_\perp^2 + m_2^2}{1-x}. \quad (21)$$

Note that the second term in the denominator of Eq. (20) comes from the multipole-type vertex function H_V defined in Eq. (9). Although we take $n = 2$ for a direct comparison with the manifestly covariant result f_V^{cov} , the qualitative result regarding the zero-mode issue is independent of the power $n(\geq 2)$, as we shall show.

A. Decay constant from longitudinal polarization

Using the plus component ($\mu = +$) of the currents with the longitudinal polarization vector ϵ_0^μ in Eq. (14), the decay constant is obtained from the relation

$$f_V^{(h=0)} = \frac{A_{h=0}^+}{M\epsilon_0^+}. \quad (22)$$

For the purpose of analyzing the zero-mode contribution to the decay constant, we denote the decay constant as $[f_V^{(h=0)}]_{\text{val}}$ (meaning the valence contribution to the decay constant) when the matrix element A_0^+ is obtained for $k^- = k_{\text{on}}^-$ in the region of $0 < x < 1$. Explicitly, it is given by

$$[f_V^{(h=0)}]_{\text{val}} = \frac{N_c}{4\pi^3} \int_0^1 \frac{dx}{(1-x)} \int d^2\mathbf{k}_\perp \chi(x, \mathbf{k}_\perp) \frac{1}{M} \times \left\{ x(1-x)M^2 + \mathbf{k}_\perp^2 + m_1m_2 + [m_2x - (1-x)m_1] \frac{[\mathbf{k}_\perp^2 + m_2^2 - (1-x)^2M^2]}{(1-x)D_{\text{con}}} \right\}. \quad (23)$$

We note that the valence contribution to the trace term in Eq. (19) comes only from the on-shell propagating part but not from the instantaneous one, i.e. $[S_0^+]_{\text{val}} = [S_0^+]_{\text{on}}$ and $[f_V^{(h=0)}]_{\text{val}} = [f_V^{(h=0)}]_{\text{on}}$.

Comparing $[f_V^{(h=0)}]_{\text{val}}$ with the manifestly covariant result f_V^{cov} , we find that $[f_V^{(h=0)}]_{\text{val}}$ is exactly the same as f_V^{cov} for the model-independent Dirac coupling, $\Gamma^\mu = \gamma^\mu$ (or $1/D = 0$). The same observation has also been made in Ref. [7]. However, $[f_V^{(h=0)}]_{\text{val}}$ is different from f_V^{cov} when the model-dependent $D = D_{\text{con}}$ term is included. In this case, the difference between the two results, $f_V^{\text{cov}} - [f_V^{(h=0)}]_{\text{val}}$, corresponds to the zero-mode contribution $[f_V^{(h=0)}]_{\text{Z.M.}}$ to the full solution $[f_V^{(h=0)}]_{\text{full}} = [f_V^{(h=0)}]_{\text{val}} + [f_V^{(h=0)}]_{\text{Z.M.}}$. For the case of $D = D_{\text{con}}$, the zero-mode contribution to $f_V^{(h=0)}$ comes from the singular p^- (or equivalently $1/x$) term in S_0^+ in the limit of $x \rightarrow 0$ when $p^- = p_{\text{on}}^-$, i.e.

$$\lim_{x \rightarrow 0} S_0^+(p^- = p_{\text{on}}^-) = 4m_1 \frac{\epsilon_0^+ p^-}{D_{\text{con}}}. \quad (24)$$

We note that the singular term in Eq. (24) comes only from the instantaneous contribution.

The necessary prescription to identify the zero-mode operator corresponding to p^- is analogous to that derived in the previous analyses of weak transition form factor calculations [4,8,14], except that there is no momentum-transfer q dependence. By replacing p^- with $-Z_2$ [4,8,14] in Eq. (24), we now identify the zero-mode operator $[S_0^+]_{\text{Z.M.}}$ corresponding to Eq. (24) as follows:

$$[S_0^+]_{\text{Z.M.}} = 4m_1 \frac{\epsilon_0^+ (-Z_2)}{D_{\text{con}}}, \quad (25)$$

where

$$Z_2 = x(M^2 - M_0^2) + m_1^2 - m_2^2 + (1-2x)M^2. \quad (26)$$

This zero-mode operator $[S_0^+]_{\text{Z.M.}}$ can be effectively included in the valence region, i.e. the full (exact) solution of the trace term is given by $[S_0^+]_{\text{full}} = [S_0^+]_{\text{val}} + [S_0^+]_{\text{Z.M.}}$. Or, equivalently, the zero-mode contribution to the decay constant, $[f_V^{(h=0)}]_{\text{Z.M.}} = [A_0^+]_{\text{Z.M.}}/(M\epsilon_0^+)$, is given by

$$[f_V^{(h=0)}]_{\text{Z.M.}} = \frac{N_c}{16\pi^3} \int_0^1 \frac{dx}{(1-x)} \int d^2\mathbf{k}_\perp \chi(x, \mathbf{k}_\perp) \frac{[S_0^+]_{\text{Z.M.}}}{M\epsilon_0^+}. \quad (27)$$

Adding Eqs. (23) and (27), we finally obtain the full result of the decay constant for the longitudinal polarization as

$$\begin{aligned}
[f_V^{(h=0)}]_{\text{full}} &= \frac{N_c}{4\pi^3} \int_0^1 \frac{dx}{(1-x)} \int d^2\mathbf{k}_\perp \chi(x, \mathbf{k}_\perp) \\
&\times \frac{1}{M} \left\{ x(1-x)M^2 + \mathbf{k}_\perp^2 + m_1 m_2 \right. \\
&\left. + x(m_1 + m_2) \frac{[\mathbf{k}_\perp^2 + m_2^2 - (1-x)^2 M^2]}{(1-x)D_{\text{con}}} \right\}. \quad (28)
\end{aligned}$$

It can be checked that Eq. (28) is identical to the manifestly covariant result of Eq. (12).

B. Decay constant from transverse polarization

Using the perpendicular components ($\mu = \perp$) of the currents with the transverse polarization vector ϵ_+^μ , the decay constant is obtained from the relation

$$f_V^{(h=1)} = \frac{\mathbf{A}_{h=1}^\perp \cdot \epsilon_+^{\perp*}}{M}. \quad (29)$$

In this case, the valence contributions to the trace term in Eq. (19) come not only from the on-shell part but also from the instantaneous one, i.e. $[S_+^\perp]_{\text{val}} = [S_+^\perp]_{\text{on}} + 2k^+ \epsilon_+^\perp \Delta_p^-$, where the latter corresponds to the instantaneous contribution. The on-shell contribution to the decay constant $f_V^{(h=1)}$ is given by

$$\begin{aligned}
[f_V^{(h=1)}]_{\text{on}} &= \frac{N_c}{4\pi^3} \int_0^1 \frac{dx}{(1-x)} \int d^2\mathbf{k}_\perp \chi(x, \mathbf{k}_\perp) \\
&\times \frac{1}{M} \left\{ \frac{\mathbf{k}_\perp^2 + \mathcal{A}^2}{2x(1-x)} - \mathbf{k}_\perp^2 + \frac{(m_1 + m_2)}{D_{\text{con}}} \mathbf{k}_\perp^2 \right\}, \quad (30)
\end{aligned}$$

where $\mathcal{A} = (1-x)m_1 + xm_2$, and the valence contribution $[f_V^{(h=1)}]_{\text{val}} = [f_V^{(h=1)}]_{\text{on}} + [f_V^{(h=1)}]_{\text{inst}}$ is given by

$$\begin{aligned}
[f_V^{(h=1)}]_{\text{val}} &= \frac{N_c}{8\pi^3} \int_0^1 \frac{dx}{(1-x)^2} \int d^2\mathbf{k}_\perp \chi(x, \mathbf{k}_\perp) \\
&\times \frac{1}{M} \left\{ (2x-1)(\mathbf{k}_\perp^2 + m_2^2) + 2(1-x)m_1 m_2 \right. \\
&\left. + (1-x)^2 M^2 + 2(1-x) \frac{(m_1 + m_2)}{D_{\text{con}}} \mathbf{k}_\perp^2 \right\}. \quad (31)
\end{aligned}$$

We find that Eq. (31) does not coincide with the manifestly covariant result. That is, the decay constant obtained from the perpendicular components of the currents with the transverse polarization receives a zero mode.

Again, the zero-mode contribution to $f_V^{(h=1)}$ can be obtained from all possible singular p^- terms in $S_+^\perp$ in the limit of $x \rightarrow 0$ when $p^- = p_{\text{on}}^-$. From Eqs. (16) and (17), we find the nonvanishing singular term as follows:

$$\lim_{x \rightarrow 0} S_+^\perp(p^- = p_{\text{on}}^-) = 2p^- \epsilon_+^\perp. \quad (32)$$

We should note that the singular term in Eq. (32) comes only from the on-shell part $[S_+^\perp]_{\text{on}}$, but not from the instantaneous part $[S_+^\perp]_{\text{inst}}$. This implies that the zero-mode contribution to $f_V^{(h=1)}$ comes only from the model-independent Dirac coupling part, $\Gamma^\mu = \gamma^\mu$. By the replacement $p^- \rightarrow -Z_2$ in Eq. (32), as previously discussed in the derivation of Eq. (25) from Eq. (24), we now obtain the corresponding zero-mode operator $[S_+^\perp]_{\text{Z.M.}}$ as

$$[S_+^\perp]_{\text{Z.M.}} = 2(-Z_2) \epsilon_+^\perp. \quad (33)$$

That is, the full solution of the trace term in the valence region is $[S_+^\perp]_{\text{full}} = [S_+^\perp]_{\text{val}} + [S_+^\perp]_{\text{Z.M.}}$. Or the zero-mode contribution to the decay constant, $[f_V^{(h=1)}]_{\text{Z.M.}} = [\mathbf{A}_{h=1}^\perp]_{\text{Z.M.}} \cdot \epsilon_+^{\perp*}/M$, is given by

$$\begin{aligned}
[f_V^{(h=1)}]_{\text{Z.M.}} &= \frac{N_c}{16\pi^3} \int_0^1 \frac{dx}{(1-x)} \\
&\times \int d^2\mathbf{k}_\perp \chi(x, \mathbf{k}_\perp) \frac{[S_+^\perp]_{\text{Z.M.}} \cdot \epsilon_+^{\perp*}}{M}. \quad (34)
\end{aligned}$$

Adding Eqs. (31) and (34), we now obtain the full result of the decay constant for the transverse polarization as

$$\begin{aligned}
[f_V^{(h=1)}]_{\text{full}} &= \frac{N_c}{4\pi^3} \int_0^1 \frac{dx}{(1-x)} \int d^2\mathbf{k}_\perp \chi(x, \mathbf{k}_\perp) \\
&\times \frac{1}{M} \left\{ xM_0^2 - m_1(m_1 - m_2) - \mathbf{k}_\perp^2 \right. \\
&\left. + \frac{(m_1 + m_2)}{D_{\text{con}}} \mathbf{k}_\perp^2 \right\}. \quad (35)
\end{aligned}$$

One can check that $[f_V^{(h=1)}]_{\text{full}}$ is the same as $[f_V^{(h=0)}]_{\text{full}}$ [Eq. (28)] as well as the manifestly covariant result f_V^{cov} . We also confirm that our $[f_V^{(h=1)}]_{\text{full}}$ is exactly the same as the one obtained by Jaus [4] [see Eq. (4.22) of Ref. [4]].

III. STANDARD LIGHT-FRONT QUARK MODEL

In the standard LFQM [3,10–12,18–20], the momentum-space meson wave function is given by

$$\Psi_{\lambda_1 \lambda_2}^{SS_z}(x, \mathbf{k}_\perp) = \mathcal{R}_{\lambda_1 \lambda_2}^{SS_z}(x, \mathbf{k}_\perp) \phi(x, \mathbf{k}_\perp), \quad (36)$$

where ϕ is the radial wave function and $\mathcal{R}_{\lambda_1 \lambda_2}^{SS_z}$ is the spin-orbit wave function that is obtained by the interaction-independent Melosh transformation [29] from the ordinary spin-orbit wave function assigned by the quantum numbers J^{PC} . The explicit form of the spin-orbit wave function of definite spin (S, S_z) is constructed out of LF helicity (λ_1, λ_2) as follows:

$$\begin{aligned} \mathcal{R}_{\lambda_1 \lambda_2}^{SS_z}(x, \mathbf{k}_\perp) &= \sum_{s_1, s_2} \langle \lambda_1 | \mathcal{R}_M^\dagger(1-x, \mathbf{k}_\perp, m_1) | s_1 \rangle \\ &\times \langle \lambda_2 | \mathcal{R}_M^\dagger(x, -\mathbf{k}_\perp, m_2) | s_2 \rangle \\ &\times \left\langle \frac{1}{2} s_1 \frac{1}{2} s_2 | SS_z \right\rangle, \end{aligned} \quad (37)$$

where $|s_i\rangle$ are the usual Pauli spinor, and $\mathcal{R}_M$ is the Melosh transformation operator:

$$\mathcal{R}_M(x_i, \mathbf{k}_\perp, m_i) = \frac{m_i + x_i M_0 - i\sigma \cdot (\hat{\mathbf{n}} \times \mathbf{k}_\perp)}{\sqrt{(m_i + x_i M_0)^2 + \mathbf{k}_\perp^2}}, \quad (38)$$

with $\hat{\mathbf{n}} = (0, 0, 1)$ being a unit vector in the z direction. The spin-orbit wave functions can also be represented in the following covariant way:

$$\mathcal{R}_{\lambda_1 \lambda_2}^{SS_z}(x, \mathbf{k}_\perp) = \frac{\bar{u}_{\lambda_1}(p_1) \Gamma v_{\lambda_2}(p_2)}{\sqrt{2[M_0^2 - (m_1 - m_2)^2]^{1/2}}}. \quad (39)$$

The vertex operator for a pseudoscalar meson is $\Gamma = \gamma_5$, and that for a vector meson is given by [3,11,12,18–20]

$$\Gamma = -\tilde{\epsilon} + \frac{\tilde{\epsilon} \cdot (p_1 - p_2)}{D_{\text{LF}}}, \quad (40)$$

where $D_{\text{LF}} = M_0 + m_1 + m_2$, and $\tilde{\epsilon}$ is the polarization vector specified in the center-of-mass frame of the quark and antiquark. We should note that while the transverse polarization vector $\tilde{\epsilon}_\perp^\mu$ of the $q\bar{q}$ system coincides with that of the meson polarization vector $\epsilon_\perp^\mu$ given by Eq. (14), the longitudinal polarization vector $\tilde{\epsilon}_0^\mu$ is different from ϵ_0^μ and is given by [3,11,12,18–20]

$$\tilde{\epsilon}_0^\mu = \frac{1}{M_0} \left[P^+, \frac{-M_0^2 + \mathbf{P}_\perp^2}{P^+}, \mathbf{P}_\perp \right]. \quad (41)$$

That is, the invariant mass M_0 instead of the physical mass M is used to define the vector meson vertex operator Γ . We also should note that $\tilde{\epsilon}_0^\mu$ is used only in the vector meson vertex operator in Eq. (40). The virtue of using M_0 is to satisfy the normalization of $\mathcal{R}_{\lambda_1 \lambda_2}^{SS_z}$ automatically regardless of any kinds of vector mesons, i.e.

$$\sum_{\lambda_1 \lambda_2} \mathcal{R}_{\lambda_1 \lambda_2}^{SS_z^\dagger} \mathcal{R}_{\lambda_1 \lambda_2}^{SS_z} = 1. \quad (42)$$

The explicit helicity components of $\mathcal{R}_{\lambda_1 \lambda_2}^{SS_z}$ for pseudoscalar and vector mesons are given in Appendix A.

For the radial wave function ϕ , we use the same Gaussian wave function for both pseudoscalar and vector mesons:

$$\phi(x, \mathbf{k}_\perp) = \frac{4\pi^{3/4}}{\beta^{3/2}} \sqrt{\frac{\partial k_z}{\partial x}} \exp(-\vec{k}^2/2\beta^2), \quad (43)$$

where β is the variational parameter fixed by the analysis of meson mass spectra [10]. The longitudinal component k_z is defined by $k_z = (x - 1/2)M_0 + (m_2^2 - m_1^2)/2M_0$, and the Jacobian of the variable transformation $\{x, \mathbf{k}_\perp\} \rightarrow \vec{k} = (\mathbf{k}_\perp, k_z)$ is given by

$$\frac{\partial k_z}{\partial x} = \frac{M_0}{4x(1-x)} \left\{ 1 - \left[\frac{m_1^2 - m_2^2}{M_0^2} \right]^2 \right\}. \quad (44)$$

The normalization of our wave function is then given by

$$\sum_{\lambda_1 \lambda_2} \int \frac{dx d^2 \mathbf{k}_\perp}{16\pi^3} |\Psi_{\lambda_1 \lambda_2}^{SS_z}(x, \mathbf{k}_\perp)|^2 = \int \frac{dx d^2 \mathbf{k}_\perp}{16\pi^3} |\phi(x, \mathbf{k}_\perp)|^2. \quad (45)$$

Using the plus component of the currents and the longitudinal polarization vector, the SLF calculation of the matrix element in Eq. (7) is

$$\begin{aligned} A_0^+ &= \sqrt{N_c} \sum_{\lambda_1 \lambda_2} \int \frac{dx d^2 \mathbf{k}_\perp}{16\pi^3} \phi(x, \mathbf{k}_\perp) \mathcal{R}_{\lambda_1 \lambda_2}^{10}(x, \mathbf{k}_\perp) \\ &\times \frac{\bar{v}_{\lambda_2}(p_2)}{\sqrt{p_2^+}} \gamma^+ \frac{u_{\lambda_1}(p_1)}{\sqrt{p_1^+}} \\ &= f_V^{\text{SLF}} M \epsilon_0^+. \end{aligned} \quad (46)$$

We again note that $\tilde{\epsilon}_0$ is used only in the calculation of the spin-orbit wave function $\mathcal{R}_{\lambda_1 \lambda_2}^{10}(x, \mathbf{k}_\perp)$. We then obtain the SLF result of the vector meson decay constant as follows [see Ref. [30] or Appendix A for the Dirac matrix elements for the helicity spinors in Eq. (46)] [3,11,12,19]:

$$f_V^{\text{SLF}} = \frac{\sqrt{2N_c}}{8\pi^3} \int_0^1 dx \int d^2 \mathbf{k}_\perp \frac{\phi(x, \mathbf{k}_\perp)}{\sqrt{\mathbf{k}_\perp^2 + \mathcal{A}^2}} \left[\mathcal{A} + \frac{2\mathbf{k}_\perp^2}{D_{\text{LF}}} \right]. \quad (47)$$

IV. CORRESPONDENCE BETWEEN THE MANIFESTLY COVARIANT MODEL AND LFQM

In this section, we shall analyze the relations between $f_V^{(h=0,1)}$ in the manifestly covariant BS model and f_V^{SLF} in the standard LFQM. The main differences between the BS model and the standard LFQM are attributed to the different spin structures of the $q\bar{q}$ system (i.e. off-shellness vs on-shellness) and the different vertex functions (χ vs ϕ). In other words, the results of the BS model allow the nonzero binding energy $E_{\text{B.E.}} = M^2 - M_0^2$, but the SLF result is obtained from the zero-binding-energy limit (i.e. $M \rightarrow M_0$).

Thus, one should take those different prescriptions into account in connecting the two different models.

For direct comparison between $f_V^{(h=0,1)}$ and f_V^{SLF} , the LF covariant vertex function χ and the D factor $D_{\text{con}} = M + m_1 + m_2$ in $f_V^{(h=0,1)}$ may be replaced with the Gaussian wave function ϕ and $D_{\text{LF}} = M_0 + m_1 + m_2$ in f_V^{SLF} via

$$\text{Type I: } \sqrt{2N_c} \frac{\chi(x, \mathbf{k}_\perp)}{1-x} \rightarrow \frac{\phi(x, \mathbf{k}_\perp)}{\sqrt{\mathcal{A}^2 + \mathbf{k}_\perp^2}}, \quad D_{\text{con}} \rightarrow D_{\text{LF}}. \quad (48)$$

We denote Eq. (48) as “type-I” correspondence between the manifestly covariant BS model and the standard LFQM. Essentially, Jaus [4] and subsequently the authors in Ref. [6] used this type-I correspondence when they connected the two different models and claimed that $[f_V^{(h=1)}]_{\text{full}}$ after applying a type-I replacement is the correct result in the LFQM, refuting effectively the SLF result f_V^{SLF} . However, one should note that this correspondence limits the replacement $M \rightarrow M_0$ only in the D factor so that the consistency of the replacement $M \rightarrow M_0$ within the LFQM is not assured. This limitation leads to a consequence of not satisfying the symmetry constraint on DAs anticipated from the LFQM, as we discuss in the following section, Sec. V.

The correspondence between χ and ϕ given by Eq. (48) has already been derived from the calculation of the zero-mode free weak transition form factors between pseudoscalar and vector (or pseudoscalar) mesons [4,6,8,14]. We also explicitly demonstrate in Appendix B that the correspondence between χ and ϕ can be obtained from the comparison of the pseudoscalar meson decay constants between the two models. On the other hand, the validity of the simple replacement $D_{\text{con}} \rightarrow D_{\text{LF}}$ for the D factor has not yet been clarified as explicitly as in the case of the vertex function replacement. In the present work, however, since we have now two exact forms of the decay constants $[f_V^{(h=0)}]_{\text{full}}$ and $[f_V^{(h=1)}]_{\text{full}}$ obtained from the covariant BS model, we are able to clearly check if type-I correspondence is valid or not. That is, if type-I correspondence is correct, then $[f_V^{(h=0)}]_{\text{full}}$ and $[f_V^{(h=1)}]_{\text{full}}$ should give the same result regardless of the SLF result f_V^{SLF} . This is not the case, as we shall show in Sec. V; e.g., $[f_V^{(h=1)}]_{\text{full}}$ differs not only with f_V^{SLF} but also with $[f_V^{(h=0)}]_{\text{full}}$ when a type-I replacement is used.

Considering that the result of f_V^{SLF} is essentially obtained from the requirement of all constituents being on their respective mass shell, we note that it is more natural to apply the replacement $M \rightarrow M_0$ in each and every term, including M in the integrand of $f_V^{(h=0)}$ and $f_V^{(h=1)}$, than to apply it only in the D factor as in the case of type-I replacement. For the self-consistency of the model, we thus use the following replacement to connect the two different models:

$$\text{Type II: } \sqrt{2N_c} \frac{\chi(x, \mathbf{k}_\perp)}{1-x} \rightarrow \frac{\phi(x, \mathbf{k}_\perp)}{\sqrt{\mathcal{A}^2 + \mathbf{k}_\perp^2}}, \quad M \rightarrow M_0, \quad (49)$$

in the integrand of the formulas for $f_V^{(h=0)}$ and $f_V^{(h=1)}$. We denote Eq. (49) as “type-II” correspondence between the covariant BS model and the standard LFQM. The essential point of this type-II replacement is to apply the replacement of $M \rightarrow M_0$ to all physical mass terms in the integrands of $f_V^{(h=0)}$ and $f_V^{(h=1)}$. Without such self-consistent correspondence as given by type II, it would not be possible to anticipate that the LFQM with effective degrees of freedom represented by the constituent quark and antiquark may satisfy the chiral symmetry consistent with the QCD. With the self-consistent type-II replacement, we find numerically that the three different forms $[f_V^{(h=0)}]_{\text{full}}$, $[f_V^{(h=1)}]_{\text{full}}$, and f_V^{SLF} indeed yield the identical result. Moreover, we find that the on-shell contribution $[f_V^{(h=1)}]_{\text{on}}$ to $[f_V^{(h=1)}]_{\text{full}}$ also gives the same result as the other three, i.e. $[f_V^{(h=1)}]_{\text{on}} = [f_V^{(h=1)}]_{\text{full}} = [f_V^{(h=0)}]_{\text{full}} = f_V^{\text{SLF}}$ in the standard LFQM. From those observations, we conclude that the type-II replacement provides the self-consistent correspondence in connecting the covariant BS model and the standard LFQM.

Although those four different forms give the same result as each other when applying type-II replacement, their quark DAs are quite different. Therefore, by checking the DAs as an important constraint of the model, we are able to further pin down the self-consistent LF covariant forms of the decay constant. The quark DA of a vector meson, $\phi_V(x, \mu)$, is the probability of finding collinear quarks up to the scale μ in the $L_z = 0$ (s-wave) projection of the meson wave function defined by

$$\phi_V(x, \mu) = \int^{|\mathbf{k}_\perp| < \mu} \frac{d^2 \mathbf{k}_\perp}{16\pi^3} \Psi_{\lambda_1 \lambda_2}^{SS_z}(x, \mathbf{k}_\perp). \quad (50)$$

The dependence on the scale μ is given by the QCD evolution [30] and can be calculated perturbatively. However, the DAs at a certain low scale can be obtained by the necessary nonperturbative input from LFQM. Moreover, the presence of the damping Gaussian wave function allows us to perform the integral up to infinity without loss of generality. The quark DA for a vector meson is constrained by [11,19]

$$\int_0^1 \phi_V(x, \mu) dx = \frac{f_V}{2\sqrt{6}}. \quad (51)$$

One may also redefine the normalized quark DA as $\Phi_V(x) = (2\sqrt{6}/f_V)\phi_V(x)$, so that $\int_0^1 dx \Phi_V(x) = 1$.

For the equal quark and antiquark bound-state meson such as ρ , we find that only two forms of the decay

constant, i.e. f_V^{SLF} from the longitudinal polarization and $[f_V^{(h=1)}]_{\text{on}}$ from the transverse one, yield the anticipated symmetric quark DA. The other two forms, i.e. $[f_V^{(h=1)}]_{\text{full}}$ and $[f_V^{(h=0)}]_{\text{full}}$, that involve the corresponding zero-mode contributions do not reproduce solely this fundamental constraint expected from the symmetry associated with the two constituent masses m_1 and m_2 but reflect also the intrinsic characteristic of the zero modes inherited from the vacuum property. The involved zero mode Z_2 [see Eq. (26)] is apparently antisymmetric under $x \leftrightarrow (1-x)$ when $m_1 = m_2$ is taken with the replacement $M \rightarrow M_0$, so that the integration of Z_2 over x vanishes as it reflects the vacuum property. Consequently, in the standard LFQM [3,11,12,18–20], we expect two self-consistent LF covariant forms of the vector meson decay constant, i.e. f_V^{SLF} given by Eq. (47) and $[f_V^{(h=1)}]_{\text{on}}$ given by Eq. (30), which provide the expected symmetric quark DAs for $m_1 = m_2$. This expectation is realized by the type-II replacement but not with the type-I replacement. The normalized quark DAs obtained from f_V^{SLF} and $[f_V^{(h=1)}]_{\text{on}}$ (with the type-II replacement) correspond to the twist-2 $\phi_{2;V}^{\parallel}(x)$ and twist-3 $\phi_{3;V}^{\perp}(x)$, respectively. Our complete results for the twist-2 and twist-3 DAs in the standard LFQM are as follows:

$$\begin{aligned}\phi_{2;V}^{\parallel}(x) &= \frac{2\sqrt{6}}{f_V} \int \frac{d^2\mathbf{k}_{\perp}}{16\pi^3} \frac{\phi(x, \mathbf{k}_{\perp})}{\sqrt{\mathbf{k}_{\perp}^2 + \mathcal{A}^2}} \left[\mathcal{A} + \frac{2\mathbf{k}_{\perp}^2}{D_{\text{LF}}} \right], \\ \phi_{3;V}^{\perp}(x) &= \frac{2\sqrt{6}}{f_V} \int \frac{d^2\mathbf{k}_{\perp}}{16\pi^3} \frac{\phi(x, \mathbf{k}_{\perp})}{\sqrt{\mathbf{k}_{\perp}^2 + \mathcal{A}^2}} \frac{1}{M_0} \left\{ \frac{\mathbf{k}_{\perp}^2 + \mathcal{A}^2}{2x(1-x)} \right. \\ &\quad \left. - \mathbf{k}_{\perp}^2 + \frac{(m_1 + m_2)}{D_{\text{LF}}} \mathbf{k}_{\perp}^2 \right\},\end{aligned}\quad (52)$$

where we use $N_c = 3$, and they satisfy the normalization given by Eq. (3). We also note that while $f_V = f_V^{\text{SLF}} = [f_V^{(h=1)}]_{\text{on}}$ (with type-II replacement), f_V used in $\phi_{2;V}^{\parallel}(x)$ and $\phi_{3;V}^{\perp}(x)$ corresponds to f_V^{SLF} and $[f_V^{(h=1)}]_{\text{on}}$, respectively. In the next section, we show our findings explicitly with the numerical calculations of the decay constants and quark DAs of vector mesons.

V. NUMERICAL RESULTS

In our numerical calculations within the standard LFQM, we use the set of the model parameters (i.e. constituent quark masses and the Gaussian parameters β) for the harmonic oscillator (HO) confining potentials given in Table I, which was obtained from the calculation of meson mass spectra using the variational principle in our LFQM [10–12].

TABLE I. The constituent quark mass (in GeV) and the Gaussian parameters β (in GeV) for the HO potential obtained from the variational principle in our LFQM [10–12]. $q = u$ and d .

Model	m_q	m_s	m_c	β_{qq}	β_{qs}	β_{qc}	β_{cc}
HO	0.25	0.48	1.8	0.3194	0.3419	0.4216	0.6998

In Table II, we show the results of the decay constants for (ρ , K^* , D^* , J/ψ) mesons obtained from $[f_{\text{full}}^{(h=0)}]$, $[f_{\text{on}}^{(h=1)}]$, $f_{\text{full}}^{(h=1)}$ after applying type-I and type-II correspondences and f_V^{SLF} and compare them with the experimental data [31]. For the type-I correspondence, the fact that the results of $f_{\text{full}}^{(h=0)}$ are different from those of $f_{\text{full}}^{(h=1)}$ implies the breakdown of the covariance of the decay constant. For the type-II correspondence, however, the three different forms $f_{\text{full}}^{(h=0)}$, $f_{\text{full}}^{(h=1)}$, and f_V^{SLF} are found to yield the same numerical results. Although the on-shell contribution $f_{\text{on}}^{(h=1)}$ to $f_{\text{full}}^{(h=1)}$ gives identical results to the full solution for both type-I and type-II replacements, it is quite obvious to use type-II correspondence in order to ensure the self-consistent covariant description of a vector meson decay constant in the standard LFQM.

While four different forms, i.e. $[f_{\text{full}}^{(h=0)}]$, $[f_{\text{on}}^{(h=1)}]$, $f_{\text{full}}^{(h=1)}$ obtained from type-II correspondence and f_V^{SLF} , give the identical results, they have different quark DAs, as we discussed in the previous section, Sec. IV. Since $f_{\text{full}}^{(h=0)}$ and $f_{\text{full}}^{(h=1)}$ involve the corresponding zero-mode contributions, they impose the intrinsic characteristic of the zero modes; i.e., they are antisymmetric under $x \leftrightarrow (1-x)$, inherited from the vacuum property. Thus, the quark DAs from $f_{\text{full}}^{(h=0)}$ and $f_{\text{full}}^{(h=1)}$ do not satisfy the expected constraint, i.e. symmetric DAs, even for $m_1 = m_2$. However, f_V^{SLF} and $f_{\text{on}}^{(h=1)}$ free from the explicit zero-mode contribution must yield the symmetric DAs for the equal quark and antiquark bound-state mesons such as ρ .

TABLE II. Decay constants (in MeV) of vector mesons obtained from $f_{\text{full}}^{(h=0)}$, $f_{\text{on}}^{(h=1)}$, and $f_{\text{full}}^{(h=1)}$ in the manifestly covariant model, but with type-I [type-II] replacement compared with f_V^{SLF} in our LFQM [10–12] and the experimental data [31].

		$f_{\text{full}}^{(h=0)}$	$f_{\text{on}}^{(h=1)}$	$f_{\text{full}}^{(h=1)}$	f_V^{SLF}	f^{exp}
ρ	I	256	299	299	215	220 (2) ^a , 209 (4) ^b
	II	215	215	215		
K^*	I	272	320	320	223	217 (5)
	II	223	223	223		
D^*	I	240	277	277	212	...
	II	212	212	212		
J/ψ	I	478	545	545	395	416 (6)
	II	395	395	395		

^aExperimental value for $\Gamma(\rho^0 \rightarrow e^+e^-)$.

^bExperimental value for $\Gamma(\tau \rightarrow \rho\nu_\tau)$.

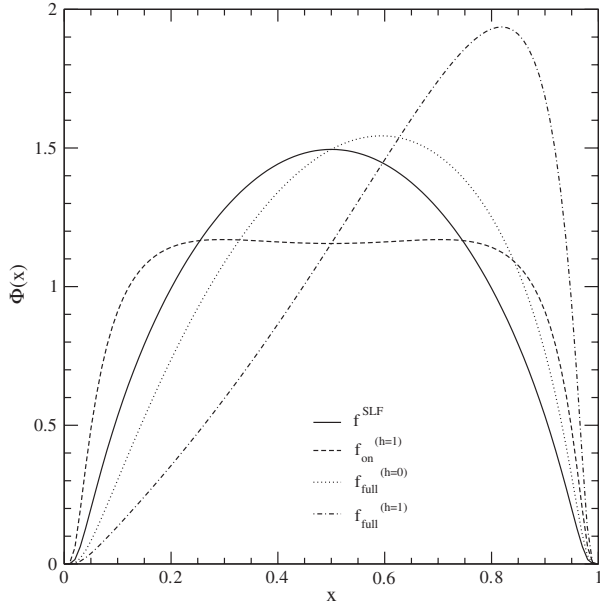


FIG. 2. Normalized quark DAs $\Phi(x)$ of a ρ meson obtained from the decay constants f^{SLF} (solid line), $f_{\text{on}}^{(h=1)}$ (dashed line), $f_{\text{full}}^{(h=0)}$ (dotted line), and $f_{\text{full}}^{(h=1)}$ (dot-dashed line). Only two of them, i.e. the results from f^{SLF} (solid line) and $f_{\text{on}}^{(h=1)}$ (dashed line), show the correct symmetric DAs.

In Fig. 2, we show the normalized quark DAs of a ρ meson obtained from the decay constants f^{SLF} (solid line), $f_{\text{on}}^{(h=1)}$ (dashed line), $f_{\text{full}}^{(h=0)}$ (dotted line), and $f_{\text{full}}^{(h=1)}$ (dot-dashed line). As one can see from Fig. 2, while the two results obtained from f^{SLF} and $f_{\text{on}}^{(h=1)}$ produce the anticipated symmetric DAs, the other two results obtained from

$f_{\text{full}}^{(h=0)}$ and $f_{\text{full}}^{(h=1)}$ show asymmetric DAs, reflecting the corresponding zero-mode contributions.

From the analysis of the quark DAs for the ρ meson, we further pin down the correct covariant descriptions of vector meson decay constants. That is, we find that $f_{\text{on}}^{(h=1)}$ with type-II replacement (but not with type-I replacement) and f^{SLF} provide self-consistent LF covariant descriptions of vector meson decay constants in the standard LFQM. Since $f^{\text{SLF}} [f_{\text{on}}^{(h=1)}]$ is obtained from using (J_W^+, ϵ_0) [$(J_W^\perp, \epsilon_+)$], the normalized quark DAs obtained from f^{SLF} and $f_{\text{on}}^{(h=1)}$ correspond to the twist-2 DA $\phi_{2;V}^\parallel(x)$ and twist-3 DA $\phi_{3;V}^\perp(x)$ given by Eq. (52), respectively.

In Fig. 3, we show the twist-2 DAs $\phi_{2;V}^\parallel(x)$ (solid line) and twist-3 DAs $\phi_{3;V}^\perp(x)$ (dashed line) of the ρ meson in the cases of finite constituent quark masses (left panel) and a massless quark respecting chiral symmetry (right panel). We should note that our LFQM predictions of twist-2 and twist-3 DAs in the chiral symmetry ($m_q \rightarrow 0$) limit remarkably reproduce the exact asymptotic DAs, i.e. $[\phi_{2;V}^\parallel(x)]_{\text{as}} = 6x(1-x)$ and $[\phi_{3;V}^\perp(x)]_{\text{as}} = (3/4)(1+\xi^2)$, where $\xi = 2x-1$ as anticipated from QCD sum rule predictions [22]. This example may show again that the LFQM prediction satisfies the chiral symmetry consistent with the QCD if one correctly implements the zero-mode link to the QCD vacuum. The quark mass correction is not large for the twist-2 $\phi_{2;V}^\parallel(x)$; however, it is very significant for the case of twist-3 $\phi_{3;V}^\perp(x)$, especially at the end-point regions. In Fig. 4, we show $\phi_{2;V}^\parallel(x)$ and $\phi_{3;V}^\perp(x)$ for K^* (left panel)

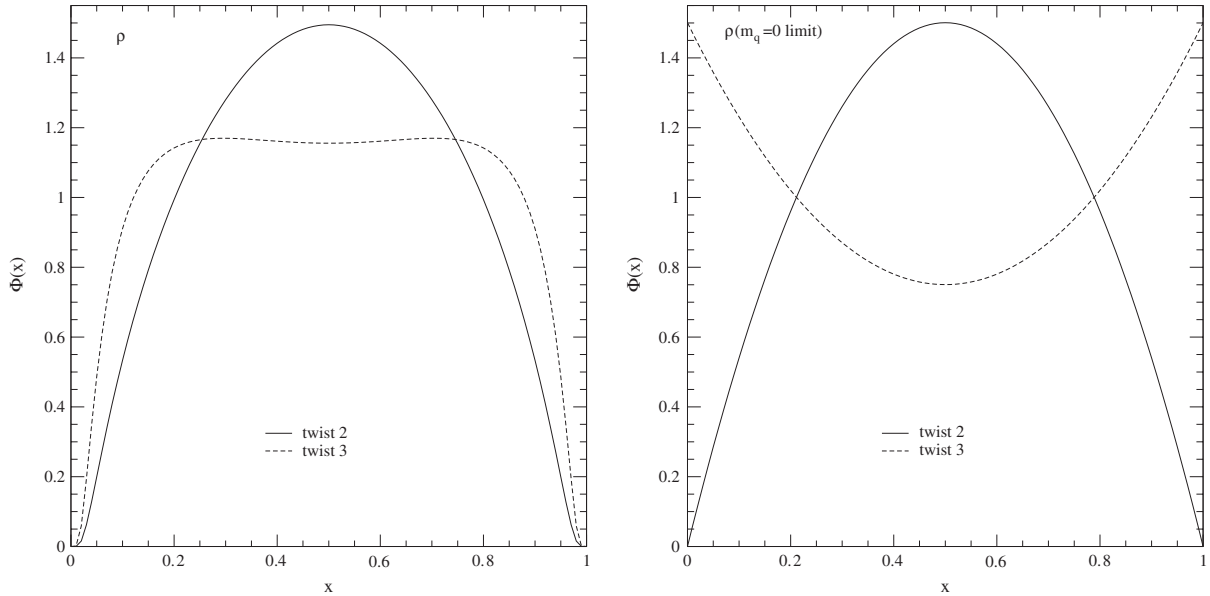


FIG. 3. The twist-2 DAs $\phi_{2;V}^\parallel(x)$ and twist-3 DAs $\phi_{3;V}^\perp(x)$ for a ρ meson with the nonzero constituent quark masses given in Table II (left panel) compared to those with the massless quark case (right panel).

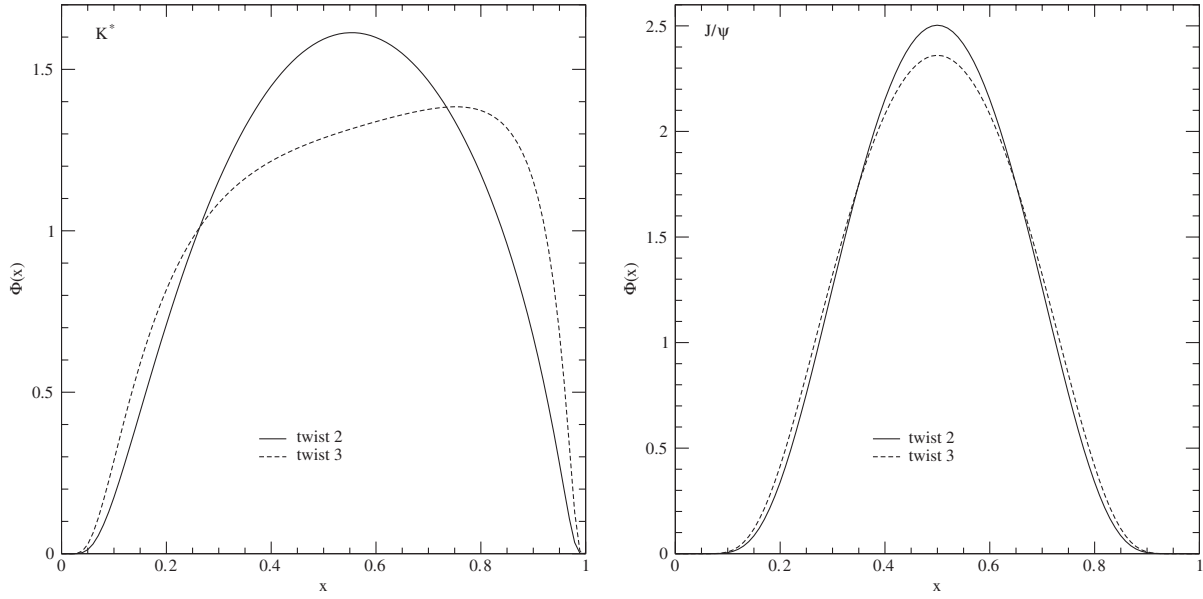


FIG. 4. The twist-2 DAs $\phi_{2;V}^{\parallel}(x)$ and twist-3 DAs $\phi_{3;V}^{\perp}(x)$ for K^* (left panel) and for J/ψ (right panel) mesons.

and J/ψ (right panel) mesons. For K^* mesons, we assign the momentum fractions x for the heavy s quark and $(1-x)$ for the light $u(d)$ quark. One can see from the DAs of ρ and K^* that the SU(3) breaking effect is more pronounced for $\phi_{3;V}^{\perp}(x)$ than for $\phi_{2;V}^{\parallel}(x)$. Comparing DAs of ρ and J/ψ , we also find that the differences between $\phi_{2;V}^{\parallel}(x)$ and $\phi_{3;V}^{\perp}(x)$ are significantly reduced for heavy quarkonium system and $\phi_{2;V}^{\parallel}(x) \approx \phi_{3;V}^{\perp}(x)$ as the constituent quark mass $m_q \rightarrow \infty$.

For comparison purposes, we compute the expectation value of the longitudinal momentum, the so-called ξ moments:

$$\langle \xi^n \rangle = \int_0^1 dx \xi^n \Phi(x), \quad (53)$$

where $\Phi = (\phi_{2;V}^{\parallel}, \phi_{3;V}^{\perp})$. In Table III, we summarize the first few ξ moments of twist-2 (T2) and twist-3 (T3) DAs

TABLE III. The ξ moments of twist-2 (T2) and twist-3 (T3) DAs for $(\rho, K^*, D^*, J/\psi)$ mesons. The numbers in parentheses for the ρ meson give the asymptotic values.

		$\langle \xi^1 \rangle$	$\langle \xi^2 \rangle$	$\langle \xi^3 \rangle$	$\langle \xi^4 \rangle$
ρ	T2	0	0.193 (0.200)	0	0.078
		(0.086)	T3	0	0.254
		(0.400)	0	0.120 (0.257)	
K^*	T2	0.085	0.177	0.040	0.067
	T3	0.125	0.227	0.067	0.101
D^*	T2	0.410	0.258	0.170	0.123
	T3	0.484	0.332	0.239	0.185
J/ψ	T2	0	0.083	0	0.016
	T3	0	0.090	0	0.019

for $(\rho, K^*, D^*, J/\psi)$ mesons obtained from our extracted DAs given by Eq. (52). The numbers in parentheses for the ρ meson give the asymptotic values.

VI. SUMMARY

In this work, we extended our previous analysis [17] of the vector meson decay constant from the exactly solvable, manifestly covariant BS model to the more phenomenologically accessible, realistic LFQM [3,10–12,18–20]. We discussed a self-consistent covariant description of the vector meson decay constant in view of the link between the chiral symmetry of QCD and the expected numerical results of the LFQM. As the zero-mode contribution is locked into a single point of the LF longitudinal momentum in the meson decay process, one of the constituents of the meson carries the entire momentum of the meson, and it is important to capture the effect from a pair creation of particles with zero LF longitudinal momenta from the strongly interacting vacuum. The LFQM with effective degrees of freedom represented by the constituent quark and antiquark may thus provide the view of an effective zero-mode cloud around the quark and antiquark inside the meson. Consequently, the constituents dressed by the zero-mode cloud may be expected to satisfy the chiral symmetry of QCD. Our numerical results were consistent with this expectation and effectively indicated that the constituent quark and antiquark in the standard LFQM [3,10–12,18–20] could be considered as the dressed constituents including the zero-mode quantum fluctuations from the QCD vacuum.

As the SLF approach within the LFQM by itself is not amenable to determining the zero-mode contribution, we

utilized the manifestly covariant model to check the existence (or absence) of the zero mode. Performing a LF calculation in the covariant BS model, we computed the decay constants using two different combinations of LF weak currents J_W^μ and polarization vectors ϵ_h , i.e. $f_V^{(h=0)}$ obtained from $(J_W^\mu, \epsilon_h) = (J_W^+, \epsilon_0)$ and $f_V^{(h=1)}$ from (J_W^μ, ϵ_+) , and checked the LF covariance of the decay constants. We found in the manifestly covariant model that both combinations gave the same result with some particular LF vertex functions if the missing zero-mode contributions were properly taken into account.

We then substituted the radial and spin-orbit wave functions with the phenomenologically accessible model wave functions provided by the LFQM and compared $f_V^{(h=0,1)}$ obtained from the BS model with the decay constant f_V^{SLF} obtained directly from the SLF approach used in the LFQM [3,10–12,18–20]. Linking the covariant BS model to the standard LFQM, we found the matching condition (i.e. type-II correspondence) between the two to give a self-consistent covariant description of the decay constant within the LFQM. Using the type-II correspondence, we were able to pin down two independent covariant forms of vector meson decay constants, one obtained from (J_W^+, ϵ_0) and the other from (J_W^μ, ϵ_+) . Although both of them yield the identical decay constant, each of them corresponds to a different twist DA: (J_W^+, ϵ_0) and (J_W^μ, ϵ_+) correspond to twist-2 and twist-3 two-particle DAs, respectively. Our twist-2 and twist-3 DAs not only satisfy the fundamental constraint of the DAs anticipated from the isospin symmetry, i.e. symmetric DAs for the equal quark and antiquark bound-state mesons (e.g. the ρ meson), but also reproduce the correct asymptotic DAs in the chiral symmetry limit. Further analysis including the chirality-odd and higher-twist DAs is under consideration.

APPENDIX A: SPIN STRUCTURE IN THE STANDARD LIGHT-FRONT QUARK MODEL

It is instructive to use the appropriate basis of Dirac spinors:

$$u_\lambda(p) = \frac{1}{\sqrt{p^+}}(\not{p} + m)u_\lambda, \quad v_\lambda(p) = \frac{1}{\sqrt{p^+}}(\not{p} - m)v_\lambda, \quad (\text{A1})$$

where

$$u_{\frac{1}{2}} = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \quad u_{-\frac{1}{2}} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}, \quad (\text{A2})$$

and $v_\lambda = u_{-\lambda}$. In this basis, the γ matrices are represented by

$$\gamma^0 = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}, \quad \gamma^i = \begin{pmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{pmatrix}, \quad (\text{A3})$$

where I is the 2×2 unit matrix and σ^i are Pauli matrices. The normalization is $\bar{u}_\lambda(p)u_\lambda(p) = -\bar{v}_\lambda(p)v_\lambda(p) = 2m$.

We then obtain the spin-orbit wave functions of pseudoscalar and vector mesons in the following matrix forms:

$$\mathcal{R}_{\lambda_1\lambda_2}^{00} = \frac{\mathcal{R}_0}{\sqrt{2}} \begin{pmatrix} -k^L & \mathcal{A} \\ -\mathcal{A} & -k^R \end{pmatrix} \quad (\text{A4})$$

and

$$\begin{aligned} \mathcal{R}_{\lambda_1\lambda_2}^{11} &= \mathcal{R}_0 \begin{pmatrix} \mathcal{A} + \frac{\mathbf{k}_\perp^2}{D_{\text{LF}}} & k^R \frac{xM_0+m_1}{D_{\text{LF}}} \\ -k^R \frac{(1-x)M_0+m_2}{D_{\text{LF}}} & -\frac{(k^R)^2}{D_{\text{LF}}} \end{pmatrix}, \\ \mathcal{R}_{\lambda_1\lambda_2}^{10} &= \frac{\mathcal{R}_0}{\sqrt{2}} \begin{pmatrix} k^L \frac{\mathcal{M}}{D_{\text{LF}}} & \mathcal{A} + \frac{2\mathbf{k}_\perp^2}{D_{\text{LF}}} \\ \mathcal{A} + \frac{2\mathbf{k}_\perp^2}{D_{\text{LF}}} & -k^R \frac{\mathcal{M}}{D_{\text{LF}}} \end{pmatrix}, \\ \mathcal{R}_{\lambda_1\lambda_2}^{1-1} &= \mathcal{R}_0 \begin{pmatrix} -\frac{(k^L)^2}{D_{\text{LF}}} & k^L \frac{(1-x)M_0+m_2}{D_{\text{LF}}} \\ -k^L \frac{xM_0+m_1}{D_{\text{LF}}} & \mathcal{A} + \frac{\mathbf{k}_\perp^2}{D_{\text{LF}}} \end{pmatrix}, \end{aligned} \quad (\text{A5})$$

where $k^{R(L)} = k^x \pm ik^y$, $\mathcal{R}_0 = \frac{1}{\sqrt{A^2 + \mathbf{k}_\perp^2}}$, and $\mathcal{M} = (1-2x)M_0 + m_2 - m_1$.

APPENDIX B: PSEUDOSCALAR MESON DECAY CONSTANT

The decay constant f_{P_s} of a pseudoscalar meson with the four-momentum P and mass M as a $q\bar{q}$ bound state is defined by the matrix element of the axial vector current

$$\langle 0 | \bar{q} \gamma^\mu \gamma_5 q | P_s(P) \rangle = i f_{P_s} P^\mu. \quad (\text{B1})$$

The matrix element $B^\mu \equiv \langle 0 | \bar{q} \gamma^\mu \gamma_5 q | P_s(P) \rangle$ is given in the one-loop approximation as a momentum integral

$$B^\mu = N_c \int \frac{d^4 k}{(2\pi)^4} \frac{H_V}{N_p N_k} S_{P_s}^\mu, \quad (\text{B2})$$

where

$$S_{P_s}^\mu = \text{Tr}[\gamma^\mu \gamma_5 (\not{p} + m_1) \gamma_5 (-\not{k} + m_2)], \quad (\text{B3})$$

which can be separated into the on-mass-shell propagating part $[S_{P_s}^\mu]_{\text{on}}$ and the off-mass-shell instantaneous part $[S_{P_s}^\mu]_{\text{inst}}$ as

$$[S_{P_s}^\mu]_{\text{on}} = \text{Tr}[\gamma^\mu \gamma_5 (\not{p}_{\text{on}} + m_1) \gamma_5 (-\not{k}_{\text{on}} + m_2)] \quad (\text{B4})$$

and

$$[S_{P_s}^\mu]^\text{inst} = -\frac{1}{2}(\Delta_k^-)\text{Tr}[\gamma^\mu(-\not{p}_\text{on} + m_1)\gamma^+] - \frac{1}{2}(\Delta_p^-)\text{Tr}[\gamma^\mu\gamma^+(-\not{k}_\text{on} + m_2)]. \quad (\text{B5})$$

For the pseudoscalar meson decay constant, we find that while the plus component of the currents is immune to the zero-mode contribution, the minus component of the current receives the zero mode.

Explicitly, the pseudoscalar decay constant $[f_{P_s}]_\text{full} = [f_{P_s}]_\text{val}$ obtained from $\mu = +$ is

$$[f_{P_s}]_\text{full} = \frac{N_c}{4\pi^3} \int_0^1 \frac{dx}{(1-x)} \int d^2\mathbf{k}_\perp \chi(x, \mathbf{k}_\perp) \mathcal{A}. \quad (\text{B6})$$

For $\mu = -$, while the valence contribution (i.e. $k^- = k_\text{on}^-$) to the trace term is given by

$$[S_{P_s}^-]^\text{val} = [S_{P_s}^-]^\text{on} + [S_{P_s}^-]^\text{inst} = 4 \left[m_2 M^2 + (m_1 - m_2) \frac{\mathbf{k}_\perp^2 + m_2^2}{1-x} \right], \quad (\text{B7})$$

we find the singular term in $S_{P_s}^-$ in the limit of $x \rightarrow 0$ when $p^- = p_\text{on}^-$ as

$$\lim_{x \rightarrow 0} S_{P_s}^-(p^- = p_\text{on}^-) = 4(m_2 - m_1)p^-. \quad (\text{B8})$$

Thus, the corresponding zero-mode operator is given by

$$[S_{P_s}^-]^\text{Z.M.} = 4(m_2 - m_1)(-Z_2). \quad (\text{B9})$$

Adding $[S_{P_s}^-]^\text{val}$ and $[S_{P_s}^-]^\text{Z.M.}$, we obtain

$$[S_{P_s}^-]^\text{tot} = [S_{P_s}^-]^\text{val} + [S_{P_s}^-]^\text{Z.M.} = 4M^2 \mathcal{A}, \quad (\text{B10})$$

and the full solution obtained from $\mu = -$ is shown to be exactly equal to the one given by Eq. (B6).

On the other hand, the SLF result of a pseudoscalar meson decay constant is given by

$$f_{P_s}^\text{SLF} = \frac{\sqrt{2N_c}}{8\pi^3} \int_0^1 dx \int d^2\mathbf{k}_\perp \frac{\phi(x, \mathbf{k}_\perp)}{\sqrt{\mathbf{k}_\perp^2 + \mathcal{A}^2}} \mathcal{A}. \quad (\text{B11})$$

Comparing Eqs. (B6) and (B11), we find that the vertex function χ in Eq. (B6) may be replaced with the Gaussian wave function ϕ in Eq. (B11) via

$$\sqrt{2N_c} \frac{\chi(x, \mathbf{k}_\perp)}{1-x} = \frac{\phi(x, \mathbf{k}_\perp)}{\sqrt{\mathcal{A}^2 + \mathbf{k}_\perp^2}}.$$

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- [1] A. Bazavov *et al.*, *Phys. Rev. D* **85**, 114506 (2012), and references therein.
 - [2] A. Khodjamirian, *Phys. Rev. D* **79**, 031503 (2009), and references therein.
 - [3] W. Jaus, *Phys. Rev. D* **41**, 3394 (1990); **44**, 2851 (1991).
 - [4] W. Jaus, *Phys. Rev. D* **60**, 054026 (1999).
 - [5] W. Jaus, *Phys. Rev. D* **67**, 094010 (2003).
 - [6] H.-Y. Cheng, C.-K. Chua, and C.-W. Hwang, *Phys. Rev. D* **69**, 074025 (2004).
 - [7] B. L. G. Bakker, H.-M. Choi, and C.-R. Ji, *Phys. Rev. D* **65**, 116001 (2002); **67**, 113007 (2003).
 - [8] H.-M. Choi and C.-R. Ji, *Nucl. Phys. A* **856**, 95 (2011); *Phys. Lett. B* **696**, 518 (2011).
 - [9] J. P. B. C. de Melo and T. Frederico, *Phys. Lett. B* **708**, 87 (2012).
 - [10] H.-M. Choi and C.-R. Ji, *Phys. Rev. D* **59**, 074015 (1999).
 - [11] H.-M. Choi and C.-R. Ji, *Phys. Rev. D* **75**, 034019 (2007).
 - [12] H.-M. Choi, *Phys. Rev. D* **75**, 073016 (2007).
 - [13] S. J. Brodsky, H.-C. Pauli, and S. Pinsky, *Phys. Rep.* **301**, 299 (1998).
 - [14] H.-M. Choi and C.-R. Ji, *Phys. Rev. D* **80**, 054016 (2009).
 - [15] H.-M. Choi and C.-R. Ji, *Phys. Rev. D* **72**, 013004 (2005).
 - [16] M. Burkardt, *Phys. Rev. D* **47**, 4628(1993); S. J. Brodsky and D. S. Hwang, *Nucl. Phys. B* **543**, 239 (1999); J. P. B. C. de Melo, J. H. O. Sales, T. Frederico, and P. U. Sauer, *Nucl. Phys. A* **631**, 574 (1998); H.-M. Choi and C.-R. Ji, *Phys. Rev. D* **58**, 071901 (1998).
 - [17] H.-M. Choi and C.-R. Ji, *Few-Body Syst.* **54**, 1633 (2013).
 - [18] H.-Y. Cheng, C.-Y. Cheung, and C.-W. Hwang, *Phys. Rev. D* **55**, 1559 (1997).
 - [19] C.-W. Hwang, *Phys. Rev. D* **81**, 114024 (2010).
 - [20] L. A. Kondratyuk and D. V. Tchekin, *Phys. At. Nucl.* **64**, 727 (2001).
 - [21] P. Ball and V. M. Braun, *Phys. Rev. D* **54**, 2182 (1996).
 - [22] P. Ball, V. M. Braun, Y. Koike, and K. Tanaka, *Nucl. Phys. B* **529**, 323 (1998).

- [23] C.-R. Ji and B. L. G. Bakker, *Int. J. Mod. Phys. E* **22**, 1330002 (2013).
- [24] R. L. Jaffe and X. Ji, *Nucl. Phys.* **B375**, 527 (1992).
- [25] P. Ball and G. W. Jones, *J. High Energy Phys.* **03** (2007) 069.
- [26] J. R. Forshaw and R. Sandapen, *J. High Energy Phys.* **10** (2011) 093.
- [27] B. L. G. Bakker, M. A. DeWitt, C.-R. Ji, and Y. Mikhchenko, *Phys. Rev. D* **72**, 076005 (2005).
- [28] C.-R. Ji, W. Melnitchouk, and A. W. Thomas, *Phys. Rev. Lett.* **110**, 179101 (2013).
- [29] H. J. Melosh, *Phys. Rev. D* **9**, 1095(1974); P. L. Chung, F. Coester, B. D. Keister, and W. N. Polyzou, *Phys. Rev. C* **37**, 2000 (1988).
- [30] G. P. Lepage and S. J. Brodsky, *Phys. Rev. D* **22**, 2157 (1980).
- [31] J. Beringer *et al.* (Particle Data Group), *Phys. Rev. D* **86**, 010001 (2012)