

Strange magnetic form factor of the nucleon in a chiral effective model at next to leading order

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(Received 12 December 2013; published 18 February 2014)

The strange magnetic form factor of the nucleon is studied in a chiral effective model motivated by the heavy baryon formalism at next-to-leading order. The one-loop contributions from kaon and intermediate octet and decuplet hyperons are included, using finite-range regularization to deal with the ultraviolet divergences. Drawing on an established connection between quenched and full QCD, this model makes it possible to predict the strange magnetic form factor under the hypothesis that for a dipole regulator mass Λ around 0.8 GeV, strangeness in the core is negligible. The strange magnetic form factor is found to be small and negative over a range of momentum transfer, while the strange magnetic moment is consistent with the best lattice QCD determinations.

DOI: [10.1103/PhysRevD.89.033008](https://doi.org/10.1103/PhysRevD.89.033008)

PACS numbers: 13.40.-f, 14.20.-c, 12.39.Fe, 11.10.Gh

I. INTRODUCTION

While our understanding of quantum chromodynamics (QCD) in the perturbative sector is excellent, in the non-perturbative sector we have much to learn. Certainly, lattice QCD has achieved a great deal of success, and the masses of the ground-state hadrons are under control [1,2]. The detailed study of excited states is beginning in earnest [3,4], and there has been some success in the calculation of elastic form factors [5–10] and even transition form factors [11,12]. However, with very few exceptions, the form factor studies, which complement the recent experimental progress at facilities such as Jefferson Lab, deal with so-called “connected contributions” in which the external current acts on a quark line running directly from the hadronic source to the sink. As discussed below, only a very few studies have directly addressed the “disconnected contributions.”

Perhaps the most famous example of a disconnected contribution is the strange quark contribution to the nucleon elastic form factors [13,14]. Its fundamental importance is associated with the fact that it is directly analogous to the vacuum polarization contribution to the Lamb shift, the correct calculation of which confirmed the validity of quantum electrodynamics. If QCD is to be confirmed unambiguously as the complete theory of QCD and our capacity to use it judged reliable, it is crucial that it successfully reproduce the measured strange quark form factors.

The discovery of the EMC spin crisis [15] in the late 1980s stimulated enormous interest in the strange quark contribution to the nucleon spin, Δs (essentially the matrix

element of $\bar{s}\gamma^\mu\gamma_5 s$ in the proton), because of speculation that the explanation might lie in an unexpectedly large strange quark spin contribution. Recent lattice QCD studies suggest that Δs is actually rather small [16], consistent with a recent model study suggesting the $SU(3)$ is likely broken at the 20% level for the axial charge g_A^8 of the nucleon [17]. Instead, it appears that the resolution of the spin crisis lies in the exchange of quark spin for orbital angular momentum of the quarks (and antiquarks) [18–20]. Of course, this leaves as a fascinating challenge the detailed partition of the nucleon spin amongst the quarks and gluons.

Although the spin crisis has faded as a motivation for detecting the role strange quarks play in the nucleon, its fundamental role as a totally disconnected contribution means that it provides a vital role to test our understanding of QCD [14]. An extremely impressive program of measurements of parity-violating electron scattering (PVES) at MIT-Bates [21,22], JLab [23–27], and Mainz [28–30] has provided the essential third constraint so that, in combination with the electric or magnetic form factors of the proton and neutron and the assumption of charge symmetry, a careful global analysis [31] has allowed a determination of the strange quark magnetic moment and charge radius. For details of the experimental programs and some related theoretical work, we refer to Ref. [32]. It is a remarkable confirmation of our growing capacity to compute with nonperturbative QCD that the results agree well with the most recent determinations from lattice QCD [33–35].

In the future, we can expect that both lattice QCD and experiment will achieve greater precision at nonzero

momentum transfer, and so map out the Q^2 dependence of the strange form factors. At the same time, in order to develop a physical understanding of these results, it will be helpful to have calculations within a variety of models, informed as far as possible by the lattice QCD results. In the past, a variety of theoretical models have been applied to the calculation of the strange nucleon form factors [14,36–46]. In 2003, Lewis *et al.* [47] used lowest-order, quenched chiral perturbation theory, together with lattice QCD simulations to calculate the strange form factors. The magnetic form factor which they obtained at $Q^2 = 0.1 \text{ GeV}^2$ was $+0.05 \pm 0.06$. Recently, by combining the constraints of charge symmetry [48,49] with new chiral extrapolation techniques and low-mass, quenched lattice QCD simulations of the individual quark contributions to the magnetic moment and form factor of the nucleon, precise nonzero values of the strange magnetic moment and form factor were obtained [33,50].

Chiral perturbation theory (χ PT) is also a powerful tool with which to study hadron properties at low energy. There has been some work on strange form factors with heavy baryon chiral perturbation theory [51,52]. However, there is an unknown low-energy constant appearing in the chiral Lagrangian which has limited the capacity to calculate the strange magnetic form factor. In other words, the quantity one wishes to predict—the strangeness vector current matrix element—is the same quantity one needs to know in order to make a prediction [53,54]. While this is the case in conventional χ PT, experience with finite-range regularization (FRR) has shown that by varying the regulator parameter, one can shift strength from the loop contributions into the core. This suggests that within FRR χ -EFT, one might identify the core contribution with the tree-level contribution and make the approximation that, for Λ around 0.8 GeV, the strangeness content of the core is negligible. In this way, full QCD results have been obtained rather successfully from quenched lattice data [33,34,50,55,56]. We should emphasize that unquenching only works for Λ around 0.8 GeV. Only then does one define a core contribution that is approximately invariant between quenched and full QCD.

In earlier work, the strange magnetic moment and form factor have been studied at leading order [33,50]. Here we extend the study to next-to-leading order in order to investigate the role of high-order terms. It is also important to see to what extent FRR suppresses the high-order contribution to the strange magnetic form factor. For the strange magnetic form factor with regulator $\Lambda = 0.8 \text{ GeV}$, the low-energy constant related to the three-quark core contribution is approximately zero. Therefore, we can make a prediction for the strange magnetic form factor with FRR without any parameter.

This work is a model calculation motivated by chiral perturbation theory. Compared with the usual chiral perturbation theory, our calculation with FRR is at a physically

motivated scale, where the residual series of analytic terms describe the three-quark core contributions. With the introduction of the dipole regulator, analytic behavior may also differ from chiral perturbation theory, to the extent that it is known. The paper is organized as follows: In Sec. II, we briefly introduce the heavy baryon chiral Lagrangian which will be used in the calculation. The strange magnetic form factor is presented in Sec. III. Numerical results are shown in Sec. IV. Finally, Sec. V presents a summary of our results.

II. CHIRAL LAGRANGIAN

There are many papers which deal with heavy baryon chiral perturbation theory—for details, see, for example, Refs. [57–59]. For completeness, we briefly introduce the formalism in this section. In heavy baryon chiral perturbation theory, the lowest chiral Lagrangian for the baryon-meson interaction which will be used in the calculation of the nucleon magnetic moments, including the octet and decuplet baryons, is expressed as

$$\begin{aligned} \mathcal{L}_v = & i\text{Tr}\bar{B}_v(v \cdot \mathcal{D})B_v + 2D\text{Tr}\bar{B}_v S_v^\mu \{A_\mu, B_v\} \\ & + 2F\text{Tr}\bar{B}_v S_v^\mu [A_\mu, B_v] - i\bar{T}_v^\mu (v \cdot \mathcal{D})T_{v\mu} \\ & + \mathcal{C}(\bar{T}_v^\mu A_\mu B_v + \bar{B}_v A_\mu T_v^\mu), \end{aligned} \quad (1)$$

where S_μ is the covariant spin operator defined as

$$S_v^\mu = \frac{i}{2}\gamma^5 \sigma^{\mu\nu} v_\nu. \quad (2)$$

Here, v^ν is the nucleon four-velocity [in the rest frame, we have $v^\nu = (1, 0)$]. D , F , and $\mathcal{C}$ are the usual coupling constants. The chiral covariant derivative, D_μ , is written as $D_\mu B_v = \partial_\mu B_v + [V_\mu, B_v]$. The pseudoscalar meson octet couples to the baryon field through the vector and axial vector combinations

$$V_\mu = \frac{1}{2}(\zeta \partial_\mu \zeta^\dagger + \zeta^\dagger \partial_\mu \zeta), \quad A_\mu = \frac{1}{2}(\zeta \partial_\mu \zeta^\dagger - \zeta^\dagger \partial_\mu \zeta), \quad (3)$$

where

$$\zeta = e^{i\phi/f}, \quad f = 93 \text{ MeV}. \quad (4)$$

The matrix of pseudoscalar fields ϕ is expressed as

$$\phi = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & K^0 \\ K^- & \bar{K}^0 & -\frac{2}{\sqrt{6}}\eta \end{pmatrix}. \quad (5)$$

B_v and T_v^μ are the new velocity-dependent fields which are related to the original baryon octet and decuplet fields B and T^μ by

$$B_v(x) = e^{im_N \not{v} x^\mu} B(x), \quad (6)$$

$$T_v^\mu(x) = e^{im_N \not{v} x^\mu} T^\mu(x). \quad (7)$$

In the chiral $SU(3)$ limit, the octet baryons will have the same mass, m_B . In our calculation, we use the physical masses for the members of the baryon octet and decuplet. The explicit form of the baryon octet is written as

$$B = \begin{pmatrix} \frac{1}{\sqrt{2}}\Sigma^0 + \frac{1}{\sqrt{6}}\Lambda & \Sigma^+ & p \\ \Sigma^- & -\frac{1}{\sqrt{2}}\Sigma^0 + \frac{1}{\sqrt{6}}\Lambda & n \\ \Xi^- & \Xi^0 & -\frac{2}{\sqrt{6}}\Lambda \end{pmatrix}. \quad (8)$$

For the baryon decuplets, there are three indices, defined as

$$\begin{aligned} T_{111} &= \Delta^{++}, & T_{112} &= \frac{1}{\sqrt{3}}\Delta^+, & T_{122} &= \frac{1}{\sqrt{3}}\Delta^0, \\ T_{222} &= \Delta^-, & T_{113} &= \frac{1}{\sqrt{3}}\Sigma^{*,+}, & T_{123} &= \frac{1}{\sqrt{6}}\Sigma^{*,0}, \\ T_{223} &= \frac{1}{\sqrt{3}}\Sigma^{*,-}, & T_{133} &= \frac{1}{\sqrt{3}}\Xi^{*,0}, \\ T_{233} &= \frac{1}{\sqrt{3}}\Xi^{*,-}, & T_{333} &= \Omega^-. \end{aligned} \quad (9)$$

From the Lagrangian of electromagnetic moments, one can get the strange quark contribution. The octet, decuplet, and octet-decuplet transition magnetic moment operators are needed in the one-loop calculation of the nucleon strange form factor. The baryon octet magnetic Lagrangian is written as

$$\mathcal{L} = \frac{e}{4m_N} (\mu_D \text{Tr} \bar{B}_v \sigma^{\mu\nu} \{F_{\mu\nu}^+, B_v\} + \mu_F \text{Tr} \bar{B}_v \sigma^{\mu\nu} [F_{\mu\nu}^+, B_v]), \quad (10)$$

where

$$F_{\mu\nu}^+ = \frac{1}{2} (\zeta^\dagger F_{\mu\nu} Q \zeta + \zeta F_{\mu\nu} Q \zeta^\dagger). \quad (11)$$

Q is the charge matrix $Q = \text{diag}\{2/3, -1/3, -1/3\}$. At the lowest order, the Lagrangian will generate the following nucleon magnetic moments:

$$\mu_p = \frac{1}{3}\mu_D + \mu_F, \quad \mu_n = -\frac{2}{3}\mu_D. \quad (12)$$

The decuplet magnetic moment operator is expressed as

$$\mathcal{L} = -i \frac{e}{m_N} \mu_C q_{ijk} \bar{T}_{v,ikl}^\mu T_{v,jkl}^\nu F_{\mu\nu}, \quad (13)$$

where q_{ijk} and $q_{ijk}\mu_C$ are the charge and magnetic moment of the decuplet baryon T_{ijk} . The transition magnetic operator is

$$\mathcal{L} = i \frac{e}{2m_N} \mu_T F_{\mu\nu} (\epsilon_{ijk} Q_i^j \bar{B}_{vm}^\mu S_v^\nu T_{v,klm}^{\nu,klm} + \epsilon^{ijk} Q_i^l \bar{T}_{v,klm}^\mu S_v^\nu B_{vj}^m). \quad (14)$$

The electromagnetic moments of the baryon octet and decuplet as well as the transition moments can be written in terms of μ_u , μ_d , and μ_s instead of μ_D , μ_F , μ_C , and μ_T [60]. For the particular choice $\mu_s = \mu_d = -\frac{1}{2}\mu_u$, one finds the following relationship:

$$\begin{aligned} \mu_D &= \frac{3}{2}\mu_u, & \mu_F &= \frac{2}{3}\mu_D, \\ \mu_C &= \mu_D, & \mu_T &= -4\mu_D. \end{aligned} \quad (15)$$

Therefore, by comparing with the quark model, one can get the strange quark contribution to the magnetic moment of baryons at tree level as [60]

$$\begin{aligned} \mu_p^s &= \mu_n^s = 0, & \mu_{\Sigma^+}^s &= \mu_{\Sigma^-}^s = \mu_{\Sigma^0}^s = \mu_s = -\frac{1}{3}\mu_D, \\ \mu_\Lambda^s &= -3\mu_s = \mu_D. \end{aligned} \quad (16)$$

Similarly, the strange quark contribution to the decuplet and transition magnetic moments can be written as [61]

$$\mu_{\Sigma^{*,+}}^s = \mu_{\Sigma^{*,0}}^s = \mu_{\Sigma^{*,-}}^s = -3\mu_s = \mu_D, \quad (17)$$

$$\mu_{\Sigma^{*,+}\Sigma^+}^s = -\mu_{\Sigma^{*,0}\Sigma^0}^s = -\mu_{\Sigma^{*,-}\Sigma^-}^s = -2\sqrt{2}\mu_s = \frac{2\sqrt{2}}{3}\mu_D. \quad (18)$$

In the above Eqs. (16) through (18), following the usual convention, the charge of the strange quark is taken to be 1. Therefore, the standard μ_s has been multiplied by -3 .

In the heavy baryon formalism, the propagators of the octet or decuplet baryon, j , are expressed as

$$\frac{i}{v \cdot k - \delta^{jN} + i\epsilon} \quad \text{and} \quad \frac{iP^{\mu\nu}}{v \cdot k - \delta^{jN} + i\epsilon}, \quad (19)$$

where $P^{\mu\nu}$ is $v^\mu v^\nu - g^{\mu\nu} - (4/3)S_v^\mu S_v^\nu$. $\delta^{ab} = m_b - m_a$ is the mass difference between the two baryons. The propagator of the K meson is the usual free propagator, i.e.

$$\frac{i}{k^2 - M_K^2 + i\epsilon}. \quad (20)$$

III. STRANGE MAGNETIC FORM FACTOR

The one-loop Feynman diagrams which contribute to the strange magnetic form factor of the nucleon are plotted in Fig. 1. The contribution from Fig. 1(a) is expressed as

$$G_M^{s(1a)} = -\frac{m_N(D+3F)^2 I_{1K}^{N\Lambda} + 9m_N(D-F)^2 I_{1K}^{N\Sigma}}{48\pi^3 f_\pi^2}. \quad (21)$$

The integration $I_{1K}^{\alpha\beta}$ is expressed as

$$I_{1K}^{\alpha\beta} = \int d\vec{k} \frac{k_y^2 u(\vec{k} + \vec{q}/2) u(\vec{k} - \vec{q}/2) (\omega_K(\vec{k} + \vec{q}/2) + \omega_K(\vec{k} - \vec{q}/2) + \delta^{\alpha\beta})}{A_K^{\alpha\beta}}, \quad (22)$$

where

$$A_K^{\alpha\beta} = \omega_K(\vec{k} + \vec{q}/2) \omega_K(\vec{k} - \vec{q}/2) (\omega_K(\vec{k} + \vec{q}/2) + \delta^{\alpha\beta}) \times (\omega_K(\vec{k} - \vec{q}/2) + \delta^{\alpha\beta}) (\omega_K(\vec{k} + \vec{q}/2) + \omega_K(\vec{k} - \vec{q}/2)) \quad (23)$$

and $\omega_K(\vec{k}) = \sqrt{m_K^2 + \vec{k}^2}$ is the energy of the kaon. In our calculation, we use finite-range regularization, and $u(\vec{k})$ is the ultraviolet regulator. The first term in Eq. (21) comes from the charged K -meson cloud with an intermediate Λ hyperon. The second term includes both charged and neutral K -meson cloud contributions with an intermediate Σ hyperon. Fig. 1(b) is the same as Fig. 1(a), but the intermediate states are decuplet baryons. Its contribution to the strange magnetic form factor is expressed as

$$G_M^{s(1b)} = \frac{m_N \mathcal{C}^2}{48\pi^3 f_\pi^2} I_{1K}^{N\Sigma^*}. \quad (24)$$

The contribution to the form factor from Fig. 1(c) is expressed as

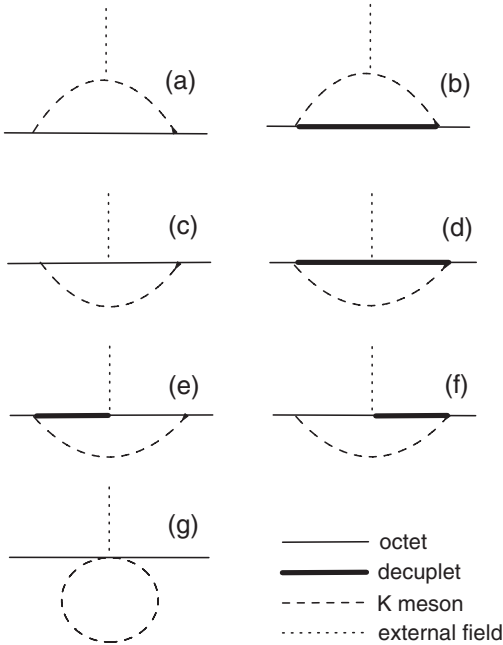


FIG. 1. The one-loop Feynman diagrams for the nucleon strange magnetic form factor. The solid, thick solid, dashed, and dotted lines are for the octet baryons, decuplet baryons, K mesons, and external fields, respectively.

$$G_M^{s(1c)} = -\frac{1}{192\pi^3 f_\pi^2} [3(D-F)^2 \mu_s I_{2K}^{N\Sigma} - (D+3F)^2 \mu_s I_{2K}^{N\Lambda}],$$

where

$$I_{2K}^{\alpha\beta} = \int d\vec{k} \frac{k^2 u(\vec{k})^2}{\omega_K(\vec{k}) (\omega_K(\vec{k}) + \delta^{\alpha\beta})^2}. \quad (25)$$

The strange quark contribution to the magnetic moment of the hyperons at tree level, expressed in terms of μ_s , is used in the one-loop calculation.

The contribution to the form factor of Fig. 1(d) is expressed as

$$G_M^{s(1d)} = -\frac{5\mathcal{C}^2 \mu_s}{288\pi^3 f_\pi^2} I_{2K}^{N\Sigma^*}. \quad (26)$$

Figures. 1(e) and 1(f) provide the following contribution to the nucleon strange magnetic form factor:

$$G_M^{s(1e+1f)} = -\frac{(D-F)\mathcal{C}\mu_s}{12\pi^3 f_\pi^2} I_{5K}^{N\Sigma\Sigma^*}, \quad (27)$$

where

$$I_{5K}^{\alpha\beta\gamma} = \int d\vec{k} \frac{k^2 u(\vec{k})^2}{\omega_K(\vec{k}) (\omega_K(\vec{k}) + \delta^{\alpha\beta}) (\omega_K(\vec{k}) + \delta^{\alpha\gamma})}. \quad (28)$$

In chiral effective theory, there exists a tadpole diagram [Fig. 1(g)] for the strange form factor. Its contribution to the strange form factor is expressed as

$$G_M^{s(1g)} = \frac{\mu_D}{32\pi^3 f_\pi^2} I_{4K}, \quad (29)$$

where

$$I_{4K} = \int d\vec{k} \frac{u(\vec{k})^2}{\omega_K(\vec{k})}. \quad (30)$$

The numerical contribution from Fig. 1(g) is $0.095\mu_D$, which leads to a positive value. However, in the previous study, we found that the tadpole contribution should not be included if we define the quenched low-energy constant in the case of FRR to be the three-quark core contribution [56]. As explained in Ref. [56], the tadpole contribution from the contact term corresponds to the contribution of Fig. 1(c) summed over an infinite set of highly excited baryon states, and phenomenologically this appears to be

TABLE I. Contributions to the strange magnetic moment of the nucleon G_M^s in units of μ_N , and the total strange magnetic moment.

Λ (GeV)	Fig. 1(a)	Fig. 1(b)	Fig. 1(c)	Fig. 1(d)	Figs. 1(e)+1(f)	G_M^s
0.6	-0.021	0.004	-0.008	0.005	-0.003	-0.024
0.7	-0.034	0.006	-0.014	0.008	-0.005	-0.039
0.8	-0.050	0.009	-0.021	0.013	-0.009	-0.058
0.9	-0.070	0.013	-0.031	0.019	-0.013	-0.082
1.0	-0.094	0.017	-0.043	0.027	-0.018	-0.111

appropriately incorporated through Eq. (44) in Ref. [56]. This equation shows that if we include the tadpole contribution, we have to determine the unknown low-energy constant a'_0 . Therefore, the total strange magnetic form factor of the nucleon can be written as

$$G_M^s(Q^2) = \sum_{i=a}^f G_M^{s(i)}(Q^2), \quad (31)$$

where with the regulator parameter $\Lambda = 0.8$ GeV, the three-quark core contribution to the strange form factor is zero.

IV. NUMERICAL RESULTS

In the numerical calculations, the parameters are chosen as $D = 0.76$ and $F = 0.50$ ($g_A = D + F = 1.26$). The coupling constant $\mathcal{C}$ is chosen to be -1.2 , which is the same as that used in Ref. [57]. The regulator form factor, $u(k)$, could be chosen to be a monopole, dipole, or Gaussian function, any of which would give similar results [62]. In our calculations, a dipole form is used:

$$u(k) = \frac{1}{(1 + k^2/\Lambda^2)^2}, \quad (32)$$

with $\Lambda = 0.8 \pm 0.2$ GeV. This choice has been widely applied in the extrapolation of lattice data for hadron mass, moments, form factors, radii, first moments of GPDs, etc. [34,62–67]. With this choice, it has been shown that reasonable physical results can be obtained from the quenched lattice data at both leading and next-to-leading order [33,34,50,55,56]. Λ around 0.8 GeV is the value required to identify a core contribution that is invariant between quenched and full QCD. This invariance of the core supports the assumption that the core contains no strangeness.

Our model calculation is motivated by chiral perturbation theory with the same chiral Lagrangian. As in the usual chiral perturbation theory, both the loop integrals and low-energy coefficients are scale dependent in a way which leads to the scale independence of the total contribution. Compared with the usual chiral perturbation theory, our calculation with FRR is at a physically motivated scale, where earlier work has suggested that the residual series of analytic terms best describe the three-quark core

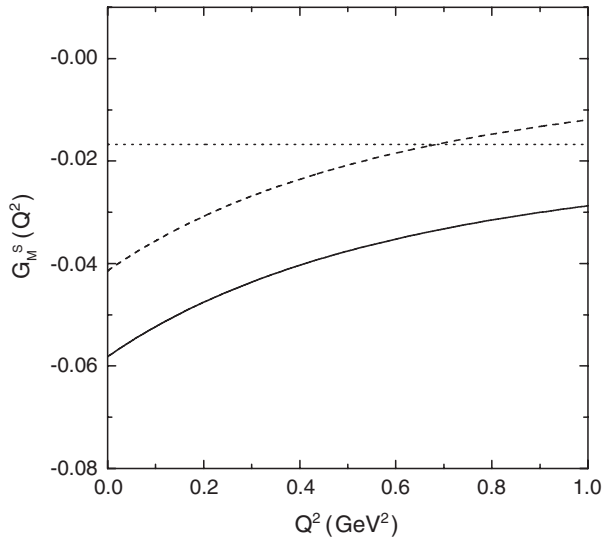


FIG. 2. The strange magnetic form factor versus Q^2 . The solid, dashed, and dotted lines are for total, leading-order, and next-to-leading-order results, respectively.

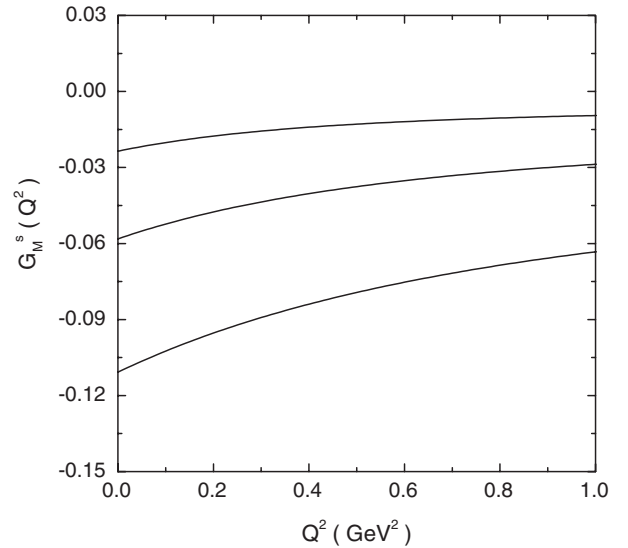


FIG. 3. The strange magnetic form factor versus Q^2 . The upper, middle, and lower lines are for $\Lambda = 0.6, 0.8$, and 1.0 GeV, respectively.

TABLE II. The strange magnetic form factor at different Q^2 . Uncertainties reflect the range of Λ considered herein.

Q^2 (GeV ²)	0	0.1	0.23	0.477	0.62
$G_M^s(Q^2)$	$-0.058^{+0.034}_{-0.053}$	$-0.052^{+0.031}_{-0.051}$	$-0.046^{+0.029}_{-0.048}$	$-0.038^{+0.024}_{-0.040}$	$-0.035^{+0.023}_{-0.040}$

contributions. From the previous extrapolation of quenched lattice data, it is found that this preferred value of Λ in the dipole regulator is around 0.8 GeV. It may not be a coincidence that this is just a little softer than the measured axial form factor of the nucleon [68]. The variation of Λ from 0.6 to 1 GeV provides an estimate of the degree of model dependence of our result. With the introduction of the dipole regulator, the analytic behavior is also different from the usual chiral perturbation theory. Though the leading and next-to-leading nonanalytic terms are the same in both cases, our results include high-order analytic and nonanalytic terms. The advantage of this formulation is that it leads to a good representation of lattice data, for a wide variety of observables, over a wide range of pion mass.

The contribution from each diagram in Fig. 1 is shown in Table I. The value of the loop contribution is small, mainly because the K -meson mass is large compared with the pion mass. The leading-order diagram [Fig. 1(a)] gives a negative contribution to the strange magnetic form factor. Figure 1(b), with intermediate decuplet hyperons, gives a positive contribution, but it is numerically 5–6 times smaller than that of Fig. 1(a). The sum of the contribution from these two diagrams is $-0.041\mu_N$ for $\Lambda = 0.8$ GeV.

The contributions for the next-to-leading-order diagrams, Figs. 1(c), 1(d), 1(e), and 1(f), are much smaller than the leading contribution. They depend on the parameter μ_s , which is the constituent strange quark contribution to the hyperon magnetic form factor. Assuming $SU(3)$ symmetry, one has $\mu_s = \mu_d = -\frac{1}{2}\mu_u = -\frac{1}{3}\mu_D$. In fact, this relation was applied in our previous investigation of nucleon magnetic form factors [56,64]. In the previous extrapolation of nucleon magnetic form factors, we found that μ_D equals $2.55\mu_N$ and $2.34\mu_N$ for full QCD and quenched extrapolation, respectively [56,64]. Therefore, $\mu_s = -0.8\mu_N$ should be a good estimate. Similar to the leading-order case, at next-to-leading order, the contributions from intermediate octet and decuplet hyperons [Figs. 1(c) and 1(d)] have the opposite sign. For $\Lambda = 0.8$ GeV, the total next-to-leading-order contribution, including transition diagrams [Figs. 1(e) and 1(f)], is $0.021\mu_s = -0.017\mu_N$.

At $Q^2 = 0$, the nucleon strange magnetic moment is $G_M^s(0) = -0.041\mu_N + 0.021\mu_s$. With the estimate $\mu_s = -0.8\mu_N$, the strange magnetic moment is $G_M^s(0) = -0.058\mu_N$. If we vary Λ from 0.6 GeV to 1 GeV, $G_M^s(0)$ will change from $-0.024\mu_N$ to $-0.111\mu_N$. Numerical results show that $G_M^s(0)$ is always negative with a large parameter range.

At $\Lambda = 0.8$ GeV, the strange magnetic form factor is shown in Fig. 2. The solid, dashed, and dotted lines are for

total, leading-order, and next-to-leading-order results, respectively. The Q^2 dependence of $G_M^s(Q^2)$ is determined by the leading-order contribution. The next-to-leading-order contribution is small and Q^2 independent, showing that FRR provides good convergence. Considering the $SU(3)$ symmetry breaking, the next-to-leading-order contribution is even smaller with the smaller μ_s .

In Fig. 3, we plot the strange magnetic form factor $G_M^s(Q^2)$ versus Q^2 at $\Lambda = 0.6, 0.8$, and 1.0 GeV. One can see that $G_M^s(Q^2)$ decreases in magnitude with increasing Q^2 . It is obvious that the strange magnetic form factor does not change sign for any of the choices of Λ when Q^2 increases.

With FRR, the strange form factor is totally determined by the loop contribution. The low-energy constants for the three-quark, tree-level contribution are zero. Therefore, we can calculate the strange magnetic form factor at relatively large Q^2 . At $Q^2 = 0$, the strange magnetic moment is $-0.058^{+0.034}_{-0.053}\mu_N$, in excellent agreement with the lattice QCD results of Refs. [33–35]. The numerical values of the strange magnetic moment and form factor at different Q^2 which the experiments are focusing on are listed in Table II. The error bars are caused by the uncertainty in Λ .

V. SUMMARY

We have calculated the nucleon strange magnetic form factor within a chiral effective model using the finite-range regularization method up to next-to-leading order. With the regulator parameter ~ 0.8 GeV, the invariance of the core contribution between quenched and full QCD indicates the low-energy constant related to the three-quark core contribution is approximately zero. This makes it possible to predict the strange magnetic form factor, as there is no low-energy constant to be specified. The strange magnetic form factor is totally determined by the FRR loop contribution. The coupling constants in the loop contribution are well known. The uncertainty arises mostly from the variation of Λ , which is allowed to run over the wide range 0.6–1.0 GeV, as suggested by numerous studies of lattice QCD data as a function of quark mass.

In contrast with the case of the nucleon magnetic form factor, here we do not need to deal with the Q^2 dependence of a tree-level contribution to the strange magnetic form factor. As a result, the strange magnetic form factor can be calculated at relatively large Q^2 . Numerical results show that the nucleon strange magnetic form factor is always negative, regardless of the variation in the regulator mass. It decreases in magnitude with increasing Q^2 , which makes it

more difficult for the experiments to measure a nontrivial value for the strange magnetic form factor at large Q^2 .

We provide a relatively precise prediction for the nucleon strange form factor. Our result is consistent with current lattice simulations. The small negative value for the strange magnetic form factor is also comparable with the existing experimental data. We look forward to a new generation of precise PVES experiments to compare with our calculated results.

ACKNOWLEDGMENTS

This work is supported in part by DFG and NSFC (CRC 110), by the National Natural Science Foundation of China (Grant No. 11035006) and by the Australian Research Council through Grants No. FL0992247 (AWT) and No. DP110101265 (DBL), and through the ARC Centre of Excellence for Particle Physics at the Terascale.

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