

Quasi-rip: A new type of rip model without cosmic doomsday

Hao Wei,* Long-Fei Wang, and Xiao-Jiao Guo

School of Physics, Beijing Institute of Technology, Beijing 100081, China

(Received 12 July 2012; published 8 October 2012)

The fate of our universe is an unceasing topic of cosmology and the human being. The discovery of the current accelerated expansion of the universe significantly changed our view of the fate of the universe. Recently, some interesting scenarios concerning the fate of the universe attracted much attention in the community, namely the so-called “little rip” and “pseudo-rip”. It is worth noting that all the big rip, little rip and pseudo-rip arise from the assumption that the dark energy density $\rho(a)$ is monotonically increasing. In the present work, we are interested to investigate what will happen if this assumption is broken, and then propose a so-called “quasi-rip” scenario, which is driven by a type of quintom dark energy. In this work, we consider an explicit model of quasi-rip in detail. We show that quasi-rip has an unique feature different from big rip, little rip and pseudo-rip. Our universe has a chance to be rebuilt from the ashes after the terrible rip. This might be the last hope in the “hopeless” rip.

DOI: [10.1103/PhysRevD.86.083003](https://doi.org/10.1103/PhysRevD.86.083003)

PACS numbers: 95.36.+x, 98.80.-k

I. INTRODUCTION

Since its discovery in 1998, the current accelerated expansion of our universe [1] has been one of the most active fields in modern cosmology. As is well known, it could be due to an unknown energy component (dark energy) or a modification to general relativity (modified gravity) [1,2]. Before 1998, it was commonly believed that in the future our universe will either expand forever or contract again into a final big crunch. However, the discovery of the current accelerated expansion of the universe significantly changed our view of the fate of the universe. In fact, many novel possibilities are under the active consideration in the community nowadays.

Today, there are many dark energy candidates in the market. Among them, the famous phantom dark energy [3,4] is very interesting. Its equation-of-state parameter (EOS) is smaller than -1 . Although phantom dark energy is consistent with the current observational data [1,3], it violates all the energy conditions. One of the consequences is that our universe will encounter a singularity at a finite time, namely the so-called big rip [4]. At this singularity, the scale factor a , energy density and pressure of our universe are all divergent. In fact, besides the traditional big bang, big crunch, and the big rip, many novel singularities have been considered in the literature, such as sudden singularities, generalized sudden singularities, quiescent singularities, big boost, big brake, big freeze, w singularities, inaccessible singularities, directional singularities (see, e.g., Refs. [5,6] and references therein for some brief reviews). These singularities arise at the price of violation of one or several energy conditions. As is well known, in Ref. [6] (see also e.g., Refs. [5,7,8]) the future singularities have been classified into four types, namely

- (i) Type I (big rip): $a \rightarrow \infty$, $\rho \rightarrow \infty$, $H \rightarrow \infty$, $|p| \rightarrow \infty$, when $t \rightarrow t_s < \infty$;
- (ii) Type II (sudden singularity): $a \rightarrow a_s$, $\rho \rightarrow \rho_s$, $H \rightarrow H_s$, $|p| \rightarrow \infty$, $\dot{H} \rightarrow \infty$, when $t \rightarrow t_s < \infty$;
- (iii) Type III (big freeze): $a \rightarrow a_s$, $\rho \rightarrow \infty$, $H \rightarrow \infty$, $|p| \rightarrow \infty$, when $t \rightarrow t_s < \infty$;
- (iv) Type IV (generalized sudden singularity): $a \rightarrow a_s$, $\rho \rightarrow \rho_s$, $H \rightarrow H_s$, $|p| \rightarrow p_s$, $\dot{H} \rightarrow \dot{H}_s$, and higher derivatives of H diverge, when $t \rightarrow t_s < \infty$,

where t_s , a_s , ρ_s , H_s , p_s , $\dot{H}_s$ are all finite constants ($a_s \neq 0$); ρ and p are energy density and pressure respectively; $H \equiv \dot{a}/a$ is the Hubble parameter, and a dot denotes a derivative with respect to the cosmic time t . In fact, the above four types can include almost all known future singularities.

Of course, singularities usually are not desirable in physics. Therefore, other possible fates of our universe are also considered in the literature, such as the cyclic/oscillatory cosmology. Recently, some interesting scenarios concerning the fate of the universe attracted much attention in the community, namely the so-called “little rip” [9] and “pseudo-rip” [10]. In Refs. [9–11], their authors showed that if the cosmic energy density will remain constant or monotonically increase in the future, then all the possible fates of our universe can be divided into four categories based on the time asymptotics of the Hubble parameter $H(t)$ [10], namely

- (i) Big rip: $H(t) \rightarrow \infty$, when $t \rightarrow t_{\text{rip}} < \infty$;
- (ii) Little rip: $H(t) \rightarrow \infty$, when $t \rightarrow \infty$;
- (iii) Cosmological constant: $H(t) = \text{const.}$;
- (iv) Pseudo-rip: $H(t) \rightarrow H_\infty < \infty$, when $t \rightarrow \infty$,

where H_∞ is a constant. Obviously, the big rip singularity is not the only fate of our universe with the phantomlike dark energy. Both little rip and pseudo-rip are nonsingular, and hence fall outside of the four categories in Ref. [6]. Similar to the big rip, the little rip dissociates *all* bound structures,

*haowei@bit.edu.cn

but the strength of dark energy is not enough to rip apart spacetime (unlike the big rip) [9,10]. On the other hand, the pseudo-rip dissociates the bound structures which are held together by a binding force at or below a particular threshold, and hence it is possible that only *some* bound structures are dissociated while the others are *not* dissociated (depending on the model parameters) [10]. In fact, the little rip is an intermediate case between the cosmological constant and the big rip [9], while the pseudo-rip is an intermediate case between the cosmological constant and the little rip [10].

It is worth noting that all the big rip, little rip and pseudo-rip scenarios arise from the assumption that the dark energy density $\rho(a)$ is monotonically increasing [9–11], i.e., the dark energy is phantomlike (its EOS $w < -1$). In the present work, we are interested to investigate what will happen if this assumption is broken. Obviously, in the case of the dark energy density $\rho(a)$ is monotonically decreasing (i.e., the dark energy is quintessencelike with an EOS $w > -1$), no rip will happen and no bound structures will be dissociated. On the other hand, in the case of the dark energy density $\rho(a)$ monotonically decreases (namely $w > -1$) in the first stage and then monotonically increases (namely $w < -1$) in the second stage (this is the case of the so-called “quintom A” dark energy in terminology of, e.g., Ref. [12] and references therein), the fate of our universe is the big rip, which is trivial in some sense. The third case is that the dark energy density $\rho(a)$ monotonically increases (namely $w < -1$) in the first stage and then monotonically decreases (namely $w > -1$) in the second stage (this is the case of the so-called “quintom B” dark energy in terminology of, e.g., Refs. [12] and references therein). It can be expected that in the first stage some or all bound structures will be dissociated (similar to the case of pseudo-rip), but then the disintegration process will stop, and the already disintegrated structures have the possibility to be recombined in the second stage. We dub it “quasi-rip”, which is the subject of the present work (here we temporarily do not consider the case of oscillatory quintom dark energy). Since the quintom-like dark energy [13] (whose EOS can cross the so-called phantom divide $w = -1$) is slightly favored by the observational data [1] (see, e.g., Ref. [12] for a comprehensive review), we note that the quasi-rip is well-motivated in fact.

This paper is organized as follow. In Sec. II, we discuss the disintegration of bound structures. In Sec. III, we present an explicit model of quasi-rip. We constrain this model with the observational data, and then clearly show the quasi-rip in this model. In Sec. IV, some concluding remarks are given.

II. THE DISINTEGRATION OF BOUND STRUCTURES

It is useful to introduce the concept of “inertial force” when we discuss the disintegration of bound structures.

According to Refs. [10,11,14], in a flat Friedmann-Robertson-Walker universe dominated by dark energy, (motivated by the well-known Newton’s second law) the inertial force F_{inert} on a mass m as seen by a gravitational source separated by a comoving distance r_0 is given by

$$\begin{aligned} F_{\text{inert}} &\equiv mr_0 \frac{\ddot{a}}{a} = mr_0 (\dot{H} + H^2) = -mr_0 \frac{4\pi G}{3} (\rho + 3p) \\ &= mr_0 \frac{4\pi G}{3} \left(2\rho + a \frac{d\rho}{da} \right), \end{aligned} \quad (1)$$

in which we have used the energy conservation equation $\dot{\rho} + 3H(\rho + p) = 0$. A bound structure dissociates when the inertial force F_{inert} (dominated by dark energy) is equal to the force F_{bound} holding together this bound structure [10,11]. For the gravitationally bound structure whose mass is M , it dissociates when

$$F_{\text{inert}} = mr_0 \frac{\ddot{a}}{a} = F_{\text{bound}} = G \frac{Mm}{r_0^2} = mr_0 \omega_0^2, \quad (2)$$

namely

$$\frac{\ddot{a}}{a} = \omega_0^2 = \frac{GM}{r_0^3}, \quad (3)$$

or equivalently

$$f(a) \equiv \frac{1}{\rho_0} \left(2\rho + a \frac{d\rho}{da} \right) = \frac{1}{\rho_0} \left(2\rho + \frac{d\rho}{d \ln a} \right) = \frac{2H_0^{-2} GM}{\Omega_0 r_0^3}, \quad (4)$$

where ω_0 is the angular velocity; ρ_0 is the present density of dark energy; $\Omega_0 \equiv (8\pi G \rho_0)/(3H_0^2)$ is the present fractional density of dark energy. Note that in this paper, the subscript “0” usually indicates the present value of corresponding quantity, and we have set $a_0 = 1$. In fact, one can check that the results Eqs. (3) and (4) are coincident with the ones in the case of phantom dark energy whose EOS w is a constant [4] (we refer to, e.g., Ref. [15] for its detailed derivation; note that the result of Ref. [4] is invalid for a nonconstant w , while our results Eqs. (3) and (4) are still valid).

However, if the bound structure is massive enough to significantly affect the local spacetime metric, it is not accurate to express F_{inert} in terms of a Friedmann-Robertson-Walker metric [9,10]. A more accurate method was presented in Ref. [15]. Following Ref. [15], in the Newtonian limit, the interpolating metric of the local spacetime is given by

$$ds^2 = \left[1 - \frac{2GM}{a(t)\eta} \right] dt^2 - a^2(t) [d\eta^2 + \eta^2(d\theta^2 + \sin^2\theta d\varphi^2)], \quad (5)$$

where η is the comoving radial coordinate. The radial equation of motion for a test particle in the Newtonian limit reads

$$\ddot{r} = \frac{\ddot{a}}{a}r + \frac{L^2}{r^3} - \frac{GM}{r^2}, \quad (6)$$

where $L = r^2\dot{\varphi} = r^2\omega = \text{const} = r_0^2\omega_0$ is the constant angular momentum per unit mass. Noting that $\omega_0^2 = GM/r_0^3$, Eq. (6) can be recast as

$$\ddot{r} = \frac{\ddot{a}}{a}r + \frac{GMr_0}{r^3} - \frac{GM}{r^2} = -\frac{dV_{\text{eff}}}{dr}, \quad (7)$$

where the time-dependent effective potential is given by

$$V_{\text{eff}} = -\frac{1}{2}\frac{\ddot{a}}{a}r^2 + \frac{GMr_0}{2r^2} - \frac{GM}{r}. \quad (8)$$

The bound structure dissociates when the minimum of the time-dependent effective potential (including the centrifugal term) disappears. Note that the corresponding radius $r_{V_{\min}}$ at which the effective potential reaches its minimum is determined by

$$\frac{\ddot{a}}{a}r_{V_{\min}}^4 - GMr_{V_{\min}} + GMr_0 = 0. \quad (9)$$

One can find (with the help of, e.g., MATHEMATICA) that Eq. (9) has a real solution only for

$$\frac{\ddot{a}}{a} \leq \frac{27}{256} \frac{GM}{r_0^3}. \quad (10)$$

Therefore, the minimum of the time-dependent effective potential disappears when

$$\frac{\ddot{a}}{a} = \frac{27}{256} \frac{GM}{r_0^3}, \quad (11)$$

which is also the time when the bound structure dissociates. Similarly, we can also recast Eq. (11) as

$$f(a) \equiv \frac{1}{\rho_0} \left(2\rho + a \frac{d\rho}{da} \right) = \frac{1}{\rho_0} \left(2\rho + \frac{d\rho}{d \ln a} \right) = \frac{27}{128} \frac{H_0^{-2} GM}{\Omega_0 r_0^3}. \quad (12)$$

As is shown in Ref. [15], the accurate disintegration time obtained here is usually earlier than the qualitative one obtained in Ref. [4] (this point can also be seen by comparing Eqs. (11) and (12) with Eqs. (3) and (4)). It is worth noting that most of the results in Ref. [15] are valid for dark energy whose EOS w is a constant, while our results obtained here are also valid for dynamical dark energy whose EOS w is not a constant.

In the present work, we consider five bound structures, namely the Coma Cluster ($M = 6 \times 10^{48}$ g, $r_0 = 9 \times 10^{24}$ cm), the Milky Way galaxy ($M = 2 \times 10^{45}$ g, $r_0 = 5 \times 10^{22}$ cm), the Solar System ($M = 2 \times 10^{33}$ g, $r_0 = 7 \times 10^{15}$ cm), the Earth ($M = 6 \times 10^{27}$ g, $r_0 = 6.4 \times 10^8$ cm), and the hydrogen atom. Note that the first three bound structures are suitable for Eqs. (11) and (12), while the Earth is suitable for Eqs. (3) and (4), since its surface material has no centrifugal term in the effective potential. Finally, the hydrogen atom is also suitable for

Eqs. (3) and (4), but in which the term GM should be replaced by a new term $q_e^2/(4\pi\epsilon_0 m_e)$, since F_{bound} is the electromagnetic force in this case. Note that $q_e = 1.6 \times 10^{-19}$ C, $m_e = 0.511$ MeV (1 MeV = 1.7827×10^{-27} g), $1/(4\pi\epsilon_0) = 9 \times 10^{18}$ g cm³ C⁻² sec⁻², and $r_0 = 5.3 \times 10^{-9}$ cm for the hydrogen atom. In Eqs. (4) and (12), $G = 6.672 \times 10^{-6}$ cm³ g⁻¹ sec⁻², $H_0^{-1} = 3.0856 \times 10^{17} h^{-1}$ sec (here h is the Hubble constant H_0 in units of 100 km/sec/Mpc). Note that Ω_0 and h will be determined by the observational data (see below).

Since the current observational data are in the epoch $a < 1$, we cannot ignore the contribution from pressureless matter in this stage, although it can be safely ignored in the above discussions when the universe is dominated by dark energy. From the Friedmann equation $3H^2 = 8\pi G\rho_{\text{tot}} = 8\pi G(\rho + \rho_m)$ and $\rho_m = \rho_{m0}a^{-3}$, we have

$$E^2 \equiv \frac{H^2}{H_0^2} = \Omega_0 \frac{\rho}{\rho_0} + \Omega_{m0} a^{-3} = \Omega_0 \frac{\rho}{\rho_0} + (1 - \Omega_0) a^{-3}, \quad (13)$$

where $\Omega_{m0} \equiv (8\pi G\rho_{m0})/(3H_0^2)$ is the present fractional density of pressureless matter. If $\rho(a)$ is given, we can use Eq. (13) to constrain the model parameters with the current observational data. On the other hand from Eqs. (12) or (4), one can find the corresponding scale factor a_* at which the bound structure dissociates. Then, noting $H = \dot{a}/a$, we can evaluate the disintegration time measuring from today ($a = 1$), namely

$$t_* - t_0 = H_0^{-1} \int_1^{a_*} \frac{da}{aE(a)} = H_0^{-1} \int_0^{\ln a_*} \frac{d \ln a}{E(\ln a)}, \quad (14)$$

where $H_0^{-1} = 9.7776 h^{-1}$ Gyr.

III. AN EXPLICIT MODEL OF QUASI-RIP

Here, we consider an explicit model of quasi-rip. In some sense, constructing a suitable model is a smart task. There are various ways to this end. For example, one can specify the scale factor a as a function of the cosmic time t (see, e.g., Refs. [5, 16, 17]), the pressure p as a function of the energy density ρ (see, e.g., Refs. [6, 18, 19]), the energy density ρ as a function of the scale factor a (see e.g., Refs. [9, 10]), or the Hubble parameter H as a function of the cosmic time t (see, e.g., Refs. [11]). It is shown in Refs. [9–11] that these ways are equivalent in fact. In our case, for convenience, we choose to specify the energy density ρ as a function of the scale factor a , similar to the cases of little rip [9] and pseudo-rip [10]. As mentioned in Sec. I, here our task is to construct an explicit function $\rho(a)$, which monotonically increases (namely $w < -1$) in the first stage and then monotonically decreases (namely $w > -1$) in the second stage. A naive idea is to use a piecewise function, which is phantomlike in the first sector (namely $\rho(a) \propto a^{-3(1+w_1)}$ with a constant EOS $w_1 < -1$) and is quintessencelike in the second sector (namely

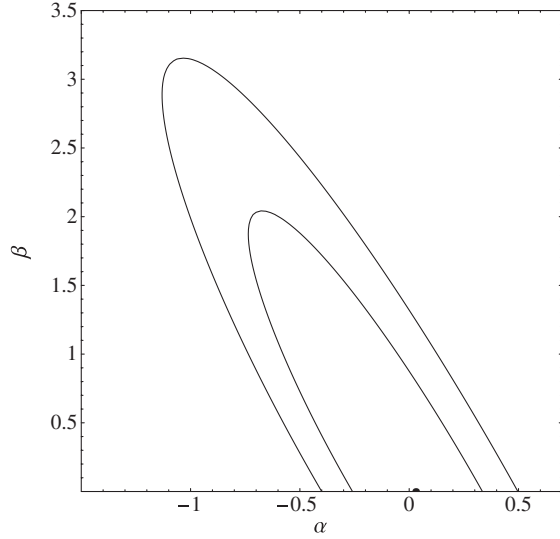


FIG. 1. The 68.3% and 95.4% confidence level contours in the $\alpha - \beta$ parameter space. The best-fit parameters are also indicated by a black solid point.

$\rho(a) \propto a^{-3(1+w_2)}$ with a constant EOS $w_2 > -1$). Then, let us refine this naive idea with a smooth function $\rho(a) \propto a^{\mu(a)}$, in which $\mu(a < a_t) > 0$ and $\mu(a > a_t) < 0$, where a_t is the transition point. Noting that it is more convenient to use $\ln a$ as the variable in Eqs. (12), (4), and (14), as well as the fact $a = e^{\ln a}$, it is appropriate to write μ as a function of $\ln a$ instead. Therefore, the simplest function $\rho(a)$ can be given by

$$\rho(a) = \rho_0 a^{\alpha - \beta \ln a} = \rho_0 e^{\ln a (\alpha - \beta \ln a)}, \quad (15)$$

where α and β are both constants. Let us have a closer observation. By requiring $d\rho/d\ln a = 0$, we find that its

extremum locates at $\ln a = \alpha/(2\beta)$. To ensure this extremum is a maximum, $d^2\rho/d(\ln a)^2$ should be negative here, therefore $\beta > 0$ is required.

Substituting Eq. (15) into Eq. (13), we have

$$\begin{aligned} E^2 &= \frac{H^2}{H_0^2} = \Omega_0 a^{\alpha - \beta \ln a} + (1 - \Omega_0) a^{-3} \\ &= \Omega_0 e^{\ln a (\alpha - \beta \ln a)} + (1 - \Omega_0) e^{-3 \ln a} \\ &= \Omega_0 (1 + z)^{-\alpha - \beta \ln(1+z)} + (1 - \Omega_0) (1 + z)^3, \end{aligned} \quad (16)$$

where $z = 1/a - 1$ is the redshift. There are three free model parameters, namely Ω_0 , α and β . With this $E(z)$, following the methodology in e.g., Refs. [20–22], we can constrain this model with the latest Union2.1 Type Ia supernovae (SNIa) dataset [23] which consists of 580 SNIa. In fact, we find that the best fit has $\chi^2_{\min} = 562.224$, and the best-fit parameters are $\Omega_0 = 0.718859$, $\alpha = 0.0294816$ and $\beta = 0.0002525$, whereas the corresponding $h = 0.698862$. In Fig. 1, we present the corresponding 68.3% and 95.4% confidence level (C.L.) contours in the $\alpha - \beta$ parameter space. Although their best-fit values are small, α and β can still be consistent with the observational data in the large 2σ C.L. region, i.e., $-1.2 \lesssim \alpha \lesssim 0.5$ and $0 \lesssim \beta \lesssim 3.2$.

Next, we show the quasi-rip in this model clearly. Substituting Eq. (15) into the reduced inertial force $f(a)$ defined in Eqs. (12) and (4), we have

$$\begin{aligned} f(a) &= \frac{\rho}{\rho_0} (2 + \alpha - 2\beta \ln a) = (2 + \alpha - 2\beta \ln a) a^{\alpha - \beta \ln a} \\ &= (2 + \alpha - 2\beta \ln a) e^{\ln a (\alpha - \beta \ln a)}. \end{aligned} \quad (17)$$

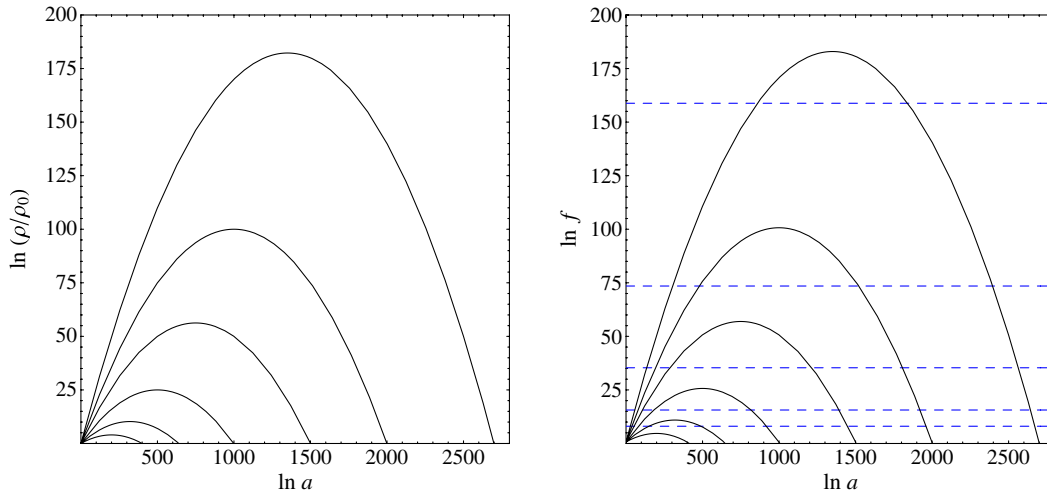


FIG. 2 (color online). $\ln(\rho/\rho_0)$ and $\ln f$ as functions of $\ln a$ for various model parameters. The curves from the innermost to the outermost are for $\alpha = 0.04$, $\alpha = 0.064$, $\alpha = 0.1$, $\alpha = 0.15$, $\alpha = 0.2$ and $\alpha = 0.27$, respectively, while β is fixed to be 0.0001. The dashed horizontal lines in the right panel indicate the corresponding thresholds to disintegrate the Coma Cluster, the Milky Way galaxy, the Solar System, the Earth and the hydrogen atom (from bottom to top). Note that Ω_0 and h are taken to be their best-fit values.

In Fig. 2, we plot $\ln(\rho/\rho_0)$ and $\ln f$ as functions of $\ln a$ for various model parameters. For convenience, we fix $\beta = 0.0001$, which is at the same order of its best-fit value, while the various values of α are taken from the 1σ C.L. region in Fig. 1. From the left panel of Fig. 2, it is easy to see that our $\rho(a)$ in Eq. (15) is the desirable one, which monotonically increases in the first stage and then monotonically decreases in the second stage. It is not surprising that the plot of $\ln f$ is very similar to the one of $\ln(\rho/\rho_0)$, since their difference is given by $\ln(2 + \alpha - 2\beta \ln a)$, which is a small quantity in fact (cf., Eq. (17)). Depending on the model parameters α and β , the inertial force can successively disintegrate the Coma Cluster, the Milky Way galaxy, the Solar System, the Earth and the hydrogen atom. In different cases, some bound structures will be disintegrated while the other bound structures will not. If none of the bound structures is disintegrated, the corresponding quasi-rip is a failed rip. From the right panel of Fig. 2, we find that the innermost one is a failed rip, the outermost one is the strongest quasi-rip which can disintegrate all the five bound structures, and the others are moderate ones which can only disintegrate one or several of the five bound structures. From Eqs. (12) or (4), we can find the corresponding $\ln a_*$ at which the bound structure dissociates. Then, we can evaluate the disintegration time measuring from today $t_* - t_0$ by using Eq. (14) with $E(\ln a)$ given in Eq. (16), and present the results in Table. I. For a fixed β , a bound structure dissociates earlier with a larger α , since the corresponding inertial force is stronger. In the case of $\alpha = 0.27$, the difference between the disintegration time of Earth and Solar System is 4349 years, while the difference between the disintegration time of hydrogen atom and Earth is only 15 years. Finally, it is easy to see that the most distinct feature of quasi-rip is that the inertial force monotonically decreases in the second stage. Eventually, it will become lower than all the thresholds to disintegrate the bound structure. Therefore, the already disintegrated structures have the possibility to be recombined in the second stage. This is the unique feature of quasi-rip different from big rip, little rip and pseudo-rip. Our universe has a chance to be rebuilt from the ashes after the terrible rip. This might be the last hope in the “hopeless” rip.

IV. CONCLUDING REMARKS

The fate of our universe is an unceasing topic of cosmology and the human being. The discovery of the current accelerated expansion of the universe significantly changed our view of the fate of the universe. Recently, some interesting scenarios concerning the fate of the universe attracted much attention in the community, namely the so-called “little rip” and “pseudo-rip”. It is worth noting that all the big rip, little rip and pseudo-rip arise from the assumption that the dark energy density $\rho(a)$ is monotonically increasing. In the present work, we are interested to investigate what will happen if this assumption is broken, and then propose a so-called “quasi-rip” scenario, which is driven by a type of quintom dark energy. In this work, we consider an explicit model of quasi-rip in detail. We show that quasi-rip has an unique feature different from big rip, little rip and pseudo-rip. Our universe has a chance to be rebuilt from the ashes after the terrible rip. This might be the last hope in the “hopeless” rip.

Some remarks are in order. First, as is shown in Sec. III, our quasi-rip model is well consistent with the current observational data. However, even in the 1σ C.L. region of $\alpha - \beta$ parameter space, the future behavior of our universe can be different enough (depending on the particular model parameters α and β). In fact, as is well known, the current observational data can be consistent with all the phantomlike, quintessencelike and quintomlike dark energy models. Therefore, the current observational data cannot tightly tell what is the true fate of our universe. Most of the possibilities (including big rip, little rip, pseudo-rip, quasi-rip, de Sitter expansion, other future singularities, and so on) are still living.

Second, the explicit model of quasi-rip considered in the present work is the simplest case. One can construct other, more complicated $\rho(a)$ to implement the quasi-rip. For example, one might construct an EOS $w(a)$ as a function of scale factor a , which is smaller than -1 when $a < a_i$ and larger than -1 when $a > a_i$. Then the corresponding $\rho(a)$ can be found from the energy conservation equation $\dot{\rho} + 3H\rho(1 + w(a)) = 0$. Of course, other smart methods to construct the desirable $\rho(a)$ are awaiting us.

Third, as mentioned in Ref. [10], in its second pseudo-rip model, the reduced inertial force $f(a)$ can also have a peak, similar to our quasi-rip model. However, we note that

TABLE I. The disintegration time measuring from today $t_* - t_0$ (in units of Gyr) for models with various α , while β is fixed to be 0.0001, and Ω_0 and h are taken to be their best-fit values.

Bound structure	$\alpha = 0.04$	$\alpha = 0.064$	$\alpha = 0.1$	$\alpha = 0.15$	$\alpha = 0.2$	$\alpha = 0.27$
Coma cluster		563.154	333.555	216.991	161.28	118.715
Milky Way			344.035	223.126	165.75	122.005
Solar system				223.281	165.857 985 136	122.083 461 362
Earth					165.857 991 583	122.083 465 711
Hydrogen atom						122.083 465 696

in the pseudo-rip model, after the peak, the reduced inertial force $f(a) \rightarrow \text{const}$ which is still higher than the corresponding threshold to disintegrate the bound structure. Therefore, the already disintegrated structures have *no* possibility to be recombined in the pseudo-rip models. On the contrary, in the quasi-rip models, the reduced inertial force $f(a)$ monotonically decreases in the second stage. Eventually, it will become lower than all the thresholds to disintegrate the bound structure. Therefore, the already disintegrated structures have the possibility to be recombined in the second stage.

Fourth, it is well known that phantom is unstable at quantum level and hence the perturbations grow large. Noting that in the present work our discussions are at classical level instead, this problem could be set aside. In fact, the quantum stability of a phantom phase has been considered in Ref. [24]. The authors of Ref. [24] studied the perturbations in the quantum-corrected effective field equation at one- and two-loop order, and they found that the system is stable. On the other hand, it is claimed in Ref. [25] that scalar perturbations can grow during a phantom phase if EOS $w < -5/3$. However, from Eq. (15) and Fig. 1, it is easy to see that the corresponding w is only slightly smaller than -1 in the phantom phase for our particular α and β (n.b. Fig. 1). So, $w > -5/3$ instead and hence our quasi-rip

model can avoid the problem raised in Ref. [25]. Further, in fact the dark energy considered in this work is not necessarily a scalar field. It could even be an effective dark energy from modified gravity, namely the so-called “geometric dark energy”. So, it might avoid the corresponding problems in the phantom phase.

Finally, in the present work, we consider only the quintom dark energy which crosses the phantom divide $w = -1$ once. In fact, we can also consider the quintom dark energy which can cross the phantom divide for many times. The most attractive quasi-rip model might be the one driven by the oscillatory quintom dark energy. In this oscillatory quasi-rip model, our universe will be destroyed and then be rebuilt again and again.

ACKNOWLEDGMENTS

We thank the anonymous referee for quite useful comments and suggestions, which helped us to improve this work. We are grateful to Professors Rong-Gen Cai and Shuang Nan Zhang for helpful discussions. We also thank Minzi Feng for kind help and discussions. This work was supported in part by NSFC under Grants No. 11175016 and No. 10905005, as well as NCET under Grant No. NCET-11-0790, and the Fundamental Research Fund of Beijing Institute of Technology.

-
- [1] E. J. Copeland, M. Sami, and S. Tsujikawa, *Int. J. Mod. Phys. D* **15**, 1753 (2006); J. Frieman, M. Turner, and D. Huterer, *Annu. Rev. Astron. Astrophys.* **46**, 385 (2008); S. Tsujikawa, [arXiv:1004.1493](#); M. Li, X. D. Li, S. Wang, and Y. Wang, *Commun. Theor. Phys.* **56**, 525 (2011); D. H. Weinberg *et al.*, [arXiv:1201.2434](#).
 - [2] A. De Felice and S. Tsujikawa, *Living Rev. Relativity* **13**, 3 (2010), <http://www.livingreviews.org/lrr-2010-3>; T. Clifton, P. G. Ferreira, A. Padilla, and C. Skordis, *Phys. Rep.* **513**, 1 (2012); T. P. Sotiriou and V. Faraoni, *Rev. Mod. Phys.* **82**, 451 (2010); S. Tsujikawa, *Lect. Notes Phys.* **800**, 99 (2010); S. Capozziello, M. De Laurentis, and V. Faraoni, [arXiv:0909.4672](#); R. Durrer and R. Maartens, [arXiv:0811.4132](#); S. Nojiri and S. D. Odintsov, *Int. J. Geom. Methods Mod. Phys.* **04**, 115 (2007); *Phys. Rep.* **505**, 59 (2011); K. Bamba, S. Capozziello, S. Nojiri, and S. D. Odintsov, [arXiv:1205.3421](#) [*Astrophys. Space Sci.* (to be published)].
 - [3] R. R. Caldwell, *Phys. Lett. B* **545**, 23 (2002).
 - [4] R. R. Caldwell, M. Kamionkowski, and N. N. Weinberg, *Phys. Rev. Lett.* **91**, 071 301 (2003).
 - [5] L. Fernandez-Jambrina, *Phys. Rev. D* **82**, 124 004 (2010).
 - [6] S. Nojiri, S. D. Odintsov, and S. Tsujikawa, *Phys. Rev. D* **71**, 063 004 (2005).
 - [7] M. P. Dabrowski and T. Denkiewicz, *AIP Conf. Proc.* **1241**, 561 (2010).
 - [8] A. V. Yurov, *Phys. Lett. B* **689**, 1 (2010).
 - [9] P. H. Frampton, K. J. Ludwick, and R. J. Scherrer, *Phys. Rev. D* **84**, 063003 (2011).
 - [10] P. H. Frampton, K. J. Ludwick, and R. J. Scherrer, *Phys. Rev. D* **85**, 083001 (2012).
 - [11] P. H. Frampton, K. J. Ludwick, S. Nojiri, S. D. Odintsov, and R. J. Scherrer, *Phys. Lett. B* **708**, 204 (2012).
 - [12] Y. F. Cai, E. N. Saridakis, M. R. Setare, and J. Q. Xia, *Phys. Rep.* **493**, 1 (2010).
 - [13] B. Feng, X. L. Wang, and X. M. Zhang, *Phys. Lett. B* **607**, 35 (2005).
 - [14] I. Brevik, E. Elizalde, S. Nojiri, and S. D. Odintsov, *Phys. Rev. D* **84**, 103 508 (2011); A. V. Astashenok, S. Nojiri, S. D. Odintsov, and A. V. Yurov, *Phys. Lett. B* **709**, 396 (2012); **713**, 145 (2012).
 - [15] S. Nesseris and L. Perivolaropoulos, *Phys. Rev. D* **70**, 123 529 (2004).
 - [16] J. D. Barrow, *Classical Quantum Gravity* **21**, L79 (2004); J. D. Barrow, *Classical Quantum Gravity* **21**, 5619 (2004); J. D. Barrow and S. Z. W. Lip, *Phys. Rev. D* **80**, 043 518 (2009).
 - [17] C. Cattoen and M. Visser, *Classical Quantum Gravity* **22**, 4913 (2005).
 - [18] S. Nojiri and S. D. Odintsov, *Phys. Rev. D* **72**, 023 003 (2005).
 - [19] H. Stefancic, *Phys. Rev. D* **71**, 084 024 (2005).
 - [20] E. Di Pietro and J. F. Claeskens, *Mon. Not. R. Astron. Soc.* **341**, 1299 (2003).

- [21] S. Nesseris and L. Perivolaropoulos, *Phys. Rev. D* **72**, 123 519 (2005); L. Perivolaropoulos, *Phys. Rev. D* **71**, 063 503 (2005).
- [22] H. Wei, *Classical Quantum Gravity* **29**, 175008 (2012); H. Wei, X.J. Guo, and L.F. Wang, *Phys. Lett. B* **707**, 298 (2012); H. Wei, *J. Cosmol. Astropart. Phys.* 04 (2011) 022; *Commun. Theor. Phys.* **56**, 972 (2011); *Phys. Lett. B* **692**, 167 (2010); *J. Cosmol. Astropart. Phys.* 08 (2010) 020; *Phys. Lett. B* **691**, 173 (2010); **687**, 286 (2010); *Eur. Phys. J. C* **62**, 579 (2009); **60**, 449 (2009); H. Wei and S.N. Zhang, *Eur. Phys. J. C* **63**, 139 (2009).
- [23] N. Suzuki *et al.* (SCP Collaboration), *Astrophys. J.* **746**, 85 (2012); The numerical data of the full Union2.1 SNIa sample are available at <http://supernova.lbl.gov/Union>.
- [24] E.O. Kahya and V.K. Onemli, *Phys. Rev. D* **76**, 043 512 (2007).
- [25] J.C. Fabris and S.V.B. Goncalves, *Phys. Rev. D* **74**, 027 301 (2006); J.C. Fabris, D.F. Jardim, and S.V.B. Goncalves, *Europhys. Lett.* **82**, 69 001 (2008).