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Adiabatic and nonadiabatic perturbations for loop quantum cosmology

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We generalize the perturbations theory of loop quantum cosmology to a hydrodynamical form and define an effective curvature perturbation on a uniform-density hypersurfaces ζ_e. As in the classical cosmology, ζ_e should be gauge-invariant and conservative on the large scale. The evolutions of both the adiabatic and the nonadiabatic perturbations for a multifluid model are investigated in the framework of the effective hydrodynamical theory of loop quantum cosmology with the inverse-triad correction. We find that, different from the classical cosmology, the evolution of the large-scale nonadiabatic entropy perturbation can be driven by an adiabatic curvature perturbation and this adiabatic source for the nonadiabatic perturbation is a quantum effect. As an application of the related formalism, we study a decay model and give out the numerical results.

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I. INTRODUCTION

The theory of cosmological perturbations has become a cornerstone of the modern cosmology. It provides the key to understanding the early evolution and the current large-scale structure of our universe. And it is also used to describe the growth of the structure in the universe, or to calculate the predicted microwave background fluctuations, and many other purposes. A widely used reference work on the cosmological perturbations can be seen in [1].

Generally speaking, the origin of the cosmological perturbations is believed to come from the quantum fluctuation during the inflation. Therefore, it is interesting to study a possible quantum gravity effect in the cosmological perturbations theory. However, the problem of finding a quantum theory of the gravitational field is still open. One of the most active of the current approaches is loop quantum gravity. Loop quantum gravity [2–4] is a mathematically well-defined, nonperturbative and background-independent quantization of general relativity. Its cosmological version, the loop quantum cosmology (LQC) [5], has achieved many successes. A major success of LQC is the resolution of the big-bang singularity [6–8]; this result depends crucially on the discreteness of the spacetime geometry. With such a result, the big-bang singularity will be avoided through a big-bounce mechanism in the high-energy region. In addition, LQC can set up suitable initial conditions for a successful inflation [9,10] as well as possibly leaving an imprint in the cosmic microwave background [10].

There are two types of quantum corrections that are expected from the Hamiltonian of loop quantum gravity. One is called "the inverse-triad correction" and the other is called "the holonomy correction" (see a review article [11]). The application of them on the scalar mode of perturbation can be found in [12], the vector mode in [13] and the tensor mode in [14]. The application of the higher-order holonomy corrections [15] to the perturbations theory of cosmology is studied in [16]. The gauge-invariant approach for the cosmological perturbations with the holonomy correction has been studied in [17].

In this paper we focus on the effective theory of LQC with the inverse-triad correction. The gauge-invariant approach for the cosmological perturbations with the inverse-triad correction has been built up in [18–20].

On the other hand, the perturbation considered in most of the works is the adiabatic perturbation, for which the density fluctuation is proportional to the pressure perturbation [21–24]. However, there are some more detailed models, such as the reheating at the end of inflation [25] or the effect of late-decaying scalar fields [26], requiring a discussion of the multicomponent system where it is useful to identify the gauge-invariant adiabatic and nonadiabatic modes.

In the classical cosmology, the most important result of the nonadiabatic perturbation is that [27,28] the nonadiabatic (entropy) perturbation evolves independently of the curvature perturbation on the large scale, but that the evolution of the large-scale curvature perturbation can be sourced by the nonadiabatic perturbation. In other words, the nonadiabatic perturbation can translate into the curvature perturbation on the large scale, while the curvature perturbation can not change to the entropy perturbation.

However, in loop quantum cosmology, the inverse-triad correction will lead to a completely different evolution compared with the classical universe. Therefore, it is interesting to discuss the relationship between the adiabatic and the nonadiabatic perturbations under the theoretical framework of LQC.

The paper is organized as follows. At first, the gauge-invariant formalism of the cosmological perturbations theory with the inverse-triad correction is reviewed briefly in Sec. II. Then in Sec. III, the perturbations theory of loop

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quantum cosmology is generalized to the hydrodynamical form. In Sec. IV, the evolution of the nonadiabatic perturbation and the relationship between the adiabatic and the nonadiabatic perturbations under the theoretical framework of LQC are analyzed in detail. As an application of the related formalism, in Sec. V we study a decay model. Finally, Sec. VI is the summary and conclusions.

II. SCALAR PERTURBATION OF LQC

In this section, we review briefly the effective theory of LQC and the gauge-invariant formalism of the cosmological perturbations theory with the inverse-triad correction. We give the background equations of LQC and the evolution equations of perturbation. In this paper, we only focus on the scalar-mode perturbation which, along with the background FRW metric, takes the form

$$ds^2 = a^2(\eta)[-(1+2\phi)d\eta^2 + (1-2\psi)\delta_{ij}dx^i dx^j], \quad (1)$$

where the scale factor a is a function of the conformal time η, the spatial indices i and j run from 1 to 3, and ϕ and ψ are the scalar modes of the metric perturbation [24].

A. Background

The universe considered in this section is fulfilled by a scalar field φ with the potential V, so the effective Friedmann equation with the inverse-triad correction of LQC can be written as [12]

$$\mathcal{H}^2 = \frac{8\pi G}{3}\alpha\left[\frac{\varphi'^2}{2\nu} + pV(\varphi)\right], \quad (2)$$

where G is the gravitational constant, $p = a^2$ is the triad variable in minisuperspace [7], $\mathcal{H} \equiv a'/a = p'/2p$ is the Hubble parameter; the prime "$\prime$" denotes the derivative with respect to the conformal time, and α and ν are the parameters characterizing the effective inverse-triad correction [20].

$$\alpha \approx 1 + \alpha_0\delta_{\mathrm{pl}}, \quad (3)$$

$$\nu \approx 1 + \nu_0\delta_{\mathrm{pl}}, \quad (4)$$

where

$$\delta_{\mathrm{pl}} \equiv \left(\frac{p_{\mathrm{pl}}}{p}\right)^{\sigma/2} = \left(\frac{a_{\mathrm{pl}}}{a}\right)^{\sigma}, \quad (5)$$

$p_{\mathrm{pl}} = a_{\mathrm{pl}}^2$ is a constant. σ is an ambiguity parameter for quantization; it depends on the geometrical minisuperspace variable that has an equidistant stepsize in the dynamics. A detailed calculation then shows that the constant coefficients α_0 and ν_0 are [29]

$$\alpha_0 = \frac{(3q-\sigma)(6q-\sigma)}{2^2 3^4}\left(\frac{\Delta_{\mathrm{pl}}}{p_{\mathrm{pl}}}\right)^2, \quad (6)$$

$$\nu_0 = \frac{\sigma(2-l)}{54}\left(\frac{\Delta_{\mathrm{pl}}}{p_{\mathrm{pl}}}\right)^2, \quad (7)$$

where $\Delta_{\mathrm{pl}} \equiv 2\sqrt{3}\pi\gamma\ell_{\mathrm{pl}}^2$ is the area gap, and γ is the Barbero-Immirzi parameter. $1/2 \le l < 1$ and $1/3 \le q < 2/3$ are other sets of the ambiguity parameters; they relate to different ways of quantizing the classical Hamiltonian.

From the definition of the Hubble parameter, it is easy to check the relationship between the derivative with respect to the conformal time and the derivative with respect to p

$$\prime = 2\mathcal{H}\frac{d}{d\ln p}, \quad (8)$$

therefore

$$\delta'_{\mathrm{pl}} = -\sigma\mathcal{H}\delta_{\mathrm{pl}}. \quad (9)$$

And the effective Klein-Gordon equation can be read [20]

$$\varphi'' + 2\mathcal{H}\left(1 - \frac{d\ln\nu}{d\ln p}\right)\varphi' + \nu pV_{,\varphi} = 0, \quad (10)$$

where $V_{,\varphi}$ means $\partial V/\partial\varphi$.

B. Gauge-invariant formalism

If one only considers the primary correction functions of α and ν, there will be anomalies in the effective constraint algebra [12–14]. To keep the consistency of the theory, one must introduce the counterterms which proportional to δ_{pl} [18,19]. Explicitly, these counterterms are [20]:

$$f = \frac{1}{\sigma}\frac{d\ln\alpha}{d\ln p} = -\frac{\alpha_0}{2}\delta_{\mathrm{pl}}, \quad (11)$$

$$f_1 = f - \frac{1}{3}\frac{d\ln\nu}{d\ln p} = \frac{1}{2}\left(\frac{\sigma\nu_0}{3} - \alpha_0\right)\delta_{\mathrm{pl}}, \quad (12)$$

$$h = 2\frac{d\ln\alpha}{d\ln p} - f = \alpha_0\left(\frac{1}{2} - \sigma\right)\delta_{\mathrm{pl}}, \quad (13)$$

$$g_1 = \frac{1}{3}\frac{d\ln\alpha}{d\ln p} - \frac{d\ln\nu}{d\ln p} + \frac{2}{9}\frac{d^2\ln\nu}{d\ln p^2} = \frac{\sigma}{2}\left(\frac{\sigma\nu_0}{9} + \nu_0 - \frac{\alpha_0}{3}\right)\delta_{\mathrm{pl}}, \quad (14)$$

$$f_3 = f_1 - g_1 = \frac{1}{2}\left[\alpha_0\left(\frac{\sigma}{3} - 1\right) - \frac{2\sigma\nu_0}{3}\left(\frac{\sigma}{6} + 1\right)\right]\delta_{\mathrm{pl}}. \quad (15)$$

In this paper, we only focus on the linear order of counterterms as in [20], for instance, $(1+f)(1+h) = 1 + f + h + O(\delta_{\mathrm{pl}}^2)$. We neglect the terms of order $O(\delta_{\mathrm{pl}}^2)$.

If we ignore the anisotropy of the universe, two metric perturbations are proportional to each other [19]

$$\phi = (1+h)\psi. \quad (16)$$

One should note that, different from the classical cosmology, there is a counterterm h in Eq. (5).

The effective dynamical equation for the metric perturbation is [19]

$$3\mathcal{H}(1+f)[\psi' + (1+f)\mathcal{H}\phi] - \alpha^2\nabla^2\psi = -4\pi G\frac{\alpha}{\nu}(1+f_3)[\varphi'\delta\varphi' - \varphi'^2(1+f_1)\phi + \nu p V_{,\varphi}\delta\varphi], \tag{17}$$

where $\delta\varphi$ is the perturbation of φ, and the effective Klein-Gordon equation for $\delta\varphi$ is [20]

$$\delta\varphi'' + 2\mathcal{H}B_1\delta\varphi' - (s^2\nabla^2 - \nu p V_{,\varphi\varphi})\delta\varphi - B_2\varphi'\psi' + 2B_3\mathcal{H}\varphi'\psi = 0, \tag{18}$$

where

$$B_1 = 1 - \frac{d\ln\nu}{d\ln p} - \frac{dg_1}{d\ln p} = 1 + B_{10}\delta_{\rm pl}, \tag{19}$$

$$B_2 = 4 + f_1 + h + 3g_1 = 4 + B_{20}\delta_{\rm pl}, \tag{20}$$

$$B_3 = (1 + f_1 + h)\frac{\nu p V_{,\varphi}}{\mathcal{H}\varphi'} - \frac{dh}{d\ln p} - \frac{df_3}{d\ln p} = -2 - \frac{\varphi''}{\mathcal{H}\varphi'} + B_{30}\delta_{\rm pl}, \tag{21}$$

and

$$B_{10} \equiv \sigma\left[\nu_0\left(\frac{\sigma}{6} + 1\right) - \frac{\alpha_0}{2}\right], \tag{22}$$

$$B_{20} \equiv \frac{\sigma}{2}\left(\frac{\sigma\nu_0}{3} + \frac{10\nu_0}{3} - 3\alpha_0\right), \tag{23}$$

$$B_{30} \equiv \sigma\left[\left(\frac{\nu_0}{6} - \alpha_0\right)\left(1 - \frac{\varphi''}{\mathcal{H}\varphi'}\right) - \nu_0\left(\frac{\sigma}{12} + 2\right) + \frac{\alpha_0}{2}(7 - \sigma)\right], \tag{24}$$

and $s^2 = \alpha^2(1 - f_3)$ is the squared propagation speed of the perturbation.

The other dynamical equations can be seen in [19].

III. EFFECTIVE HYDRODYNAMICAL PERTURBATION

One can notice that the dynamical equations of the perturbation Eq. (17) and (18) are all based on the matter of scalar field. For discussing the multicomponent model, it is convenient to generalize the model to a hydrodynamical form, in which the universe is fulfilled by a general perfect fluid.

To our knowledge, the theory of the cosmological perturbations with the inverse-triad corrections based on the general perfect fluid model has not been built up. However, by using the equivalence between the dynamics of the scalar field and the perfect fluid, we can generalize the dynamical equations of the scalar field to a general perfect fluid model. This equivalence has been reviewed in [30] and discussed in [31].

In some model of the classical cosmology, the scalar field can be viewed as a sort of the perfect fluid [22]. If we rewrite the dynamical equations for the metric perturbation and the matter perturbation to the hydrodynamical forms, then we can find that they are also legitimate for a more general perfect fluid. This implies that the scalar field has the general feature of the perfect fluid, and there is an equivalence between the dynamics of the scalar field and the perfect fluid.

Certainly, a fluid is different from a scalar field, so whether this equivalence is generally valid is still an open question. But in most cosmological model, the dynamics of the perfect fluid is exactly equivalent to those of a derivatively coupled scalar field, playing the role of the fluid potential velocity [30].

Although the equivalence between the scalar field and the perfect fluid has not been studied under the framework of LQC, we can assume that the quantum corrections which come from LQC will not influence this equivalence, i.e., this equivalence is also legitimate in the theory of LQC.

So, in order to get the dynamical equations of perturbations for general perfect fluid, we adopt a simple strategy. First, we rewrite the effective equations Eq. (17) and (18) to hydrodynamical forms, then we define an effective density ρ_e, an effective pressure P_e, and their perturbations $\delta\rho_e$ and δP_e. And then we represent the evolution equations by these effective quantities. Finally, we assume that these hydrodynamical equations are also legitimate for the more general perfect fluid.

Therefore, there are two requirements for our hydrodynamical form.

(i) The classical limit of the effective density, the effective pressure, and their perturbations should coincide with the classical density and pressure of fluid.

(ii) The classical limit of the effective hydrodynamical perturbation equations with inverse-triad corrections of LQC should coincide with the classical equations [1].

A. Effective hydrodynamical equations

At first, we give the background equations in fluid form. The effective Friedmann equation in fluid form can be seen in [18]

$$\mathcal{H}^2 = \frac{8\pi G}{3}(\alpha p)\rho_e. \tag{25}$$

Compared with Eq. (2), we have $\rho_e \equiv \frac{\varphi'^2}{2p\nu} + V(\varphi)$, where the subscript "e" means the "effective."

From Eq. (10) and (25), one can obtain the effective continuity equation

$$\rho_e' + 3\mathcal{H}\left(1 - \frac{1}{3}\frac{d\ln\nu}{d\ln p}\right)(\rho_e + P_e) = 0, \tag{26}$$

where $P_e \equiv \frac{\varphi'^2}{2p\nu} - V(\varphi)$ is the effective pressure.

Equation (17) inspires us to define an effective density perturbation as follows:

$$\delta\rho_e \equiv \frac{1}{p\nu}[\varphi'\delta\varphi' - (1 + f_1)\varphi'^2\phi] + V,_\varphi \delta\varphi, \tag{27}$$

and we define an effective pressure perturbation analogously

$$\delta P_e \equiv \frac{1}{p\nu}[\varphi'\delta\varphi' - (1 + f_1)\varphi'^2\phi] - V,_\varphi \delta\varphi. \tag{28}$$

Under these definitions, Eq. (17) changes to

$$\begin{aligned} &3\mathcal{H}(1 + f)[\psi' + (1 + f)\mathcal{H}\phi] - \alpha^2\nabla^2\psi \\ &\quad = -4\pi G\alpha p(1 + f_3)\delta\rho_e. \end{aligned} \tag{29}$$

From Eqs. (10), (18), and (27), it can be verified that $\delta\rho_e$ is suitable for equation as follows:

$$\begin{aligned} &\delta\rho_e' + 3\mathcal{H}\left(1 - \frac{1}{3}\frac{d\ln\nu}{d\ln p} - \frac{1}{3}\frac{dg_1}{d\ln p}\right)(\delta\rho_e + \delta P_e) \\ &\quad - 3(1 + g_1)(\rho_e + P_e)\psi' - s^2\frac{\nabla^2\varphi'\delta\varphi}{p\nu} = 0. \end{aligned} \tag{30}$$

On the large-scale limit, where the ∇^2 term tends to vanish, Eq. (30) changes to

$$\delta\rho_e' + 3\mathcal{H}D_1(\delta\rho_e + \delta P_e) - 3D_2(\rho_e + P_e)\psi' = 0, \tag{31}$$

where

$$D_1 \equiv 1 - \frac{1}{3}\frac{d\ln\nu}{d\ln p} - \frac{1}{3}\frac{dg_1}{d\ln p}, \tag{32}$$

$$D_2 \equiv 1 + g_1. \tag{33}$$

At this point, the definitions of ρ_e, P_e, $\delta\rho_e$ and δP_e as well as Eq. (29) and (31) satisfy the two requirements on the large scales mentioned above.

At least on the large scale, we can rewrite the theory of the gauge-invariant perturbation of LQC to the hydrodynamical form. From now on, we assume that these hydrodynamical equations are valid not only for the scalar field but also for a general perfect fluid, and we restrict our discussion to the large-scale limit.

B. Curvature perturbation on uniform-density hypersurfaces

It is convenient for the cosmological applications to introduce a curvature perturbation on a uniform-density hypersurface ζ, which first introduced by Bardeen, Steinhardt, and Turner [32] as a conserved quantity for the adiabatic perturbation on the large scale [33].

In the classical cosmology, the definition of ζ is

$$-\zeta \equiv \psi + \mathcal{H}\frac{\delta\rho}{\rho'}. \tag{34}$$

We introduce a LQC correction D_3 into this definition

$$-\zeta_e \equiv \psi + D_3\mathcal{H}\frac{\delta\rho_e}{\rho_e'}. \tag{35}$$

But we do not fix the form of D_3 right now.

For the classical cosmology, ζ is a conservative and a gauge-invariant quantity on the large scale. Therefore, it is natural to require that the effective curvature perturbation ζ_e has also the similar properties.

The evolution equation of ζ_e can be obtained by taking the time derivative of Eq. (35)

$$\begin{aligned} \zeta_e' = -\psi' - \Bigg[&D_3'\mathcal{H}\frac{\delta\rho_e}{\rho_e'} + D_3\Bigg(\mathcal{H}'\frac{\delta\rho_e}{\rho_e'} \\ &+ \mathcal{H}\frac{\delta\rho_e'}{\rho_e'} - \mathcal{H}\delta\rho_e\frac{\rho_e''}{\rho_e'^2}\Bigg)\Bigg]. \end{aligned} \tag{36}$$

From Eq. (26) we have

$$\begin{aligned} &\rho_e'' + 3[\mathcal{H}'(1 - f + f_1) + \mathcal{H}(f_1 - f)'](\rho_e + P_e) \\ &\quad + 3\mathcal{H}(1 - f + f_1)(\rho_e' + P_e') = 0. \end{aligned} \tag{37}$$

Substituting Eq. (37) to Eq. (36) we have

$$\begin{aligned} \zeta_e' = -\psi' - \frac{1}{\rho_e'}\Bigg\{&D_3'\mathcal{H}\delta\rho_e + D_3\mathcal{H}'\delta\rho_e + D_3\mathcal{H}\delta\rho_e' \\ &+ 3D_3\mathcal{H}\delta\rho_e\Bigg[\frac{\mathcal{H}'}{\rho_e'}(1 - f + f_1)(\rho_e + P_e) \\ &+ \frac{\mathcal{H}}{\rho_e'}(f_1 - f)'(\rho_e + P_e) + (1 - f + f_1)\mathcal{H}(1 + c_{se}^2)\Bigg]\Bigg\}, \end{aligned} \tag{38}$$

where $c_{se}^2 \equiv P_e'/\rho_e'$ is an effective adiabatic sound speed. By using Eq. (31) and (32) we have

$$\begin{aligned} \zeta_e' = -\frac{1}{\rho_e'}\Bigg\{&3\mathcal{H}(\rho_e + P_e)\psi'[D_2D_3 - (1 - f + f_1)] \\ &+ \left[D_3' - \frac{D_3(f_1 - f)'}{1 - f + f_1}\right]\mathcal{H}\delta\rho_e + 3D_3\mathcal{H}^2(\delta\rho_e + \delta P_e) \\ &\times(1 - f + f_1 - D_1) - 3\mathcal{H}^2\delta P_{\text{nad}}\Bigg\}, \end{aligned} \tag{39}$$

where $\delta P_{\text{nad}} \equiv \delta P_e - c_{se}^2\delta\rho_e$ is a nonadiabatic pressure perturbation.

One can notice that if we set $D_3 = 1 - f + f_1$ and choose suitable σ, ℓ and q to make $g_1 = 0$ ($D_2 = 1$ and $D_1 = D_3$), we will find a simple relationship

$$\zeta_e' = -\frac{\mathcal{H}\delta P_{\text{nad}}}{(1 - f + f_1)(\rho_e + P_e)}. \tag{40}$$

So, the definition of ζ_e could be

$$-\zeta_e \equiv \psi + (1 - f + f_1)\mathcal{H}\frac{\delta\rho_e}{\rho_e'}. \tag{41}$$

Under this definition, ζ_e is a conserved quantity on the large scale when we neglect the nonadiabatic perturbation (when $g_1 = 0$). This is the same as the classical cosmology. However, this definition of ζ_e is not necessarily gauge-invariant. We discuss this topic next.

Back to the scalar model, the gauge transformations for $\delta\varphi$, ψ and ϕ are [19]

$$\delta\varphi \to \delta\varphi + \varphi'(1 + f_1)\xi^0, \tag{42}$$

$$\psi \to \psi - \mathcal{H}(1 + f)\xi^0, \tag{43}$$

$$\phi \to \phi + \mathcal{H}\xi^0 + (\xi^0)', \tag{44}$$

where ξ^0 is the 0-component of the infinitesimal coordinate transformations ξ^μ.

From the definition Eq. (27) of $\delta\rho_e$, we can obtain both the gauge transformation for $\delta\rho_e$ and the gauge transformation for ζ_e as, respectively,

$$\delta\rho_e \to \delta\rho_e + \frac{\varphi'^2\mathcal{H}\xi^0}{p\nu}\left[\frac{f_1'}{\mathcal{H}} - (1 + f_1)\left(3 - 2\frac{d\ln\nu}{d\ln p}\right)\right], \tag{45}$$

and

$$\zeta_e \to \zeta_e + \mathcal{H}\xi^0(1 + f) + \mathcal{H}\xi^0(1 - f + f_1) \times \frac{\frac{f_1'}{\mathcal{H}} - (1 + f_1)(3 - 2\frac{d\ln\nu}{d\ln p})}{3 - \frac{d\ln\nu}{d\ln p}}. \tag{46}$$

If we require that ζ_e is gauge-invariant, we need

$$(1 + f) + (1 - f + f_1)\frac{\frac{f_1'}{\mathcal{H}} - (1 + f_1)(3 - 2\frac{d\ln\nu}{d\ln p})}{3 - \frac{d\ln\nu}{d\ln p}} = 0. \tag{47}$$

From Eqs. (11)–(15), then Eq. (47) can be reduced to

$$9\nu_0 + \sigma\nu_0 = 3\alpha_0. \tag{48}$$

This condition is the same as $g_1 = 0$. So, when $g_1 = 0$, ζ_e, which we have defined in this paper, is a gauge-invariant and a conserved quantity on the large scale.

IV. NON-ADIABATIC PERTURBATIONS

A general thermodynamic system can be fully described by three variables: (ρ, P, S), where ρ is the energy density, P the pressure, and S the entropy. However, only two of them are independent. If we choose ρ and S as two independent variables, the pressure can be expressed as $P \equiv P(S, \rho)$. Then, the pressure perturbation can be expanded into a Taylor series as

$$\delta P = \frac{\partial P}{\partial S}\delta S + \frac{\partial P}{\partial \rho}\delta\rho. \tag{49}$$

This can be recast in a more familiar form

$$\delta P = \delta P_{\rm nad} + c_s^2\delta\rho, \tag{50}$$

where $c_s^2 \equiv \frac{\partial P}{\partial \rho}|_S$ is the adiabatic sound speed. If the system is adiabatic, which means $\delta S = 0$, we can find that $\delta P = c_s^2\delta\rho$. Therefore, $c_s^2\delta\rho$ is the adiabatic part of δP and $\delta P_{\rm nad} \equiv \frac{\partial P}{\partial S}|_\rho\delta S$ is the nonadiabatic part of it [28].

One can notice that, when the system is adiabatic, $\delta P = \frac{\partial P}{\partial \rho}\delta\rho$ means that P is only a function of ρ, i.e., $P = P(\rho)$. Therefore, $P = P(\rho)$ can be seen as the adiabatic condition. By extension, for a thermodynamic quantity $\mathcal{G}$, its adiabatic condition is that it is only the function of ρ, i.e., $\mathcal{G} = \mathcal{G}(\rho)$.

A. Interacting fluids

In this section, we will consider a multifluid model. The number of the interacting fluids in universe is arbitrary. Each fluid has an energy-momentum tensor $T^{\mu\nu}_{(\alpha)}$; the symbol α here denotes a different fluid, μ or ν denotes a spacetime index, the inverse-triad correction of LQC is not included. The total energy-momentum tensor $T^{\mu\nu} = \sum_\alpha T^{\mu\nu}_{(\alpha)}$, is covariantly conserved, but, for energy, we allow transfer between the fluids

$$\nabla_\mu T^{\mu\nu}_{(\alpha)} = Q^\nu_\alpha, \tag{51}$$

where Q^ν_α is the quantity of the energy transferring in α fluid. The total energy-momentum tensor is conservation, i.e., $\nabla_\mu T^{\mu\nu} = 0$, so it requires that $\sum_\alpha Q^\nu_\alpha = 0$.

Total energy density and total pressure are

$$\rho_e = \sum_\alpha \rho_{e(\alpha)}, \qquad P_e = \sum_\alpha P_{e(\alpha)}. \tag{52}$$

The continuity equation for each individual fluid is thus [34]

$$\rho'_{e(\alpha)} + 3\mathcal{H}\left(1 - \frac{1}{3}\frac{d\ln\nu}{d\ln p}\right)(\rho_{e(\alpha)} + P_{e(\alpha)}) = Q_\alpha, \tag{53}$$

where Q_α is the time component of the energy transferring vector Q^0_α and we assume the inverse-triad correction for all fluids is the same.

The perturbation for energy transferring can be written as $Q_\alpha\phi + \delta Q_\alpha$ [34]. So the evolution equation of the density perturbation for each individual fluid is

$$\delta\rho'_{e(\alpha)} + 3\mathcal{H}D_1(\delta\rho_{e(\alpha)} + \delta P_{e(\alpha)}) - 3D_2(\rho_{e(\alpha)} + P_{e(\alpha)})\psi' = Q_\alpha\phi + \delta Q_\alpha. \tag{54}$$

Analogous to the definition of ζ_e, we can define the curvature perturbation for each individual fluid

$$-\zeta_{e(\alpha)} \equiv \psi + (1 - f + f_1)\mathcal{H}\frac{\delta\rho_{e(\alpha)}}{\rho'_{e(\alpha)}}. \tag{55}$$

It can be proved easily that the total curvature perturbation ζ_e is a weighted sum of the individual perturbation

$$\zeta_e = \sum_\alpha \frac{\rho'_{e(\alpha)}}{\rho'_e}\zeta_{e(\alpha)}. \tag{56}$$

The difference between any two curvature perturbations describes a relative entropy (or isocurvature) perturbation

$$\begin{aligned}\mathcal{S}_{\alpha\beta} &= 3(\zeta_{e(\alpha)} - \zeta_{e(\beta)}) \\ &= -3(1-f+f_1)\mathcal{H}\left(\frac{\delta\rho_{e(\alpha)}}{\rho'_{e(\alpha)}} - \frac{\delta\rho_{e(\beta)}}{\rho'_{e(\beta)}}\right).\end{aligned} \tag{57}$$

B. Evolution equations

In the multifluid model, the total nonadiabatic pressure perturbation $\delta P_{\rm nad}$ may be split into two parts:

$$\delta P_{\rm nad} \equiv \delta P_{\rm intr} + \delta P_{\rm rel}, \tag{58}$$

where $\delta P_{\rm intr} \equiv \sum_\alpha \delta P_{{\rm intr}(\alpha)}$, and $\delta P_{{\rm intr}(\alpha)}$ is the intrinsic nonadiabatic pressure perturbation of each fluid; its definition is

$$\delta P_{{\rm intr}(\alpha)} \equiv \delta P_{e(\alpha)} - c^2_{e\alpha}\delta\rho_{e(\alpha)}, \tag{59}$$

where $c^2_{e\alpha} \equiv P'_{e(\alpha)}/\rho'_{e(\alpha)}$ is the effective sound speed of each fluid. It is related to c^2_{se} by

$$c^2_{se} = \sum_\alpha \frac{\rho'_{e(\alpha)}}{\rho_e} c^2_{e\alpha}. \tag{60}$$

From the Eq. (58), one can notice that the first part of $\delta P_{\rm nad}$ comes from the intrinsic nonadiabatic perturbation of each individual fluid. And the other part $\delta P_{\rm rel}$ should come from the relative entropy perturbation $\mathcal{S}_{\alpha\beta}$. From Eqs. (57) and (58) one can represent the $\delta P_{\rm rel}$ by $\mathcal{S}_{\alpha\beta}$

$$\delta P_{\rm rel} = -\frac{1+f-f_1}{6\mathcal{H}\rho'_e}\sum_{\alpha,\beta}\rho'_{e(\alpha)}\rho'_{e(\beta)}(c^2_{e\alpha} - c^2_{e\beta})\mathcal{S}_{\alpha\beta}. \tag{61}$$

If $P_{e(\alpha)} = P_{e(\alpha)}(\rho_{e(\alpha)})$, the intrinsic nonadiabatic pressure perturbation $\delta P_{{\rm intr}(\alpha)}$ and $\delta P_{\rm intr}$ will vanish. Even so, the $\delta P_{\rm rel}$ will not be vanished, because it depends on the curvature perturbation of the individual fluid $\zeta_{e(\alpha)}$.

The evolution of $\zeta_{e(\alpha)}$ can be obtained from Eq. (55)

$$\begin{aligned}\zeta'_{e(\alpha)} &= \frac{3\mathcal{H}^2(1-f+f_1)^2\delta P_{{\rm intr}(\alpha)}}{\rho'_{e(\alpha)}} - (1-f+f_1) \\ &\quad\times \frac{\mathcal{H}Q_\alpha}{\rho'_{e(\alpha)}}\left\{\frac{\psi'}{(1-f+f_1)\mathcal{H}} + \frac{\delta\rho_{e(\alpha)}}{\rho'_{e(\alpha)}}\left[-\frac{Q'_\alpha}{Q_\alpha}\right.\right. \\ &\quad\left.\left. + \frac{\mathcal{H}'}{\mathcal{H}} + \frac{(f_1-f)'}{1-f+f_1}\right] + \phi + \frac{\delta Q_\alpha}{Q_\alpha}\right\}.\end{aligned} \tag{62}$$

Similar to what is done in [28], we define an effective nonadiabatic perturbation of energy transferring by

$$\begin{aligned}\delta Q_{{\rm nad}(\alpha)} \equiv Q_\alpha&\left\{\frac{\psi'}{(1-f+f_1)\mathcal{H}} + \frac{\delta\rho_{e(\alpha)}}{\rho'_{e(\alpha)}}\left[-\frac{Q'_\alpha}{Q_\alpha} + \frac{\mathcal{H}'}{\mathcal{H}}\right.\right. \\ &\left.\left. + \frac{(f_1-f)'}{1-f+f_1}\right] + \phi + \frac{\delta Q_\alpha}{Q_\alpha}\right\}.\end{aligned} \tag{63}$$

Different from the classical situation, there is an effective quantum term in the nonadiabatic perturbation of energy transferring. It means that LQC quantum corrections will affect the energy transferring between different fluids. This influence is nonadiabatic. If there is no energy transferring between the different fluids, i.e., $Q_\alpha = 0$, then $\delta Q_{{\rm nad}(\alpha)} = 0$. If $\delta P_{{\rm intr}(\alpha)}$ vanishes also, the $\zeta_{e(\alpha)}$ will conserve on the large scale. However, speaking in general, $\delta Q_{{\rm nad}(\alpha)} \neq 0$. Therefore, $\zeta_{e(\alpha)}$ is not conservation on the large scale.

There are two source for $\zeta_{e(\alpha)}$; one is an intrinsic nonadiabatic perturbation, and the other is a nonadiabatic perturbation of energy transferring. The nonadiabatic perturbation of energy transferring can also be split into two parts as the same of $\delta P_{\rm nad}$

$$\delta Q_{{\rm nad}(\alpha)} = \delta Q_{{\rm intr}(\alpha)} + \delta Q_{{\rm rel}(\alpha)}, \tag{64}$$

where the definition of the intrinsic part is

$$\delta Q_{{\rm intr}(\alpha)} \equiv \delta Q_\alpha - (Q'_\alpha/\rho'_{e(\alpha)})\delta\rho_{e(\alpha)}. \tag{65}$$

If $Q_\alpha = Q_\alpha(\rho_{e(\alpha)})$, then $\delta Q_{{\rm intr}(\alpha)} = 0$. Thus, by using Eq. (29) and (63), one can obtain (notice that $g_1 = 0 \Rightarrow f_1 = f_3$)

$$\delta Q_{{\rm rel}(\alpha)} = \frac{Q_\alpha\rho'_e}{2\rho_e}\left(\frac{\delta\rho_{e(\alpha)}}{\rho'_{e(\alpha)}} - \frac{\delta\rho_e}{\rho'_e}\right) + \frac{Q_\alpha\mathcal{H}\delta\rho_{e(\alpha)}}{\rho'_{e(\alpha)}} + Q_\alpha\mathcal{E}_\alpha, \tag{66}$$

where

$$\mathcal{E}_\alpha \equiv (2f-f_1)\left[(\sigma+1)\mathcal{H}\frac{\delta\rho_{e(\alpha)}}{\rho'_{e(\alpha)}} + \zeta_{e(\alpha)}\right]. \tag{67}$$

$\mathcal{E}_\alpha$ is an effective quantum term which vanishes only in the classical limit. One should notice that it depends on the curvature perturbation of the individual fluid $\zeta_{e(\alpha)}$.

The first term of Eq. (66) can be represented by $\mathcal{S}_{\alpha\beta}$

$$\frac{Q_\alpha\rho'_e}{2\rho_e}\left(\frac{\delta\rho_{e(\alpha)}}{\rho'_{e(\alpha)}} - \frac{\delta\rho_e}{\rho'_e}\right) = -\frac{(1+f-f_1)Q_\alpha}{6\mathcal{H}\rho_e}\sum_\beta \rho'_{e(\beta)}\mathcal{S}_{\alpha\beta}. \tag{68}$$

If we represent our discussion by cosmic time, then the second term of Eq. (66) can be absorbed into the first term [28]. From Eq. (61), (68), and (66), we have

$$\delta P_{\rm rel} = \frac{2\rho_e}{\rho'_e}\sum_\alpha \rho'_{e(\alpha)}c^2_{e(\alpha)}\left(\frac{\delta Q_{rel(\alpha)}}{Q_\alpha} - \frac{\mathcal{H}\delta\rho_{e(\alpha)}}{\rho'_{e(\alpha)}} + \mathcal{E}_\alpha\right). \tag{69}$$

We find that there is also an effective quantum term in $\delta P_{\rm rel}$.

From the definition of $\mathcal{S}_{\alpha\beta}$, we know that

$$\mathcal{S}'_{\alpha\beta} = 3(\zeta'_{e(\alpha)} - \zeta'_{e(\beta)}). \tag{70}$$

By using Eq. (62) and (68), we can obtain the evolution of the entropy perturbation

$$\mathcal{S}'_{\alpha\beta} = \mathcal{A}_{\rm intr(\alpha\beta)} + \mathcal{B}_{\rm iso(\alpha\beta)} + \mathcal{C}_{\rm quan(\alpha\beta)}, \tag{71}$$

where

$$\begin{aligned}\mathcal{A}&_{\rm intr(\alpha\beta)}\\ &= 3(1-f+f_1)\mathcal{H}\bigg(\frac{3\mathcal{H}(1-f+f_1)\delta P_{\rm intr(\alpha)} - \delta Q_{\rm intr(\alpha)}}{\rho'_{e(\alpha)}}\\ &\quad - \frac{3\mathcal{H}(1-f+f_1)\delta P_{\rm intr(\beta)} - \delta Q_{\rm intr(\beta)}}{\rho'_{e(\beta)}}\bigg),\end{aligned} \tag{72}$$

$$\mathcal{B}_{\rm iso(\alpha\beta)} = -\sum_\gamma \frac{\rho'_{e(\gamma)}}{2\rho_e}\bigg(\frac{Q_\alpha}{\rho'_{e(\alpha)}}\mathcal{S}_{\alpha\gamma} - \frac{Q_\beta}{\rho'_{e(\beta)}}\mathcal{S}_{\beta\gamma}\bigg), \tag{73}$$

$$\mathcal{C}_{\rm quan(\alpha\beta)} = -3(1-f+f_1)\mathcal{H}\bigg(\frac{Q_\alpha}{\rho'_{e(\alpha)}}\mathcal{E}_\alpha - \frac{Q_\beta}{\rho'_{e(\beta)}}\mathcal{E}_\beta\bigg) \tag{74}$$

$$\begin{aligned}&= -3(2f-f_1)\mathcal{H}\bigg[\frac{1}{3}\bigg(\frac{Q_\alpha}{\rho'_{e(\alpha)}}\sum_\gamma \frac{\rho'_{e(\gamma)}}{\rho'_e}\mathcal{S}_{\alpha\gamma}\\ &\quad - \frac{Q_\beta}{\rho'_{e(\beta)}}\sum_\gamma \frac{\rho'_{e(\gamma)}}{\rho'_e}\mathcal{S}_{\beta\gamma}\bigg) + \zeta_e\bigg(\frac{Q_\alpha}{\rho'_{e(\alpha)}} - \frac{Q_\beta}{\rho'_{e(\beta)}}\bigg)\\ &\quad + (\sigma+1)\mathcal{H}\bigg(\frac{Q_\alpha\delta\rho_{e(\alpha)}}{\rho'^2_{e(\alpha)}} - \frac{Q_\beta\delta\rho_{e(\beta)}}{\rho'^2_{e(\beta)}}\bigg)\bigg].\end{aligned} \tag{75}$$

The relationship

$$\zeta_{e(\alpha)} = \zeta_e + \frac{1}{3}\sum_\gamma \frac{\rho'_{e(\gamma)}}{\rho'_e}\mathcal{S}_{\alpha\gamma} \tag{76}$$

is used in the last equation.

As we can see, there are three sources for the entropy perturbation. The first is the intrinsic nonadiabatic perturbation $\mathcal{A}_{\rm intr(\alpha\beta)}$, the second is the entropy perturbation of others fluid $\mathcal{B}_{\rm iso(\alpha\beta)}$, and the third comes from the effective quantum correction $\mathcal{C}_{\rm quan(\alpha\beta)}$.

There are also three parts to the quantum source. The first part comes from the entropy perturbation of others fluid. The second part comes from the adiabatic curvature perturbation ζ_e. Speaking in general, the third term of Eq. (75) cannot be represented by $\mathcal{S}_{\alpha\beta}$ completely, so it depends on the adiabatic curvature perturbation, too.

From Eq. (74), we know that the quantum source for the entropy perturbation comes from $\mathcal{E}_\alpha$. And from Eq. (66) one can find that $\mathcal{E}_\alpha$ affects the energy transferring between different fluids. So, the quantum source is caused actually by the energy transferring due to quantum effect, and this quantum energy transferring is related to the adiabatic curvature perturbation $\zeta_{e(\alpha)}$. Therefore, the adiabatic curvature perturbation has a quantum influence on the energy transferring between different fluids, and this influence affects the evolution of nonadiabatic entropy.

So, we can come to a conclusion that the nonadiabatic entropy on the large scale can be driven by the adiabatic curvature perturbation. This conclusion is different from the classical cosmology, and this adiabatic source for the nonadiabatic perturbations reflects the quantum nature of the system.

However, from Eq. (75) we can see that, if the following condition

$$\frac{Q_\alpha}{\rho'_{e(\alpha)}} = \frac{Q_\beta}{\rho'_{e(\beta)}} = \mathcal{Z}, \quad \forall\ \alpha, \beta \tag{77}$$

can be satisfied, then the quantum adiabatic source will vanish, where $\mathcal{Z}$ is the same for all fluids. Under this condition, the second term of Eq. (75) is equal to zero, and the third term can be represented by the entropy perturbation $\mathcal{S}_{\alpha\beta}$. This implies that the evolution of the entropy perturbation only depends on the nonadiabatic perturbation, and itself obeys a homogeneous second-order equation on the super-Hubble scales. It is the same as the classical conclusion.

One needs to notice that, due to energy conservation, i.e., $\sum_\alpha Q_\alpha = 0$, the condition (77) leads to $\sum_\alpha Q_\alpha = \mathcal{Z}\sum_\alpha \rho'_{e(\alpha)} = \mathcal{Z}\rho'_e = 0$. This means that there are two possible scenarios. First, $\mathcal{Z} = 0$, which indicates that there is no energy transferring between different fluids. As we have discussed above, the quantum source for the entropy perturbation comes from the energy transferring between different fluids. So, for the model in which the energy transferring does not exist, the quantum adiabatic source will vanish, and the entropy perturbation can evolve independently as in the classical cosmology. Second, $\rho'_e = 0$, which shows that $\rho_e = \Lambda$ where Λ is a constant. From Eq. (26) we know that in this situation, $P_e = -\Lambda$. It is nothing but a cosmological constant. So for the cosmology model in which the matter in universe behaves like a cosmological constant, the quantum adiabatic source will also be vanished.

V. DECAY MODEL

After deriving a general formalism of the adiabatic and the nonadiabatic perturbations for loop quantum cosmology, in this section we apply it to a simple decay model, that is, a specific case of the nonrelativistic matter ϑ decaying into radiation ς. This process could be used in the curvaton scenario [26,35–37]. In the model considered here, we assume that the energy density is dominated by the radiation ρ_ς and it is unperturbed, i.e., $\delta\rho_\varsigma \simeq 0$.

We assume that the matter ϑ is unstable. It can decay into a radiation ς with a decay rate Ξ. In our discussion, the decay rate Ξ is treated as a constant, and the energy transfers from the pressureless fluid to the radiation fluid. We will give the evolving equations for the adiabatic and the nonadiabatic perturbations and solve them numerically. At the same time, we will give the results of the classical perturbations theory, used as a comparison.

A. Background

The energy transferring from ϑ to ς is described by

$$Q_\vartheta = -Q_\varsigma = -\Xi\rho_\vartheta. \tag{78}$$

And the energy conservation equations are

$$\rho'_\vartheta = -\rho_\vartheta\left(3\mathcal{H} - \mathcal{H}\frac{d\ln\nu}{d\ln p} + \Xi\right), \tag{79}$$

$$\rho'_\varsigma = -4\mathcal{H}\left(1 - \frac{1}{3}\frac{d\ln\nu}{d\ln p}\right)\rho_\varsigma + \Xi\rho_\vartheta, \tag{80}$$

$$\rho'_e = -\mathcal{H}\left(1 - \frac{1}{3}\frac{d\ln\nu}{d\ln p}\right)(3\rho_\vartheta + 4\rho_\varsigma). \tag{81}$$

Also the Friedmann equation is

$$\mathcal{H}^2 = \frac{8\pi G}{3}\alpha p(\rho_\vartheta + \rho_\varsigma). \tag{82}$$

It is convenient to introduce the dimensionless density parameters Ω_ϑ, Ω_ς and the reduced decay rate g:

$$\Omega_\vartheta \equiv \frac{\rho_\vartheta}{\rho_e}, \qquad \Omega_\varsigma \equiv \frac{\rho_\varsigma}{\rho_e}, \qquad g \equiv \frac{\Xi}{\Xi + \mathcal{H}}. \tag{83}$$

After that, the Eqs. (79)–(82) can be rewritten as

$$\partial_{\ln a}\Omega_\vartheta = \Omega_\vartheta\left[\left(1 - \frac{1}{3}\frac{d\ln\nu}{d\ln p}\right)\Omega_\varsigma - \frac{g}{1-g}\right], \tag{84}$$

$$\partial_{\ln a}\Omega_\varsigma = \Omega_\vartheta\left[\frac{g}{1-g} - \left(1 - \frac{1}{3}\frac{d\ln\nu}{d\ln p}\right)\Omega_\varsigma\right], \tag{85}$$

$$\partial_{\ln a} g = \frac{g(1-g)}{2}\left[\left(1 - \frac{1}{3}\frac{d\ln\nu}{d\ln p}\right)(4 - \Omega_\vartheta) + 1 + \frac{d\ln\alpha}{d\ln p}\right] \tag{86}$$

From the definitions of α and ν we note that:

$$\frac{d\ln\alpha}{d\ln p} \propto \frac{d\ln\nu}{d\ln p} \propto \exp(-\sigma\ln a). \tag{87}$$

So the Eqs. (84)–(86) can be solved by numerical method.

However, there is a constraint to this system:

$$\Omega_\vartheta + \Omega_\varsigma = 1. \tag{88}$$

Therefore, there are only two independent dynamical equations. The solutions with a fixed initial condition are illustrated in Fig. 1 and 2. We can see that the expansion of universe and the decay of matter ϑ are both faster than classical evolution.

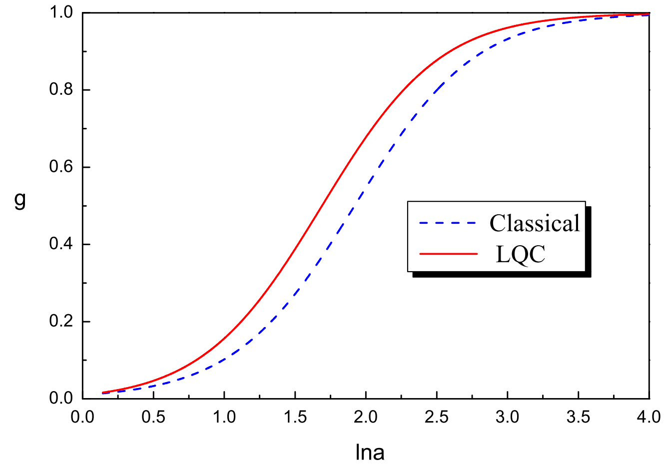

FIG. 1 (color online). The evolution of the reduced decay rate g as a function of $\ln a$. Here, the initial density Ω_ϑ, the initial decay rate g, and the parameter σ are taken as, respectively, $\Omega_\vartheta = 10^{-1}$, $g = 10^{-2}$, and $\sigma = 2$. The classical evolution is represented by the dashed line.

B. Perturbations

Both ϑ and ς have fixed equations of state, hence they meet the intrinsic adiabatic condition, i.e., $\delta P_{\mathrm{intr}(\vartheta)} = \delta P_{\mathrm{intr}(\varsigma)} = 0$. However, there is a nonzero entropy perturbation $\mathcal{S}_{\vartheta\varsigma}$, so the curvature perturbation ζ_e is not a conserved quantity on the large scale. From Eq. (40) and (61) we have

$$\zeta'_e = \frac{(1 + f - f_1)\mathcal{H}}{3\rho_e'^2}\rho'_\vartheta\rho'_\varsigma\mathcal{S}_{\vartheta\varsigma}. \tag{89}$$

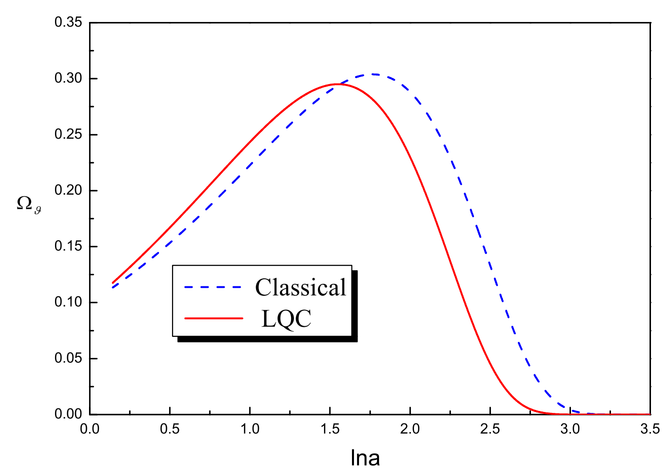

FIG. 2 (color online). The evolution of the dimensionless density parameter Ω_ϑ as a function of $\ln a$. Here, the initial density Ω_ϑ, the initial decay rate g, and the parameter σ are taken as, respectively, $\Omega_\vartheta = 10^{-1}$, $g = 10^{-2}$, and $\sigma = 2$. The classical evolution is represented by the dashed line.

The perturbed energy transferring is given by

$$\delta Q_{\vartheta} = -\delta Q_{\varsigma} = -\Xi \delta \rho_{\vartheta}, \tag{90}$$

where we assume Ξ is fixed by microphysics, i.e., $\delta \Xi = 0$.

The energy transferring of ϑ is determined only by its energy density, therefore $\delta Q_{\mathrm{intr}(\vartheta)} = 0$. However, for radiation ς, its energy transferring depends on the decay of ϑ, so $\delta Q_{\mathrm{intr}(\varsigma)} \neq 0$. We can find that

$$\delta Q_{\mathrm{intr}(\varsigma)} = \delta Q_{\varsigma} - \frac{Q'_{\varsigma}}{\rho'_{\varsigma}} \delta \rho_{\varsigma} = -\frac{\Xi \rho'_{\vartheta}}{3\mathcal{H}(1 - f + f_1)} \mathcal{S}_{\vartheta\varsigma}. \tag{91}$$

Under our hypothesis, $\delta \rho_{\varsigma} \simeq 0$, we have

$$\mathcal{S}_{\vartheta\varsigma} \simeq -3(1 - f + f_1)\mathcal{H} \frac{\delta \rho'_{\vartheta}}{\rho'_{\vartheta}}. \tag{92}$$

Therefore, we can obtain the evolution equation for the entropy perturbation

$$\mathcal{S}'_{\vartheta\varsigma} = \mathcal{A}_{\mathrm{intr}(\vartheta\varsigma)} + \mathcal{B}_{\mathrm{iso}(\vartheta\varsigma)} + \mathcal{C}_{\mathrm{quan}(\vartheta\varsigma)}, \tag{93}$$

where

$$\mathcal{A}_{\mathrm{intr}(\vartheta\varsigma)} = -\Xi \frac{\rho'_{\vartheta}}{\rho'_{\varsigma}} \mathcal{S}_{\vartheta\varsigma}, \tag{94}$$

$$\mathcal{B}_{\mathrm{iso}(\vartheta\varsigma)} = \frac{\Xi \rho_{\vartheta}}{2\rho_e} \left(\frac{\rho'_{\varsigma}}{\rho'_{\vartheta}} - \frac{\rho'_{\varsigma}}{\rho'_{\vartheta}} \right) \mathcal{S}_{\vartheta\varsigma}, \tag{95}$$

$$\begin{aligned} \mathcal{C}_{\mathrm{quan}(\vartheta\varsigma)} = & -3\mathcal{H}(2f - f_1)\Xi \rho_{\vartheta} \left(\frac{1}{\rho'_{\vartheta}} + \frac{1}{\rho'_{\varsigma}} \right) \zeta_e \\ & - \mathcal{H}(2f - f_1)\Xi \rho_{\vartheta} \left[\frac{\rho'_{\vartheta}}{\rho'_{\varsigma}\rho'_e} - \frac{\rho'_{\varsigma}}{\rho'_{\vartheta}\rho'_e} \right. \\ & \left. + (\sigma + 1) \frac{1}{\rho'_{\vartheta}} \right] \mathcal{S}_{\vartheta\varsigma}. \end{aligned} \tag{96}$$

Equation (89) and (93) form a closed system of the first-order equations for the evolution of the adiabatic perturbation ζ_e and the entropy perturbation $\mathcal{S}_{\vartheta\varsigma}$ on the large scale.

In the ensuing discussion, we will focus on the evolution of the ζ_e. In our model, the initial entropy perturbation is positive. From Eq. (89), we know that ζ_e should increase as the universe is expanding. However, as ϑ decays into ς, the entropy perturbation must vanish, and the ζ_e becomes a conserved quantity on the large scale. The numerical result is shown in Fig. 3.

From Fig. 3, we can see that the final ζ_e is bigger than the classical one. We know that, in the classical theory, the evolution of the entropy perturbation is independent, and the evolution of ζ does not impact on the entropy perturbation. However, in LQC, the evolution of the entropy is affected by ζ_e. When ζ_e increases, There is an additional quantum adiabatic source for entropy perturbations. This will directly affect the evolution of ζ_e itself, so that the entropy perturbation is also the source of adiabatic curvature perturbation. This make the ζ_e increase to values greater than those in the classical cosmology. So we can see the different final value of ζ in Fig. 3.

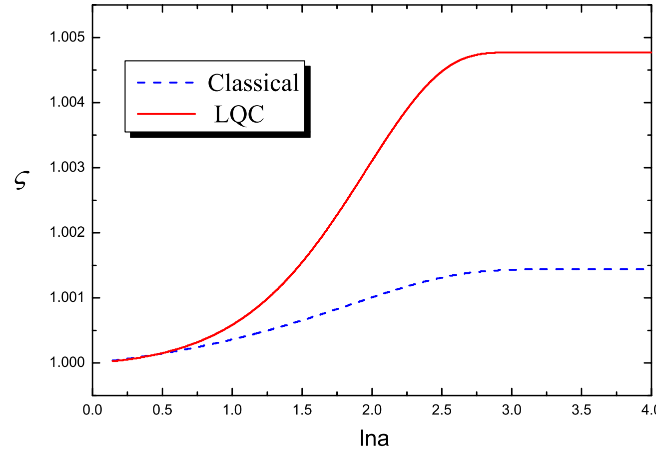

FIG. 3 (color online). The evolution of the adiabatic perturbation ζ as a function of $\ln a$. Here, the initial values of the related parameters are taken as: the initial density $\Omega_{\vartheta} = 10^{-1}$, the initial decay rate $g = 10^{-2}$, the initial adiabatic perturbation $\zeta = 1$, the initial entropy perturbation $\mathcal{S}_{\vartheta\varsigma} = 10^{-2}$ and the parameter $\sigma = 2$. The classical evolution is represented by the dashed line.

VI. SUMMARY AND CONCLUSIONS

What we study in this paper are the adiabatic and the nonadiabatic cosmological perturbations with the inverse-triad correction of LQC.

In order to discuss the general universe model, we need a more general form of the perturbations theory of LQC. The complete theory should come from the analysis of the effective Hamiltonian, as in [18,19]. However, the gauge-invariant form of this theory has yet to be addressed. So, we have to rewrite the perturbations theory of LQC to be a hydrodynamical formalism. In principle, this formalism can only be applied to the universe fulfilled by the scalar field (taking the scalar field as a special fluid). Despite this, in this paper, by use of the equivalence between the dynamics of the scalar field and the perfect fluid, we assume this effective form can also be applied to a more general fluid.

In the classical theory for the cosmological perturbations, the curvature perturbation on the uniform-density hypersurfaces ζ is gauge-invariant and conservative on the large scale. So, in our definition of the effective curvature perturbation ζ_e, it is required to have similar properties. The requirements of the gauge invariance and the conservation both lead to one of the counterterms $g_1 = 0$, i.e., Eq. (48). Therefore, Eq. (48) can be seen as a restriction to the space of ambiguity parameters (σ, q, ℓ).

In a further discussion, we generalize the hydrodynamical form of theory to a multifluids model. There are interactions between the different fluids in this model. So there will be a nonadiabatic entropy perturbation which reflects the difference of the curvature perturbation between the different fluids.

In the classical theory, the entropy perturbation can evolve into an adiabatic curvature perturbation at late time on the large scale, and itself can evolve independently. In other words, there is not any adiabatic source for the nonadiabatic entropy perturbation.

However, we find that in our discussion, in the effective theory of LQC, there will be a quantum adiabatic source for the nonadiabatic entropy perturbation. The adiabatic curvature perturbation has a influence on the energy transferring between different fluids, this influence is of quantum nature. This quantum effects impact the evolution of the nonadiabatic entropy perturbations. Therefore, there will be an adiabatic source, and this source is of quantum nature.

We also find that, in a more general model of the universe, this source does not disappear, except for some special situation in which $Q_\alpha/\rho'_{e(\alpha)}$ is the same for all fluids. This situation corresponds to two cosmological model. One is the model in which the energy transferring between different fluids can be ignored. The other is the model in which the matter in universe behaves like a cosmological constant. In these two special models, the quantum adiabatic source for nonadiabatic entropy perturbations will be vanished, and the entropy perturbations evolves independently, as in the classical cosmology. From this we know that the reason for the emergence of the quantum adiabatic source is an asynchronous change of $Q_\alpha/\rho'_{e(\alpha)}$. This is similar to the entropy perturbation which comes from the asynchronous change of $\delta\rho_{e(\alpha)}/\rho'_{e(\alpha)}$.

As an application of the related formalism, we study a simple decay model in which a nonrelativistic matter ϑ decaying into a radiation ς. We find that the final value of ζ_e is bigger than its counterpart in the classical theory.

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