

Testing dark matter warmness and quantity via the reduced relativistic gas modelJulio C. Fabris,¹ Ilya L. Shapiro,² and A. M. Velasquez-Toribio¹¹*Departamento de Física—CCE, Universidade Federal do Espírito Santo Vitória, CEP: 29060-900, ES, Brazil*²*Departamento de Física—ICE, Universidade Federal de Juiz de Fora, CEP: 36036-330, MG, Brazil*

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We use the framework of a recently proposed model of reduced relativistic gas to obtain the bounds for Ω 's of dark matter and dark energy (in the present case, a cosmological constant), taking into consideration an arbitrary warmness of dark matter. An equivalent equation of state has been used by Sakharov to predict the oscillations in the matter power spectrum. Two kinds of tests are accounted for in what follows, namely, the ones coming from the dynamics of the conformal factor of the homogeneous and isotropic metric and also the ones based on linear cosmic perturbations. The reduced relativistic gas model demonstrated its high effectiveness, permitting to explore a large volume in the space of mentioned parameters in a rather economic way. Taking together the results of such tests as supernova type Ia (Union2 sample), $H(z)$, cosmic microwave background (R factor), baryon acoustic oscillation, and large-scale structure data (2dFGRS data), we confirm that Λ CDM is the most favored model. At the same time, for the 2dFGRS data alone we found that an alternative model with a very small quantity of dark matter is also viable. This output is potentially relevant in view of the fact that the large-scale structure data is the only test which cannot be affected by the possible quantum contributions to the low-energy gravitational action.

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I. INTRODUCTION

According to the standard estimates of the energy balance of the present-day Universe, the relative energy densities of dark energy and dark matter (DM) are close to $\Omega_{\Lambda}^0 = 0.7$ and $\Omega_{\text{DM}}^0 = 0.25$, respectively, while the visible (more precisely, baryonic) matter is represented by a modest less than 5% of the total energy density [1,2]. Both dark energy and DM represent somehow mysterious components, but for different reasons. The main candidate to be dark energy is a cosmological constant (CC), however the cosmological constant problem is considered one of the most difficult problems of modern fundamental physics. The main CC problem is due to the conflict between the overall value of CC which is likely causing an acceleration of the present-day Universe, and several much greater contributions to it which can be evaluated at the particle physics scale (see [3] for a review). At the moment, no solution of this problem is known. On the other hand, there is another difficulty, which is known as a problem of coincidence. The problem is to explain why the Universe started to accelerate only recently. This problem can be solved or at least alleviated in two distinct ways. First, one can replace the CC term by some artificially designed substance (e.g. quintessence) which can adjust its energy density according to the expansion of the Universe. Indeed, this approach is essentially worsening the situation with the main CC problem, because the amount of the requested fine-tuning becomes even greater and goes beyond the original one, which is due to the hierarchy between the particle physics scale and cosmic scale [4]. The second approach is much less explored, despite that it looks much

simpler. It assumes changing the energy balance of the Universe such that in the new model Ω_{Λ}^0 becomes closer to unity. In this case the acceleration starts earlier and the coincidence problem becomes partially resolved.

The unique way to increase Ω_{Λ}^0 is by the expense of Ω_{DM}^0 , and at first sight this idea does not look realistic. The well-known reason is that the DM is required by a well-established set of observational data (see, e.g., books [5,6] and reviews [7–10]). At the same time it is interesting to verify the mentioned possibility without prejudice, especially taking the possible warmness of the DM into account [11,12]. The exploration of the wide set of observational data is a complicated task. In particular, one has to choose such a model which could take care of possible DM warmness and, at the same time, keep the amount of requested calculations under control. It is also desirable to be able to implement some new physical input into this model, but this case will be dealt with at the next stages of investigation. All in all, we need a sufficiently simple, but yet physically reasonable model for warm dark matter (WDM). Fortunately, a very useful simplified model of this sort, called reduced relativistic gas (RRG), has been suggested recently in [13,14]. An important historical note has to be made. After the first version of this paper was submitted, we have learned from [15] that the equivalent simplified model, interpolating between radiation and matter epochs, had been used by A. D. Sakharov in the famous work [16], where the oscillations in the matter power spectrum were discovered for the first time.

The idea of RRG is to treat WDM as an approximately Maxwell-distributed ideal gas of massive particles. Because the large-scale description of the Universe

concerns mainly the equation of state of the DM, it is not supposed to be sensitive to the noncollisional nature of the DM particles and to their anisotropic distribution at the astrophysical scale. Hence, there cannot be much difference between the RRG description from one side and the one based on relativistic Fermi distribution from the other side. The same is expected to be true, up to some extent, for the more complete approach based on the Boltzmann-Einstein system of equations [6].

The expression “approximately Maxwell” concerning the RRG model means we take an approximation where all DM particles have equal kinetic energies. As was shown in [13], qualitatively and numerically the equation of state suffers only a very small modification from this approximation. At the same time, the model based on RRG admits a simple analytical form for the equation of state and even enables one to analytically solve the Friedmann equation. The analysis of cosmic perturbations in the framework of the RRG model and LSS data provided [14] an upper bound on the DM warmness which is very close to the one obtained from much more complicated analysis based on the standard Boltzmann-Einstein system [17–20]. Thus, the RRG model represents a really useful tool for exploring the new cosmological models with the WDM. At the same time, it looks interesting to check this model on such sets of observational data as cosmic microwave background (CMB), baryon acoustic oscillation (BAO), and others.

There is also a special motivation for us to explore the possibility of alternative values of Ω_{DM} with the WDM, which is related to the potentially relevant quantum effects of vacuum. The backgrounds of the cosmological and astrophysical applications of the theory with renormalization-group based quantum corrections were established in [21,22]. In the recent paper [23] it was shown, from general considerations based on covariance and dimensional arguments, that the form of these quantum corrections can be defined up to a single free parameter ν . The recent analysis of the galaxies rotation curves in [24] in the theory without DM, but with the mentioned quantum corrections to Newton’s law, was surprisingly successful. It was shown that such quantum corrections can compete with CDM in explaining the rotation curves. The small amount of sufficiently warm DM does not affect these results and, therefore, we can think about weakening the standard restrictions to the DM in the theory with quantum corrections. On the other side, the cosmological model based on the same renormalization-group quantum corrections has been considered in [21,22]. The analysis of cosmic perturbations in this model [22] (see also [25]) demonstrated that the quantum contributions almost do not affect the power spectrum, such that their impact on the LSS (2dFGRS) data is very weak. Therefore, if such quantum corrections do exist, the LSS data alone should manifest such a possibility even in the zero-order approximation—that means without taking into account the

quantum effects. At the same time, we should expect the quantum corrections to be relevant for other test, especially for lensing, supernova, $H(z)$, and CMB. In order to take this into account, one has to consider the 2dFGRS test separately. The positive result for the zero-order approximation is the case when the LSS part alone should be compatible with an alternative model, with smaller Ω_{DM} , while the full set of tests should uniformly indicate to the Λ CDM as a unique possible option *without* taking quantum effects into account. This is the only possible output which can actually leave the chance for relevant effects of quantum corrections at the astrophysical and cosmological scale.

We will present in this paper the RRG model of WDM, characterized by the dark matter fractional density measured today Ω_{dm0} and also by the parameter b , which specifies the degree of warmness of the DM component. The model will be confronted against background observational data [supernova type Ia, baryon acoustic oscillation, cosmic microwave background R parameter, and the age of the Universe as a function of z , $H(z)$] and the matter power spectrum at the linear regime. We will see that the background tests, in general, favor a scenario of the Λ CDM model, with $b \approx 0$, and $\Omega_{\text{dm0}} \approx 0.25$, even if for specific tests some particularities may appear. However, the matter power spectrum alone favors a model with a very small value of Ω_{dm0} , giving room for some degree of warmness. In view of the discussion presented above this result can be seen as a sign for a possibility of a new concordance scenario if the quantum corrections are taken into account.

The paper is organized as follows. In Sec. II we present a brief necessary review of the RRG model. The contents of this section is essentially the same as [14] and we include it only for making all considerations in the paper more consistent. In Sec. III, we discuss linear perturbations of matter including the baryon component. The observational data which are used in our analysis are presented in Sec. IV. Finally, in Sec. V we draw our conclusions and discuss further perspectives for exploring an alternative concordance model with quantum corrections.

II. BACKGROUND NOTIONS OF THE RRG MODEL FOR WDM

In this section we present the necessary elements of the RRG model [13,14].

Equation of state and zero-order cosmology

Consider the cosmological model which is based on the relativistic gas of massive particles representing WDM, plus baryonic matter, radiation, and a cosmological constant. As a simplification, it is assumed that the relativistic gas is composed by particles with equal kinetic energy, or equal speed $\beta = v/c$. By elementary means one can obtain the following equation of state, which links the pressure P and energy density ρ of such gas:

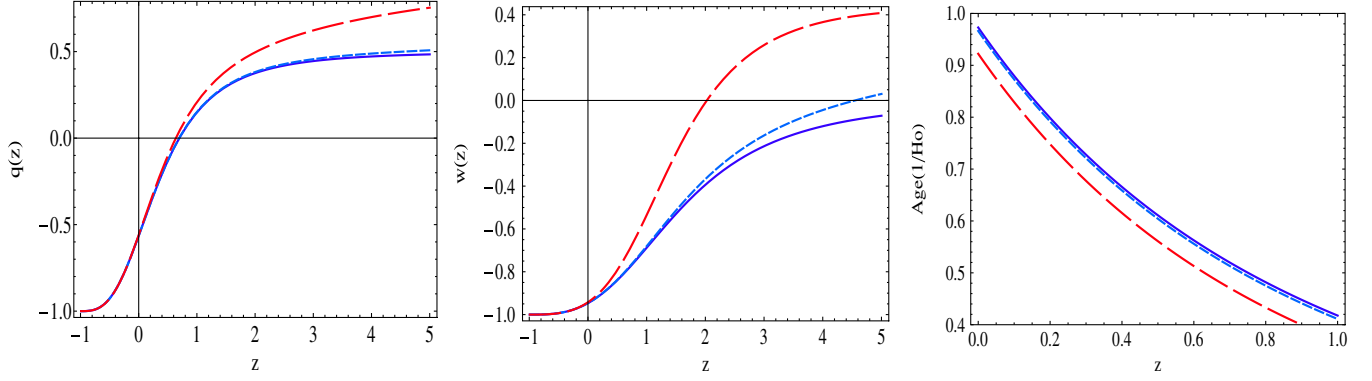


FIG. 1 (color online). The deceleration parameter (left), equation of state (center), and age of the Universe (right) for different values of the parameter $b = 0, 0.01, 0.2$ (from top to bottom of the first plot). Here $w(z)$ is an accumulated parameter corresponding to the sum of the cosmological constant term and the matter content. In all cases here we used the values $\Omega_{b0} = 0.04$ and $\Omega_{dm0} = 0.25$. The Λ CDM model corresponds to the value $b = 0$.

$$P = \frac{\rho}{3} \left[1 - \left(\frac{mc^2}{\varepsilon} \right)^2 \right] = \frac{\rho}{3} \left(1 - \frac{\rho_d^2}{\rho^2} \right). \quad (1)$$

In this formula ε is the kinetic energy of the individual particle, $\varepsilon = mc^2/\sqrt{1 - \beta^2}$. Furthermore, $\rho_d = \rho_{d0}^2(1 + z)^3$ is the mass (static energy) density. One can use one or another form of the equation of state (1), depending on the situation. For example, in order to explore the evolution of the conformal factor $a = a(t)$ in this model, one can take the second expression in (1) and using the conservation law find that the relative energy density for the relativistic gas is given by the expression

$$\Omega_{dm}(a) = \frac{\Omega_{dm0}}{a^3 \sqrt{1 + b^2}} \sqrt{1 + \frac{b^2}{a^2}}, \quad (2)$$

where Ω_{dm0} is the present-day dark matter density, and b is related to particle speed. Indeed, $b \approx 0$ means that the particles are nonrelativistic. Furthermore, for small velocities one can easily obtain the relation $b \approx \beta$ [14]. Finally, by using the equations presented above and the ones for other components, the Hubble parameter can be presented in the form

$$H^2(a, \Omega_{dm0}, \Omega_{b0}, b, \Omega_{r0}) = H_0^2 \left[\frac{\Omega_{dm0}}{a^3 \sqrt{1 + b^2}} \sqrt{1 + \frac{b^2}{a^2}} + \frac{\Omega_{b0}}{a^3} + \frac{\Omega_{r0}}{a^4} + \Omega_{\Lambda0} \right], \quad (3)$$

where we assume that

$$\Omega_{\Lambda0} = 1 - \Omega_{dm0} - \Omega_{b0} - \Omega_{r0}$$

and use the value $\Omega_{r0} h^2 = 2.42 \times 10^{-5}$. In this work we consider only the space-time manifolds with the flat space section.

If taken alone, the RRG model provides a natural interpolation between the radiation-dominated and matter-dominated Universe regimes. Because of the adiabatic expansion, the relativistic gas is cooling down and the

Universe can look as filled by ultrarelativistic gas at the early stage of expansion and almost as a dust at the final stage of this expansion. One can find more details about the RRG model in [13,14].

In order to illustrate the effect of the warmth of the matter content of the Universe, in Fig. 1 we display the deceleration parameter $q(z)$, the equation of state $w(z)$, and the age Universe in H_0 units. One can see that for $b \approx 0.1$ the function plot goes away from the Λ CDM plot with ($b = 0$). The state function is the function more sensitive to the value of parameter b .

III. EQUATIONS FOR LINEAR PERTURBATIONS

Following the approach developed in our previous paper [14], the dynamics of density perturbations has been analyzed in the synchronous gauge. The calculation of these perturbations is rather a standard routine; the only exception is the variation of the equation of state (1), where one has to use the first form of this equation, taking

$$\delta P = \frac{\delta \rho}{3} \left[1 - \left(\frac{Mac^2}{\varepsilon} \right)^2 \right] = \frac{\delta \rho}{3} \left(1 - \frac{\rho_d^2}{\rho^2} \right). \quad (4)$$

The reason is that in the framework of the RRG model one has to provide kinetic energies of all particles to be equal and, therefore, we have no right to change the ratio mc^2/ε . In other words, the variations of the energy density and of the rest energy density, $\delta \rho_d$ and $\delta \rho$, should be always proportional. By using this rule we arrive at the following system of equations for the linear perturbations [14]:

$$\begin{aligned} \delta_b'' + \left(\frac{2}{a} + \frac{f'}{f} \right) \delta_b' &= -\frac{3}{2} \frac{\Omega_b}{a^3 f^2} \delta_b - \frac{3}{2} \frac{\Omega_{dm}}{f^2} (1 + 3w) \delta_{dm}, \\ \delta_{dm}' + (1 + w) \left(\frac{v}{f} - \delta_b' \right) &= 0, \\ (1 + w) \left[v' + (2 - 3w) \frac{v}{a} \right] &= \frac{k^2}{k_0^2} \frac{\omega \delta_{dm}}{a^2 f}, \end{aligned} \quad (5)$$

where v is the peculiar velocity, δ_b is the baryonic matter density fluctuation, and δ_{dm} is the relative dark matter density fluctuation. Other relevant definitions are

$$f = f(a) = a \sqrt{\Omega_{\text{dm}} + \frac{\Omega_{b0}}{a^3} + 1 - \Omega_{b0} - \Omega_{\text{dm}0}},$$

$$w = w(a) = \frac{1 - r(a)}{3} \quad \text{and} \quad r(a) = \frac{1 + b^2}{1 + (b^2/a)^2}, \quad (6)$$

where Ω_{dm} is given by Eq. (2). The system above differs from the CDM model, because in our model the peculiar velocity and the state function $w(a)$ are nonzero. Additionally, the CDM model interacts only through gravity and can be considered as a pressureless perfect fluid, unlike our model which has a component of pressure given by (4). The solution of the system (5) was performed using a MATHEMATICA-based code. To determine the power spectrum we use the Bardeen-Bond-Kaiser-Szalay (BBKS) transfer function [26]. It is important to note that we also use the transfer function described in Refs. [27,28] which is adapted for WDM particles, which depends on some specific inputs, obtaining the same results as when the BBKS transfer function is employed.

For the scale invariant spectrum, favored by the primordial inflationary scenario, the BBKS transfer function is given by

$$T(k) = \frac{\ln(1 + 2.34q)}{2.34q} [1 + 3.89q + 16.1q^2 + 5.64q^3 + 6.71q^4]^{-1/4}$$

where $q(k) = \frac{k}{h\Gamma\text{Mpc}^{-1}}$

and $\Gamma = \Omega_{m0}h \exp\left(-\Omega_{b0} - \frac{\Omega_{b0}}{\Omega_{m0}}\right).$ (7)

We have used the function $T(k)$ to impose the initial conditions of the system (5). The power spectrum is defined as usual; at $z = 0$ we have

$$P(k) = |\delta(k)|^2 = AT(k) \left[\frac{g(\Omega_{m0})}{\Omega_T} \right]^2 k, \quad (8)$$

where A is a normalization constant of the spectrum, which can be fixed from the spectrum of anisotropy of the cosmic microwave background radiation. The expression for $g(\Omega)$ is given by

$$g(\Omega) = \frac{5\Omega}{2(\Omega^{4/7} + 1.01(\frac{\Omega}{2} + 1) - 0.75)}. \quad (9)$$

The analysis of the linear density perturbations enables us to obtain the upper bound for the warmness of the DM in a very economic way [14]. In Fig. 2 we compare the 2dFGRS data with the power spectrum of our model for the usual Λ CDM energy balance case. We used the best fit obtained using the χ^2 in the next section. One can see that

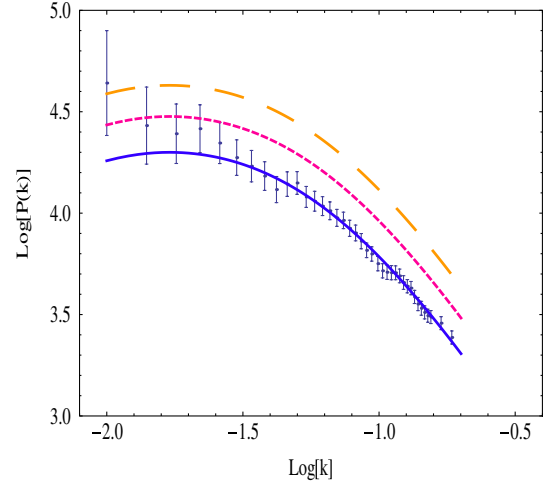


FIG. 2 (color online). Power spectrum using $\Omega_{b0} = 0.044$ and $\Omega_{\text{dm}0} = 0.24$, for three different values of $b = 10^{-4}$, 5.0×10^{-3} , 10^{-2} (from bottom to top). It is easy to see that the last two values for b provide the linear power spectrum, at small scale, which is incompatible with the 2dFGRS data.

if $b \approx 10^{-3}$, the predicted power spectrum is outside of the region of data shown in the plot. A detailed analysis of observational constraints is given in Sec. V.

IV. CONSTRAINTS FROM OBSERVATIONAL DATA

Nowadays there is a large number of observational tests focused on different aspects of the cosmological models. It is expected that the crossing of these different observational tests may lead to an expressive constraint on the free parameters of a given model, in some cases confirming the viability of the proposed scenario or ruling it out. Here we consider essentially two kinds of tests: those focusing on the background functions, specially $H(z)$ and also the perturbative tests, which involve more profoundly the description of the matter/energy content of the model. Concerning the tests related to the background behavior only, we will consider the supernova type Ia data (Union2 sample), BAO, the position of the first acoustic peak in the CMB spectrum (the R parameter), and the age of the old objects. In what concerns the perturbative tests, we will restrict ourselves to the matter power spectrum associated with linear perturbations.

The analysis of statistics begins with the χ^2 functions constructed following the general expression

$$\chi^2 = \sum_{i=1}^N \frac{(\rho_i^{\text{th}} - \rho_i^{\text{ob}})^2}{\sigma_i^2}, \quad (10)$$

where N is the total number of observational data, ρ_i^{th} are corresponding theoretical predictions, and ρ_i^{ob} represent the observational values, with an error bar given by σ_i . The probability distribution function is constructed from the χ^2 function as

$$P(x^j) = A e^{-\chi^2(x^j)/2}, \quad (11)$$

where x^j are the set of free parameters of the model and A is a normalization factor. Within the RRG model, there are two free parameters, $\Omega_{\text{dm}0}$ and b . In what follows we describe the results for each of these tests.

A. The CMB shift parameter

We used the shift parameter, R , which relates the angular diameter distance to the last scattering surface with the angular scale of the first acoustic peak in the WMAP7 power spectrum, and is given by (for $k = 0$) [29]

$$R(\Omega_{\text{dm}0}, \Omega_{b0}, b) = \sqrt{\Omega_m} \int_0^{1090} \frac{dz}{E(\Omega_{\text{dm}0}, \Omega_{b0}, b)} \\ = 1.725 \pm 0.018, \quad (12)$$

where $E(\Omega_{\text{dm}0}, \Omega_{b0}, b) = H(\Omega_{\text{dm}0}, \Omega_{b0}, b)/H_0$. It is important to point out that the measured value of R is model independent and, therefore, can be used as a test of cosmology. In general, the shift parameter alone is insufficient for this purpose. In Ref. [30] it is shown how to derive the R parameter in a model-independent way. In order to include the CMB shift parameter in the analysis, it is needed to integrate up to the matter-radiation decoupling ($z \approx 1090$), so that radiation is no longer negligible and it should be properly taken into account. For our model we used the minimization of the χ^2 statistic according to

$$\chi_R^2 = \frac{[1.725 - R(\Omega_{\text{dm}0}, \Omega_{b0}, b)]^2}{0.018^2}. \quad (13)$$

We used $\Omega_{b0} = 0.04$. In Fig. 3, one can see the results for our model. It is clear that this test alone cannot severely restrict the space of parameters. In particular, this test does not allow us to determine an upper limit for the parameter b for fixed Ω 's.

B. BAO

The primordial baryon-photon acoustic oscillations leave a signature in the correlation function as observed

by Eisenstein *et al.* [31]. This signature provides us with a standard ruler which can be used to constrain the following quantity:

$$A(\Omega_{\text{dm}0}, \Omega_{b0}, b) = \sqrt{\Omega_{m0}} [E(z_1, \Omega_{\text{dm}0}, \Omega_{b0}, b)]^{-1/3} \\ \times \left[\frac{1}{z_1} \int_0^{z_1} \frac{dz}{E(z, \Omega_{\text{dm}0}, \Omega_{b0}, b)} \right]^{2/3}, \quad (14)$$

where the observed value of A is $A_{\text{obs}} = 0.469 \pm 0.017$ and $z_1 = 0.35$ is the redshift of the Sloan Digital Sky Survey (SDSS) sample. The χ^2 statistics may be computed from

$$\chi_{\text{BAO}}^2(\Omega_{\text{dm}0}, \Omega_{b0}, b) = \frac{[0.469 - A(\Omega_{\text{dm}0}, 0.044, b)]^2}{0.017^2}. \quad (15)$$

Here we will use the $\Omega_{b0} = 0.044$ value to build the confidence regions. The results are presented in Fig. 4. We can see that BAO does not impose an upper limit for the parameter b , but it restricts the value of $\Omega_{\text{dm}0}$.

C. $H(z)$

The observational Hubble parameter depends on the redshift z in the form

$$H(z) = -\frac{1}{1+z} \frac{dz}{dt}. \quad (16)$$

Then, $H(z)$ can be obtained as far as dz/dt is known. Jimenez *et al.* [32] demonstrated the feasibility of the method while Simon *et al.* [33] and Stern *et al.* [34] obtained $H(z)$ in the range of $0.09 < z < 1.8$. Another way to get values of $H(z)$ can be by using the BAO peak position as a standard ruler in the radial direction. Using this prescription in Ref. [35] the authors determined $H(z)$ in $z = 0.43$ and $z = 0.24$. We employ the 11 data values of [33,34] and two data values of [35]. The best-fit values for the model parameters from observational Hubble data are determined by minimizing the quantity

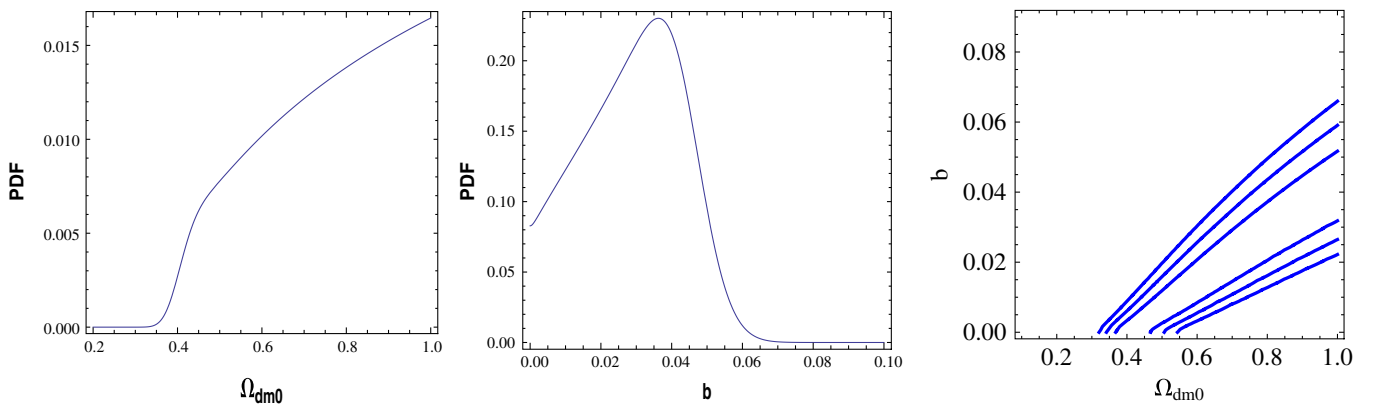
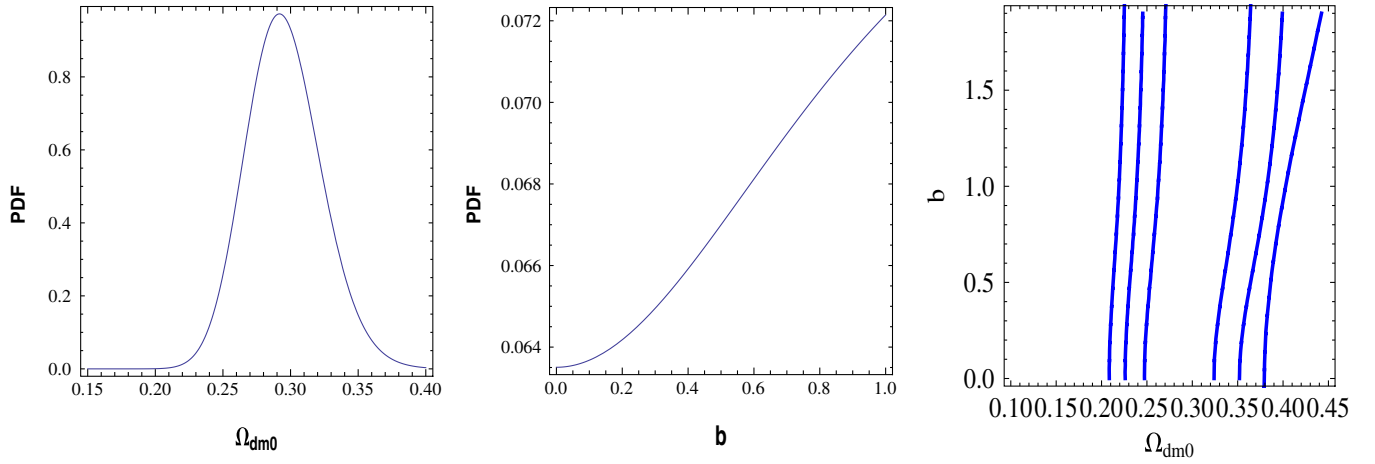


FIG. 3 (color online). The PDFs using CMB. The confidence level with 1σ , 2σ , and 3σ (right panel).

FIG. 4 (color online). The PDFs using BAO. The confidence level with 1σ , 2σ , and 3σ (right panel).

$$\chi_H^2(\Omega_{\text{dm}0}, \Omega_{b0}, b, H_0) = \sum_i^{13} \frac{[H_{\text{obs},i} - H_{\text{th}}(\Omega_{\text{dm}0}, \Omega_{b0}, b, H_0)]^2}{\sigma_i^2}. \quad (17)$$

Here we use the value $\Omega_{b0} = 0.04$. The free parameters are $\Omega_{\text{dm}0}$, H_0 , and b . However, in our case, we have found that the one-dimensional probability distribution function in H_0 is quite narrow and the values H_0 are well approximated by the value $H_0 = 70.5$ (minimizing the χ_H^2 considering the three free parameters). Therefore, one has only two free parameters, namely b and the $\Omega_{\text{dm}0}$ parameter. The results for this test are presented in Fig. 5. In general, this test provides the results which are similar to the ones of BAO. In particular, this is because the parameter b remains weakly constrained, but the restrictions are strong for the value of $\Omega_{\text{dm}0}$.

D. Compilation Union2

The supernovae Ia data give us the distance modulus μ to each supernova, that is given by

$$\mu \equiv m - M = 5 \log_{10} \left[\frac{d_L}{\text{Mpc}} \right] + 25, \quad (18)$$

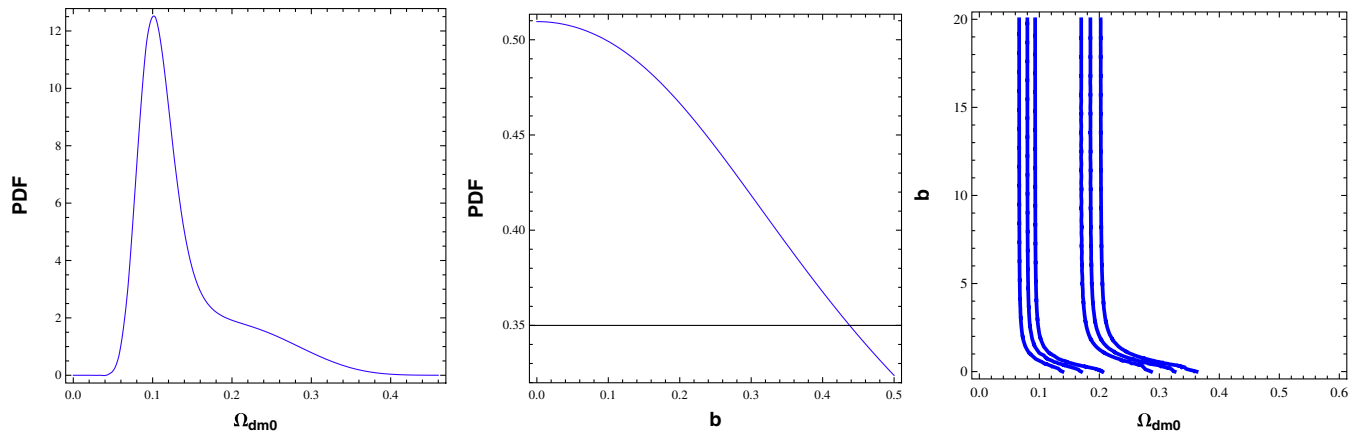
where M and m are the absolute and apparent magnitudes, respectively. The distance modulus can also be written as

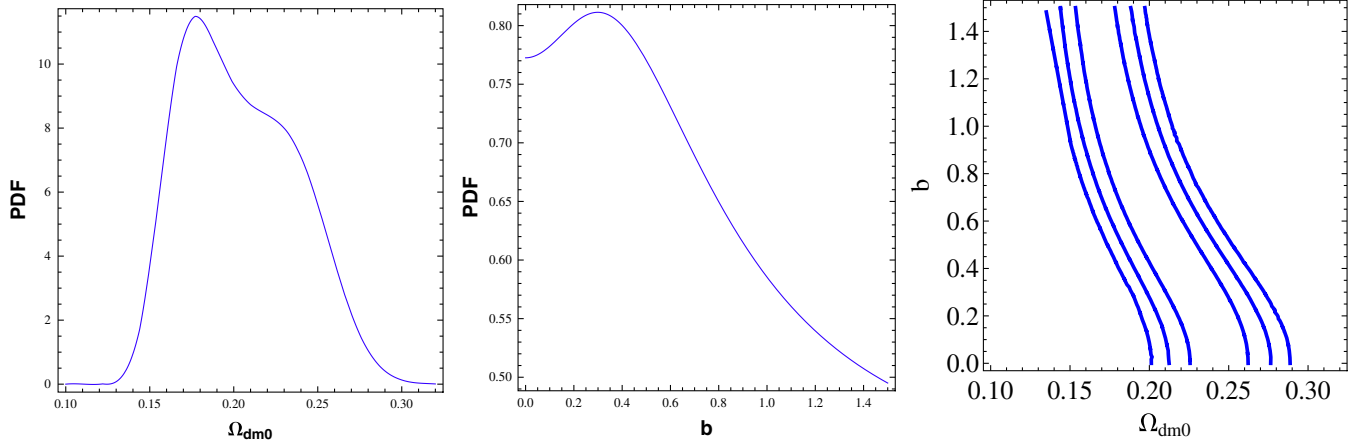
$$\mu = 5 \log_{10} D_L(z) + \mu_0, \quad (19)$$

where $D_L = \frac{H_0 d_L}{c}$ is the Hubble-free luminosity distance and μ_0 is the zero point offset (which is an additional model-independent parameter) defined by

$$\mu_0 = 5 \log_{10} \left(\frac{c H_0^{-1}}{\text{Mpc}} \right) + 25 = 42.38 - 5 \log_{10} h. \quad (20)$$

In the present paper we used the Union2 data set including 557 data values of Amanullah *et al.* [36], that includes the intermediate- z data observed during the first season of the Sloan Digital Sky Survey (SDSS)-II supernova survey [37] and the high z data from the Union compilation [38].

FIG. 5 (color online). The PDFs using $H(z)$ data. The confidence level with 1σ , 2σ , and 3σ (right panel).

FIG. 6 (color online). The PDFs using SNIa Union 2. The confidence level with 1σ , 2σ , and 3σ (right panel).

In our case the χ^2_{SNIa} is given by

$$\chi^2_{\text{SNIa}}(p_i) = \sum_{i=1}^n \frac{(\mu_{\text{the}}(p_i, z_i) - \mu_{\text{obs}}(z_i))^2}{\sigma_{\text{obs},i}^2}. \quad (21)$$

The χ^2 function can be minimized with respect to the μ_0 parameter, as it is independent of the data points and the data set. The σ_{obs} are provided in [36] and include both the observational and intrinsic magnitude scatter. We assumed a Gaussian and uncorrelated probability distribution for the data. Expanding the equation above with respect to μ_0 , we obtain

$$\chi^2(p_i)_{\text{SNIa}} = A(p_i) - 2\mu_0 B(p_i) + \mu_0^2 C(p_i), \quad (22)$$

which has a minimum for $\mu_0 = B(p_i)/C(p_i)$, giving

$$\chi^2_{\text{SNIa,min}} = \bar{\chi}^2_{\text{SNIa}} = A(p_i) - \frac{B^2(p_i)}{C(p_i)}, \quad (23)$$

where

$$A(p_i) = \sum_i^n \frac{(\mu_{\text{th}} - \mu_{\text{obs}}(p_i, \mu_0 = 0))^2}{\sigma_i^2}, \quad (24)$$

$$B(p_i) = \sum_i^n \frac{\mu_{\text{th}} - \mu_{\text{obs}}(p_i, \mu_0 = 0)}{\sigma_i}, \quad (25)$$

$$C(p_i) = \frac{1}{\sigma_i^2}. \quad (26)$$

Let us note that the new $\bar{\chi}^2_{\text{SNIa}}$ is independent on μ_0 and can be minimized with respect to the parameters of our theoretical model. The results are shown in the Fig. 6. We can see that the parameters b and $\Omega_{\text{dm}0}$ are relatively constrained. In particular, the parameter b has a maximum likelihood value for $b \approx 0.4$ (see Table I).

E. 2dFGRS

We used the 2dFGRS data [39] to compare the power spectrum of our model and constrain the free parameters. To do this, we minimize χ^2 defined as

$$\chi^2_{2\text{dF}}(\Omega_{\text{dm}0}, \Omega_{b0}, b) = \sum_{i=1}^{39} \frac{[P((k_i, \Omega_{\text{dm}0}, \Omega_{b0}, b) - P_{\text{obs}}(k_i, \Omega_{\text{dm}0}, \Omega_{b0}, b)]^2}{\sigma_i^2}. \quad (27)$$

Here $P(k)$ is given by Eq. (8) and the number of free parameters is two, since we fixed the value of $\Omega_{b0} = 0.04$. In Fig. 7 we present the probability density function (PDF) for our model. The impact that has the model on the linear power spectrum is dominated by the b parameter that is associated with the speed of dark matter particles. In the

TABLE I. In this table we compile the results of observational constraints using different data sets. The statistical error is 68.3%(1 σ) and the results in brackets correspond to an error of 2 σ .

Data	χ^2_{min}	$\Omega_{\text{dm}0}$	b
2dFGRS	21.495	$0.0069^{+0.1604[+0.3400]}_{-0.0069[-0.0069]}$	$0.001^{+0.002[+0.003]}_{-0.001[-0.001]}$
SNIa	542.45	$0.21^{+0.02}_{-0.01}$	$0.48^{+0.31}_{-0.10}$
$R + \text{BAO} + H(z)$	8.449	$0.28^{+0.07}_{-0.05}$	$\approx 0^{+0.006}_{-0.000}$
$R + \text{BAO} + H(z) + \text{SNIa}(\text{Base})$	547.383	$0.26^{+0.03}_{-0.05}$	$0.00064^{+0.00027}_{-0.00064}$
$R + \text{BAO} + H(z) + 2\text{dFGRS}$	24.24	$0.28^{+0.07}_{-0.05}$	$\approx 0^{+0.001}_{-0.000}$
$R + \text{BAO} + H(z) + \text{SNIa} + 2\text{dFGRS}$	563.271	$0.26^{+0.05}_{-0.04}$	$\approx 0.0001^{+0.003}_{-0.0001}$

upper left panel we can see that if $\Omega_{\text{dm}0}$ is close to Λ CDM value, then the b value is close to zero. But if the value of b is, for example, 10^{-4} , then the dark matter fraction decreases significantly. The corresponding data can be seen in Table I.

F. Combining the data sets

We considered that the observational data are independent, so we defined the χ^2_{total} as

$$\chi^2_{\text{total}} = \bar{\chi}^2_{\text{SNIa}} + \chi^2_{\text{BAO}} + \chi^2_R + \chi^2_H + \chi^2_{2\text{dF}}. \quad (28)$$

The best-fit values for the model can be determined by minimizing the total χ^2 . For Gaussian distributed measurements, the χ^2 function is directly related to the maximum

likelihood estimator. The likelihood function is determined as

$$L(\Omega_{\text{dm}0}, b) = L_0 \exp\left(-\frac{\chi^2_{\text{total}}}{2}\right), \quad (29)$$

where L_0 is a normalization constant. In order to constrain the parameters of interest, we marginalize over the other parameters. In Figs. 8–10. We display the results of the χ^2_{total} . An important issue that needs to be considered is the difference between the PDFs of the background tests (BAO + R + H + SNIa) and linear perturbations. In the case of the $\Omega_{\text{dm}0}$ parameter, we see that the low probability in the range of $\Omega_{\text{dm}0} < 0.1$ eliminates high probability of this sector in case of perturbations (Fig. 7). Therefore, the

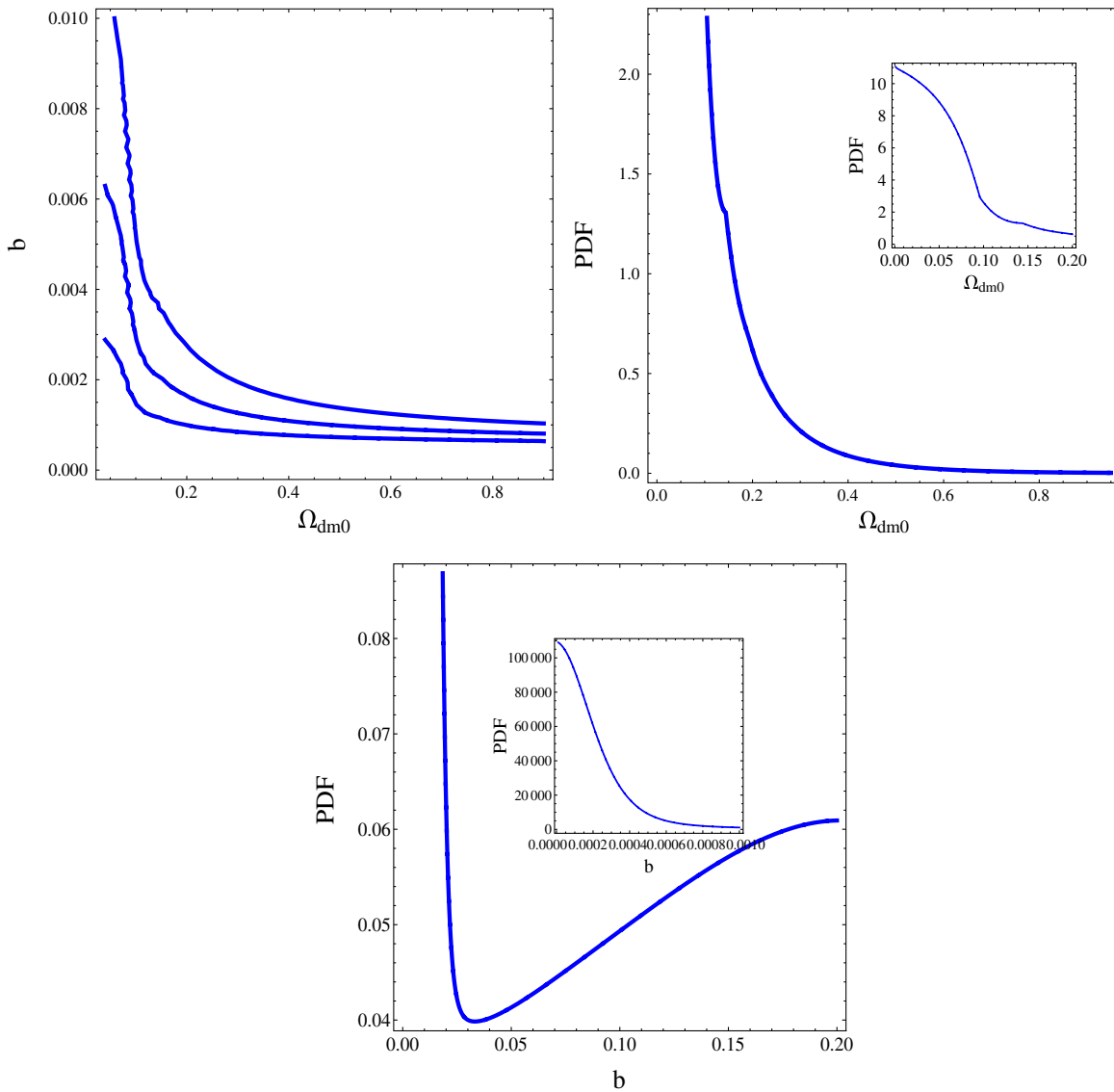


FIG. 7 (color online). The probability density function using 2dFGRS for $\Omega_{\text{dm}0}$ (upper right panel) and b (bottom central panel). The confidence level with 1σ , 2σ , and 3σ . To build the PDF we marginalize the free parameters considering the intervals $\Omega_{\text{dm}0} \in [0.5; 0.95]$ and $b \in [0.001; 0.4]$, for each case. As we can see, the Λ CDM model is included; i.e., $\Omega_{\text{dm}0} = 0.25$ corresponds to a $b \approx 0$.

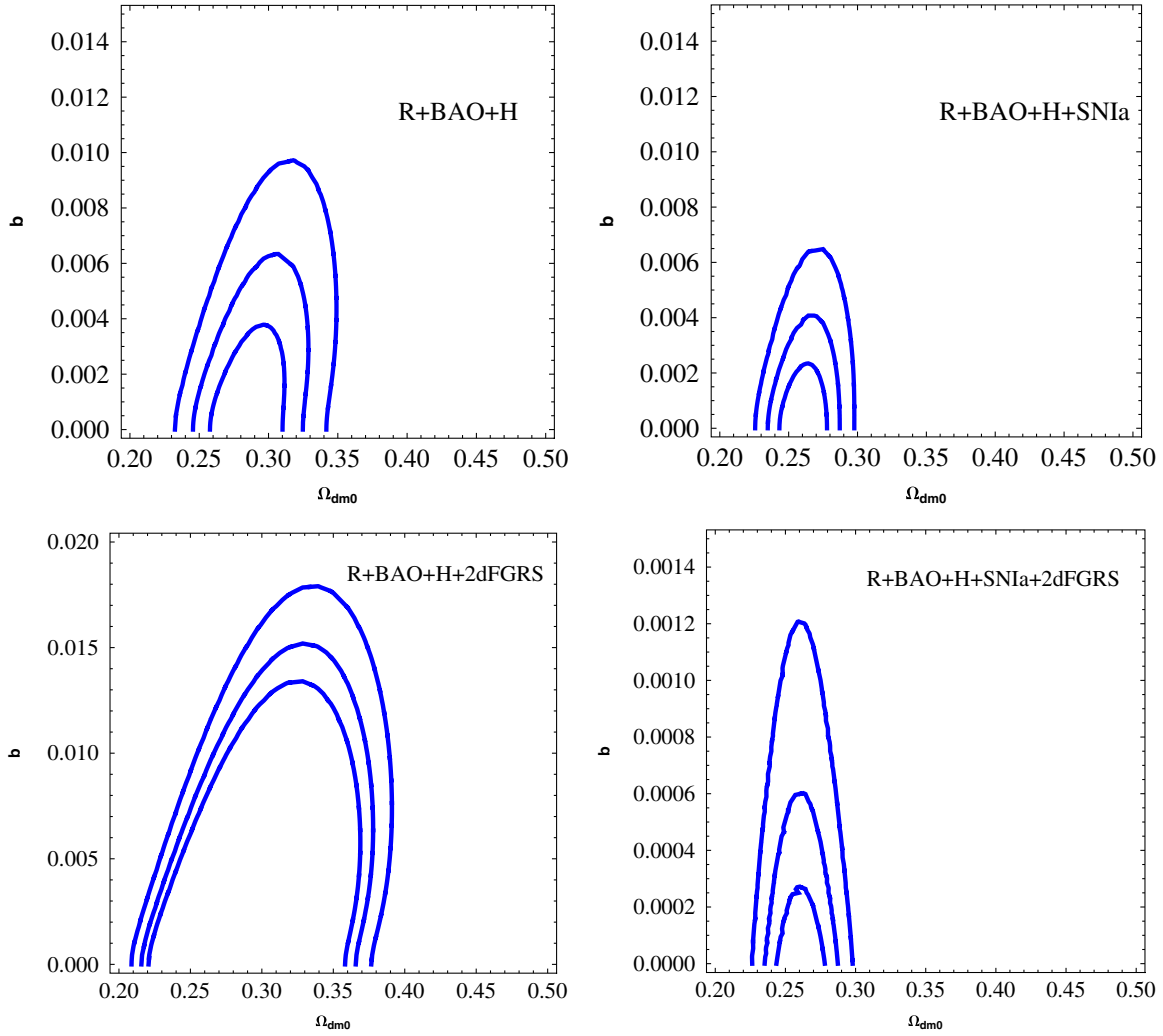


FIG. 8 (color online). Confidence regions at 1σ , 2σ , and 3σ levels from inner to outer, respectively, on the $(\Omega_{\text{dm}0}, b)$ plane for our relativistic model in the flat case. The effect of supernovae type Ia is not apparent when we consider only tests of background ($H + R + \text{BAO}$), but it is important when combined with power spectrum data; for example, the constraints on the $\Omega_{\text{dm}0}$ parameter are more narrow and the upper limit for the b parameter decreases significantly (an order of magnitude) compare the two panels on the right.

results are close to the Λ CDM model. The effects of structure formation may be critical to study the characteristics of models that include warm dark matter, where the speed of the particles is no longer negligible, as in the case of CDM. In the Table I we compile the results of observational constraints at 1σ and, for the power spectrum, we give also the estimations at 2σ .

V. RESULTS AND DISCUSSION

The reduced relativistic gas model enables one to determine the equation of state which interpolates between the radiation-dominated and the matter-dominated eras. It can represent warm dark matter, characterized by the parameter b , where $v \approx bc$. The main difference between WDM and CDM comes from the behavior at small scales, even during the relativistic phase. WDM is composed of lighter particles with respect to the CDM particles, and density

fluctuations are suppressed at small scales, presenting the effect of a free stream. In our case, such small-scale effects are not relevant, because we are interested in scales corresponding to the linear regime, and transition is smooth, not taking into account the details of the decoupling of the warm dark matter particles. In such a case, the main effect of our model is due to the nonzero, even though small, value of the velocity of the particles. For our purposes, we can ignore this small-scale process and take into account only the information encoded in the transfer function for our relevant scales. For very large scales, the WDM and CDM transfer functions are essentially the same, and small differences appear only at intermediate scales, as we could expect. This is in agreement with our results which are essentially the same for the WDM and CDM transfer function. Hence, the dark component would have a small (but nonzero) velocity today.

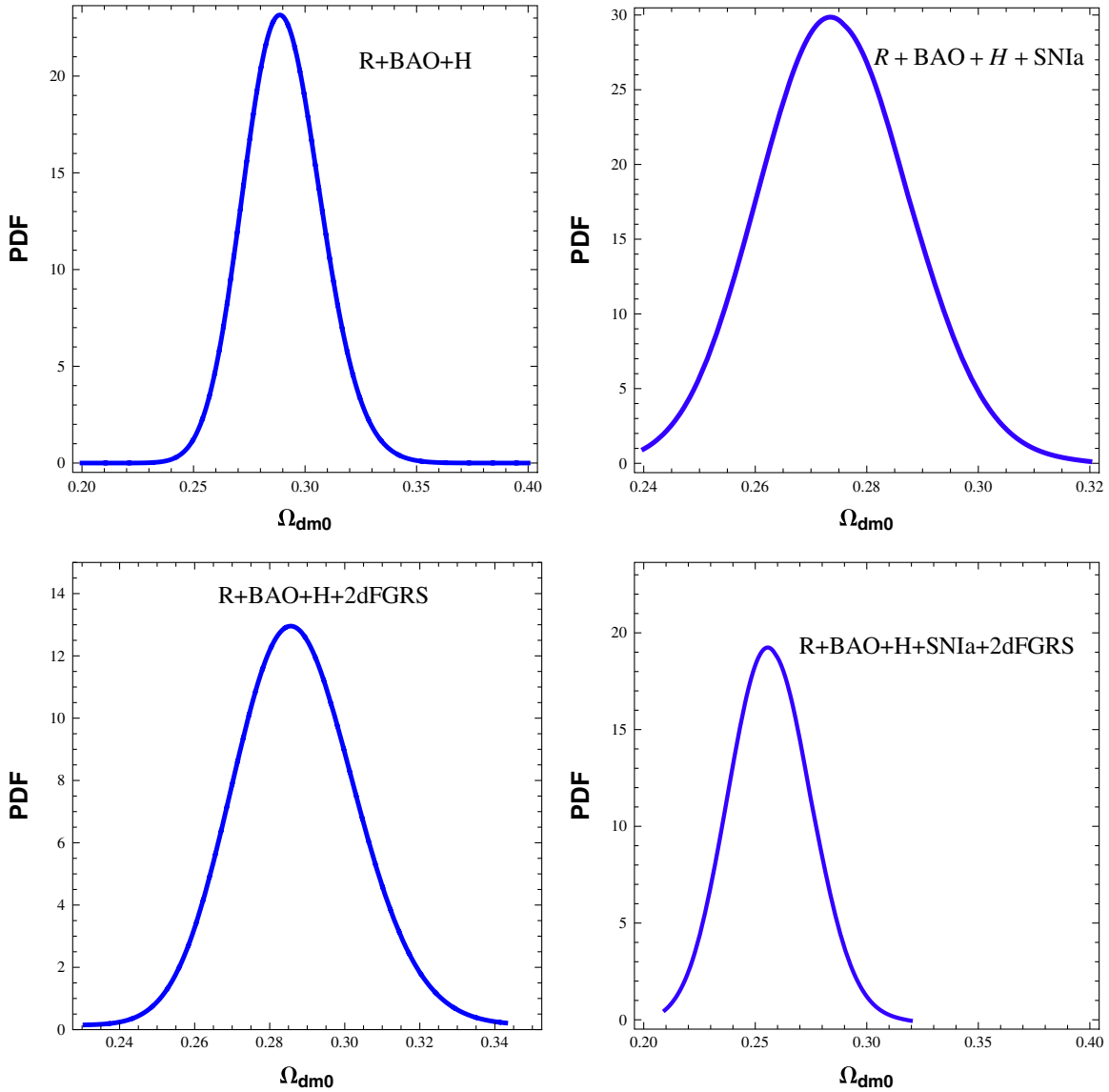


FIG. 9 (color online). The one-dimensional probability distribution function for the $\Omega_{\text{dm}0}$ parameter. The parameter b was marginalized in the range $[0, 1]$.

The model has been tested using four background observational tests: supernova type Ia (SNIa) (Union2 sample), $H(z)$, CMB (R factor), and BAO. Moreover, a detailed study of structure formation at linear level has been performed using the 2dFGRS data for the matter power spectrum. The different tests have been crossed in order to obtain a more clear forecast for the free parameters, which are essentially the velocity parameter for the dark matter particles b and the dark matter ratio to the critical density $\Omega_{\text{dm}0}$. All the analysis has been performed using the flat-universe prior.

In general, the background tests predict a $\Omega_{\text{dm}0}$ very similar to the value obtained in the Λ CDM model, except for the CMB R factor which predicts a universe essentially dominated by dark matter. For the velocity parameter b , the background tests predict a maximum probability for a

finite value at $b \ll 1$, except for BAO, where a large value for b is predicted. But the most striking feature of the observational tests concerns the formation of structure, for which the maximum probability for $\Omega_{\text{dm}0}$ occurs at a zero value. This seems to be a consequence of the restriction of the analysis to a linear level, since a certain amount of dark matter is necessary in order to have the formation of structure process. In any case, the results concern a probability distribution: a small amount of dark matter is certainly admitted, but much less than that predicted by the Λ CDM model. For the parameter b , the PDF is concentrated around zero, but there is another region of much smaller, albeit nonzero, probabilities from $b \sim 0.04$ on. We remember that the estimations of warm dark matter in dwarf spiral galaxies predict $b \sim 10^{-3}$ [40]. In this context, our model seems to be a possible alternative for

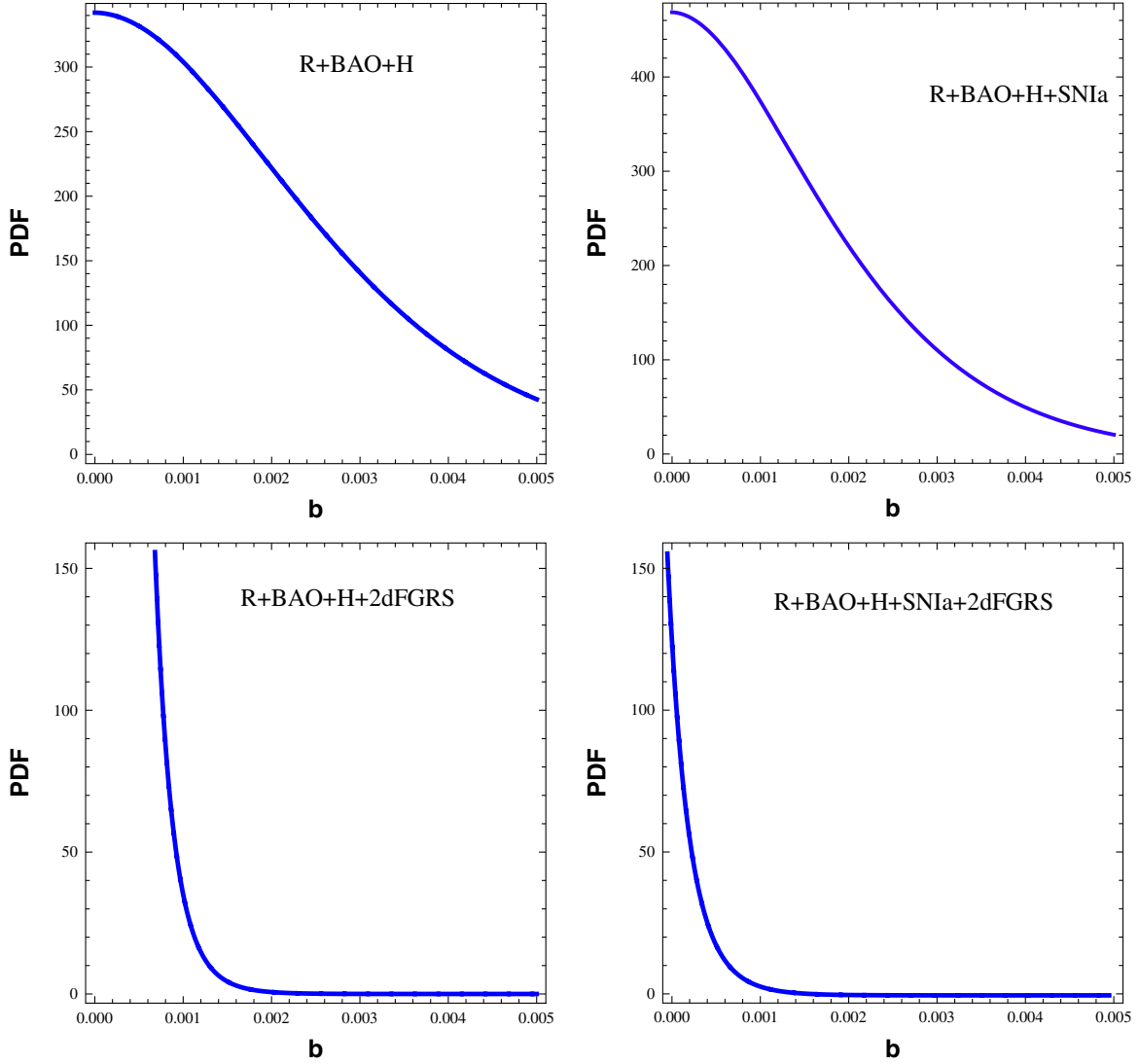


FIG. 10 (color online). The one-dimensional probability distribution function for the b parameter. The $\Omega_{\text{dm}0}$ was marginalized in the range $[0.05, 0.95]$.

understanding the formation of structures. Additionally as we had seen in the RRG, the dispersion velocity is a free parameter predictable directly and not derived.

The crossing of the background tests with the ones related to cosmic perturbations leads to scenarios similar to the Λ CDM model but with a quite large dispersion in the estimations for b . Here we must distinguish some special features. When the SNIa test is not considered, the range of allowed values for b becomes larger. The addition of the structure formation to the background tests does not change the scenario considerably, in spite of the fact that the test alone would lead to a very small value of $\Omega_{\text{dm}0}$. This occurs because for small $\Omega_{\text{dm}0}$ the PDF is essentially zero for the background tests: the crossing of perturbative and background tests renders this possibility very unlikely. However, considering an error of 2σ (see Table I) our value of $\Omega_{\text{dm}0}$ includes the value of the Λ CDM model $\Omega_{\text{dm}0} = 0.229 \pm 0.015$ [29].

We can summarize the main results of this work in the following way. The background tests result, without the SN data, is centered in the usual values characterizing the Λ CDM model; that means $b \approx 0$ and $\Omega_{\text{dm}0} \sim 0.26$, but with a non-negligible dispersion. The inclusion of the SN data sharpens the confidence region, leading to stronger evidence in favor of the Λ CDM model. In both cases, the inclusion of the LSS test enlarges the dispersion mainly in the parameter b direction. For the one-dimensional estimations, those related to the dark matter density change very little, while for the b parameter, the inclusion of SN in the background tests compresses the PDF distribution near $b = 0$, and the inclusion of the LSS tests accentuates this effect. This seems to be in contradiction with the previous two-dimensional description, but it must be seen as an effect of the marginalization on the density parameter.

In the literature the mechanisms most used to study warm dark matter have been based in a distribution

function of the form $f_x(v) = \beta/(\epsilon^{p/\alpha T_\gamma} + 1)$ [12], where T_γ is the photon temperature, $v = p/(p^2 + m_x^2)^{1/2}$, and three parameters α , β , and m_x . The power spectrum today and the relation between pressure and energy density is given with a function of this distribution. In this context the transfer function can be approximated by the fitting function $T(k) = [1 + (\alpha k)^{2.24}]^{-4.46}$ [27], where α is a function of free parameters of WDM. We have used this function with $m_x = 1$ keV and $m_x = 0.25$ keV and observe that the obtained results are very close to those obtained with the BBKS function, therefore our results are not affected significantly by the scale of mass.

It is important to remember that the standard model, Λ CDM, appears to be in conflict with observations on subgalactic scales. There are two major conflicts between Λ CDM models and observations in the local Universe. First, the inner mass density profiles of simulated dark matter halos are more cuspy than inferred from the rotation curves of dwarfs and low surface brightness galaxies. Currently, this conflict seems well established [41]. Therefore, the Λ CDM models in their current form cannot provide the unknown mass that surrounds galaxies. The second problem is related to the N-body simulations of Λ CDM models that predict large numbers of low mass halos greatly in excess when compared to the observed number of satellite galaxies in the local group. These and other issues suggest that it is necessary to investigate

different theoretical possibilities. In this paper we have shown that, in part of linear perturbations alone, our RRG model for WDM provides results that are different from the standard Λ CDM model. It is possible, in principle, that the parameter estimations from other observational data can be modified by the quantum corrections in the form discussed in [21,22], such that eventually we may converge to an alternative cosmic concordance model. The present investigation can be seen as a first step toward this program.

It is clear that to better quantify these qualitative expectations, one needs to take into account the nonlinear effects, including the dynamics of the collapse and the halo concentrations, e.g., by using the algorithm suggested in Ref. [42], etc. Another important thing to do is to consider the other sets of observational tests, like the integrated Sachs-Wolfe effect, in the first order in quantum corrections and see whether they are compatible with the small Ω_{odm} and small WDM model.

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