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SU(3)-Symmetric Coupling of Mesons to Baryons and the Strong D/F Ratio*

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Pseudoscalar meson–baryon–baryon (*PBB*) couplings are analyzed in both the Klein-Gordon (KG) and Duffin-Kemmer-Petiau (DKP) formalisms. Present determinations of these coupling constants indicate that both the pseudoscalar (ps) and pseudovector (pv) KG couplings are in relatively poor agreement with SU(3), whereas the pv DKP coupling is in excellent agreement with SU(3). Furthermore, the value $\beta = D/(D+F) = 0.71 \pm 0.07$ extracted from the DKP coupling is close to the canonical SU(6) value $\beta = 0.6$.

It is well known that the coupling of the $J^P = \frac{1}{2}^+$ baryons (B) and $J^P = 0^-$ pseudoscalar (P) mesons can be expressed in a number of ways. To start with, the pseudoscalar mesons can be coupled to baryons using either pseudoscalar (ps) or pseudovector (pv) coupling, and for each of these possibilities the mesons can be assumed to be described by either the Klein-Gordon (KG) or Duffin-Kemmer-Petiau (DKP) equation.[1,2] These possibilities generate the five alternative *PBB* couplings given in Table I.[3] The SU(3) indices have been suppressed but are easily inserted. For example, the KG ps coupling can be written in the explicit form

$$g_1 \overline{B} \gamma_5 B \varphi = (F_1 f_{ijk} + D_1 d_{ijk}) \overline{B}^i \gamma_5 B^j \varphi^k , \qquad (1)$$

with two parameters F_1 and D_1. Since $(g_1)_{\pi NN} = D_1 + F_1$, it is both convenient and conventional to parametrize the *PBB* couplings in terms of $(g_1)_{\pi NN}$ and $\beta_1 = D_1/(D_1 + F_1)$. Analogous statements exist for the other couplings. The coupling constants are then extracted from experiment as described below.

The statement that a particular coupling scheme obeys SU(3) means[4] that on a plot of the predicted coupling constants the experimental points fall along a vertical line corresponding to a unique value of β. Such plots[5] are given in Figs. 1 and 2 for KG ps and DKP pv couplings. It is seen that the DKP pv coupling gives the better agreement with SU(3) and that the derived value of $\beta = 0.71 \pm 0.07$ with a χ^2 of only 0.25 is in excellent agreement with the SU(6) value $\beta = 0.6$ ($D/F = \frac{3}{2}$). The other possibilities are not shown but none gives a better fit than does the KG ps which has a χ^2 of 8.9, 35 times larger than the DKP pv χ^2.

Although the theory as outlined above is quite straightforward, some care must be exercised in the experimental determination of the coupling constants.

The relevant experimental quantities are the residues at a pole of an appropriate $PB \to P'B'$ scattering amplitude, and their determination is *independent* of the way in which the *PBB* couplings are expressed. The residue can be expressed as any function of the masses, X, times the product of "coupling constants." However, it is convenient to choose the X so that the coupling constants correspond exactly to g_1 of the KG ps theory in Eq. (1). In this way, one gives a precise meaning to the word "coupling constant" and can then speak of SU(3) invariance of a given theory. In general the coupling constants when expressed in terms of the parameters g_i will contain mass factors. For convenience, "effective coupling constants" have been tabulated in Table I. One simply inserts $\hat{g}$ into the KG ps result to obtain the residues for a particular theory.

If we assume the validity of SU(2), then there exist 12 independent *PBB* coupling constants. To date five of these have been determined,[6] namely πNN, $\pi\Lambda\Sigma$, $\pi\Sigma\Sigma$, $KN\Sigma$, and $KN\Lambda$. There has been little controversy over the first four of these, but the $KN\Lambda$ coupling constant has been the source of

TABLE I. The elementary PBB couplings in the KG and DKP formalisms. In the DKP formalism $\bar{u}$ denotes the Lorentz-invariant spinor $\bar{u}=(0,0,0,0,1)$. M and M' denote the masses of the initial and final baryons, and μ is the meson mass. The couplings proportional to g_4 and g_5 are designated as DKP pv and DKP pv′, respectively. For a simple comparison of the couplings one inserts $\hat{g}$ into the KG ps result to obtain the respective dispersion-theoretic pole terms. The DKP couplings $\bar{u}\psi$ and $\bar{u}\beta^\lambda\psi$ can be derived from the couplings $P\psi$ and $P_\lambda\psi$, where $P\equiv\beta_1^2\beta_2^2\beta_3^2\beta_4^2$ and $P_\lambda\equiv P\beta_\lambda$ are the DKP spin-0 scalar and vector projection operators.[3]

	KG	DKP
pseudoscalar	$g_1\bar{B}\gamma_5B\varphi$ $\hat{g}=g_1$	$g_3\bar{B}\gamma_5B\bar{u}\psi$ $\hat{g}=g_3\sqrt{\mu}$
pseudovector	$ig_2\bar{B}\gamma_5\gamma_\lambda B\partial^\lambda\varphi$ $\hat{g}=g_2(M+M')$	$ig_4\bar{B}\gamma_5\gamma_\lambda B\bar{u}\beta^\lambda\psi$ $\hat{g}=g_4(M+M')/\sqrt{\mu}$
(pseudovector)′		$ig_5\bar{B}\gamma_5\gamma_\lambda B\bar{u}\partial^\lambda\psi$ $\hat{g}=g_5\sqrt{\mu}(M+M')$

some difficulty. Attempts to determine it from experiment fall into three categories: dispersion theory, photoproduction, or extrapolation and, historically, have yielded a wide range of results.[7] Photoproduction results appear to be model-dependent and are not mutually consistent. Extrapolation methods are a serious alternative but are not emphasized here because of well-known difficulties in the analogous extrapolation techniques in production processes. Certainly analyses which fit calculated dispersion integrals to simple poles are highly suspect. Thus we believe the dispersion-theory approach to be the most reliable at present. Therefore, we quote the values for $g_{KN\Lambda}$ from the two latest evaluations from forward dispersion relations.[8] However, we do not claim to give the final answer to the experimental questions. We simply urge the reader to exercise critical judgment before quoting an older result for the $KN\Lambda$ coupling. Some analyses, for example, have been shown to be inadequate for coupling-constant determinations and we consequently feel that results based on these analyses are of historical interest only.

Fortunately, experimental analyses[8] of $\bar{K}N$ have settled down to the point where there exist at present several semi-independent and mutually consistent determinations[8] of enough information to calculate the $KN\Lambda$ coupling constant, and these lead to mutually compatible results. From a theoretical point of view there are a number of methods for extracting the PBB coupling constants from the experimental data, each using different theoretical assumptions and each emphasizing different energy regions and partial waves. To list

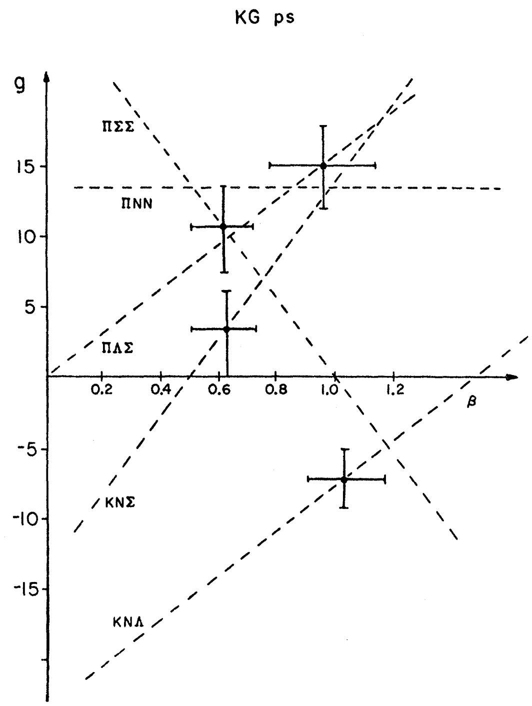

FIG. 1. Meson-baryon coupling constants in the KG ps formalism. The dashed lines give the SU(3) predictions as a function of $\beta=D/(D+F)$, and the points are the latest experimental values. The canonical SU(6) value for β is 0.6.

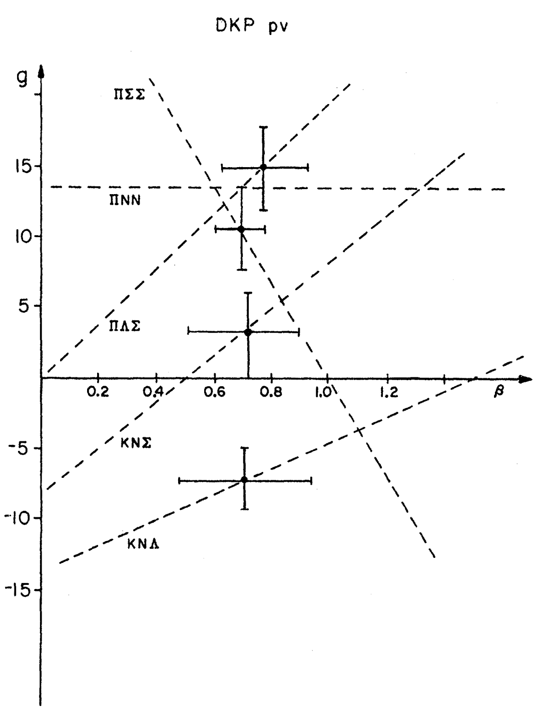

FIG. 2. Meson-baryon coupling constants in the DKP pv formalism. Conventions and scale are those of Fig. 1.

a few, there are forward dispersion relations, invariant-amplitude dispersion relations, continuous-moment sum rules, finite-energy sum rules, superconvergent dispersion relations, and several other unique methods without names at present. Fortunately all dispersion-theoretic methods yield compatible results for a particular coupling constant if given the same experimental input. Therefore, the situation rests completely on the experimental input.

Except for the πNN coupling, all relevant information comes from systems related to $\bar{K}N$ states. There is a region between $\pi\Lambda$ and $\bar{K}N$ threshold accessible at present only through fits to low-energy $\bar{K}N$ and KN data, and this information plays a crucial role in any dispersion-theoretic approach. It has been shown in several ways that experimental parametrizations which do not have the proper threshold and unitarity behavior are essentially useless for coupling-constant determinations. Only K-matrix parametrizations are free of this difficulty. The pioneering analysis was done by Kim,[9] and certainly without this approach attempts to determine the coupling constants would be fruitless today. However, there have been at least four more recent analyses which show that the parameters of Kim's fit should be revised.[8] In particular, this fit is not uniquely determined, and does not agree with later data on the neutral channels in cross section, angular distribution, or branching ratio. It has been further argued[10] that the S-wave effective ranges which play an important role in this fit are determined by the region far above threshold where D waves are important. Thus their use may be dangerous. The other analyses lead to unique fits and are compatible with the $KN\Lambda$ data which we use.

As mentioned, there has been no controversy over the other four couplings. There have been updated calculations as more data have become available, but all changes have been within the quoted uncertainties. We use the latest values[6,8] of $g^2/4\pi$ determined for the five constants:

$$\pi NN = 14.64 \pm 0.2 \equiv 14.64,$$

$$KN\Lambda = 4.0 \pm 2.4,$$

$$KN\Sigma = 0.9 \pm 1.5,$$

$$\pi\Sigma\Lambda = 17.8 \pm 7,$$

$$\pi\Sigma\Sigma = 9.0 \pm 5.$$

In conclusion we can summarize our results as follows: (a) Of the five couplings listed in Table I, DKP pv is the most compatible with SU(3); (b) furthermore, the derived value $\beta = 0.71 \pm 0.07$ with $\chi^2 = 0.25$ is surprisingly close to the SU(6) value $\beta = 0.6$ ($D/F = \frac{3}{2}$); (c) the relatively poor agreement ($\chi^2 = 8.9$) between KG ps and SU(3) is not simply a consequence of our critical analysis of the various coupling constants: Even if one arbitrarily chose published results it would be difficult to make all of the conventional KG ps coupling constants agree with SU(3).

Of the remaining seven PBB coupling constants, $K\Xi\Lambda$, ηNN, and $\pi\Xi\Xi$ appear to be the most accessible experimentally. It is interesting to note that both ηNN and $K\Xi\Lambda$ would vanish identically for $\beta = 0.75$, and hence we predict both of these constants to be extremely small. (In fact very preliminary analysis indicates that $K\Xi\Lambda$ is indeed very small.) This could be a significantly different prediction between the DKP and KG formalisms: Agreement between KG theory and SU(3) most likely can be achieved only if the experimental determinations of $g(\pi\Lambda\Sigma)$ and $g(KN\Lambda)$ are incorrect and give too high a β. (See Fig. 1.) Then the "correct" KG value of β would be near 0.6 and the $K\Xi\Lambda$ and ηNN couplings could not vanish in the way predicted by the DKP formalism.

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[1]E. Fischbach, F. Iachello, A. Lande, M. M. Nieto, and C. K. Scott, Phys. Rev. Lett. 26, 1200 (1971); E. Fischbach, M. M. Nieto, H. Primakoff, C. K. Scott, and J. Smith, Phys. Rev. Lett. 27, 1403 (1971).

[2]N. G. Deshpande and P. C. McNamee, Phys. Rev. D 5, 1012 (1972).

[3]For a discussion of the DKP projection operator couplings see E. Fischbach, M. M. Nieto, and C. K. Scott, Prog. Theor. Phys. 48, 574 (1972).

[4]The meaning of SU(3) invariance in the presence of symmetry-breaking effects requires a longer discussion than space permits. In the present paper the structure constants f_{ijk} and d_{ijk} are assumed to have their exact SU(3)-symmetric values, whereas the masses of the particles are given by their actual physical values in both the theoretical and experimental analyses.

[5]R. D. Field, Jr. and J. D. Jackson, Phys. Rev. D 4, 693 (1971). This is the only reference which is sensitive to the relative signs of the coupling constants. Although not definitive, it favors a relative positive sign for $\pi\Sigma\Lambda$ and $KN\Sigma$. We follow the usual practice of resolving sign ambiguities in favor of SU(3).

[6]G. Ebel, A. Müllensiefen, H. Pilkuhn, F. Steiner, D. Wegener, M. Gourdin, C. Michael, J. L. Petersen, M. Roos, B. R. Martin, G. Oades, and J. J. de Swart, Nucl. Phys. B33, 317 (1971).
[7]See, for example, T. A. Lasinski in *Hyperon Resonances-70,* edited by E. C. Fowler (Moore, Durham, North Carolina, 1970), p. 85.
[8]A. D. Martin and G. G. Ross, Nucl. Phys. B16, 479 (1970); B. R. Martin and M. Sakitt, Phys. Rev. 183, 1345 (1969); D. Berley, S. P. Yamin, R. R. Kofler, A. Mann, G. W. Meisner, S. S. Yamamoto, J. Thompson, and W. Willis, Phys. Rev. D 1, 1996 (1970); Y.-A. Chao, R. W. Kraemer, D. W. Thomas, and B. R. Martin, Nucl. Phys. B56, 46 (1973).
[9]J. K. Kim, Phys. Rev. Lett. 19, 1074 (1967).
[10]A. D. Martin, N. M. Queen, and G. Violini, Phys. Lett. 29B, 311 (1969); Nucl. Phys. B10, 481 (1969).

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Gauge Theories Based on Quarks and the $\eta \to 3\pi$ Problem*

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If chiral $U(3)_L \times U(3)_R$ symmetry is broken only in the $(3, 3^*) + (3^*, 3)$ manner (ignoring weak and electromagnetic interactions), as is the case for the class of renormalizable gauge theories based on quarks and the simple quark-gluon model, then it is shown that the contribution of the U_3 term to the $\eta \to 3\pi$ decay treated to all orders in perturbation theory leads to a grossly incorrect slope and perhaps a suppressed magnitude contrary to usual expectations. Exceptions to the above result might have arisen if either of the interesting relations $\sqrt{2}\,\epsilon_0 + \epsilon_8 = 0$ or $\sqrt{2}\,\epsilon_0 + \epsilon_8 = \pm\sqrt{3}\,\epsilon_3$ were satisfied exactly, which, however, does not seem to be permitted by nature. Thus $\eta \to 3\pi$ decay remains a major problem for the class of models mentioned above, even in the presence of the U_3 term.

I. INTRODUCTION

It is interesting to observe that renormalizability of the weak, electromagnetic, and strong interactions (a phenomenon which seems possible to be realized in the context of gauge theories) together with the requirement that the strong interactions be parity-conserving imposes strong constraints on the chiral property of the hadronic world. Since the weak and electromagnetic currents must necessarily be generated by at least a subset of what one calls the $SU(3)_L \times SU(3)_R$ gauges (in fact the observed SU(3) symmetry is defined by the algebra of their charges), it follows from the above requirements that the strong interactions must be invariant at least under the $SU(3)_L \times SU(3)_R$ group. If the basic fermions in theory, however, are quarks or quarklike objects [belonging to the three-dimensional representation of SU(3)], the minimal symmetry group of the strong interactions, with the requirements mentioned above, becomes in fact[1] the bigger group $G_U \equiv U(3)_L \otimes U(3)_R$. The said symmetry is allowed to be broken only by the quark mass terms (which may arise in such a scheme through spontaneous symmetry breaking). Thus, G_U *as a symmetry, may be broken badly by the Lagrangian* (after shifting of the Higgs scalars); *however, it may only be broken in the* $(3,3^*)+(3^*,3)$ manner (ignoring, of course, the weak and electromagnetic interactions). To summarize, it appears to be a *necessary feature*[2] *of renormalizable gauge theories with quarks*[3] that the Lagrangian should have the chiral structure given by

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}' + \mathcal{L}_{\rm em} + \mathcal{L}_{\rm wk}\,, \qquad (1)$$

where $\mathcal{L}_0$ is invariant under G_U and $\mathcal{L}'$ is noninvariant under G_U, but may be expressed as a sum of terms belonging to a single $(3,3^*)+(3^*,3)$ representation of G_U (we have not exhibited the leftover Higgs boson interaction terms, since these do not affect the discussion to follow).

Insofar as the chiral property of the Lagrangian given by Eq. (1) is a necessary feature of a large class of renormalizable gauge theories, it is of interest to examine the consequences of such a structure on the hadronic matrix elements. The purpose of this note is to point out that a Lagrangian with the said chiral behavior imposes strong constraints on the mechanism for $\eta \to 3\pi$ decay, so much so that $\eta \to 3\pi$ decay remains a puzzle *even* in the presence of a U_3 term in the Lagrangian.

We recall that the assumption of current algebra and strong PCAC (partially conserved axial-vector current) implies that the second-order contribution