No-Pole S-Matrix Fit to the $\Delta(1236)$ Resonance*

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Recently measurements of π-N 3-3 phase shifts have been made to an accuracy of about 0.1°. We show that the 14 experimental points can be fitted with a 4-parameter no-pole S matrix (having all the usually accepted analytic properties) to a precision of 1°. Whereas this is unacceptable as a fit to the precisely determined Δ^{++}, it is a better accuracy than that for which all the other elementary particle resonances are known. We conclude that one should associate a resonance with a Breit-Wigner form (implying an S-matrix pole) because of the physics in the problem, such as a symmetry model, and not because of the ease in fitting certain experimental data. Thus we stress the need for high-accuracy data to help settle the problem of resonances (such as the Z^*'s) whose dynamic origin might be questionable.

Experimentally, many "resonances" are seen corresponding to peaks in cross sections or to phase shifts going up through $\frac{1}{2}\pi$. One says that unstable particles are present as intermediate states. Usually one fits the resonant cross section and/or phase shifts with a Breit-Wigner form which then associates an S-matrix pole in the complex energy plane with the unstable particle. It was pointed out a few years ago[1] that, in principle, there does not exist a one-to-one correspondence between unstable particles and poles of the S matrix. Some resonances could, in fact, be described by an S matrix having the usually accepted analytic properties without any associated pole in the complex plane, while other resonances will have poles associated with them.

The purpose of this paper is to explore the possibility of fitting a resonance with a simple S matrix with no poles, involving just a few parameters. It is in fact obvious that, allowing an arbitrary number of parameters, one can fit a given set of data to any desired accuracy.

Recently there has been an extremely accurate determination[2] of the pion-nucleon phase shift at 14 energy points in the 3-3 resonance region, having errors of the order of 0.1°, which is an order of magnitude more accurate than previous data for the 3-3 resonance and is even a greater improvement in accuracy for all the other elementary particle resonances. It has been pointed out[3] that five different resonance fits to δ_{33} yield an S-matrix pole in the complex energy plane at a nearly unique value of $1211-50i$ MeV. It was necessary to use 4-parameter formulas in order to get a χ^2 of $\lesssim 1$ per point.

We wanted to test the no-pole description just on this so-well-determined resonance. To fit the accurate data of Ref. 2, following the work of Ref. 1, we chose the simple expression

$$\delta_{33}(W)=\frac{\pi}{2}\left(\frac{E_R}{E}\right)^{1/2}\frac{\int_{-\eta}^{X(W)}(1-t^2)^n(1-t^2/\eta^2)dt}{\int_{-\eta}^{0}(1-t^2)^n(1-t^2/\eta^2)dt}\,, \tag{1}$$

where n is a positive integer, $0<\eta<1$, $E=W-M_N-M_\pi$, and W is the center-of-mass energy. $X(W)$ is a function of energy chosen in such a way that the S matrix obtained from (1) possesses the following properties:

(a) $S(W)=-1$ at $E=E_R$, i.e., we have a resonance there.

(b) $S(W)$ is analytic in the complex energy plane cut from $-\infty$ to W_0 (dynamical left-hand branch cut) and from M_N+M_π to $+\infty$ (physical kinematical cut).

(c) $S(W)$ does not possess any other singularities (in particular there are no poles) in any sheet of the complex energy plane.

(d) $S(W)$ possesses the correct threshold behavior ($\delta_{33}\sim k^3$) and approaches unity as $W\to\infty$ along any direction and in any sheet.

We were able to find two rather good functions:

$$X_1(W)=-\eta\,\frac{\ln[(E+\mu)/(E_R+\mu)]}{\ln[\mu/(E_R+\mu)]}\,, \tag{2}$$

$$X_2(W)=\frac{\eta}{\ln(E_R/\mu)}\left[\ln\frac{E+\mu}{E+E_R^2/\mu}-\ln\frac{E_R+\mu}{E_R+E_R^2/\mu}\right], \tag{3}$$

with $\mu = M_N + M_\pi - W_0$, for which all the above requirements on the S matrix are satisfied.

We have studied the phase shift of Eq. (1) for both (2) and (3) for $X(W)$ for various values of the four parameters n, η, E_R, and W_0. We find that the results are rather insensitive to n. With the choice (2) for $X(W)$, we easily find a range for the parameters (for a given $n \geq 4$) for which the calculated values are off from each experimental point by 1°. With the choice (3) for $X(W)$, we were able to get within 4° of the experimental values.[4] (See Fig. 1.)

Since we have been able to get within a few degrees per point for the Δ^{++} resonance over the energy range covered in the experiment of Ref. 2 using the very simple phase shift of Eq. (1), it seems reasonable to conclude that one should be able to fit accurately all the other elementary-particle resonances, which are known to much less accuracy, with a simple no-pole S matrix. Since the very same statement holds true also for the Breit-Wigner-like description, it is left open whether for a given elementary particle resonance, other than the Δ^{++}, the pole or the no-pole description applies.

We conclude that one should associate a resonance with a Breit-Wigner form, (implying an S-matrix pole) because of the physics in the problem, such as a symmetry model or a dynamical scheme, and not because of the ease in fitting certain experimental data. Therefore, if a resonance does not fit in a given symmetry scheme, great caution must be used in rejecting the scheme before examining other possibilities, such as that it might belong to the no-pole category. Thus far, however, while the no-pole S matrix given by Eq. (1) obeys all the fundamental principles required of it, we have no idea of the underlying physics. Thus it would be of strong interest to try to find a dynamical scheme which could produce this no-pole resonance in order to see how simple or complicated the mechanism must be. Finally, noting how the high accuracy of the data of Ref. 2 ruled out the simple no-pole form as given by (1), we stress the value and need for high-accuracy data over a wide energy range in helping to settle the origin of any resonance (such as the Z^*'s) whose dynamical origin might be questionable.

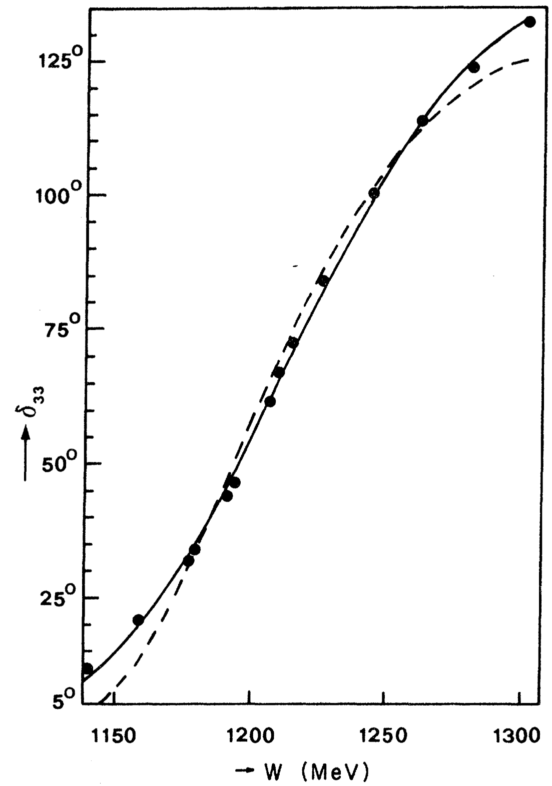

FIG. 1. The theoretical phase shift δ_{33} calculated using $X_1(W)$ (continuous line) and $X_2(W)$ (dashed line). The 14 experimental points are indicated by dots. The parameters for the X_1 case are $n = 7$, $\eta = 0.656$, $W_R = 1233.4$ MeV, and $W_0 = 218$ MeV. For the X_2 case $n = 14$, $\eta = 0.92$, $W_R = 1230.4$ MeV, and $W_0 = 838$ MeV.

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[4]We report the results obtained both from the choice $X_1(W)$ and from the choice $X_2(W)$ for $X(W)$, even though with $X_1(W)$ the fit looks much better, since outside the considered energy interval $\delta_{33}^{[X_1]}$ exhibits unpleasant oscillations while $\delta_{33}^{[X_2]}$ has a very smooth behavior. This

is due to the fact that $X_1(W)$ maps a finite energy interval onto $-\eta \le x \le \eta$, while $X_2(W)$ maps the whole kinematical cut $M_N + M_\pi \le W < +\infty$ onto the same interval. There results a phase shift which can be better controlled over the entire energy axis on varying the parameters. The second fit is therefore more satisfactory at higher energies, and, on the other hand, not bad in the interesting energy region.