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# Electromagnetic Decay Rates of Neutral Mesons*

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A simple model combining vector dominance, SU(3) symmetry, and the $\omega$-$\phi$ and $\eta$-$\eta'$ mixing hypotheses is in reasonable agreement with all measured electromagnetic decay rates of neutral vector and pseudoscalar mesons. In particular, this model predicts the width of the $\eta'$ meson to be between 0.7 and 1.7 MeV.

The combination of SU(3) symmetry with vector-meson dominance[1] forms a powerful tool for predicting the electromagnetic decay rates of the neutral vector and pseudoscalar mesons. This theoretical approach succeeded in correctly predicting the unexpectedly large branching ratio $\omega \to \pi^0\gamma/\omega \to \text{all}$ (Ref. 2) and the small (compared to simple counting of powers of $\alpha$) branching ratio $\eta \to \pi^+\pi^-\gamma/\eta \to \gamma\gamma$. The model, however, predicted the width of the decay $\eta \to \gamma\gamma$ to be ~200 eV, while the measured width turned out to be ~1000 eV. In order to remedy the situation, $\omega$-$\phi$ mixing and $\eta$-$\eta'$ mixing were introduced.[3] Since then, many extensive studies have been done along this line.[4] They have found generally reasonable agreement with experiment. For the past year or so, there has been an accumulation of new data and better measurement of some of the well-known rates. In this paper, we report the result of the fit we made to all known data with the model combining vector-meson dominance, SU(3) symmetry, and the $\omega$-$\phi$ and $\eta$-$\eta'$ mixing hypotheses. We find the model particularly attractive in relating several decays of the $\eta$ meson. Although the model is simple and naive as compared with more sophisticated versions,[5] it still agrees quantitatively with experiment.

## SU(3) STRUCTURE OF *VVP* COUPLING AND OCTET-SINGLET MIXING

The vector-vector-pseudoscalar vertex is written in Lorentz space in terms of an effective Lagrangian

$$L_{\text{int}} = G\epsilon^{\mu\nu\lambda\kappa}\partial_\mu V_\nu^{(1)}\partial_\lambda V_\kappa^{(2)}P, \tag{1}$$

where $V$ and $P$ stand for vector- and pseudoscalar-meson fields, respectively. In momentum space, one can write it as

$$\epsilon_\kappa^{(2)}\langle V^{(1)}|\, J^{(2)\kappa}(0)|\, P\rangle = G\epsilon^{\mu\nu\lambda\kappa}p_\mu^{(1)}\epsilon_\nu^{(1)}p_\lambda^{(2)}\epsilon_\kappa^{(2)}, \tag{2}$$

where $\epsilon_\mu^{(i)}$ and $p_\mu^{(i)}$ ($i = 1$ and 2) are the polarizations and momenta of the vector mesons, and $J^{(2)\kappa}(0)$ is the source function of the vector field $V^{(2)}$. The coupling constant $G$ is a Lorentz invariant (the state vectors are so normalized).

If SU(3) symmetry holds for the strong interaction, we can write three independent couplings for $VVP$:

$$\begin{aligned} A_1 &= \frac{2}{\sqrt{18}}\, G_1\, \mathrm{Tr}(V_8V_8P_8), \\ A_2 &= \frac{2}{\sqrt{18}}\, G_2\, \mathrm{Tr}(V_8V_8)\eta_1, \\ A_3 &= \frac{1}{\sqrt{18}}\, G_3\, \mathrm{Tr}(V_8P_8)\omega_1, \end{aligned} \tag{3}$$

where $V_8$ and $P_8$ are the vector and pseudoscalar octets, $\omega_1$ and $\eta_1$ are the singlets, and $G_1$, $G_2$, and $G_3$ are SU(3)-invariant constants.

In addition, we can introduce SU(3)-breaking interactions into the Hamiltonian which transform like $\lambda_8$. Then we can write five more independent couplings relevant to the radiative decays:

$$\begin{aligned} A_4 &= \frac{\sqrt{2}}{3\sqrt{3}}\, G_4\, \mathrm{Tr}(V_8V_8\lambda_8P_8), \\ A_5 &= \frac{\sqrt{2}}{3\sqrt{3}}\, G_5\, \mathrm{Tr}(V_8V_8)\,\mathrm{Tr}(\lambda_8P_8), \\ A_6 &= \frac{\sqrt{2}}{6\sqrt{3}}\, G_6\, \mathrm{Tr}(V_8P_8)\,\mathrm{Tr}(\lambda_8V_8), \\ A_7 &= \frac{\sqrt{2}}{3\sqrt{3}}\, G_7\, \mathrm{Tr}(V_8V_8\lambda_8)\eta_1, \\ A_8 &= \frac{1}{3\sqrt{6}}\, G_8\, \mathrm{Tr}(V_8\lambda_8P_8)\omega_1. \end{aligned} \tag{4}$$

These eight couplings describe all SU(3)-breaking effects to first order in $\lambda_8$. However, because of the closeness of the masses of $\omega_1$ and $\omega_8$ (and also $\eta_1$ and $\eta_8$), some of the higher-order SU(3)-breaking effects must be taken into account through the mixing of the particle states. The approximate validity of the Gell-Mann-Okubo mass formula for $P_8$ indicates that $\eta$-$\eta'$ mixing is small. The mixing angle $\chi$ defined by

$$\begin{aligned} \eta_1 &= \eta'\cos\chi + \eta\sin\chi, \\ \eta_8 &= -\eta'\sin\chi + \eta\cos\chi \end{aligned} \tag{5}$$

is approximately $|\chi| = 10°$ if one chooses the quadratic version of the mass formula, and $|\chi| = 23°$ in case of the linear mass formula. For $V_8$ the mixing is more significant because of the approximate degeneracy between the octet and the singlet. The linear and quadratic mass formulas give the mixing angle $\theta$, defined by

$$\omega_1 = \omega\cos\theta + \phi\sin\theta,$$
$$\omega_8 = -\omega\sin\theta + \phi\cos\theta, \tag{6}$$

as $|\theta| = 40°$ and 37°, respectively. The abnormally small production rate of the $\phi$ in inelastic non-strange particle collisions is ensured if $\theta \sim \tan^{-1}(\frac{1}{2})^{1/2} \sim 35°$. In the language of the quark model, the $\phi$ is made purely of the strange quark-antiquark pair in this case. We will consider Eqs. (5) and (6) as relations in field operators so that the quadratic mass formula is implicitly assumed. For the sake of completeness, we give our phase convention for $\omega_1$ and $\omega_8$ (same as for $\eta_1$ and $\eta_8$):

$$V_8 = \begin{pmatrix} (\frac{1}{2})^{1/2}\rho^0 - (\frac{1}{6})^{1/2}\omega_8 & \rho^+ & K^{*+} \\ \rho^- & -(\frac{1}{2})^{1/2}\rho^0 - (\frac{1}{6})^{1/2}\omega_8 & K^{*0} \\ K^{*-} & \bar{K}^{*0} & (2/\sqrt{6})\omega_8 \end{pmatrix}.$$

## VECTOR-MESON DOMINANCE

We will assume vector-meson dominance of the electromagnetic current. First of all, we assume that the electromagnetic current is an SU(3) octet with no mixture of a singlet. The photon therefore transforms like

$$\rho - (\tfrac{1}{3})^{1/2}\omega_8 \propto \tfrac{1}{3}(2\mathcal{P}\bar{\mathcal{P}} - \mathcal{N}\bar{\mathcal{N}} - \lambda\bar{\lambda}). \tag{7}$$

In the vector-meson-dominance model, the $\rho\gamma$, $\omega\gamma$, and $\phi\gamma$ transitions are described by the transition strengths $f_{\rho\gamma}$, $f_{\omega\gamma}$, and $f_{\phi\gamma}$, which obey the relation

$$f_{\rho\gamma} : f_{\omega\gamma} : f_{\phi\gamma} = 1 : \frac{1}{\sqrt{3}}\sin\theta : -\frac{1}{\sqrt{3}}\cos\theta \tag{8}$$

if SU(3) symmetry holds, and

$$f_{\rho\pi\pi}\, f_{\rho\gamma} = e, \tag{9}$$

with

$$f_{\rho\pi\pi}^2/4\pi = 2.43 \tag{10}$$

from $\Gamma(\rho\to\pi\pi) = 125$ MeV.

The photon-vector-meson transition coupling $f_{V\gamma}$ is a dimensionless constant defined by

$$\langle 0|J_\mu(0)|V\rangle = \epsilon_\mu m_V^2 f_{V\gamma}. \tag{11}$$

We follow the simplest and most popular convention that $f_{V\gamma}$ obeys SU(3) symmetry.

To check the validity of vector-meson dominance, we compare the widths of $\rho^0\to\pi^0\gamma$ and $\pi^0\to 2\gamma$. From SU(2) symmetry combined with vector-meson dominance, we have

$$\frac{\Gamma(\rho^0\to\pi^0\gamma)}{\Gamma(\pi^0\to\gamma\gamma)} = \frac{4}{3}\frac{1}{f_{\rho\gamma}^2}\frac{1}{m_{\pi^0}^3}\left(\frac{m_\rho^2 - m_{\pi^0}^2}{2m_\rho}\right)^3$$
$$= 9.2\times 10^3. \tag{12}$$

This prediction will test directly the vector-meson-dominance model since it rests only on SU(2) symmetry.

## CALCULATION OF DECAY RATES

The width for the decay of one meson of four-momentum $p_0$ into $n$ particles of four-momenta $p_1, p_2, \ldots p_n$ is

$$\Gamma_{fi} = \frac{(2\pi)^7}{(2\pi)^{3(n+1)}2m_0} \times \int |T_{fi}|^2\,\delta^4\Big(\sum_{i=1}^n p_i - P_0\Big)\prod_{i=1}^n \frac{d\vec{p}_i}{2\omega_i}. \tag{13}$$

Here $m_0$ is the mass of the decaying particle, $\omega_i$ is the energy of the $i$th decay product, and $T_{fi}$ is the $T$-matrix element for the reaction. Then for two-body decays, we have

$$\Gamma(P\to\gamma\gamma) = \frac{G^2 f^4 m_P^3}{64\pi},$$
$$\Gamma(P\to V\gamma) = \frac{G^2 f^2}{4\pi}\left(\frac{m_P^2 - m_V^2}{2m_P}\right)^3, \tag{14}$$
$$\Gamma(V\to P\gamma) = \frac{G^2 f^2}{12\pi}\left(\frac{m_V^2 - m_P^2}{2m_V}\right)^3 \quad (f\equiv f_{\rho\gamma}),$$

where $P$ ($V$) stands for a member of the pseudoscalar (vector) nonet, and $G$ is a linear combination of the $G_i$. The coefficients of the constants $G_i$ are shown in Table I. For decays with more than two particles in the final state, we calculated the decay width by numerical integration.

For the decay $\eta\to\pi^+\pi^-\gamma$,

$$\Gamma(\eta\to\pi^+\pi^-\gamma) = G^2 f^2 \frac{f_{\rho\pi\pi}^2}{24\pi^3}\int_0^{E_\gamma\,\max}\left(1 - \frac{4m_\pi^2}{W^2}\right)^{1/2} \times\left(\frac{W^2}{4} - m_\pi^2\right) m_\eta E_\gamma^3 F(W^2)\,dE_\gamma \tag{15}$$

TABLE I. Coefficients of the $G_i$.

| Decay | $G_1$ | $G_2$ | $G_3$ | $G_4$ | $G_5$ | $G_6$ | $G_7$ | $G_8$ |
|---|---|---|---|---|---|---|---|---|
| $\pi^0 \to \gamma\gamma$ | $\frac{4}{3\sqrt{2}}$ | 0 | 0 | 0 | 0 | $\frac{2\sqrt{2}}{\sqrt{3}}$ | 0 | 0 |
| $\eta \to \gamma\gamma$ | $-\frac{4}{3\sqrt{6}}\cos\chi$ | $\frac{8}{3}\sin\chi$ | 0 | $-\frac{4\sqrt{2}}{9}\cos\chi$ | $-\frac{8\sqrt{2}}{3}\cos\chi$ | $-\frac{2\sqrt{2}}{3}\cos\chi$ | $-\frac{4}{3\sqrt{3}}\sin\chi$ | 0 |
| $\rho \to \pi\gamma$ | $\frac{2}{3\sqrt{2}}$ | 0 | 0 | 0 | 0 | $\frac{\sqrt{2}}{\sqrt{3}}$ | 0 | 0 |
| $\rho \to \eta\gamma$ | $-\frac{2}{\sqrt{6}}\cos\chi$ | $2\sin\chi$ | 0 | 0 | $-2\sqrt{2}\cos\chi$ | 0 | 0 | 0 |
| $\omega \to \pi\gamma$ | $\frac{2}{\sqrt{6}}\sin\theta$ | 0 | $\cos\theta$ | 0 | 0 | $\sqrt{2}\sin\theta$ | 0 | 0 |
| $\omega \to \eta\gamma$ | $\frac{2}{3\sqrt{2}}\sin\theta\cos\chi$ | $\frac{2}{\sqrt{3}}\sin\theta\sin\chi$ | $-\frac{1}{\sqrt{3}}\cos\theta\cos\chi$ | $-\frac{4\sqrt{2}}{3\sqrt{3}}\sin\theta\cos\chi$ | $-\frac{2\sqrt{2}}{\sqrt{3}}\sin\theta\cos\chi$ | $-\frac{2\sqrt{2}}{\sqrt{3}}\sin\theta\cos\chi$ | $-\frac{4}{3}\sin\theta\sin\chi$ | $\cos\theta\cos\chi$ |
| $\phi \to \pi\gamma$ | $-\frac{2}{\sqrt{6}}\cos\theta$ | 0 | $\sin\theta$ | 0 | 0 | $-\sqrt{2}\cos\theta$ | 0 | 0 |
| $\phi \to \eta\gamma$ | $-\frac{2}{3\sqrt{2}}\cos\theta\cos\chi$ | $-\frac{2}{\sqrt{3}}\cos\theta\sin\chi$ | $-\frac{1}{\sqrt{3}}\sin\theta\cos\chi$ | $\frac{4\sqrt{2}}{3\sqrt{3}}\cos\theta\cos\chi$ | $\frac{2\sqrt{2}}{\sqrt{3}}\cos\theta\cos\chi$ | $\frac{2\sqrt{2}}{\sqrt{3}}\cos\theta\cos\chi$ | $\frac{4}{3}\cos\theta\sin\chi$ | $\sin\theta\cos\chi$ |
| $\eta' \to \gamma\gamma$ | $\frac{4}{3\sqrt{6}}\sin\chi$ | $\frac{8}{3}\cos\chi$ | 0 | $\frac{4\sqrt{2}}{9}\sin\chi$ | $\frac{8\sqrt{2}}{3}\sin\chi$ | $\frac{2\sqrt{2}}{3}\sin\chi$ | $-\frac{4}{3\sqrt{3}}\cos\chi$ | 0 |
| $\eta' \to \omega\gamma$ | $-\frac{2}{3\sqrt{2}}\sin\theta\sin\chi$ | $\frac{2}{\sqrt{3}}\sin\theta\cos\chi$ | $\frac{1}{\sqrt{3}}\cos\theta\sin\chi$ | $\frac{4\sqrt{2}}{3\sqrt{3}}\sin\theta\sin\chi$ | $\frac{2\sqrt{2}}{\sqrt{3}}\sin\theta\sin\chi$ | $\frac{2\sqrt{2}}{\sqrt{3}}\sin\theta\sin\chi$ | $-\frac{4}{3}\sin\theta\cos\chi$ | $-\cos\theta\sin\chi$ |
| $\eta' \to \rho\gamma$ | $\frac{2}{\sqrt{6}}\sin\chi$ | $2\cos\chi$ | 0 | 0 | $2\sqrt{2}\sin\chi$ | 0 | 0 | 0 |
| $\eta' \to \phi\gamma$ | $\frac{2}{3\sqrt{2}}\cos\theta\sin\chi$ | $-\frac{2}{3}\cos\theta\cos\chi$ | $\frac{1}{\sqrt{3}}\sin\theta\sin\chi$ | $-\frac{4\sqrt{2}}{3\sqrt{3}}\cos\theta\sin\chi$ | $-\frac{2\sqrt{2}}{\sqrt{3}}\cos\theta\sin\chi$ | $-\frac{2\sqrt{2}}{\sqrt{3}}\cos\theta\sin\chi$ | $\frac{4}{3}\cos\theta\cos\chi$ | $-\sin\theta\sin\chi$ |

where

$$W^2 = m_\eta{}^2 - 2m_\eta E_\gamma\,, \quad F(W^2) = \frac{1}{(m_\rho{}^2 - W^2)^2}\,, \tag{16}$$

and $E_\gamma$ is the energy of the $\gamma$ in the $\eta$ rest system. For the decay $\eta \to \pi^+\pi^-\pi^0\gamma$,

$$\Gamma(\eta \to \pi^+\pi^-\pi^0\gamma) = G^2 f^2 \frac{1}{2(2\pi)^8} \int \frac{f_{\omega 3\pi}{}^2 M_{3\pi}{}^6}{m_\eta{}^2 (m_\omega{}^2 - M_{3\pi}{}^2)^2} (\vec{p}_\gamma \cdot \vec{q})^2 \delta^4(p_+ + p_- + p_0 + p_\gamma - p_\eta) \frac{d^3p_+}{2E_+} \frac{d^3p_-}{2E_-} \frac{d^3p_0}{2E_0} \frac{d^3p_\gamma}{2E_\gamma}\,, \tag{17}$$

where $M_{3\pi}$ is the invariant mass of the three pions, and $\vec{q} \equiv \vec{p}_+ \times \vec{p}_-$ in the rest system of the three pions. In all the decays, we assumed the outgoing $\rho$ and $\omega$ to have zero width.

## FIT TO DATA

We have introduced ten parameters: $G_1, \ldots G_8$, $\chi$, and $\theta$. We will, however, ignore the SU(3)-breaking couplings $G_4$, $G_5$, $G_6$, $G_7$, and $G_8$, and retain only $G_1$, $G_2$, $G_3$, $\chi$, and $\theta$, because there are not enough data to determine the ten parameters. We hope that most of the SU(3) breaking is included in the particle mixing.

Current experimental data are summarized in the first column of Table II.[6,7] Unless otherwise specified, the value of the meson decay rate is the weighted average from the Particle Data Group.[8]

We used five measured decay widths and two branching ratios to fit the five parameters $G_1$, $G_2$, $G_3$, $\chi$, and $\theta$. The upper limits were used only to see if the results of the fit were consistent with these limits. We minimized the quantity

$$\chi^2 = \sum_{i=1}^{n} \frac{[(\Gamma_{\mathrm{exp}})_i - (\Gamma_{\mathrm{th}})_i]^2}{(\sigma_{\mathrm{exp}})_i{}^2}\,, \tag{18}$$

where $\Gamma_i$ indicates the width or the branching ratio for the $i^{\mathrm{th}}$ decay, and $\sigma_{\mathrm{exp}}$ is the experimental error.

The Particle Data Group gives the weighted average value of $\Gamma(\pi^0 \to \gamma\gamma)$ as $7.84 \pm 0.92$ eV, while a recent measurement of the $\pi^0$ lifetime through the Primakoff effect gives $11.7 \pm 1.2$ eV.[9] Since we do not know the reason for this discrepancy, we used two values in the fit: $\Gamma(\pi^0 \to \gamma\gamma) = 7.84 \pm 0.92$ eV and $11.7 \pm 1.2$ eV.

Among the five parameters we keep, $\chi$ and $\theta$ are fairly severely restricted; $\chi$ must be small and $\theta$ must be around 30° to 40°. Furthermore, the strong suppression of $\phi$ production in non-strange-particle reactions indicates

$$G_1/G_3 \sim \sqrt{6}\,\tan\theta. \tag{19}$$

TABLE II. Experimental and fitted decay rates.

| Decay width or branching ratio | Experimental value | Fit: $\Gamma(\pi^0 \to \gamma\gamma) = 7.84$ eV | Fit: $\Gamma(\pi^0 \to \gamma\gamma) = 11.7$ eV |
|---|---|---|---|
| $\Gamma(\pi^0 \to \gamma\gamma)$ | $(7.84 \pm 0.92) \times 10^{-6}$ MeV | $7.43 \times 10^{-6}$ MeV | |
| | $(11.7 \pm 1.2) \times 10^{-6}$ MeV | | $10.41 \times 10^{-6}$ MeV |
| $\Gamma(\eta \to \gamma\gamma)$ | $0.001 \pm 0.00022$ MeV | 0.0012 MeV | 0.0013 MeV |
| $\Gamma(\eta \to \pi^+\pi^-\gamma)/\Gamma(\eta \to \gamma\gamma)$ [a] | $0.124 \pm 0.016$ | 0.166 | 0.174 |
| $\Gamma(\eta \to \pi^+\pi^-\pi^0\gamma)$ [a] | $<1.5 \times 10^{-6}$ MeV | $3 \times 10^{-8}$ MeV | $1 \times 10^{-8}$ MeV |
| $\Gamma(\eta' \to \gamma\gamma)$ | ... | 0.0314 MeV | 0.015 MeV |
| $\Gamma(\eta' \to \pi^+\pi^-\gamma)/\Gamma(\eta' \to \gamma\gamma)$ | $17.24 \pm 3.27$ | 15.9 | 14.0 |
| $\Gamma(\eta' \to \pi^+\pi^-\pi^0\gamma)$ | ... | 0.047 MeV | 0.031 MeV |
| $\Gamma(\rho \to \pi^0\gamma)$ | $<0.25$ MeV | 0.067 MeV | 0.094 MeV |
| $\Gamma(\omega \to \pi^0\gamma)$ | $0.78 \pm 0.16$ MeV | 0.73 MeV | 0.81 MeV |
| $\Gamma(\omega \to \eta\gamma)$ | $<0.21$ MeV | 0.013 MeV | 0.008 MeV |
| $\Gamma(\phi \to \pi^0\gamma)$ [b] | $0.011 \pm 0.004$ MeV | 0.011 MeV | 0.011 MeV |
| $\Gamma(\phi \to \eta\gamma)$ [b] | $0.092 \pm 0.029$ MeV | 0.061 MeV | 0.081 MeV |

[a] Reference 6.

[b] Reference 7.

TABLE III. Fitted parameters.

| | $\Gamma(\pi^0 \to \gamma\gamma) = 7.84$ eV | $\Gamma(\pi^0 \to \gamma\gamma) = 11.7$ eV |
|---|---|---|
| $G_1$ | $(3.98 \pm 0.12) \times 10^{-3}$ | $(4.71 \pm 0.13) \times 10^{-3}$ |
| $G_2$ | $(5.24 \pm 0.56) \times 10^{-3}$ | $(3.95 \pm 0.39) \times 10^{-3}$ |
| $G_3$ | $(2.50 \pm 0.07) \times 10^{-3}$ | $(2.48 \pm 0.06) \times 10^{-3}$ |
| $\chi$ | $-0.270 \pm 0.030$ | $-0.363 \pm 0.036$ |
| $\theta$ | $0.658 \pm 0.014$ | $0.737 \pm 0.013$ |

Nevertheless, we left the five parameters free to vary in the region $|G_1|, |G_2|, |G_3| < 0.1$ and $|\theta|$, $|\chi| < \pi$. The fitted parameters are shown in Table III, and the calculated widths and branching ratios are presented in Table II. $\chi^2$ for the fit with $\Gamma(\pi^0 \to \gamma\gamma) = 7.84$ eV is 9.4, and for the fit with $\Gamma(\pi^0 \to \gamma\gamma) = 11.7$ eV, it is 14.4. In both fits, only one minimum in $\chi^2$ is found.

## SUMMARY

We find no obvious discrepancy between our simple model and the data in the best fits given in Table II. The values of $G_1/G_3$, $\chi$, and $\theta$ are all in good agreement, including signs, with those determined from other considerations such as a quark model. Considering various ambiguities associated with SU(3) corrections due to meson mass differences, we believe that at this moment the model of vector-meson dominance and SU(3) symmetry with $\omega$-$\phi$ and $\eta$-$\eta'$ mixing is quantitatively satisfactory.

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