We have considered the process of double beta decay as occurring through a $\Delta Q=2$ current allowed by the Konopinski-Mahmoud scheme of lepton conservation. Previously obtained limits on the coupling strength have been $G' \lesssim 10^{-3} G_V$. In comparison, the value just obtained, $G' \simeq 10 G_V$, is based on the assumption that the model under consideration explains double beta decay entirely. A word should be said about this assumption: We know that double beta decay can occur through conventional $\Delta Q=1$ currents.[2,3] Therefore, the large value of G' obtained leads us to conclude that the $\Delta Q=2$ process, if it exists, is totally overshadowed in double beta decay. Thus, our result is not inconsistent with previous lower estimates of G', but rather tells us that whereas $\Delta Q=2$ currents may explain other processes [such as $\mu^- + Z \to e^+ + (Z-2)$ and $K^+ \to \mu^+ e^+ \pi^-$] in which there are no competing mechanisms, $\Delta Q=2$ currents do not contribute significantly to two-neutrino double beta decay.

We are pleased to acknowledge helpful conversations with Professor Henry Primakoff.

---

*Work supported in part by the National Research Council of Canada under Grant No. A5926.

[1]H. Primakoff and S. P. Rosen, Phys. Rev. D 5, 1784 (1972).
[2]E. J. Konopinski and H. M. Mahmoud, Phys. Rev. 92, 1045 (1953).
[3]S. P. Rosen and H. Primakoff, in *Alpha-, Beta-, and Gamma-Ray Spectroscopy*, edited by K. Siegbahn (North-Holland, Amsterdam, 1965), Vol. II; T. D. Lee and C. S. Wu, Annu. Rev. Nucl. Sci. 15, 381 (1965).
[4]E. J. Konopinski, *Theory of Beta Radioactivity* (Oxford Univ. Press, London, 1966).
[5]H. Primakoff and S. P. Rosen, Rep. Prog. Phys. 22, 121 (1959).
[6]N. Takaoka and K. Ogata, Z. Naturforsch. 21A, 84 (1966).
[7]B. Pontecorvo, Phys. Lett. 26B, 630 (1968).
[8]H. Primakoff and S. P. Rosen, Phys. Rev. 184, 1925 (1969).
[9]B. Pontecorvo, Zh. Eksp. Teor. Fiz. 53, 1717 (1967) [Sov. Phys.—JETP 26, 984 (1968)].
[10]L. S. Kisslinger, Phys. Rev. Lett. 26, 998 (1971).
[11]M. D. Shuster and M. Rho, Phys. Lett. 42B, 45 (1972).
[12]D. A. Bryman, M. Blecher, K. Gotow, and R. J. Powers, Phys. Rev. Lett. 28, 1469 (1972).
[13]E. W. Beier, D. A. Buchholz, A. K. Mann, and S. H. Parker, Phys. Rev. Lett. 29, 678 (1972).
[14]A. K. Kerman and L. S. Kisslinger, Phys. Rev. 180, 1483 (1969); J. S. Vincent *et al.*, Phys. Rev. Lett. 24, 236 (1970); H. Arenhövel, M. Danos, and H. T. Williams, Phys. Lett. 31B, 109 (1970).
[15]T. Kirsten, O. A. Schaeffer, E. Norton, and R. W. Stoenner, Phys. Rev. Lett. 20, 1300 (1968); T. Kirsten, W. Gentner, and O. A. Schaeffer, Z. Physik 202, 273 (1967).
[16]R. K. Bardin, P. J. Gollon, J. D. Ullman, and C. S. Wu, Phys. Lett. 26B, 112 (1967).

PHYSICAL REVIEW D VOLUME 8, NUMBER 1 1 JULY 1973

Simple Statistical Model for Multiple-Particle Production in Heavy Nuclei*

D. C. Peaslee†
Laboratory for Nuclear Science and Department of Physics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139
(Received 8 January 1973; revised manuscript received 5 March 1973)

A simple statistical model is set up to compare with the apparently different results of previous calculations on the production of high-energy particles in a heavy nucleus by multiperipheral collisions. The discrepancy is resolved in terms of a high sensitivity of the model to the production number n in a primary collision. This sensitivity is greatly reduced by the realistic assumption of $s<1$ for the survival of reproductive effectiveness per stage of an intranuclear cascade. Flimsy cosmic-ray data suggest $s \sim 0.6$ in the 100-GeV region.

I. INTRODUCTION

It has recently been emphasized[1,2] that measurement of multiparticle production in heavy nuclei would discriminate among production mechanisms. Two different mechanisms involved are an isobar or fireball model and a multipheripheral model. In the first case, production goes through some compound intermediate state, which is not much affected by the host nucleus. The particle yield on heavier nuclei is then grossly the same as for a proton target, say within a factor of 2. For multiperipheral models the multiplicative effect of intranuclear cascades is important, and much stronger A dependence of the production can be expected. The predicted curves of this dependence[1,2] appear to differ by about an order of magnitude, however, and in any case it is desirable to have a number of calculations on different bases with which to compare ultimate measurements.

Accordingly, the present note estimates mean multiparticle production on a statistical model, rather like that of Ref. 1, but more schematic. Two new modifications are introduced: The energy partition of produced particles is logarithmic ($dn \sim dE/E$) rather than uniform[2] or some other power law,[1] and intranuclear loss in effectiveness of cascade particles is allowed for. The incompatibility of results from Refs. 1 and 2 is concluded to be only apparent, reflecting a strong dependence of total statistical production N on the single-event multiplicity n. The results are not strongly dependent on the energy partition law of produced particles, but the A dependence is very sensitive to the independent survival rate s for successive generations of cascade particles. With currently expected[3] values of n, measurement of the A dependence in the 100-GeV range should in fact mainly reveal s; a value $s \sim 0.6$ is suggested by old cosmic-ray data which are not incompatible with recent calculations.[4]

II. THE MODEL

Collision processes in the heavy nucleus are assumed pointlike and incoherent throughout. The average number of particles produced in the first collision with incident energy E is

$$n^{(1)} = n(E) = a\ln(E/E_0), \tag{1}$$

where E_0 is a threshold energy. Equation (1) implies that the produced particles do not share $(E - E_0)$ equally, but that the jth produced particle has energy $\Delta E_j \approx E_j/a$, where E_j is energy remaining from the original E after producing the first j particles. Thus, $\Delta E_j = E_{j-1} - E_j \approx Ee^{-(j-1)/a}$ $(1 - e^{-1/a})$, or, since $E_0 = E^{-n/a}$,

$$\Delta E_j = E_0^* e^{(n+1-j)/a}, \tag{2}$$

where $E_0^* = E_0(1 - e^{-1/a})$.

When the jth emitted particle multiplies at a secondary collision, it produces an average number

$$\begin{aligned} n_j &= a\ln(\Delta E_j/E_0) \\ &= (n+1-j) - \delta \end{aligned} \tag{3}$$

of particles, where $\delta = -a\ln(1 - e^{-1/a}) \approx \frac{1}{2}$ for physical values of a. Thus the total production in the second collision is

$$n^{(2)} = \sum_{j=1}^{n} n_j = \tfrac{1}{2}(n^2 - \tfrac{1}{4}) \approx \tfrac{1}{2} n^2 \tag{4}$$

for $\delta = \frac{1}{2}$. Similarly,

$$n^{(3)} \approx n^3/3!, \ldots, \; n^{(k)} \approx n^k/k!\,.$$

Note that this type of multiplication allows (through δ) for degradation of the energy with successive collisions, so that for $k \gg n$ there is practically no production: The particle energies are all too near the threshold. Qualitatively this must be a realistic feature, but we shall see that it does not sufficiently damp the heavy-nucleus production.

The incident colliding particles at each stage are conserved but use up all their energy in production of secondaries at that collision, so that after collision their energies have fallen to E_0 and they do not engage in further production. The total number of particles in the cascade after k collisions is thus

$$N_k = 1 + n + \frac{n^2}{2!} + \cdots + \frac{n^k}{k!}\,. \tag{5}$$

We assume a constant mean free path λ or corresponding cross section σ for collision, independent of energy, and in the forward direction only (no sideward deviation of the shower). The mean number of collisions in a nucleus of mass number A is

$$\begin{aligned} m &= \frac{4}{3}\left(\frac{r_0}{\lambda}\right) A^{1/3} \\ &= \left(\frac{\sigma}{\pi r_0^2}\right) A^{1/3}\,. \end{aligned} \tag{6}$$

Here r_0 is the nuclear radius parameter for high-energy reactions. The average number of produced particles is

$$\begin{aligned} N(m, n) &= \sum_k N_k P_k^m \\ &= e^n - e^{-m} \sum_{x=1}^{\infty} \left(\frac{n}{m}\right)^{x/2} I_x(2\sqrt{mn}), \end{aligned} \tag{7}$$

where $P_k^m = m^k e^{-m}/k!$ is the Poisson distribution and I_x is the Bessel function of imaginary argument.

Formula (7) approaches the asymptotic $N \to e^n \sim e^a$, which in principle allows a direct measurement of the parameter a. An extreme of power-law dependence on E was also anticipated in Ref. 1, so that this feature does not seem too directly related to the energy partition for $n(E)$ in the primary collision process. Unfortunately, however, the present asymptotic region is approached only for $m \gtrsim n$; the likely values attainable in practice appear to be $m \ll n$.

III. LOSS OF CASCADE EFFECTIVENESS

For particle production inside a nucleus there is little real absorption of the cascade. There is a loss in reproductive effectiveness per particle, however, because of rescattering and shadowing of the cascade particles by each other.[4] We call this the "survival of effectiveness" or simply "survival" of the cascade particles.

A crude assumption is that the productive effectiveness of each particle remains unimpaired for just l collisions and then drops to zero. Then Eq. (5) is replaced by

$$N_k^l = N_k - N_{k-l-1}$$
$$= \frac{n^k}{k!} + \frac{n^{k-1}}{(k-1)!} + \cdots + \frac{n^{k-l}}{(k-l)!}, \tag{8}$$

and hence

$$N^l(m, n) = \sum_k N_k^l P_k^m$$
$$= e^{-m} \sum_{x=0}^{l} \left(\frac{m}{n}\right)^{x/2} I_x(2\sqrt{mn}). \tag{9}$$

Figure 1 shows N^l for $l=4$ to compare with the limiting form N^∞. The asymptotic power-law region is never reached in these cases. For relatively small n, or hence small E, it might be possible to observe a *falloff* of production with increasing nuclear size and hence m.

Perhaps a more realistic procedure is to introduce a survival-of-effectiveness factor $s<1$ as the probability that an average particle in the cascade after the $(k-1)$st collision will be effective, producing more secondaries at the kth. Allowing this factor for the incident particle as well and neglecting any k dependence of s, we have

$$N_k' \approx s^k N_k \tag{10}$$

and hence

$$e^m N'(m, n) = \sum_{x=0}^{\infty} \left(\frac{m'}{n}\right)^{x/2} I_x(2\sqrt{m'n}), \tag{11}$$

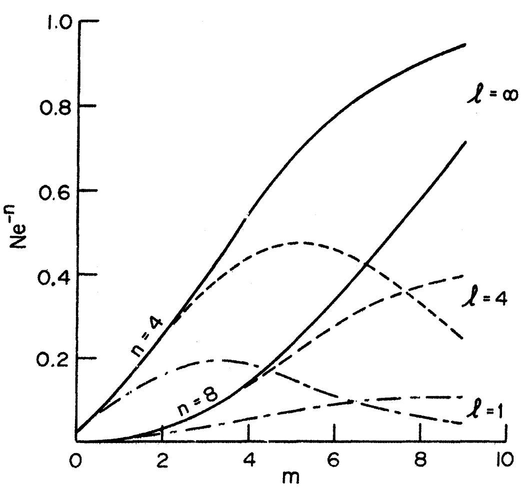

FIG. 1. Curves of $e^{-n} N^l(m,n)$ for moderate n.

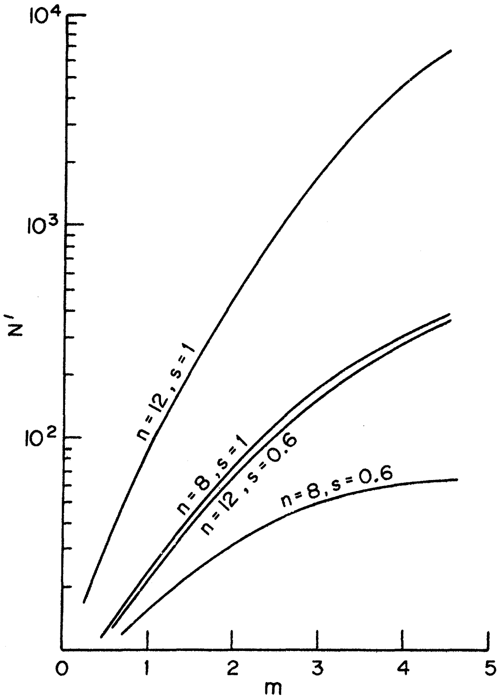

FIG. 2. Curves of $N'(m,n)$ for large n.

where $m' = ms$. Of course the same treatment can be applied to N^l; the resulting N' is not very strongly dependent on l. Curves in Fig. 2 are of $N'^l(m, n)$ for $l=3$; the variation between $l=\infty$ and $l=1$ is about a factor of 2 greater or smaller.

Because N' depends explicitly on both m and m', Fig. 2 is labeled by the independent indices n and s. It is clear that very moderate losses in survival of the cascade have strong effects on N'. As in the case of Fig. 1, for sufficiently small s and large m the curves of N' against m will turn downward, but for large n (~10) this region seems unlikely to be attained in practice.

IV. COMPARISONS

In Fig. 3 the curves of Fig. 1 are replotted to give (N/n) as a function of A. The parameters chosen are[2]

$$a = 0.63, \quad E_0 = 1.7 \text{ GeV}, \quad m = 0.5\, A^{1/3},$$

the last corresponding to $\sigma \approx 30$ mb. The curves of Fig. 3 closely resemble those in Fig. 1 of Ref. 2. The variation with A is relatively weak and may in our case be positive or negative, depending on the degree of internal loss in the cascade.

In Fig. 4 the results from Fig. 2 are used to obtain the solid curve of N vs A for $n=10$, $l=\infty$, $m=0.5\, A^{1/3}$, $s=1$. This curve is similar to that in Fig. 2 of Ref. 1, and the dashed curves in each

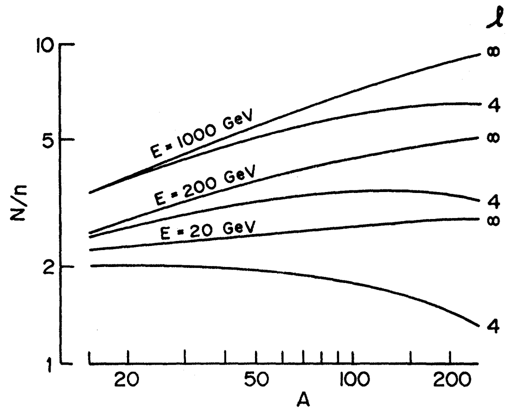

FIG. 3. N/n as a function of A for small n (see text).

diagram, which refer to truncated cascades, are qualitatively analogous to each other.

From these satisfactory comparisons we conclude that the models in Refs. 1 and 2 are physically more or less equivalent. The apparent discrepancy in their results reflects the model's great sensitivity to the mean single-nuclear multiplicity n. This sensitivity is greatly mitigated by the additional assumption of loss in the internal cascade.

The most recent data[3] suggest that $n \approx 7-12$ in the few-hundred-GeV range. In this case values for emulsion nuclei in old cosmic-ray observation[5,6] would suggest on comparison with Fig. 2 that $s \approx 0.6$, with a relatively strong A dependence for low A, flattening out for large A. The energy dependence is more pronounced for large A, however. It will be of interest to explore both E and A dependence sufficiently to see if anything so simple occurs as a constant survival parameter s. Note that a direct estimate[4] of s for $n \approx 7$ yields $s \approx 0.5-0.6$ for the "coherent" case.

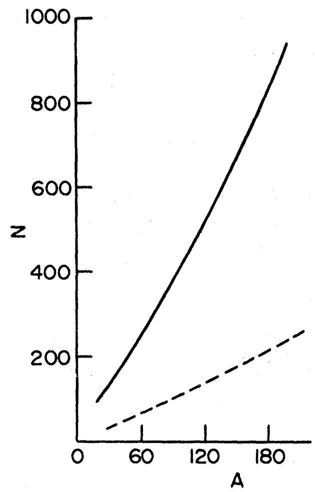

FIG. 4. N vs A for $l=\infty$ (dashed curve for $l=1$) and $n=10$, $m=0.5A^{1/3}$, $s=1$.

ACKNOWLEDGMENTS

The author wishes to thank Professor W. Busza and Professor L. Rosenson for helpful discussions.

*This work is supported in part through funds provided by the Atomic Energy Commission under Contract No. AT 11-1-3069.

†On leave from Australian National University, Canberra.

[1]P. M. Fishbane and J. S. Trefil, Phys. Rev. D 3, 238 (1971).

[2]A. Dar and J. Vary, Phys. Rev. D 6, 2412 (1972).

[3]G. Charlton *et al.*, Phys. Rev. Lett. 29, 515 (1972), and earlier references cited therein.

[4]P. M. Fishbane, J. L. Newmeyer, and J. S. Trefil, Phys. Lett 41B, 153 (1972). An even more significant loss of reproductive effectiveness by the cascade particles will be occasioned by their coherence in dissociated systems D, as discussed by L. Van Hove, Nucl. Phys. B46, 75 (1972).

[5]I. Z. Artykov, V. S. Barashenkov, and S. M. Eliseev, Nucl. Phys. B6, 11 (1968), and references cited therein.

[6]E. Lohrmann, M. W. Teucher, and M. Schein, Phys. Rev. 122, 672 (1961).