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Lepton Scattering in the Georgi-Glashow Theory*

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The effects of the physical scalar on the three reactions, $e^+e^- \to \mu^+\mu^-$, $e^{\pm}e^- \to e^{\pm}e^-$, are calculated in the Georgi-Glashow theory. A relation is found between the contribution of the scalar to these cross sections and the mass of the heavy lepton which can be used to completely test the theory, regardless of the masses of the particles, and without resorting to experiments involving hadrons.

The various unified gauge theories[1] offer extremely attractive (i.e., convergent) ways of combining the weak and electromagnetic interactions of leptons. But it also seems fair to say that no attractive model exists for the hadrons. Thus it is very important to have experimental tests of the theories which are independent of the hadrons, reasoning that a theory may fail the hadronic tests but still be completely correct for leptons.

Experimental tests of the leptonic parts of those theories which involve neutral vector mesons have been proposed. These involve neutrino scattering,[2] if the neutrinos couple to the neutral vector meson, and/or corrections to electromagnetic processes due to the neutral vector mesons. In particular, in the latter category, several groups[3] have considered the process $e^+e^- \to \mu^+\mu^-$ and pointed out that the neutral vector meson will give a contribution which is asymmetric in the cosine of the scattering angle, θ, whereas the lowest-order electromagnetic contribution contains only even powers of $\cos\theta$. Also, the parity-violating part of the interaction will give the muons a nonzero polarization. Further, these effects are enhanced by the polarization of the initial particles which occurs if the initial particles are held in storage rings.

The Georgi-Glashow theory[4] does not involve neutral vector mesons, and thus the tests mentioned above do not apply. However, the leptons do couple to the physical scalar with a coupling constant which may be large. Thus one might hope to see the effect of the scalar in a process like $e^+e^- \to \mu^+\mu^-$. To this end we calculate here the corrections to the three processes $e^+e^- \to \mu^+\mu^-$, $e^-e^{\pm} \to e^-e^{\pm}$ due to the scalar.

The relevant part of the Hamiltonian is

$$-e\bar{\psi}_e\gamma^\alpha\psi_e A_\alpha - e\frac{m_{X^+} - m_e}{2m_W}\bar{\psi}_e\psi_e\phi + \{\psi_e \to \psi_\mu,\, m_{X^+} \to m_{Y^+}\},$$

where ψ_e is the electron field, ϕ is the physical scalar, and m_{X^+} (m_{Y^+}) is the mass of the heavy lepton associated with electrons (muons). The mass of the intermediate vector meson, m_W, is less than 53 GeV.

Depending on which of the three processes is being considered the relevant diagrams are photon and ϕ poles in various channels. Normally we would expect the ϕ to make the largest contribution, relative to the photon, to the process $e^+e^- \to \mu^+\mu^-$. This process has the photon (and ϕ) only in the s channel, and thus the matrix element will never be evaluated close to the photon pole. However, when the matrix element for this reaction is squared, the cross term between the photon and the ϕ parts of the matrix element is of order m_e/E relative to the square of the photon or ϕ parts. This is not true for the other processes (because the cross term can involve the graph with the photon in one channel times the graph with the ϕ in a different channel) and therefore the other processes may give a larger relative ϕ contribution.

We calculate the cross sections in the center-of-mass system, as is appropriate for colliding beams. The incident electron beam has four-momentum and polarization

$$p_1^\mu = (E, 0, 0, p),$$

$$s^\mu = (0, s, 0, 0),$$

while the other beam is along the negative z axis and has polarization $-s^\mu$ if it is a positron but $+s^\mu$ if it is an electron. The final electron has four-momentum

$$p_3^\mu = (E, p\sin\theta\cos\phi, p\sin\theta\sin\phi, p\cos\theta).$$

We shall neglect the electron and muon masses. If we sum over the polarization of one of the final particles the other particle in the final state will not be polarized. This is different from Ref. 3, where the second particle in the final state was polarized due to parity violation in the coupling of the neutral vector meson. To lowest order in α the differential cross section for the process $e^+e^- \to \mu^+\mu^-$ is

$$\frac{d\sigma}{d\Omega}=\frac{\alpha^2}{16E^2}[1+z^2-s^2(1-z^2)\cos 2\phi]+\frac{\alpha^2}{16}\frac{m_{X^+}{}^2 m_{Y^+}{}^2}{m_W{}^4}\frac{E^2}{(4E^2-m_\phi{}^2)^2}(1-s^2) \qquad (1)$$

where $z=\cos\theta$. The second term is the contribution of the ϕ. It is easy to see that the ratio of the ϕ contribution to the photon contribution is a maximum at $s^2=0$, $z=0$. The effect of the ϕ, rather than being enhanced by the polarization of the initial particles, actually goes to zero as s^2 approaches unity. In the limit $m_\phi \gg E$, the ratio at $s^2=0=z$ is plotted in Fig. 1 (curve a) as a function of x, where

$$x=\frac{(m_{X^+}m_{Y^+})^{1/2}}{m_\phi m_W}E\,. \qquad (2)$$

For this process the ratio is simply x^4.

The differential cross section for $e^+e^-\to e^+e^-$ is

$$\frac{d\sigma}{d\Omega}=\frac{\alpha^2}{16E^2(1-z)^2}[(3+z^2)^2+(1-z^2)^2s^2\cos 2\phi]+\frac{\alpha^2}{64}\frac{E^2m_{X^+}{}^4}{m_W{}^4}\left\{\frac{4(1-s^2)}{(4E^2-m_\phi{}^2)^2}+\frac{2(1-z)(1-s^2)}{(4E^2-m_\phi{}^2)[2E^2(1-z)+m_\phi{}^2]}+\frac{(1-z)^2}{[2E^2(1-z)+m_\phi{}^2]^2}\right\}$$
$$-\frac{\alpha^2}{32}\frac{m_{X^+}{}^2}{m_W{}^2}\frac{1}{2E^2(1-z)+m_\phi{}^2}\{(1-z)^2-s^2(1-z^2)\cos 2\phi\}-\frac{\alpha^2}{4}\frac{m_{X^+}{}^2}{m_W{}^2}\frac{1}{(1-z)(4E^2-m_\phi{}^2)}(1-s^2)\,. \qquad (3)$$

The first term is the photon contribution, the second term is the square of the ϕ contribution, and the last two terms are the cross terms. Again the relative ϕ contribution is largest at $s^2=0$ and $z=0$, and here it goes to zero in the forward direction ($z=1$) because of the photon pole. Curve b of Fig. 1 shows the ratio of the ϕ contribution to the photon contribution for this reaction when $s^2=0$, $z=0$, and $m_\phi\gg E$. For this process x is given by

$$x=\frac{m_{X^+}E}{m_\phi m_W}\,. \qquad (4)$$

We can see from Fig. 1 that this process is considerably better than the other two processes for detecting the effects of the ϕ.

Actually if x is less than 0.5 the relative ϕ contribution is slightly greater (~25%) at $s^2\approx 1$, $z\approx -1$ than at $s^2=0$, $z=0$. Taking z negative means that one must measure the charge of the final particles, which, of course, makes the experiment more difficult. Further, if x is much greater than 0.5 the relative ϕ contribution is smaller at $z\approx -1$, $s^2\approx 1$ than at $z=0$, $s^2=0$, but in either case the effects of the ϕ should be easily seen. If one is restricted to $s^2\approx 1$ then the ϕ will not be seen, regardless of the value of x, unless the measurement is made at $z<0$. When $s^2\approx 1$ the most favorable reaction is e^-e^- elastic scattering.

The differential cross section for the process $e^-e^-\to e^-e^-$ is given by

$$\frac{d\sigma}{d\Omega}=\frac{\alpha^2}{4E^2}\left[\frac{(3+z^2)^2}{(1-z^2)^2}-s^2\cos 2\phi\right]+\frac{\alpha^2}{64}\frac{m_{X^+}{}^4}{m_W{}^4}E^2\left\{\frac{(1+z)^2}{[2E^2(1+z)+m_\phi{}^2]^2}+\frac{(1-z)^2}{[2E^2(1-z)+m_\phi{}^2]^2}+\frac{(1-z^2)(1-s^2\cos 2\phi)}{[2E^2(1+z)+m_\phi{}^2][2E^2(1-z)+m_\phi{}^2]}\right\}$$
$$+\frac{\alpha^2}{16}\frac{m_{X^+}{}^2}{m_W{}^2}\left\{\frac{1}{(1+z)[2E^2(1-z)+m_\phi{}^2]}[(1-z)^2+s^2(2-z)]+(z\to -z)\right\}. \qquad (5)$$

For this process the maximum ϕ contribution occurs at $s^2\approx 1$, $\cos 2\phi=1$, and $z=0$. For these values and for $m_\phi\gg E$ the ratio of the ϕ contribution to the photon contribution is shown in Fig. 1 (curve c). x is again given by (4). There is only a slight dependence on $\cos 2\phi$, and z can be varied between ±0.4 without much change.

We have also calculated a ratio of integrated cross sections

$$\int_{\Delta\Omega}\left(\frac{d\sigma}{d\Omega}\right)_\phi d\Omega\Big/\int_{\Delta\Omega}\left(\frac{d\sigma}{d\Omega}\right)_\gamma d\Omega\,, \qquad (6)$$

where we have taken $\Delta\Omega$ to include all azimuthal angles, $0\le\phi\le 2\pi$, and $-\frac{1}{2}\le z\le\frac{1}{2}$. Again the ratio is a maximum at $s^2\approx 1$ for electron-electron scattering, and $s^2=0$ for the other two reactions. The dashed lines in Fig. 1 are the ratio (6) for the three processes in the limit $m_\phi\gg E$.

Because we have neglected the lepton mass, Eqs. (3) and (5) and Fig. 1 for $e^-e^\pm\to e^-e^\pm$ also hold for $\mu^-\mu^\pm\to\mu^-\mu^\pm$ if we replace m_{X^+} in (4) by m_{Y^+}.

We have not discussed the possibility of seeing the ϕ directly as an s-channel resonance. At

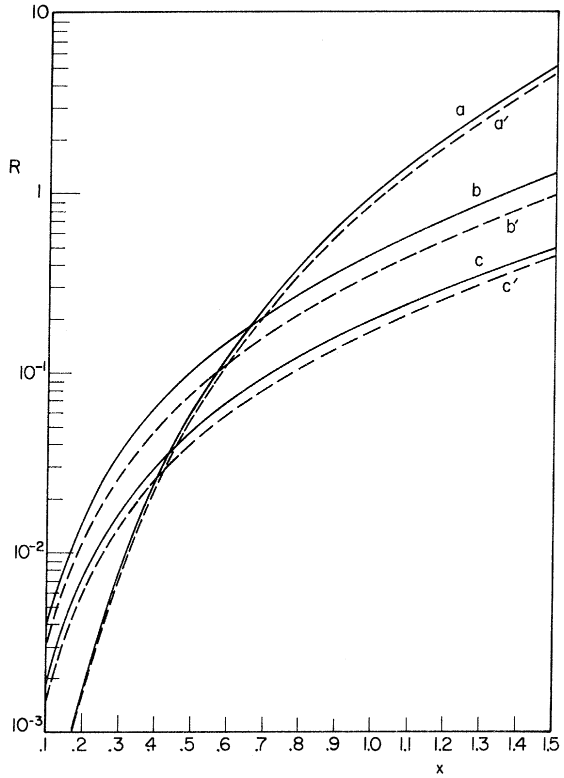

FIG. 1. The ratio R of the contribution of ϕ to the cross section to the usual quantum-electrodynamic cross section in the limit $m_\phi >> E$. The graph a shows the ratio of differential cross sections for the process $e^+e^- \to \mu^+\mu^-$ evaluated at $z=0$, $s^2=0$. x is the quantity $(m_{X^+} m_{Y^+})^{1/2} E/(m_\phi m_W)$. The graph a' is the ratio of cross sections integrated over all azimuthal angles and over z from $-\frac{1}{2}$ to $+\frac{1}{2}$ for the same process. The graphs b and b' are the ratios of the differential and integrated cross sections for the process $e^+e^- \to e^+e^-$ at $s^2=0$. In b the ratio is again evaluated at $z=0$. The graphs c and c' are the same ratios for the process $e^-e^- \to e^-e^-$ at $s^2=1$. In c the ratio is evaluated at $z=0$. For the graphs b, b', c, c' the quantity x is $m_{X^+}E/(m_\phi m_W)$. These four graphs remain the same if the processes with electrons are replaced by the same processes for muons and m_{X^+} is replaced by m_{Y^+} in x.

present beam energies this is only possible if $m_\phi \lesssim 7$ GeV [consider, for example, the second term in (1)], and it has been discussed by Primack and Quinn.[5] If m_ϕ is greater than 7 GeV, then, from Fig. 1, the effect of the ϕ cannot be detected unless the mass of the heavy lepton, m_ϕ, and m_W, have values such that x is greater than 0.2 or 0.3.

Some restrictions on the mass of the heavy lepton come from the calculation of the correction to the muon magnetic moment.[5,6] Two graphs are important in this calculation: the graph with a heavy lepton across the vertex, called $(a_\mu)_c$ by Primack and Quinn,[5] and the graph with a ϕ across the vertex, $(a_\mu)_d$. Unfortunately these contributions have opposite signs; however, if the second contribution is small compared to the first, then by comparing with experiment the mass of the heavy lepton can be bounded. This is shown clearly in Fig. 4 of Ref. 6. The ratio of the two contributions is given by[5]

$$\frac{(a_\mu)_d}{|(a_\mu)_c|} \leqslant \frac{m_Y + m_\mu}{m_\phi^2}\left(\ln\frac{m_\phi^2}{m_\mu^2} - 1.17\right). \tag{7}$$

If this ratio is small, then m_{Y^+} will have an upper bound independent of m_ϕ. If this ratio is *not* small, then m_{Y^+}/m_ϕ will have a *lower* bound and our variable x may be sufficiently large that the effects of the ϕ can be seen in $\mu^+\mu^-$ elastic scattering. Let us state this result more precisely.

If (7) is less than $1/\beta$, where β is any number greater than one, then, by interpolating Fig. 4 of Ref. 6, m_{Y^+} must be less than[7]

$$m_{Y^+} \lesssim 7.5\,\frac{\beta}{\beta-1}\,.$$

On the other hand, if (7) is greater than $1/\beta$, and m_ϕ is greater than 7 GeV, then

$$x \gtrsim \frac{0.18}{\beta}\left(\frac{E}{1\ \text{GeV}}\right).$$

So, for example, if $\beta=3$, then either (a) the mass

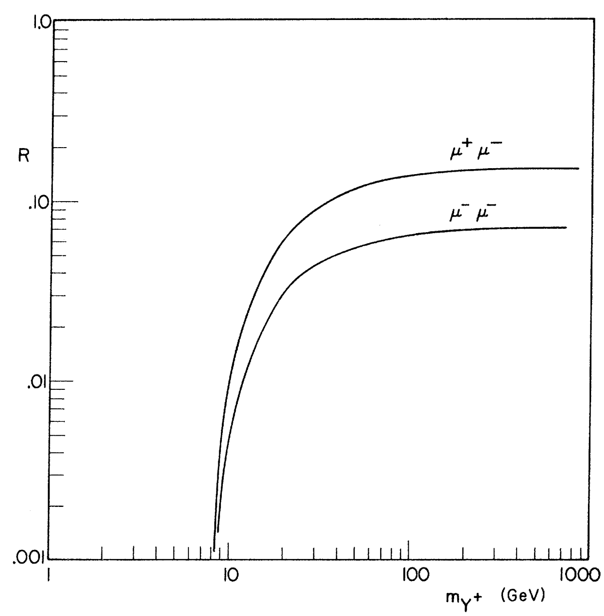

FIG. 2. The relation between the mass of the heavy lepton and R, the ratio of the ϕ and photon contribution to the differential cross sections for $\mu^+\mu^-$ and $\mu^-\mu^-$ scattering. The curves are meant to indicate that *either* m_{Y^+} has a value to the left of the curve *or* R has a value above the curve. We are assuming here that m_ϕ is greater than 7 GeV.

of the heavy lepton, m_{Y^+}, is less than 12 GeV; or (b) the mass of the ϕ is small enough that it can be seen as a direct-channel resonance; or (c) x is greater than 0.2 and the ϕ will contribute more than 1% to the differential cross section for $\mu^+\mu^-$ elastic scattering at $z=0$, $s^2=0$, and $E=3.5$ GeV (or to the integrated cross section); or (d) some combination of (a), (b), and (c) is simultaneously true.

Thus, if $\mu^+\mu^-$ elastic scattering were to be measured to an accuracy of 1%, and no effect of the ϕ seen, then the mass of the heavy lepton would have to be less than 12 GeV for the theory to be viable.

The general relation between the upper bound on m_{Y^+} and the lower bound on the contribution of the ϕ to the $\mu^+\mu^-$ or $\mu^-\mu^-$ differential cross section is shown in Fig. 2. If the theory is correct and if m_ϕ is greater than 7 GeV then *either* m_{Y^+} has a value to the left of the curve in Fig. 2 *or* the amount of ϕ contribution is greater than the value of the curve.

Of course these results are not as interesting as they would be if the scattering processes were for electrons. Unfortunately comparison with the experimental values of the electron magnetic moment does not allow similar statements to be made. A similar relation between m_{Y^+} and electron scattering certainly does hold if $m_{X^+} \approx m_{Y^+}$. In any case the statement in terms of muon scattering offers a complete test of the theory, independent of hadrons, and regardless of the values of the masses of the particles, although the experiments may be difficult.

I would like to thank Professor Vishnu S. Mathur for numerous discussions.

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[1]For a review of these theories and a complete set of references see B.W. Lee, in *Proceedings of the Sixteenth International Conference on High Energy Physics, National Accelerator Laboratory, Batavia, Ill., 1972,* edited by J. D. Jackson and A. Roberts (NAL, Batavia, Ill., 1973); B. Zumino, Cargèse Summer Institute Lectures, 1972 (unpublished).

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[7]The number 7.5 comes from assuming that the weak contribution to the muon magnetic moment must be $\geq -3\times10^{-7}$. For a discussion of this number see Refs. 5 and 6.