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Proton-Proton Elastic Scattering Including Spin

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A model for the description of proton-proton scattering at infinite energies is presented which includes the spin of the interacting particles in a natural way, by an appropriate generalization of the eikonal. Results which agree with current experimental data on several points are presented. In the limit that spin is neglected, previous results given by Chou and Yang are reproduced. An application to $p\bar{p}$ scattering is also discussed.

I. INTRODUCTION

At the highest energies currently attainable some cross sections are observed to approach non-vanishing limits. This phenomenon is generally known as diffraction scattering, and is observed, for example, in the elastic scattering of protons by protons. The elastic scattering cross section may be obtained from a model well known from classical physics. The absorption due to the presence of inelastic channels changes the amplitude of that part of the incident wave that has traversed the target. This then gives rise to the analog of a diffraction pattern by the application of Kirchhoff's formulation of Huygens's principle.

One of the more successful models of this type is due to Chou and Yang,[1] who make a particular assumption as to the form of the absorption which obtains in proton-proton (p-p) scattering. As is usual in current treatments of interactions at high energies, these authors neglect the possible effects of spin. The results of their analysis predict the existence of true zeros in the elastic scattering cross section for proton-proton collisions at infinite energy.

It will be the purpose of this paper to consider a model of proton-proton elastic scattering at infinite energy which includes the effects of spin in a natural way, and which agrees with one version of the model of Chou and Yang mentioned above, in the limit that spin is neglected. We find that our model is in reasonably close agreement with current experimental data.

II. DIFFRACTION SCATTERING

If we view the proton-proton scattering process from the laboratory frame, we may describe the incident wave by a function of the general form

$$\chi^{(1)} e^{ikz}, \tag{2.1}$$

where $\chi^{(1)}$ is the Pauli spinor for the incident particle (termed particle I), so that the initial state may be written

$$\chi^{(1)} \chi^{(2)} e^{ikz}, \tag{2.2}$$

where $\chi^{(2)}$ is the spinor of the target particle (II).

Inelastic processes modify the initial state, so that the final state may be written

$$S_{\sigma\tau;\sigma'\tau'}(\vec{b})\, \chi^{(1)}_{\sigma'} \chi^{(2)}_{\tau'} e^{ikz}, \tag{2.3}$$

where $\vec{b}$ is the impact parameter. $S_{\sigma\tau;\sigma'\tau'}(\vec{b})$ is the effective S-matrix element for the process, and allows for spin transfer between the colliding particles, as well as for absorption; it may be considered the survival amplitude of the state.

Parity invariance implies a restriction on the form of S (Ref. 2)

$$(i\sigma_2)_{\sigma\sigma'}(i\sigma_2)_{\tau\tau'} S_{\sigma'\tau';\xi'\eta'}(\vec{b}) \times (-i\sigma_2)_{\xi'\xi}(-i\sigma_2)_{\eta'\eta} = S_{\sigma\tau;\xi\eta}(\vec{b}^R), \tag{2.4}$$

where, if $\vec{b}=(b_1, b_2)$, $\vec{b}^R=(b_1, -b_2)$.

Time-reversal invariance implies[2]

$$[S_{\sigma\tau;\xi\eta}(-\vec{b}^R)]^* = S_{\sigma\tau;\xi\eta}(\vec{b}). \tag{2.5}$$

We may represent S in the form

$$\hat{S}(\vec{b}) = e^{-\hat{\chi}(\vec{b})}, \tag{2.6}$$

where the caret represents an operator in the two-particle state space. This would correspond to the choice of a purely absorptive potential in the optical potential formalism.[3]

Then the restrictions (2.4) and (2.5) above require $\hat{\chi}(\vec{b})$ to be of the form[4]

$$\hat{\chi}(\vec{b}) = d(b)\,\hat{1} + e(b)\,\vec{\sigma}^1\cdot\hat{p}\,\vec{\sigma}^2\cdot\hat{p} + f(b)\,\vec{\sigma}^1\cdot\hat{q}\,\vec{\sigma}^2\cdot\hat{q} + g(b)\,\vec{\sigma}^1\cdot\hat{r}\,\vec{\sigma}^2\cdot\hat{r} + h(b)\,(\vec{\sigma}^1+\vec{\sigma}^2)\cdot\hat{p}, \tag{2.7}$$

where $d(b), f(b), \ldots, h(b)$ are scalar functions of b, the magnitude of $\vec{b}$. $\vec{\sigma}^1$ ($\vec{\sigma}^2$) operates in the one-particle subspace of particle I (II). At infinite energy, the unit vectors $\hat{r}$, $\hat{q}$, and $\hat{p}$ defined by

$$\hat{r} \equiv \frac{\vec{p}_I}{|\vec{p}_I|}, \quad \hat{q} \equiv -\frac{\vec{b}}{|\vec{b}|}, \quad \hat{p} \equiv \hat{r} \times \hat{q} ,$$

are an orthonormal basis in coordinate space.

It is plausible, in the light of current understanding of the strong interactions, that diffraction scattering at high energies is due to the "shadow" of the inelastic processes. Using Kirchhoff's formulation of Huygens's principle, one obtains the scattering amplitude $\hat{a}(\vec{k})$,

$$\hat{a}(\vec{k}) = \frac{1}{2\pi} \int [\hat{S}(\vec{b}) - 1]\, e^{i\vec{k}\cdot\vec{b}} d^2 b$$

$$= \frac{1}{2\pi} (e^{-\hat{\chi}(\vec{b})} - 1)\, e^{i\vec{k}\cdot\vec{b}}\, d^2 b , \tag{2.8}$$

where $\vec{k}$ is the transverse momentum transfer to particle I at high energies; $|\vec{k}| \cong (-t)^{1/2}$. The differential cross section for elastic scattering is then given by

$$\frac{d\sigma}{dt} = \pi\, |\hat{a}(\vec{k})|^2 . \tag{2.9}$$

Denoting the two-dimensional Fourier transform of an arbitrary operator $\hat{f}$ by

$$\langle \hat{f}(\vec{b}) \rangle \equiv \frac{1}{2\pi} \int \hat{f}(\vec{b})\, e^{i\vec{k}\cdot\vec{b}} d^2 b \equiv \hat{f}(\vec{k}) , \tag{2.10}$$

we may rewrite (2.8) as

$$\hat{a}(\vec{k}) = \langle \hat{S}(\vec{b}) - 1 \rangle . \tag{2.11}$$

III. THE CHOU-YANG MODEL

We collect here some of the results of Chou and Yang for comparison with the model presented below. These authors note that when a high-energy collision of two particles is observed from the center-of-momentum frame, both of the particles are Lorentz-contracted along the direction of their relative motion. In this situation, only the effective density in the transverse direction should have practical importance. This effective density may be defined in terms of the particle density effective in contributing to the absorptive process $\rho(\vec{b}, z)$ as

$$D(\vec{b}) \equiv \int_{-\infty}^{+\infty} \rho(\vec{b}, z)\, dz , \tag{3.1}$$

where $+z$ is the coordinate on the direction of P_I.

The contribution to absorption at impact parameter $\vec{b}$, from the matter located at $\vec{b}'$, is then taken to be of the form

$$\eta D_I(\vec{b}')\, D_{II}(\vec{b}' - \vec{b}) , \tag{3.2}$$

where η is a constant to be determined by comparison with experiment, and the subscripts I and II refer to particles I and II.

The total opacity for impact parameter $\vec{b}$ is found by integrating over all $\vec{b}'$:

$$\chi(\vec{b}) = \eta \int D_I(\vec{b}')\, D_{II}(\vec{b}' - \vec{b})\, d^2 b' \tag{3.3}$$

(see Fig. 1).

We have as a theorem for any two suitably well-behaved functions f and g

$$\int f(\vec{b}') g(\vec{b}' - \vec{b})\, d^2 b' = \int \tilde{f}(\vec{k})\, \tilde{g}(-\vec{k})\, e^{i\vec{k}\cdot\vec{b}} d^2 k , \tag{3.4}$$

where $\tilde{f}(\vec{k})$ denotes the two-dimensional Fourier transform of f. Using this, we may rewrite Eq. (3.3) as

$$\chi(\vec{b}) = \eta \langle \langle D_I(\vec{k}) \rangle \langle D_{II}(-\vec{k}) \rangle \rangle . \tag{3.5}$$

Chou and Yang make various assumptions as to the form of $\langle D_I \rangle$ and $\langle D_{II} \rangle$ for p-p scattering:

$$\langle D_{I,II} \rangle = F_1(k^2) , \tag{3.6a}$$

$$\langle D_{I,II} \rangle = G_m(k^2)/\mu$$

$$= \frac{F_1(k^2) + 2MF_2(k^2)}{\mu} . \tag{3.6b}$$

F_1 and F_2 are the usual electromagnetic form factors of the proton as extracted from electron-proton elastic scattering using the one-photon-exchange approximation, M is the proton mass, and μ is the proton's magnetic moment.

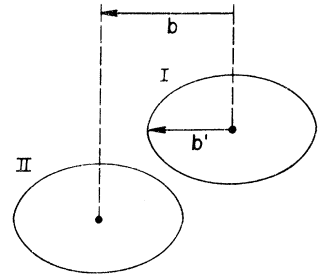

FIG. 1. Schematic representation of the collision of two particles viewed from the center-of-momentum system. The impact parameter b is perpendicular to the direction of relative motion.

IV. MODEL OF SCATTERING INCLUDING SPIN

The picture of interaction at high energies given by Chou and Yang is an intuitively reasonable one, and one would expect that a model of proton-proton scattering including spin would result in an expression for the opacity of the form

$$\hat{\chi}(\vec{b}) = \int \hat{D}_{\rm I}(\vec{b})\, \hat{D}_{\rm II}(\vec{b}'-\vec{b})\, d^2b' \ , \tag{4.1}$$

where $\hat{D}_{\rm I}$ ($\hat{D}_{\rm II}$) are suitably defined spin operators in the space of particle I (II). We may introduce our assumption as to the form of the D's in several ways.

We find it convenient to begin by defining an amplitude for absorption $\hat{a}_0(\vec{k})$ by

$$\hat{\chi}(\vec{b}) \equiv \langle \hat{a}_0(\vec{k})\rangle \ , \tag{4.2}$$

and we choose our model by defining the matrix elements of in the center-of-momentum frame as

$$(p_{\rm II}', s'|(p_{\rm I}', r'|\hat{a}_0(\vec{k})\,|p_{\rm II}, s)\,|p_{\rm I}, r) \equiv \lim_{\text{energy}\to\infty} \eta(p_{\rm II}', s'\,|\hat{J}^{\mu}_{\rm II}(0)\,|p_{\rm II}, s)(p_{\rm I}', r'\,|\hat{J}_{{\rm I}\mu}(0)\,|p_{\rm I}, r), \tag{4.3}$$

where the matrix elements of $\hat{J}(0)$ are defined by

$$(p', s'\,|\hat{J}^{\mu}(0)\,|p, s) \equiv \bar{u}^{s'}(p')[F_1(q^2)\gamma^{\mu} + i\sigma^{\mu}{}_{\nu}(p'-p)^{\nu} F_2(q^2)]\, u^{s}(p) \ . \tag{4.4}$$

The letters r, s are spinor indices; $q^{\mu} = (p'-p)^{\mu}$ for I in the case of p-p scattering.

Equivalently, at infinite energy, one may take the amplitude $\hat{a}_0(\vec{k})$ as given by the product of effective currents for right-moving particle (I) and a left-moving particle (II).

$$\begin{aligned}(p_{\rm II}', s'|\,(p_{\rm I}', r'|\hat{a}_0(\vec{k})|\,p_{\rm II}, s)|\,p_{\rm I}, r) &\equiv \lim_{\text{energy}\to\infty} \eta(p_{\rm I}', s'\,|\hat{J}^0_{\rm I}(0)+\hat{J}^3_{\rm II}(0)\,|p_{\rm I}, s)\,(p_{\rm II}', r'\,|\hat{J}^0_{\rm II}(0)-\hat{J}^3_{\rm II}(0)\,|p_{\rm II}, r)\\ &= \eta[F_1(q^2)\,\hat{1} - ikF_2(q^2)\,\vec{\sigma}^1\cdot\hat{n}]\ [F_1(q^2)\,\hat{1} - ikF_2(q^2)\,\vec{\sigma}^2\cdot\hat{n}]\\ &\equiv \eta\,\hat{D}_{\rm I}(k)\,\hat{D}_{\rm II}(-k)\ ,\end{aligned} \tag{4.5}$$

where

$$\begin{aligned}\hat{D}_{\rm I}(k) &\equiv F_1(q^2)\,\hat{1} - ikF_2(q^2)\,\vec{\sigma}^1\cdot\hat{n}\ ,\\ \hat{D}_{\rm II}(k) &\equiv F_1(q^2)\,\hat{1} + ikF_2(q^2)\,\vec{\sigma}^2\cdot\hat{n}\ .\end{aligned} \tag{4.6}$$

One may consider (4.3) as formally coming from two assumptions; that the matrix element for absorption in $pp\to pp$ can be expressed as the space integral of a density formed by the inner product of the effective currents for each particle, $\hat{J}_{\rm I}^{\mu}(x)\,\hat{J}_{{\rm II}\,\mu}(x)$, and that their matrix element then factorizes.

The assumptions above lead to an expression for $\hat{a}_0(\vec{k})$ of the form

$$\begin{aligned}\hat{a}_0(\vec{k}) = {}& F_1{}^2(q^2) - ikF_1(q^2)\,F_2(q^2)\,(\vec{\sigma}^1+\vec{\sigma}^2)\cdot\hat{n}\\ &- k^2 F_2{}^2(q^2)\,\vec{\sigma}^1\cdot\hat{n}\,\vec{\sigma}^2\cdot\hat{n}\ .\end{aligned} \tag{4.7}$$

Using (3.4) we have

$$\begin{aligned}\hat{\chi}(\vec{b}) &= \langle \hat{a}_0(\vec{k})\rangle\\ &= \int \hat{D}_{\rm I}(\vec{b}')\,\hat{D}_{\rm II}(\vec{b}'-\vec{b})\,d^2b',\end{aligned} \tag{4.8}$$

where

$$\begin{aligned}\hat{D}_{\rm I}(\vec{b}) &\equiv \frac{1}{2\pi}\int \hat{D}_{\rm I}(\vec{k})\,e^{-i\vec{k}\cdot\vec{b}}\,d^2k\\ &\equiv \lambda(b^2)\,\hat{1} - \mu(b^2)\,\vec{\sigma}^1\cdot\hat{p}\ ,\end{aligned} \tag{4.9}$$

and similarly for II. $\lambda(b^2)$ and $\mu(b^2)$ are scalar functions of b^2, and are the appropriate Bessel transforms of $F_1(q^2)$ and $k\,F_2(q^2)$.

This then establishes the connection of the present model with that of Chou and Yang; the current model reduces to that of Chou and Yang using the assumption (3.6a) when spin is neglected. This may be done formally by setting $F_2 = 0$. Combining (2.6) and (2.11), we have

$$\hat{a}(\vec{k}) = \langle e^{-\hat{\chi}(\vec{b})} - 1\rangle \tag{4.10a}$$

$$= \left\langle \hat{\chi}(\vec{b}) - \frac{1}{2!}\,\hat{\chi}(\vec{b})\,\hat{\chi}(\vec{b}) + \frac{1}{3!}\,\hat{\chi}(\vec{b})\,\hat{\chi}(\vec{b})\,\hat{\chi}(\vec{b}) - \cdots\right\rangle, \tag{4.10b}$$

$$= \langle\hat{\chi}(\vec{b})\rangle - \frac{1}{2!}\langle\hat{\chi}(\vec{b})\,\hat{\chi}(\vec{b})\rangle + \frac{1}{3!}\langle\hat{\chi}(\vec{b})\,\hat{\chi}(\vec{b})\,\hat{\chi}(\vec{b})\rangle - \cdots \tag{4.10c}$$

One can derive an expression for $\hat{\chi}(\vec{b})$ using (4.5) in (4.1) and performing the implied Fourier transform using covariant techniques of integration, or, alternatively, evaluate (4.10) above. We find that $\hat{\chi}(\vec{b})$ is of the form

$$\hat{\chi}(\vec{b}) = d(b)\,\hat{1} + e(b)\,\vec{\sigma}^1\cdot\hat{p}\,\vec{\sigma}^2\cdot\hat{p} + h(b)\,(\vec{\sigma}^1+\vec{\sigma}^2)\cdot\hat{p}\,, \tag{4.11}$$

where $d(b)$, $e(b)$, and $h(b)$ are appropriate scalar functions of b, and depend on the functional form of F_1 and F_2.

V. CALCULATION OF ELASTIC CROSS SECTION

In order to evaluate the Fourier transforms in (4.10c), we need $\hat{\chi}(\vec{b}), \hat{\chi}(\vec{b})\,\hat{\chi}(\vec{b}), \ldots, \hat{\chi}^n(\vec{b})$. We can show that the algebra formed by the set of operators $\hat{1}$, $\vec{\sigma}^1\cdot\hat{p}\,\vec{\sigma}^2\cdot\hat{p}$, $\vec{\sigma}^1\cdot\hat{q}\,\vec{\sigma}^2\cdot\hat{q}$, $\vec{\sigma}^1\cdot\hat{r}\,\vec{\sigma}^2\cdot\hat{r}$, and $(\vec{\sigma}^1+\vec{\sigma}^2)\cdot\hat{p}$ is closed. Any power of $\hat{\chi}(\vec{b})$, say $\hat{\chi}^n(\vec{b})$, may be represented as $[\hat{\chi}^{n-1}(\vec{b})]\,[\hat{\chi}(\vec{b})]$. Each of these factors is a linear combination of operators above, so $\hat{\chi}^n(\vec{b})$ is a (different) linear combination of the same operators, with expansion coefficients which are certain bilinear combinations of those for the factors. One can thus find the expansion coefficients for any power of $\hat{\chi}(\vec{b})$. A computer may be programmed to do this, and then to take the Fourier transforms indicated in (4.10c). We may write the final $\hat{a}(\vec{k})$ calculated in this way as

$$\begin{aligned}\hat{a}(\vec{k}) = {} & \alpha(k)\,\hat{1} + B(k)\,\vec{\sigma}^1\cdot\hat{n}\,\vec{\sigma}^2\cdot\hat{n} \\ & + i\gamma(k)\,(\vec{\sigma}^1+\vec{\sigma}^2)\cdot\hat{n} + \delta(k)\,\vec{\sigma}^1\cdot\hat{m}\,\vec{\sigma}^2\cdot\hat{m} \\ & + \epsilon(k)\,\vec{\sigma}^1\cdot\hat{l}\,\vec{\sigma}^2\cdot\hat{l}\;.\end{aligned} \tag{5.1}$$

The vectors $\hat{n}$, $\hat{m}$, $\hat{l}$ arise from the vectors $\hat{p}$, $\hat{q}$, $\hat{r}$ under the action of the Fourier transformation. Its action on these vectors may be formally expressed by $\hat{p}\to\hat{n}$, $\hat{q}\to\hat{m}$, $\hat{r}\to\hat{l}$.

We may then use (2.9) to obtain the total elastic cross section from this result. Further details of the computation may be found in Ref. 5.

VI. DISCUSSION

Experimental data on elastic p-p scattering exhibit structure at the highest energies currently available. The structure may be roughly described as an unusually abrupt increase in slope at about $|t| = 1.1$ $(\mathrm{GeV}/c)^2$, followed by an abrupt decrease around $|t| = 2.3$ $(\mathrm{GeV}/c)^2$, this situation obtaining at an incident momentum in the lab frame $P_{\mathrm{inc}} = 24$ GeV/c.

Figure 2 displays the predictions of this model for the quantities $\alpha^2, \beta^2, \ldots, \epsilon^2$ as functions of $|t|$, as well as the cross section they imply. The "renormalized" cross section is plotted here, which is related to the true cross section by

$$\left.\frac{d\sigma}{dt}\right|_{\mathrm{true}} = \left.\frac{d\sigma}{dt}\right|_{\mathrm{renormalized}} \times \pi \times 3.89\times10^{-28}\;.$$

Figure 3 compares our prediction with the experimental results for p-p elastic scattering taken from data of Allaby *et al.*[6] In the region from $|t| = 0$ to $|t| = 8$ $(\mathrm{GeV}/c)^2$, data taken at $P_{\mathrm{inc}} = 24$ GeV/c were used. We note that while we do not reproduce these $P_{\mathrm{inc}} = 24$ GeV/c data exactly, nevertheless our amplitude β, representing double spin flip, rises sharply from a zero at about $|t| = 1.1$ $(\mathrm{GeV}/c)^2$ flattening the normally concave upwards cross section, and thereby providing a slight "break" in the slope in this region. Other component amplitudes rise abruptly from zero at different values of $|t|$, but they are numerically too small to affect the predicted cross section at these points in a noticeable way. Here η is set equal to 10.5 to achieve good agreement with the forward scattering data.

Although the results are not presented here, we have further modified the present model by inserting a multiplicative weighting factor varying between 0 and + 1 in front of the spin-flip amplitudes only [$e(b)$ and $h(b)$ in Eq. (4.11)] and reperforming the calculation indicated above. This formal device of course, regenerates the above results when the weighing factor is set equal to unity. As the factor varies from 1 to 0, the breaks in the cross section become more pronounced, turning eventually into dips, and shift their position. Finally, in the limit that the weighting factor is set equal to zero, the results of Chou and Yang using assumption (3.6a) are reproduced.

Comparing our results with those of Chou and Yang,[1] we see that there are differences on two points. In the work of the above authors, where spin is not considered, it is predicted that the total elastic differential cross section will go through true zeros as energy becomes infinite, whereas in the model presented here zeros are predicted *not* to occur at infinite energy.

There is also a difference in that our model qualitatively predicts only one relatively rapid change in slope, occurring between $|t| = 1$ and 2 $(\mathrm{GeV}/c)^2$. Although other "structure" is present in the amplitudes α^2, β^2, ϵ^2, at other values of $|t|$, as noted above they are numerically too small to make a noticeable difference in the cross section. No second break has yet been observed, experimentally.[6]

The model presented here evidently makes

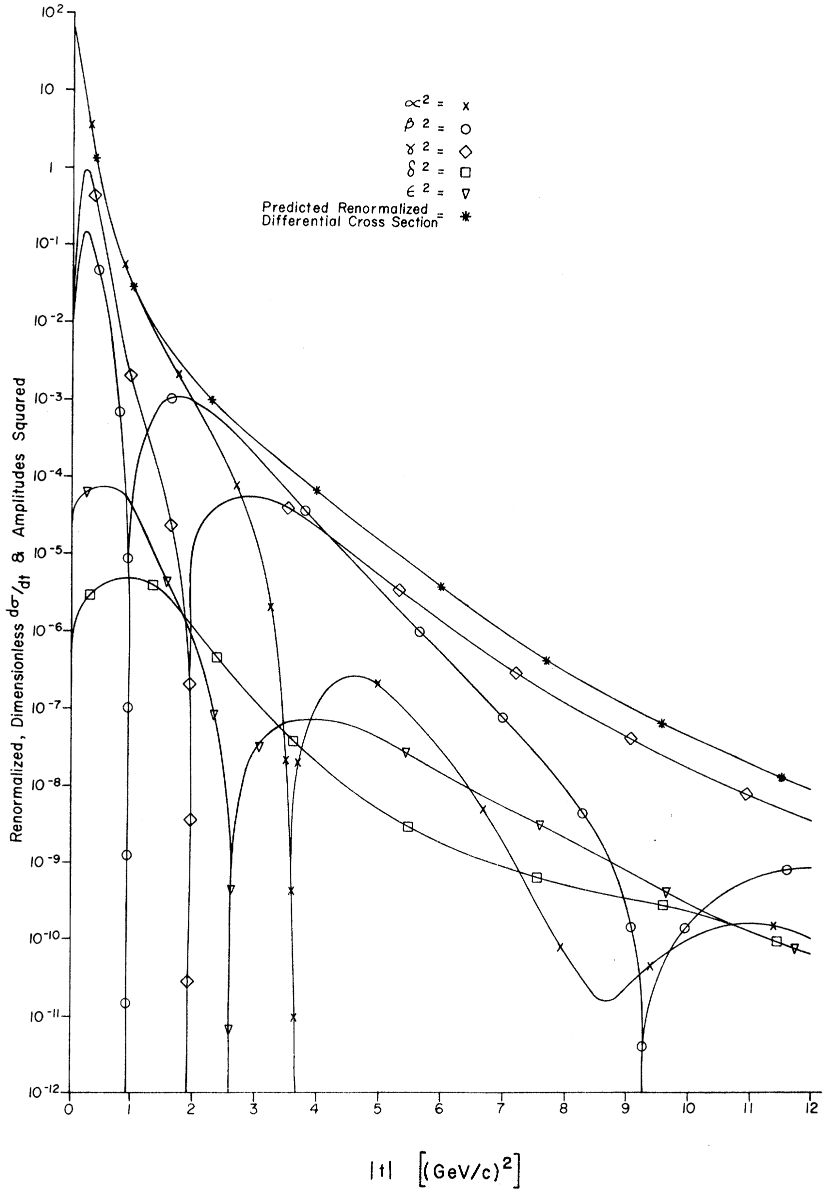

FIG. 2. "Renormalized" (dimensionless) predicted cross section and component amplitudes squared plotted versus $|t|$.

definite predictions as to the relative magnitudes of the amplitudes for various spin-flip processes at any given value of $|t|$. To date, the most detailed experimental investigation of polarization in p-p elastic scattering has been confined to the measurement of the "polarization parameter." This is predicted to be zero by our model, in agreement with current measurements within experimental error.[7]

This model may be applied to p-$\bar{p}$ scattering in a straightforward manner, under the assumption that the only change in the fundamental interaction is an increased value of η, to match the increased value of the forward cross section for p-$\bar{p}$ scattering. We find that a choice of $\eta = 11.5$ is appropriate for the p-$\bar{p}$ case. This leads to a steeper initial falloff of the forward cross section than for the p-p case. The crossover point, at which p-$\bar{p}$ falls below p-p, is about $|t| = 0.5$ (GeV/c)^2, in agreement with the current experimental value.[8]

Recently, a series of experiments on p-p elastic scattering has been reported by Beznogikh *et al.*[9] for very small values and over a very limited range of $|t|$. Their value of P_{inc} ranges between 9 and 70 GeV/c, while the range of $|t|$ is 0.0007 $\leq |t| \leq 0.02$ (GeV/c)^2. Other workers, most re-

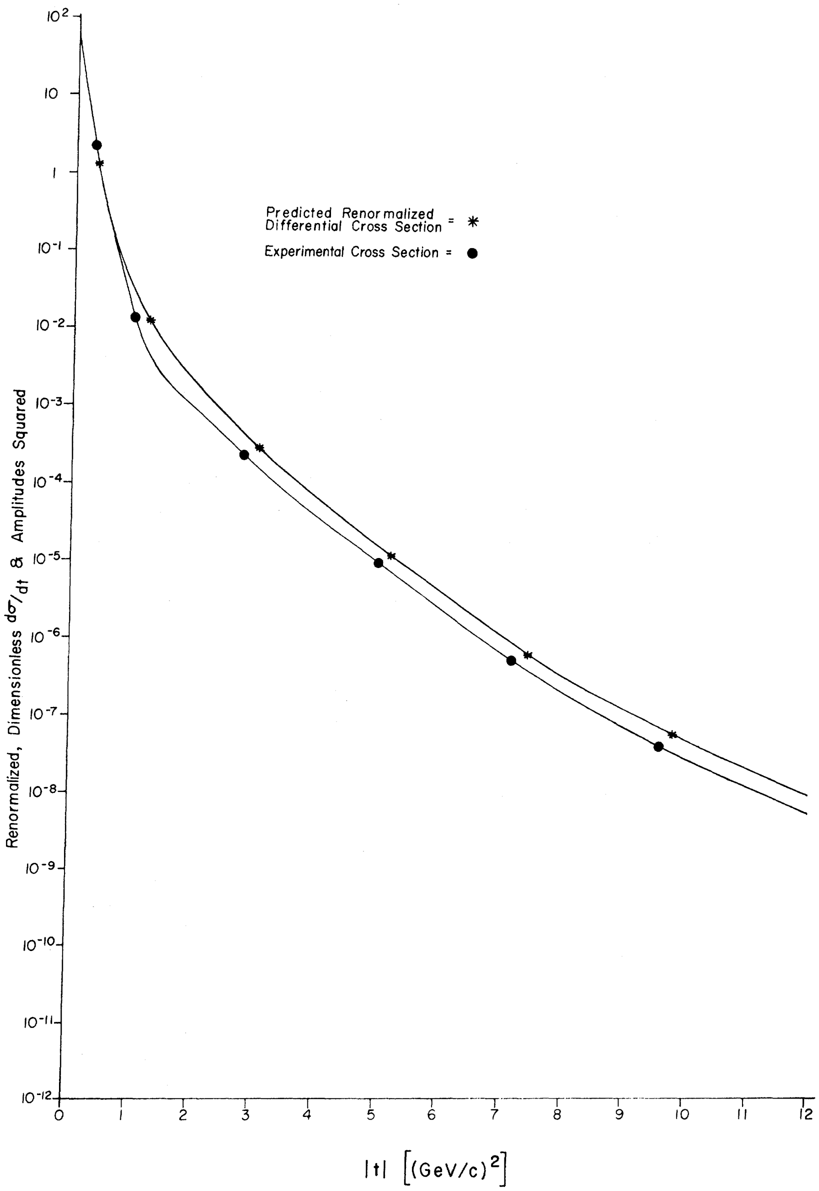

FIG. 3. Predicted "renormalized" (dimensionless) cross section plotted versus $|t|$, compared with data taken from Ref. 6.

cently Barbiellini *et al.*,[10] have made measurements at very high energies, over a somewhat larger range of $|t|$, centered at larger values: $0.060 \leq |t| \leq 0.308$ $(\mathrm{GeV}/c)^2$, and at P_{inc} up to 1496 GeV/c. Parametrizing the elastic differential cross section by $e^{b(t)t}$, where $b(t)$ is a real positive function of $|t|$, Barbiellini *et al.* find evidence for a "break" in the cross section manifested by an increase in the parameter b for decreasing $|t|$ at fixed P_{inc}. For example, at $P_{\mathrm{inc}} = 1496$ GeV/c, they find $b = 10.80$ for $0.168 \leq |t| \leq 0.308$ $(\mathrm{GeV}/c)^2$, and $b = 12.40$ for $0.060 \leq |t| \leq 0.112$ $(\mathrm{GeV}/c)^2$, the "break" thus lying between 0.112 and 0.168 $(\mathrm{GeV}/c)^2$. Naturally, this implies a somewhat higher value for the forward cross section than that given by use of the optical theorem at current energies, the increase being estimated by these authors at about 20%, or about 95 mb/$(\mathrm{GeV}/c)^2$. However, the data of Beznogikh *et al.*, taken at even smaller values of $|t|$ (but at $P_{\mathrm{inc}} = 70$ GeV/c maximum), may imply a forward cross section of more than 1500 mb/$(\mathrm{GeV}/c)^2$.

Considering our model in the light of these results, we see from our discussion of p-$\bar{p}$ scattering given above that the steepness of falloff of the cross section in the forward direction depends on our parameter η, which is in turn determined only by the forward cross section, and increases with it. Using $\eta = 10.5$ as above, we have $b(t) \cong 9.0$ for $|t| \cong 0$. On the other hand, increasing η to about 50, for example, our model yields $b(t) \cong 11.2$ for $|t| \cong 0$, a forward cross section of 424 mb/(GeV/$c)^2$, and the first break near $|t| = 0.3$ (GeV/$c)^2$.

Within the framework of the model presented here, it is clear that definitive comparison with experiment must await measurements of the forward cross section at energies which are known to be asymptotic, accumulation of data over a wide range of $|t|$ at such energies, and more detailed measurements of polarizations.

We note that the conclusions presented above are sensitive to the details of F_1 and F_2, the proton's electromagnetic form factors as determined in p-p scattering using a one-photon-exchange approximation. The physically intuitive picture presented above depends on the form factors being an accurate reflection of the intrinsic structure of the proton itself; that is, not too dependent on the details of the interaction in p-p scattering. This assumption may fail at larger values of $|t|$, in which case the "true" structure of the proton at large $|t|$ may require other methods for its determination.

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