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Radiative Decay of Vector Mesons in an Algebraic Model for Hadrons

A. Böhm*† and Jan S. Nilsson

Institute of Theoretical Physics Fack, S-402 20 Göteborg 5, Sweden

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The decay rates for vector meson→pseudoscalar meson+photon have been calculated using the same assumptions as previously used for the decay $V \to e\bar{e}$. The results are compared with experimental data.

Recently the leptonic decay rates of vector mesons $\Gamma(V \to e\bar{e})$ have been calculated in the framework of an algebraic model.[1] The decay was postulated to be described by the charge components of a vector and axial-vector octet of transition operators V_μ^α and A_μ^α, where $\mu = 0, 1, 2, 3$ and $\alpha = 0, 8, \pm 1, \pm 2, \pm 3$. Within the framework of the algebraic method these transition operators must be elements of the algebra of observables,[2] and they are assumed to be the simplest algebraic expressions with the right symmetry properties that can be formed from the generators: E_α(charges), F_α(axial charges), P_μ, $L_{\mu\nu}$ (momentum, angular momentum, boosts), Γ_μ (generalized Dirac vector), and the discrete operators C (charge conjugation), P (space inversion), and T (time reversal). This leads to the expressions[2]

$$V_\mu^\alpha = V_\mu^{(0)\alpha} + V_\mu^{(1)\alpha},$$

$$V_\mu^{(0)\alpha} = \{P_\mu, E_\alpha\}, \quad V_\mu^{(1)\alpha} = \tfrac{1}{2}\{\{\Gamma_\mu, \rho\}, E_\alpha\},$$

$$A_\mu^\alpha = A_\mu^{(0)\alpha} + A_\mu^{(1)\alpha},$$

$$A_\mu^{(0)\alpha} = \{P_\mu, F_\alpha\}, \quad A_\mu^{(1)\alpha} = \tfrac{1}{2}\{\{\Gamma_\mu, \rho\}, F_\alpha\}.$$

For the case that $[\rho, \Gamma_\mu] = 0$, $V_\mu^{(1)\alpha}$ can be written $V_\mu^{(1)\alpha} = \{\rho\Gamma_\mu, E_\alpha\} = \Gamma_\mu\{\rho, E_\alpha\}$ and $A_\mu^{(1)\alpha}$ can be written in a similar form. ρ is a quantity with dimension MeV which must appear for dimensional reasons. Experimental data as well as theoretical considerations in Ref. 1 lead to the requirement that ρ not be an SU(3) scalar. The assumption

$$\rho = \zeta M,$$

where M is the mass operator and ζ is a constant on the subspace of strangeness-zero mesons[3] is supported by the experimental data for $V \to e\bar{e}$.

We will now use the same method with the same assumption as in Ref. 1 to calculate the radiative decay of vector mesons.

For the radiative decay of vector mesons

$$V \to P + \gamma$$

(V = vector meson, P = pseudoscalar meson) the decay rate is given by

$$\Gamma = \sum_{\overline{\text{Pol}}} \int\int \frac{d^3k}{2E_\gamma}\, \frac{d^3p_P}{2E_P c_P}\, \delta(E_V - E_P - E_\gamma)\, |\langle \gamma, P, \vec{k}, \lambda, \vec{p}_P | T | \phi_V \rangle|^2$$
$$= \sum_{\overline{\text{Pol}}} \int\int \frac{d^3k}{2E_\gamma}\, \frac{d^3p_P}{2E_P c_P}\, \delta(E_V - E_P - E_\gamma) \left| \int \frac{d^3p_V}{2E_V c_V} \phi_V(\vec{p}_V) \langle \gamma P; \vec{k}, \lambda, \vec{p}_P | T | \vec{p}_V, s, s_3, V \rangle^2 \right|. \tag{1}$$

$\vec{k}$, λ are the photon momentum and polarization; $\vec{p}_P$ is the pseudoscalar-meson momentum; $\vec{p}_V$, s, s_3 are the vector-meson momentum, spin, and spin-3 component; $|\phi_V(\vec{p}_V)|^2$ is the expectation value for the momentum $\vec{p}_V$ in the ensemble of decaying vector mesons; $\sum_{\overline{\text{Pol}}}$ means summation over the photon polarizations and averaging over the vector-meson polarizations. The number c_α is due to our normalization of the generalized hadron states $|\vec{p}_\alpha, \alpha\rangle$ (Appendix B of Ref. 4). The transition operator T is split as in Ref. 1 or 4 into a leptonic part L^λ and a hadronic part H_λ:

$$T = L^\lambda H_\lambda. \tag{2}$$

In Ref. 1 we considered two possibilities for H_λ. We will consider here the following three possibilities for H_λ:

$$H_\lambda = G \mathcal{J}_\lambda^{\text{el}}, \tag{3a}$$

$$[P_\mu P^\mu, H_\lambda] = G\, \tfrac{1}{2}\{\mathcal{J}_\lambda^{\text{el}}, \rho^2\}, \tag{3b$_1$}$$

$$[P_\mu P^\mu, H_\lambda] = G\, \tfrac{1}{4}\{\{\mathcal{J}_\lambda^{\text{el}}, \rho\}, \rho\}, \tag{3b$_2$}$$

$$[P_\mu, [P^\mu, H_\lambda]] = G\, \tfrac{1}{2}\{\mathcal{J}_\lambda^{\text{el}}, \rho^2\}, \tag{3c$_1$}$$

$$[P_\mu, [P^\mu, H_\lambda]] = G\, \tfrac{1}{4}\{\{\mathcal{J}_\lambda^{\text{el}}, \rho\}, \rho\}. \tag{3c$_2$}$$

The electromagnetic transition operator $\mathcal{J}^{\text{el}}$ is given by

$$\mathcal{J}_\mu^{\mathrm{el}}=\{(P_\mu+\rho\Gamma_\mu),[H_1+(1/\sqrt{3})H_2+G_1+(1/\sqrt{3})G_2]\}. \tag{4}$$

If ρ is an operator in case b, then ansatz (6b) of Ref. 1 can be expressed in several forms; therefore we have in cases (3b) and (3c) two different subcases ($3b_1$), ($3b_2$) and ($3c_1$), ($3c_2$). The ansatz (3a) is in analogy to the treatment of the weak interaction; ansatz (3c) is suggested by the photon propagator term in the usual treatment of electromagnetic interaction; for ansatz (3b) no theoretical justification seems to exist, but it is this ansatz that leads to best agreement with the experimental data.

To calculate

$$A=\langle\gamma,P,\vec{k},\lambda,\vec{p}_P|T|\vec{p}_V,V\rangle, \tag{5}$$

we follow the calculation of the corresponding matrix elements in Refs. 1 and 4, and insert a complete system of hadron states $|\alpha\vec{p}_\alpha\rangle$

$$A=\sum_\alpha\int\frac{d^3p}{2E_\alpha c_\alpha}\langle\gamma,P,\vec{k},\lambda,\vec{p}_P|L^\lambda|\vec{p}_\alpha,\alpha\rangle \times\langle\alpha,\vec{p}_\alpha|H_\lambda|\vec{p}_V,V\rangle. \tag{6}$$

In order to know precisely what this complete system of hadron states is, we would have to know the representation containing the vector mesons V and the pseudoscalar mesons P of the algebra that describes our one hadron model and which is generated by the operators P_μ, $L_{\mu\nu}$, Γ_μ, E_α, F_α, C, P, T, and perhaps others. Not only do we not know the representation, but also the algebra is not completely known to us. In fact, the main purpose of the calculation is to discover some further properties of this algebra of which we already know that it contains $A_{2,L_{\mu\nu},P_\mu,\Gamma_\mu}$,[5] $E(\mathrm{SU}(3)_{E_\alpha})$, and probably $E(\mathrm{SL}(3,C)_{E_\alpha F_\alpha})$. Therefore, the spectrum of $\alpha=(I,I_3,\ldots s^P,C_n\ldots)$ cannot be derived and we can only hope that the intermediate states we choose are in the spectrum of α.[6] As L^ν is not to change the quantum numbers α, only those matrix elements of L^ν are different from zero for which α is the same as in the final state, i.e.,

$$\alpha=(I_P,I_{3P}Y=0,\{8\},C_n=-1,s^P=0^-\ldots) =P', \tag{7}$$

and we assume that there is only one such state. This also involves the assumption that the pseudoscalar mesons are purely octet and there is no mixing. [Any reasonable assumption of mixing will not change the conclusion of this paper. With a pseudoscalar mixing angle θ_P one would have, instead of Eq. (24c),

$$0.164\pm16\%=|d-a|\cos\theta_P,$$

so that $\theta_P=30°$ will be within the experimental limits for $\phi\to\eta\gamma$. With such an angle all predictions would remain within the experimental bound and the CERN-Bologna value for $\Gamma(\phi\to\eta\gamma)$ would be in even worse disagreement with our model.] Therewith,

$$A=\int\frac{d^3p_{P'}}{2E_{P'}c_{P'}}\langle\gamma,P,\vec{k},\lambda,\vec{p}_P|L^\gamma|\vec{p}_{P'}P'\rangle \times\langle P',\vec{p}_P'|H_\gamma|\vec{p}_V,V\rangle. \tag{8}$$

For the calculation of the hadronic matrix elements we use the results of Ref. 1. With (4) we obtain

$$\langle P'|\mathcal{J}_\mu^{\mathrm{el}}|V\rangle=\langle P',\vec{p}_P'|\{\rho,G_1+(1/\sqrt{3})G_2\}\Gamma_\mu|\vec{p}_V,V\rangle. \tag{9}$$

All the other terms of (4) give zero contribution to the above matrix element: $\langle P'|V_\mu^{(0)Q}|V\rangle=0$, $\langle P'|A_\mu^{(0)Q}|V\rangle=0$ because P_μ does not change spin; $\langle P'|V^{(1)Q}|V\rangle=0$ because H_i does not change parity and changes charge parity [cf. (7a) and (5a) of Ref. 2]. Only $A_\mu^{(1)Q}$ has transformation properties under the C-P-T-extended Poincaré group [cf. (10b), (12b) or (5c), (7b), (14) of Ref. 2] which leads to $\langle P'|A_\mu^{(1)Q}|V\rangle\neq0$. This result (9), making the axial charges responsible for the decay $V\to P\gamma$, might appear strange. It is, however, the only simple possibility within the framework of this algebraic model and is the *only* possibility if one assumes the existence of P' in the spectrum of α. If P' would not be in the spectrum of α, a much more complicated mechanism for $V\to P\gamma$ would have to be assumed in order to get $\langle\gamma P|T|V\rangle\neq0$. As we shall see in the following it is just this result (9) which leads to conservation of parity in the process $V\to P\gamma$.

The calculation of (9) is straightforward following the lines of Ref. 1:

$$\langle P'|\mathcal{J}_\mu^{\mathrm{el}}|V\rangle=(\rho_{P'}+\rho_V)\langle(P')|\tilde{G}|(V)\rangle\langle n=n_P,s=0|\Gamma_\nu|n=1,s=1,s_3=\lambda\rangle \times L_\mu^{-1\nu}(p_{P'})2c_{P'}E_{P'}(\vec{p}_{P'})\delta^3(\vec{p}_{P'}-(m_P/m_V)\vec{p}_V). \tag{10}$$

Here ρ_α are the eigenvalues of the operator ρ in the state α; $\langle(P')|\tilde{G}|(V)\rangle$ is the matrix element of the operator

$$G=G_1+\frac{1}{\sqrt{3}}G_2 \tag{11}$$

reduced to the space of internal quantum numbers,

$$(P')=(I=I_{(P)}, I_3=I_{3(P)}, Y=0, \{8\}),$$

$$(V)=(I=I_V, I_{3V}, Y=0, \{1, 8\}),$$

and n_P is the principal quantum number n for the pseudoscalar mesons. As the pseudoscalar mesons are also to be described by the generalized Dirac representation,[5] the only possible value for n_P is $n_P=0$ as $s=0$. Then[1]

$$L_\mu^{-1\nu}(p_V)\langle n=0, s=0|\Gamma_\nu|n=1, s=1, s_3=\lambda\rangle = f e_\mu(p_V,\lambda), \tag{12}$$

where

$$f=\left(\frac{1-c^2}{2\sqrt{3}}\right)^{1/2}, \tag{13}$$

c is the number that characterizes the generalized Dirac representation, and $e_\nu(p_V,\lambda)$ is the vector-meson polarization vector.[1]

Consequently, if H_μ is given by (3a), we have

$$\langle P', \vec{p}_{P'}|H_\mu|\vec{p}_V V\rangle = G(\rho_{P'}+\rho_V)\langle (P')|\tilde{G}|(V)\rangle f e_\mu(p_V,\lambda) 2c_{P'}E_{P'}(\vec{p}_{P'})\delta^3(\vec{p}_{P'}-(m_{P'}/m_V)\vec{p}_V); \tag{14a}$$

if H_μ is given by ($3b_1$),

$$\langle P', \vec{p}_{P'}|H_\mu|\vec{p}_V, V\rangle = \frac{(\rho_{P'}+\rho_V)(\rho_{P'}^2+\rho_V^2)}{(p_V^2-p_{P'}^2)} G\langle (P')|\tilde{G}|(V)\rangle f e_\mu(p_V,\lambda) 2c_{P'}E_{P'}(\vec{p}_{P'})\delta^3(\vec{p}_{P'}-(m_{P'}/m_V)\vec{p}_V); \tag{14b$_1$}$$

if H_μ is given by ($3b_2$),

$$\langle P'|H_\mu|V\rangle = \frac{(\rho_{P'}+\rho_V)^3}{(p_V^2-p_{P'}^2)} G\langle (P')|\tilde{G}|(V)\rangle f e_\mu(p_V,\lambda) 2c_{P'}E_{P'}(\vec{p}_{P'})\delta^3(\vec{p}_{P'}-(m_P/m_V)\vec{p}_V); \tag{14b$_2$}$$

and if H_μ is given by ($3c_1$), we have

$$\langle P'|H_\mu|V\rangle = \frac{(\rho_{P'}+\rho_V)(\rho_{P'}^2+\rho_V^2)}{(p_V-p_{P'})^2} G\langle (P')|\tilde{G}|(V)\rangle f e_\mu(p_V,\lambda) 2c_{P'}E_{P'}(\vec{p}_{P'})\delta^3\left(\vec{p}_{P'}-\frac{m_P}{m_V}\vec{p}_V\right) \tag{14c$_1$}$$

and a similar expression for ($3c_2$). We insert this into (8) and obtain the following expression for A:

$$A=\langle \gamma, P; \vec{k}, \lambda_{(\gamma)}, \vec{p}_P|L^\nu|(m_P/m_V)\vec{p}_V, P'\rangle e_\nu(p_V, \lambda_{(V)}) g, \tag{15}$$

where g is

$$g = Gf\langle (P)|\tilde{G}|(V)\rangle \times \begin{cases} (\rho_{P'}+\rho_V) & \text{for case a,} \\ (\rho_{P'}+\rho_V)(\rho_{P'}^2+\rho_V^2)(m_V^2-m_P^2)^{-1} & \text{for case b}_1, \\ (\rho_{P'}+\rho_V)^3(m_V^2-m_P^2)^{-1} & \text{for case b}_2, \\ (\rho_{P'}+\rho_V)(\rho_{P'}^2+\rho_V^2)(m_V-m_P)^{-2} & \text{for case c}_1, \\ (\rho_{P'}+\rho_V)^3(m_V-m_P)^{-2} & \text{for case c}_2. \end{cases} \tag{16}$$

The leptonic part L^ν for the process considered here is determined in complete analogy to the leptonic part for the lepton pairs in Refs. 1 and 4 and does not require any new principles. However, its particular form is new as we have not considered photons before. It is also determined in this case from the requirement that it does not contain any dynamics and has a purely kinematical function, a principle which is actually transcribed from the theory of radiation. In this way we obtain the following analog of Eq. (4) of Ref. 1 or Eq. (3.11) of Ref. 4:

$$\langle \gamma, P, \vec{k}, \lambda, \vec{p}_P|L^\nu|P', (m_P/m_V)\vec{p}_V\rangle = \frac{a\sqrt{c_P}}{2m_P}\frac{a\sqrt{c_{P'}}}{2m_{P'}}\epsilon^{\nu\mu\rho\sigma}\epsilon_\mu(k,\lambda)k_\rho p_{(P)\sigma}\delta^3(\vec{k}+\vec{p}_P-\vec{p}_V), \tag{17}$$

where $\epsilon_\mu(k, \lambda)$ is the photon polarization vector and the constant a has been determined in Ref. 4. Equation (17) inserted into (15) gives

$$A = \frac{a\sqrt{c_P}}{2m_P}\,\frac{a\sqrt{c_V}}{2m_V}\,g\,\epsilon^{\nu\mu\rho\sigma}e_\nu(p_V, \lambda_V)\epsilon_\mu(k, \lambda_{(\gamma)})k_\rho p_{(P)\sigma}\,\delta^3(\vec{k}+\vec{p}_P-\vec{p}_V), \tag{18}$$

where we have used[4]

$$\frac{c_{P'}}{m_{P'}^2} = \frac{c_V}{m_V^2}.$$

To obtain the decay rate we insert (18) into (1) and again make the usual assumption:

$$|\phi_V(\vec{p}_V)|^2 = 2E_V(p_V)c_V\delta^3(\vec{p}_V-\vec{q}), \quad \vec{q}=0,$$

i.e., that the ensemble of decaying vector mesons is exactly at rest.

$$\Gamma = \sum_{\text{Pol}} \int\int \frac{d^3k}{2E_\gamma}\,\frac{d^3p_P}{2E_P}\,\delta(E_V-E_P-E_\gamma)\delta^3(\vec{q}-\vec{k}-\vec{p}_P)\,\frac{1}{2E_V}\,|\epsilon^{\nu\mu\rho\sigma}e_\nu(q, \lambda_{(V)})\epsilon_\mu(k, \lambda_{(\gamma)})k_\rho p_{(P)\sigma}|^2\,\frac{a^4}{16m_P^2m_V^2}g^2. \tag{19}$$

Under the integral we have the standard expression; performing the integration and summation, and averaging over the polarizations we obtain the result:

$$\Gamma(V\to P\gamma) = \frac{a^4}{16m_P^2m_V^2}g^2\,\frac{\pi}{24}m_V^3\left(1-\frac{m_P^2}{m_V^2}\right)^3. \tag{20}$$

If we write (20) in the standard form[7]

$$\Gamma(V\to P\gamma) = |g_{VP\gamma}|^2\,\frac{\alpha}{24}m_V^3\left(1-\frac{m_P^2}{m_V^2}\right)^3, \tag{20$'$}$$

then we have for $g_{VP\gamma}$

$$\begin{aligned} g_{VP\gamma} &= \left(\frac{\pi}{\alpha}\right)^{1/2}\frac{a^2}{4m_Pm_V}g \\ &= \left(\frac{\pi}{\alpha}\right)^{1/2}\frac{a^2}{4}Gf\langle(P)|\tilde{G}|(V)\rangle\rho_{PV}, \end{aligned} \tag{21}$$

where

$$\rho_{PV} = \begin{cases} \dfrac{\rho_P+\rho_V}{m_Pm_V} & \text{for case a,} \\[2ex] \dfrac{(\rho_P+\rho_V)^3}{(m_V^2-m_P^2)m_Pm_V} & \text{for case b}_2, \\[2ex] \dfrac{(\rho_P+\rho_V)(\rho_P^2+\rho_V^2)}{(m_V^2-m_P^2)m_Vm_P} & \text{for case b}_1, \\[2ex] \dfrac{(\rho_P+\rho_V)^3}{(m_V-m_P)^2m_Vm_P} & \text{for case c}_2, \\[2ex] \dfrac{(\rho_P+\rho_V)(\rho_P^2+\rho_V^2)}{(m_V-m_P)^2m_Vm_P} & \text{for case c}_1. \end{cases} \tag{21$'$}$$

Experimental decay rates are known for the following radiative decays of vector mesons[7-9]:

$$\omega\to\pi^0\gamma, \quad \phi\to\pi^0\gamma, \quad \phi\to\eta\gamma.$$

For $\phi\to\eta\gamma$ there are two different experiments which give results in disagreement with each other.[8,9] Whereas the CERN-Bologna value is in better agreement with previous theoretical considerations, the Orsay value[9,10] is in better agreement with our model.

The experimental values of $g_{VP\gamma}$ calculated from the experimental decay rate (20′) are the following:

$|g_{\omega\pi\gamma}| = 2.82\ \text{GeV}^{-1} \pm 10\%$
from world average (Particle Properties Table),

$|g_{\omega\pi\gamma}| = 2.44\ \text{GeV}^{-1} \pm 13\%$
from latest Orsay data,[8]

$|g_{\phi\pi\gamma}| = 0.184\ \text{GeV}^{-1} \pm 18\%$ (22)
from latest Orsay data,[8]

$|g_{\phi\eta\gamma}| = 0.89\ \text{GeV}^{-1} \pm 16\%$
from latest Orsay data,[8]

$|g_{\phi\eta\gamma}| = 1.66\ \text{GeV}^{-1} \pm 12\%$
from the CERN-Bologna data.[9]

The SU(3) matrix elements are given by

$$\langle(\eta)|\tilde{G}|(\phi)\rangle = \frac{1}{\sqrt{3}}\frac{1}{\sqrt{5}}\langle 1||N||1\rangle_2\cos\theta - \frac{1}{\sqrt{3}}\langle 1||N||0\rangle\sin\theta,$$
$$\langle(\pi^0)|\tilde{G}|(\phi)\rangle = \frac{1}{\sqrt{5}}\langle 1||N||1\rangle_2\cos\theta + \langle 1||N||0\rangle\sin\theta,$$
$$\langle(\eta)|\tilde{G}|(\omega)\rangle = -\frac{1}{\sqrt{3}}\frac{1}{\sqrt{5}}\langle 1||N||1\rangle_2\sin\theta - \frac{1}{\sqrt{3}}\langle 1||N||0\rangle\cos\theta, \qquad (23)$$
$$\langle(\pi^0)|\tilde{G}|(\omega)\rangle = -\frac{1}{\sqrt{5}}\langle 1||N||1\rangle_2\sin\theta + \langle 1||N||0\rangle\cos\theta,$$
$$\langle(\pi^0)|\tilde{G}|(\rho^0)\rangle = \frac{1}{\sqrt{3}}\frac{1}{\sqrt{5}}\langle 1||N||1\rangle_2 .$$

Here $\langle 1||N||0\rangle$ is the singlet-octet reduced matrix element for the F_α and $\langle 1||N||1\rangle_2$ is the symmetric octet-octet reduced matrix element of F_α $[(1/\sqrt{5})(\frac{3}{2})^{1/2}\langle 1||N||1\rangle_2 = D]$. (The antisymmetric octet-octet reduced matrix element does not occur here because the antisymmetric Clebsch-Gordan coefficients are zero for the cases under consideration.) Even if we fix θ to be the SU(6) value $\theta = 35.2°$, which we shall do for the following, we are left with two arbitrary parameters $\langle 1||N||1\rangle_2$ and $\langle 1||N||0\rangle$ to fit the three meager experimental data (22). We write (23) in the following way:

$$\langle(\eta)|\tilde{G}|(\phi)\rangle = \frac{1}{\sqrt{3}}(D'-A),$$
$$\langle(\pi^0)|\tilde{G}|(\phi)\rangle = (D'+A),$$
$$\langle(\eta)|\tilde{G}|(\omega)\rangle = -\frac{1}{\sqrt{6}}(D'+2A),$$
$$\langle(\pi^0)|G|(\omega)\rangle = -\frac{1}{\sqrt{2}}(D'-2A), \qquad (23')$$
$$\langle(\pi^0)|\tilde{G}|(\rho^0)\rangle = -\frac{1}{\sqrt{2}}D',$$
$$\langle(\eta^0)|\tilde{G}|(\rho^0)\rangle = (\tfrac{3}{2})^{1/2}D',$$

where we have introduced

$$D' = (\tfrac{2}{15})^{1/2}\langle 1||N||1\rangle_2,$$
$$A = (\tfrac{1}{3})^{1/2}\langle 1||N||0\rangle .$$

We shall now show that the experimental data (22) rule out case a but fit both cases b_1 and b_2, if we use for the operator ρ the assumption that fitted the decay rates of $V\to e\bar{e}$,[1] i.e., $\rho=\zeta M$. In case c the ansatz ($3c_2$) can also be ruled out; however, ansatz ($3c_1$) leads to disagreement only if one uses the Orsay data, as the CERN-Bologna value for $\phi\to\eta\gamma$ gives a good fit with ansatz ($3c_1$). We use the Orsay values and consider first case b_2. From Table I and Eq. (23′) we obtain

$$\begin{aligned}
&\text{(a)}\ \ 0.280 = |-d+2a| \quad (\omega\to\pi\gamma)\ \text{(Orsay)},\\
&\text{(b)}\ \ 0.0166 = |d+a| \quad (\phi\to\pi\gamma)\ \text{(Orsay)},\\
&\text{(c)}\ \ 0.164 = |d-a| \quad (\phi\to\eta\gamma)\ \text{(Orsay)},
\end{aligned} \qquad (24)$$

where we have introduced

$$d = D'/C,\quad a = A/C,\quad C = \left(\frac{\alpha}{\pi}\right)^{1/2}\frac{4}{a^2 G f \zeta^3}.$$

Equations (24) are three equations for the two unknowns a and d, and we can check the consistency of them. Because of (24b) we set $a=-d+\epsilon$ and have

$$|d+a| = |\epsilon| = 0.0166 \pm 18\%;$$

we choose $d>0$ and have to distinguish the two cases: $\epsilon>0$, $\epsilon<0$. For $\epsilon>0$ we calculate $3d$ from (24a) and $2d$ from (24c), and obtain

$$\frac{3d}{2d} = 1.75 \pm 33\%,$$

and for $\epsilon<0$ we calculate from (24a) and (24c):

$$\frac{3d}{2d} = 1.69 \pm 33\%.$$

Thus case b_2 fits the experimental data within the experimental errors for

$$\operatorname{sgn} g_{\phi\eta\gamma} = \operatorname{sgn} g_{\phi\pi\gamma} \qquad (\epsilon>0)$$

as well as for

$$\operatorname{sgn} g_{\phi\eta\gamma} = -\operatorname{sgn} g_{\phi\pi\gamma} \qquad (\epsilon<0).$$

The same is true for case b_1; the corresponding numbers are

$$|\epsilon| = 0.021 \pm 18\%,$$
$$\frac{3d}{2d} = 1.30 \pm 33\% \ \text{ for } \epsilon>0,$$
$$\frac{3d}{2d} = 1.19 \pm 33\% \ \text{ for } \epsilon<0.$$

For case a we obtain

$$|\epsilon| = 0.022 \pm 18\%,$$
$$\frac{3d}{2d} = 0.77 \pm 33\% \ \text{ for } \epsilon>0,$$
$$\frac{3d}{2d} = 0.67 \pm 33\% \ \text{ for } \epsilon<0.$$

Therewith we have seen that case a, i.e., the ansatz (3a) for the electromagnetic transition oper-

TABLE I. Fit of the experimental data for cases (3a), $(3b_1)$, and $(3b_2)$.

	Case a	Case b_1	Case b_2	Case c_1	Case c_2
	$\frac{1}{\xi}\rho_{PV}(a)$ (GeV^{-1})	$\frac{1}{\xi^3}\rho_{PV}(b_1)$ (GeV^{-1})	$\frac{1}{\xi^3}\rho_{PV}(b_2)$ (GeV^{-1})	$\frac{1}{\xi^3}\rho_{PV}(c_1)$ (GeV^{-1})	$\frac{1}{\xi^3}\rho_{PV}(c_2)$ (GeV^{-1})
$\omega\to\pi\gamma$	8.68	9.20	12.30	13.06	17.40
$\phi\to\pi\gamma$	8.40	8.74	11.06	11.40	14.43
$\phi\to\eta\gamma$	2.81	5.10	9.36	17.01	31.22
$\omega\to\eta\gamma$		9.06	17.6		
$\rho^0\to\pi^0$			12.5±7%		
$\xi^0\to\eta\gamma$			19.0±40%		

	Case a	Case b_1	Case b_2	Case c_1	Case c_2	
$\omega\to\pi\gamma$	$\left\lvert\frac{g^{\exp}_{VP\gamma}\xi^3}{\rho_{PV}(a)}\right\rvert$	$\left\lvert\frac{g^{\exp}_{VP\gamma}\xi^3}{\rho_{PV}(b_1)}\right\rvert$	$\left\lvert\frac{g^{\exp}_{VP\gamma}\xi^3}{\rho_{PV}(b_2)}\right\rvert$	$\left\lvert\frac{g^{\exp}_{VP\gamma}\xi^3}{\rho_{PV}(c_1)}\right\rvert$	$\left\lvert\frac{g^{\exp}_{VP\gamma}\xi^3}{\rho_{PV}(c_2)}\right\rvert$	
$\phi\to\pi\gamma$						
$\phi\to\eta\gamma$	0.24	0.307	0.238±10%	0.187	0.164	world average
		0.265	0.199±13%	0.224	0.140	Orsay
$\omega\to\eta\gamma$	0.022	0.021	0.0166±18%	0.0161	0.0127	
$\rho^0\to\pi^0$	0.60	0.326	0.177±12%	0.0975	0.0532	CERN-Bologna
$\xi^0\to\eta\gamma$	0.32	0.175	0.095±16%	0.0523	0.0285	Orsay

ator H_v, is ruled out by the experimental data for the radiative decay of vector mesons.

For case c_2 with the Orsay values of Table I we obtain

$$|\epsilon| = 0.0127,$$

$$\frac{3d}{2d} = 3.89 \pm 25\% \quad \text{for } \epsilon > 0,$$

$$\frac{3d}{2d} = 4.01 \pm 25\% \quad \text{for } \epsilon < 0.$$

For case c_1 the Orsay values lead to a similar result:

$$\frac{3d}{2d} \approx 3.5 \pm 25\%.$$

Thus, if the Orsay values are correct, case c can be safely ruled out. However, with the CERN-Bologna value for $\phi\to\eta\gamma$, a good fit is obtained for ansatz $(3c_1)$. The numbers are (using the world average value for $\omega\to\pi\gamma$)

$$|\epsilon| = 0.0161,$$

$$\frac{3d}{2d} = 1.52 \pm 25\% \quad \text{for } \epsilon > 0,$$

$$\frac{3d}{2d} = 1.57 \pm 25\% \quad \text{for } \epsilon < 0.$$

Ansatz $(3c_2)$ is also ruled out by the CERN-Bologna value.

Whereas the experimental data for the decays $V\to e\bar{e}$ could not distinguish between the assumptions (3a) and (3b) for H_v but already required that ρ was an operator, the experimental data for the decays $V\to P\gamma$ clearly rule out the assumption (3a). Unfortunately, because of the uncertain experimental data, assumption c cannot be excluded. The experimental situation seems to favor assumption (3b), for which no theoretical justification can be given. In the following we calculate some predictions for case b.

For case b_2 we calculate

$$\begin{aligned} d &= 0.097 \pm 16\% \\ &\qquad \text{for } \epsilon > 0 \quad \text{and} \quad a = -0.080 \pm 0.013, \\ d &= 0.078 \pm 16\% \\ &\qquad \text{for } \epsilon < 0 \quad \text{and} \quad a = -0.095 \pm 0.013. \end{aligned} \tag{25}$$

From these values we obtain the reduced matrix elements:

$$\begin{aligned} A &= -0.080\,C \pm 14\% \quad \text{for } \operatorname{sgn} g_{\phi\eta\gamma} \\ &= \operatorname{sgn} g_{\phi\pi\gamma}\,, \\ A &= -0.095\,C \pm 13\% \quad \text{for } \operatorname{sgn} g_{\phi\eta\gamma} \\ &= -\operatorname{sgn} g_{\phi\pi\gamma}\,. \end{aligned} \tag{25'}$$

With the values for d and ϵ we can predict the branching ratios

$$\frac{\Gamma(\omega\eta\gamma)}{\Gamma(\omega\pi\gamma)}, \quad \frac{\Gamma(\rho^0 \to \pi^0\gamma)}{\Gamma(\omega \to \pi\gamma)}, \quad \frac{\Gamma(\rho^0 \to \eta\gamma)}{\Gamma(\rho^0 \to \pi^0\gamma)}\,.$$

We obtain

$$\left|\frac{g_{\omega\eta\gamma}}{g_{\omega\pi\gamma}}\right| = \frac{\rho_{\eta\omega}}{\rho_{\pi\omega}}\,\frac{1}{\sqrt{3}}\left|\frac{d+2a}{d-2a}\right|$$

$$= 0.19 \pm 18\% \quad \text{for case } b_2 \text{ with } \epsilon > 0$$

$$(= 0.33 \pm 10\% \quad \text{for case } b_2 \text{ with } \epsilon < 0)$$

and therewith

$$\frac{\Gamma(\omega \to \eta\gamma)}{\Gamma(\omega \to \pi\gamma)} = 0.005 \pm 36\% \approx 1\% \quad \text{for case } b_2\,, \;\; \epsilon > 0$$

$$(= 0.016 \pm 10\% \approx 2\% \quad \text{for case } b_2\,, \;\; \epsilon < 0)\,. \tag{26}$$

In case b_1 we obtain

$$\frac{\Gamma(\omega \to \eta\gamma)}{\Gamma(\omega \to \pi\gamma)} = \begin{cases} 0.9\% & \text{for } \epsilon > 0 \\ 1.3\% & \text{for } \epsilon < 0\,. \end{cases}$$

The experimental limit for this branching ratio is[11,12]

$$\left.\frac{\Gamma(\omega \to \eta\gamma)}{\Gamma(\omega \to \pi\gamma)}\right|_{\exp} < 7.5\%\,.$$

For the $\rho^0 \to \pi^0\gamma$ decay we obtain

$$\left|\frac{g_{\rho^0\pi^0\gamma}}{g_{\omega\pi\gamma}}\right| = \frac{\rho_{\pi\rho}}{\rho_{\pi\omega}}\left|\frac{-(1/\sqrt{2})d}{-(1/\sqrt{2})(d-2a)}\right|$$

$$= \begin{cases} 0.35 \pm 12\% & \text{for } \epsilon > 0 \text{ in case } b_2 \\ 0.28 \pm 12\% & \text{for } \epsilon < 0 \text{ in case } b_2 \end{cases}$$

and therewith

$$\frac{\Gamma(\rho^0 \to \pi^0\gamma)}{\Gamma(\omega \to \pi^0\gamma)} = \begin{cases} 0.124 \pm 24\% & \text{for } \epsilon > 0 \\ 0.073 \pm 24\% & \text{for } \epsilon < 0\,. \end{cases}$$

So the predicted decay rate for $\rho^0 \to \pi^0\gamma$ is (using $\Gamma(\omega \to \pi^0\gamma) = 0.80$ MeV)

$$\Gamma(\rho^0 \to \pi^0\gamma) = \begin{cases} 99 \text{ keV} \pm 45\% & \text{for } \operatorname{sgn} g_{\phi\eta\gamma} = \operatorname{sgn} g_{\phi\pi\gamma} \\ 58 \text{ keV} \pm 45\% & \text{for } \operatorname{sgn} g_{\phi\eta\gamma} = -\operatorname{sgn} g_{\phi\pi\gamma}\,. \end{cases}$$

The experimental limit for this decay rate is

$$\Gamma(\rho^0 \to \pi^0\gamma) < 250 \text{ keV}\,.$$

In the same way as above we calculate

$$\frac{\Gamma(\rho^0 \to \eta\gamma)}{\Gamma(\rho^0 \to \pi\gamma)} = 0.88 \pm 14\%$$

and therewith predict

$$\Gamma(\rho^0 \to \eta\gamma) = \begin{cases} 87 \text{ keV} \pm 60\% & \text{for } \operatorname{sgn} g_{\phi\eta\gamma} = \operatorname{sgn} g_{\phi\pi\gamma} \\ 52 \text{ keV} \pm 60\% & \text{for } \operatorname{sgn} g_{\phi\eta\gamma} = -\operatorname{sgn} g_{\phi\pi\gamma}\,. \end{cases}$$

The recently obtained experimental limit for this decay rate is[11]

$$\Gamma(\rho^0 \to \eta\gamma) \leq 570 \text{ keV}\,.$$

In the above calculation we have used the Orsay value for $\Gamma(\omega \to \pi\gamma)$. The results with the world average value lie within the errors and do not change our conclusions.

Finally, we point out the disagreement of assumption (3b) with the value of the CERN-Bologna group for $\Gamma(\phi \to \eta\gamma)$: From (24a) and (24b) one calculates

$$d - a = 0.191 \pm 0.031 \quad (0.224) \quad \text{for } \epsilon > 0\,,$$

$$d - a = 0.181 \pm 0.031 \quad (0.214) \quad \text{for } \epsilon < 0\,.$$

The numbers in parenthesis are obtained if we take the world average values instead of (24a). From the experimental value in the Table I we calculate

TABLE II. Comparison of experimental and predicted data for the radiative decay of vector mesons.

	Experimental	Predicted
$\lvert g_{\omega\pi\gamma}\rvert$	2.44 $\text{GeV}^{-1} \pm 13\%$	input
$\lvert g_{\phi\pi\gamma}\rvert$	0.184 $\text{GeV}^{-1} \pm 18\%$	input
$\lvert g_{\phi\eta\gamma}\rvert$	0.89 $\text{GeV}^{-1} \pm 16\%$ [a] 1.66 $\text{GeV}^{-1} \pm 12\%$ [b]	1.14 $\text{GeV}^{-1} \pm 25\%$
$\Gamma(\omega \to \eta\gamma)/\Gamma(\omega \to \pi\gamma)$	<7.5%	$(1 \pm 0.3)\%$
$\Gamma(\rho^0 \to \pi\gamma)$	<250 keV	99 keV ± 45%
$\Gamma(\rho^0 \to \eta\gamma)$	<570 keV	87 keV ± 60%

[a] See Ref. 8.
[b] See Ref. 9.

$|d-a| = 0.303 \pm 12\%$.

Using the ansatz b_1, the disagreement is worse. Ansatz a is also ruled out with the CERN-Bologna value for $\Gamma(\phi \rightarrow \eta\gamma)$.

We summarize the comparison of the predictions of our model using assumption (3b) with the experimental data in Table II. (The predicted numbers are those obtained for $\text{sgn}g_{\phi\eta\gamma} = \text{sgn}g_{\phi\pi\gamma}$.) Here we have used ansatz $(3b_2)$; ansatz $(3b_1)$ gives similar values. Ansatz $(3c_1)$, though not included in the comparison of Table II, fits the CERN-Bologna value very well but is excluded by the Orsay values.

We observe that the experimental data for the radiative decays are really not so good as to support some theoretical considerations just by themselves. However, they are good enough to definitely rule out ansatz (3a), which could not be done in Ref. 1, because each ansatz, (3a), (3b), and (3c), leads to the same results for $V \rightarrow e\bar{e}$.

In conclusion, our algebraic model gives a description of the leptonic and radiative decays of vector mesons if we assume the transition operator H_λ is given by (3b) or by $(3c_1)$ and that $\rho \sim M$. These are the two assumptions that have not been used in previous applications of the algebraic model and therefore constitute new information about the algebra of observables describing the physical system of hadrons which we wish to construct empirically.

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