

Single transverse-spin asymmetry for D -meson production in semi-inclusive deep inelastic scattering

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We study the single transverse-spin asymmetry for open charm production in the semi-inclusive lepton-hadron deep inelastic scattering. We calculate the asymmetry in terms of the QCD collinear factorization approach for D mesons at high enough $P_{h\perp}$ and find that the asymmetry is proportional to the twist-three trigluon correlation function in the proton. With a simple model for the trigluon correlation function, we estimate the asymmetry for both COMPASS and eRHIC kinematics and discuss the possibilities of extracting the trigluon correlation function in these experiments.

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I. INTRODUCTION

Single transverse-spin asymmetries (SSAs) have received much attention in recent years, both experimentally and theoretically. Large SSAs have been consistently observed in various experiments at different collision energies [1–3]. As a consequence of the parity and time-reversal invariance of the strong interaction, the SSA is directly connected to the transverse motion of partons inside a polarized hadron. Understanding the QCD dynamics behind the measured asymmetries should have the profound impact on our knowledge of strong interaction and hadron structure [4].

Two QCD mechanisms for generating SSAs have been proposed [5–7] and been applied [7–14] extensively in phenomenological studies. Both of these mechanisms connect the SSA to the parton's transverse motion inside a transversely polarized hadron. One mechanism relies on the transverse momentum dependent (TMD) factorization for the observed polarized cross sections and explicitly expresses the SSA in terms of “asymmetric” TMD parton distributions, known as the Sivers functions [5]. The other follows the QCD collinear factorization approach for cross sections when all observed momentum scales are much larger than the nonperturbative hadronic scale $1/\text{fm} \sim \Lambda_{\text{QCD}}$ and attributes the SSA to the twist-three transverse-spin-dependent multiparton correlation functions [6,7]. Unlike the first mechanism, in which the SSA measures the TMD parton distribution and the spin dependence of the parton's transverse motion at a given momentum, the twist-three transverse-spin-dependent correlation functions reveal the net spin dependence of the parton's transverse motion when its transverse momentum is integrated. Naturally, the moment of the spin-dependent TMD parton distributions could be related to the twist-three multiparton correlation functions [15]. Although two mechanisms each have their own kinematic domain of validity, they describe the same physics in the region where they overlap [16].

Most studies in both mechanisms have been concentrated on the SSAs generated by either the quark Sivers function [9–13] or the twist-three quark-gluon correlation function [7,8,14,17], which is defined as

$$T_F(x, x) = \int \frac{dy_1^- dy_2^-}{4\pi} e^{ixP^+ y_1^-} \langle P, s_\perp | \bar{\psi}(0) \gamma^+ \times [\epsilon^{\perp \sigma n \bar{n}} F_\sigma^+(y_2^-)] \psi(y_1^-) | P, s_\perp \rangle, \quad (1)$$

with the gauge links suppressed. Possibilities of accessing the transverse motion of gluons, or the gluon Sivers functions, have also been investigated recently [13,18]. Likewise, the spin dependence of the gluon's transverse motion in the QCD collinear factorization approach is represented by the twist-three trigluon correlation function, $T_G(x, x)$, defined as

$$T_G(x, x) = \int \frac{dy_1^- dy_2^-}{2\pi} e^{ixP^+ y_1^-} \frac{1}{xP^+} \langle P, s_\perp | F^+_\alpha(0) \times [\epsilon^{\perp \sigma n \bar{n}} F_\sigma^+(y_2^-)] F^{\alpha+}(y_1^-) | P, s_\perp \rangle. \quad (2)$$

Its contribution to the SSA was first studied by Ji in the context of direct-photon production in hadronic collision [19]. Although direct-photon production provides a nice possibility to access $T_G(x, x)$, the extraction of $T_G(x, x)$ could be difficult due to the contribution from the quark-initiated subprocesses and the limited knowledge on $T_F(x, x)$. In this paper, we study the single transverse-spin asymmetry for open charm production in the semi-inclusive deep inelastic scattering (SIDIS) and argue that the SSA in SIDIS is a clean observable to extract the twist-three transverse-spin-dependent trigluon correlation function, $T_G(x, x)$.

The D -meson production at large enough transverse momentum $P_{h\perp}$ in SIDIS is dominated by the photon-gluon fusion subprocess, $\gamma^* + g \rightarrow c + \bar{c}$, since the intrinsic charm contribution [20] is less relevant at large $P_{h\perp}$ and the photon-charm subprocess, $\gamma^* + c \rightarrow c + g$, is suppressed by the small charm quark distribution at the collision energy that we are interested in this paper. We

calculate the SSA for D -meson production in lepton-proton SIDIS in terms of the QCD collinear factorization approach and find that the asymmetry is directly proportional to the diagonal part of the twist-three trigluon correlation function in the polarized proton, $T_G(x, x)$, because the photon-gluon fusion subprocess at this order does not have the so-called “hard-pole” contribution to the asymmetry [16]. Therefore, the measurement of SSA for D -meson production in SIDIS is a direct measurement of the trigluon correlation function, $T_G(x, x)$. With a simple model for the trigluon correlation function, obtained under a similar assumption guiding the modeling of the quark-gluon correlation function $T_F(x, x)$, we find that the asymmetry for both COMPASS [21] and eRHIC [22] kinematics is sizable and could be measured experimentally. Recently, the COMPASS experiment successfully measured the gluon polarization, ΔG , in a longitudinally polarized proton based on the photon-gluon fusion process by tagging charmed mesons [21]. This certainly makes the measurement of SSA of open charm production in the same experimental setting and the extraction of the transverse-spin-dependent trigluon correlation function, $T_G(x, x)$, promising.

We find that the SSA of D -meson production in SIDIS has a minimum at $z_h \sim 0.5$, and it increases as z_h is moving away from this central value. This increase of the SSA away from $z_h \sim 0.5$ has the same physics origin as the observed increase of the magnitude of SSA in hadronic pion production as a function of increasing x_F (or rapidity y), and is a prediction of the twist-three formalism of the QCD collinear factorization approach to the SSA.

We also find that the twist-three gluonic contribution to the SSA has both similarity and difference from the twist-three fermionic contributions. Both gluonic and fermionic twist-three contributions to the SSA have the so-called “nonderivative” and “derivative” terms, which correspond to the terms that are proportional to the twist-three correlation functions and the derivative of the correlation functions, respectively. As noticed in Refs. [17,23], the fermionic nonderivative and derivative terms can be combined together and the dependence on the nonperturbative twist-three correlation function, $T_F(x, x)$, is proportional to a simple combination, $T_F(x, x) - xT'_F(x, x)$. However, our explicit calculation shows that the nonderivative and derivative gluonic contribution cannot be combined into the same simple form due to the difference in the partonic hard parts of these two terms.

The same approach discussed in this paper can be applied for studying the SSA in open charm production in hadronic collisions, which is dominated by the gluon-gluon fusion if $T_G(x, x)$ is not too small comparing to the $T_F(x, x)$ [24–26]. With the extraction of the trigluon correlation function, $T_G(x, x)$, and the knowledge of $T_F(x, x)$, we enter a new era of exploring nonperturbative physics beyond the parton distribution functions (PDFs) which have been well studied in the past 30 years.

The rest of our paper is organized as follows. In Sec. II, we present our calculation of the SSAs for open charm production in SIDIS. We first introduce the relevant kinematics of open charm production in SIDIS and present the formula for the unpolarized cross section. We then derive the twist-three formula for the SSA in the QCD collinear factorization approach and express the asymmetry in terms of the trigluon transverse-spin-dependent correlation function, $T_G(x, x)$. We close this section by a discussion on the calculation of the color factor of the partonic hard part which depends on the color contraction of three gluon fields in the definition of the trigluon correlation function. In principle, there could be two gauge invariant trigluon correlation functions, $T_G(x, x)$ and $\tilde{T}_G(x, x)$ defined later, due to two independent ways to neutralize the color of the three gluon fields in the matrix elements. We point out that only one of them, $T_G(x, x)$, could be related to the gluon Sivers function [18]. In Sec. III, we estimate the production rate of open charm mesons in SIDIS for both COMPASS and eRHIC kinematics. We choose a simple ansatz for the trigluon correlation function, $T_G(x, x)$, and present our predictions for the SSAs of open charm production at the existing COMPASS experiment and the planned eRHIC experiment. Finally, we conclude our paper in Sec. IV.

II. CALCULATION OF SINGLE-SPIN ASYMMETRY

We start this section by specifying our notation and kinematics of SIDIS. We consider the scattering process of an unpolarized lepton e on a polarized hadron p

$$e(\ell) + p(P, s_\perp) \rightarrow e(\ell') + h(P_h) + X, \quad (3)$$

where $s_\perp$ is the transverse-spin vector defined below, h represents the observed D -meson with momentum P_h and mass m_h . For the collision energy that we are interested in this paper, we work in the approximation of one-photon exchange and define the virtual photon momentum $q = \ell - \ell'$ and its invariant mass $Q^2 = -q^2$. We adopt the usual SIDIS variables:

$$\begin{aligned} S_{ep} &= (P + \ell)^2, & x_B &= \frac{Q^2}{2P \cdot q}, \\ y &= \frac{P \cdot q}{P \cdot \ell} = \frac{Q^2}{x_B S_{ep}}, & z_h &= \frac{P \cdot P_h}{P \cdot q}. \end{aligned} \quad (4)$$

It is also convenient to introduce the “transverse” component of the virtual photon momentum q as

$$q_t^\mu = q^\mu - \frac{q \cdot P_h}{P \cdot P_h} P^\mu - \frac{q \cdot P}{P \cdot P_h} P_h^\mu, \quad (5)$$

which is a spacelike vector and orthogonal to both P and P_h . We define

$$\tilde{q}_\perp^2 \equiv -q_t^\mu q_{t\mu} = Q^2 \left[1 + \frac{1}{x_B} \frac{q \cdot P}{P \cdot P_h} \right] - \frac{m_h^2}{z_h^2}. \quad (6)$$

To completely specify the kinematics, we will work in the so-called *hadron frame* [27], where the virtual photon and the polarized proton are taken to have only one spatial component that is in the z -direction:

$$P^\mu = P^+ \bar{n}^\mu, \quad q^\mu = -x_B P^+ \bar{n}^\mu + \frac{Q^2}{2x_B P^+} n^\mu, \quad (7)$$

where the light-cone momentum component is defined as $P^\pm = (P^0 \pm P^3)/\sqrt{2}$ and $\bar{n}^\mu = (1^+, 0^-, 0_\perp)$, $n^\mu = (0^+, 1^-, 0_\perp)$ are two lightlike vectors with $\bar{n} \cdot n = 1$. The momentum of the final-state D -meson can be written as

$$P_h^\mu = \frac{x_B P^+}{z_h Q^2} m_{h\perp}^2 \bar{n}^\mu + \frac{z_h Q^2}{2x_B P^+} n^\mu + P_{h\perp}^\mu, \quad (8)$$

where $m_{h\perp}^2 = m_h^2 + P_{h\perp}^2$ with $P_{h\perp} = \sqrt{\vec{P}_{h\perp}^2}$. From Eq. (6) one can show that $q_\perp \equiv \sqrt{\vec{q}_\perp^2} = P_{h\perp}/z_h$ in this hadron frame, independent of mass m_h .

In this hadron frame, usually, one chooses the coordinate system such that the virtual photon has a vanishing energy component, corresponding to $P^+ = Q/\sqrt{2}x_B$, and P_h lies in the xz -plane (known as the *hadron plane*), as shown in Fig. 1. The lepton momenta, ℓ and ℓ' , define the *lepton plane* and can be expressed in terms of variables ψ and ϕ as follows [27]:

$$\begin{aligned} \ell^\mu &= \frac{Q}{2} (\cosh\psi, \sinh\psi \cos\phi, \sinh\psi \sin\phi, -1), \\ \ell'^\mu &= \frac{Q}{2} (\cosh\psi, \sinh\psi \cos\phi, \sinh\psi \sin\phi, +1), \end{aligned} \quad (9)$$

where ϕ is the azimuthal angle between the hadron and lepton plane, as indicated in Fig. 1, and

$$\cosh\psi = \frac{2x_B S_{ep}}{Q^2} - 1 = \frac{2}{y} - 1. \quad (10)$$

We parametrize the transverse-spin vector of the initial proton $s_\perp$ as

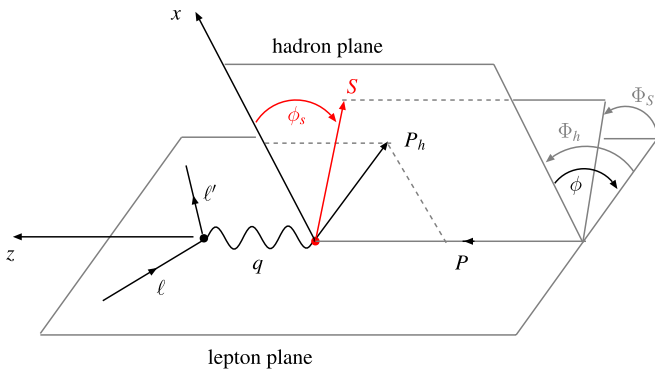


FIG. 1 (color online). Kinematics of the SIDIS process in the hadron frame.

$$s_\perp = (0, \cos\phi_s, \sin\phi_s, 0), \quad (11)$$

where ϕ_s is the azimuthal angle of $s_\perp$ measured from the hadron plane, as shown in Fig. 1. If one uses the lepton plane as the reference to define the azimuthal angle of $s_\perp$ as Φ_S , and that of the hadron plane as Φ_h , one has the relation $\phi_s = \Phi_S - \Phi_h$ and $\phi = -\Phi_h$.

The single transverse-spin asymmetry is defined as

$$\begin{aligned} A_N &= \frac{\sigma(s_\perp) - \sigma(-s_\perp)}{\sigma(s_\perp) + \sigma(-s_\perp)} \\ &= \frac{d\Delta\sigma(s_\perp)}{dx_B dy dz_h dP_{h\perp}^2 d\phi} / \frac{d\sigma}{dx_B dy dz_h dP_{h\perp}^2 d\phi}. \end{aligned} \quad (12)$$

In the following subsections, we will first review the unpolarized cross section at leading order, and then derive the single transverse polarized cross sections, $\Delta\sigma(s_\perp)$.

A. Unpolarized cross section

The unpolarized differential SIDIS cross section may be calculated from the formula

$$\frac{d\sigma}{dx_B dy dz_h dP_{h\perp}^2 d\phi} = \frac{\pi\alpha_{em}^2}{Q^4} L_{\mu\nu}(\ell, q) W^{\mu\nu}(P, q, P_h), \quad (13)$$

where $L_{\mu\nu}$ and $W^{\mu\nu}$ are the leptonic and hadronic tensors, respectively. The leptonic tensor is given by

$$L_{\mu\nu}(\ell, q) = 2(\ell_\mu \ell'_\nu + \ell'_\mu \ell_\nu - g_{\mu\nu} Q^2/2). \quad (14)$$

The hadronic tensor has the following expression in QCD:

$$\begin{aligned} W^{\mu\nu}(P, q, P_h) &= \frac{1}{4z_h} \sum_X \int \frac{d^4\xi}{(2\pi)^4} e^{iq\cdot\xi} \langle P | J_\mu(\xi) | X P_h \rangle \\ &\quad \times \langle X P_h | J_\nu(0) | P \rangle, \end{aligned} \quad (15)$$

where J^μ is the quark electromagnetic current and X represents all other final-state hadrons other than the observed open charm meson h .

The hadronic tensor can be further decomposed in terms of five parity and current conserving tensors $\mathcal{V}_i^{\mu\nu}$ [27]:

$$W^{\mu\nu} = \sum_{i=1}^5 \mathcal{V}_i^{\mu\nu} W_i, \quad (16)$$

where the W_i are structure functions which may be projected out from $W^{\mu\nu}$ by $W_i = W_{\rho\sigma} \tilde{\mathcal{V}}_i^{\rho\sigma}$, with the corresponding inverse tensors $\tilde{\mathcal{V}}_i$. Both $\mathcal{V}_i$ and $\tilde{\mathcal{V}}_i$ can be constructed from four orthonormal basis vectors:

$$\begin{aligned}
T^\mu &= \frac{1}{Q}(q^\mu + 2x_B P^\mu), \\
X^\mu &= \frac{1}{q_\perp} \left[\frac{P_h^\mu}{z_h} - q^\mu - \left(1 + \frac{q_\perp^2 + m_h^2/z_h^2}{Q^2} \right) x_B P^\mu \right], \quad (17) \\
Y^\mu &= \epsilon^{\mu\nu\rho\sigma} Z_\nu X_\rho T_\sigma, \quad Z^\mu = -\frac{q^\mu}{Q},
\end{aligned}$$

with normalization $T^2 = 1$ and $X^2 = Y^2 = Z^2 = -1$, which are reduced to those in [27] when $m_h = 0$. The tensor $\mathcal{V}_5$ does not contribute to the cross section when it is contracted with a symmetric $L_{\mu\nu}$, the other four tensors and their inverse are given as [27]:

$$\begin{aligned}
\mathcal{V}_1^{\mu\nu} &= X^\mu X^\nu + Y^\mu Y^\nu, & \mathcal{V}_2^{\mu\nu} &= g^{\mu\nu} + Z^\mu Z^\nu, \\
\mathcal{V}_3^{\mu\nu} &= T^\mu X^\nu + T^\nu X^\mu, & \mathcal{V}_4^{\mu\nu} &= X^\mu X^\nu - Y^\mu Y^\nu,
\end{aligned} \quad (18)$$

$$\begin{aligned}
\tilde{\mathcal{V}}_1^{\mu\nu} &= \frac{1}{2}(2T^\mu T^\nu + X^\mu X^\nu + Y^\mu Y^\nu), \\
\tilde{\mathcal{V}}_2^{\mu\nu} &= T^\mu T^\nu, \\
\tilde{\mathcal{V}}_3^{\mu\nu} &= -\frac{1}{2}(T^\mu X^\nu + T^\nu X^\mu), \\
\tilde{\mathcal{V}}_4^{\mu\nu} &= \frac{1}{2}(X^\mu X^\nu - Y^\mu Y^\nu).
\end{aligned} \quad (19)$$

The contraction of $L_{\mu\nu}$ and $\mathcal{V}_i^{\mu\nu}$ leads to various angular distributions. Let $\mathcal{A}_i = L_{\mu\nu} \mathcal{V}_i^{\mu\nu} / Q^2$, we have

$$\begin{aligned}
\mathcal{A}_1 &= 1 + \cosh^2 \psi, & \mathcal{A}_2 &= -2, \\
\mathcal{A}_3 &= -\cos \phi \sinh 2\psi, & \mathcal{A}_4 &= \cos 2\phi \sinh^2 \psi.
\end{aligned} \quad (20)$$

We can then write the cross section in Eq. (13) as

$$\frac{d\sigma}{dx_B dy dz_h dP_{h\perp}^2 d\phi} = \frac{\pi \alpha_{em}^2 y}{Q^2} \sum_{i=1}^4 \mathcal{A}_i W_i. \quad (21)$$

At large $P_{h\perp} \sim Q$, the collinear factorization is expected to be valid, and W_i can be factorized into a convolution of the parton distribution function, the fragmentation function for the produced D -meson, and a short-distance partonic hard part. The lowest-order (LO) contribution to the partonic hard part comes from the photon-gluon fusion subprocess $\gamma^* + g \rightarrow Q(p_c) + \bar{Q}(p_{\bar{c}})$, which gives the leading order cross section as

$$\frac{d\sigma}{dx_B dy dz_h dP_{h\perp}^2 d\phi} = \sigma_0 \int_{x_{\min}}^1 \frac{dx}{x} \int \frac{dz}{z} G(x) D(z) \delta\left(\frac{P_{h\perp}^2}{z_h^2} - \frac{(1-\hat{x})(1-\hat{z})}{\hat{x}\hat{z}} Q^2 + \hat{z}^2 m_c^2\right) \left(\frac{1}{2}\right) \sum_{i=1}^4 \mathcal{A}_i \hat{W}_i, \quad (22)$$

where $\sigma_0 = e_c^2 \alpha_{em}^2 \alpha_s y / (8\pi z_h^2 Q^2)$, $\hat{x} = x_B/x$, $\hat{z} = z_h/z$, and e_c and m_c are the fractional charge and mass of the charm quark, respectively. The $P_{h\perp}^2/z_h^2$ in the δ -function could be replaced by $q_\perp^2$, and the $1/2$ is the color factor. In Eq. (22) $G(x)$ is the unpolarized gluon distribution function with gluon momentum fraction x , and $D(z)$ is the fragmentation function for the charm quark to become a D -meson with $z = P \cdot P_h / P \cdot p_c$. We have suppressed the dependence on the factorization and renormalization scales for simplicity. We used $P_{h\perp} \approx z p_{c\perp}$ inside the δ -function, which fixes the z integration. The lower limit of x integration $x_{\min}$ is given by

$$x_{\min} = \begin{cases} x_B \left[1 + \frac{P_{h\perp}^2 + m_c^2}{z_h(1-z_h)Q^2} \right], & \text{if } z_h + \sqrt{z_h^2 + \frac{P_{h\perp}^2}{m_c^2}} \geq 1; \\ x_B \left[1 + \frac{2m_c^2}{Q^2} \left(1 + \sqrt{1 + \frac{P_{h\perp}^2}{z_h^2 m_c^2}} \right) \right], & \text{if } z_h + \sqrt{z_h^2 + \frac{P_{h\perp}^2}{m_c^2}} \leq 1. \end{cases} \quad (23)$$

The short-distance parts $\hat{W}_i$ are calculated from the photon-gluon scattering and are given by

$$\begin{aligned}
\hat{W}_1 &= 2 \left[\frac{\hat{u}}{\hat{t}} + \frac{\hat{t}}{\hat{u}} - \frac{2\hat{s}Q^2}{\hat{t}\hat{u}} + \frac{4\hat{x}^2\hat{s}}{Q^2} \right] + 4m_c^2 \left[\frac{Q^2 - 2\hat{t}}{\hat{t}^2} + \frac{Q^2 - 2\hat{u}}{\hat{u}^2} - \frac{2\hat{x}^2}{Q^2} \left(\frac{\hat{u}}{\hat{t}} + \frac{\hat{t}}{\hat{u}} + 2 \right) \right] - 8m_c^4 \left[\frac{1}{\hat{t}} + \frac{1}{\hat{u}} \right]^2, \\
\hat{W}_2 &= \frac{16\hat{x}^2}{Q^2} \left[\hat{s} - m_c^2 \left(\frac{\hat{u}}{\hat{t}} + \frac{\hat{t}}{\hat{u}} + 2 \right) \right], \\
\hat{W}_3 &= 4\hat{x}\hat{z} \frac{q_\perp}{Q} (\hat{u} - \hat{t}) \left[\frac{\hat{s} - Q^2}{\hat{t}\hat{u}} - 2m_c^2 \left(\frac{1}{\hat{t}} + \frac{1}{\hat{u}} \right)^2 \right], \\
\hat{W}_4 &= 8\hat{z}^2 q_\perp^2 \left[\frac{Q^2}{\hat{t}\hat{u}} + m_c^2 \left(\frac{1}{\hat{t}} + \frac{1}{\hat{u}} \right)^2 \right],
\end{aligned} \quad (24)$$

where $\hat{s}$, $\hat{t}$, $\hat{u}$ are defined at the partonic level as

$$\begin{aligned}\hat{s} &\equiv (xP + q)^2 = \frac{1 - \hat{x}}{\hat{x}} Q^2, \\ \hat{t} &\equiv (p_c - q)^2 - m_c^2 = -\frac{1 - \hat{z}}{\hat{x}} Q^2, \\ \hat{u} &\equiv (xP - p_c)^2 - m_c^2 = -\frac{\hat{z}}{\hat{x}} Q^2,\end{aligned}\quad (25)$$

which are different from some definitions used in the literature. We found that this definition makes the expression of $\hat{W}_i$ for massive quark production simpler. Taking $m_c = 0$ in Eqs. (24) and (25), one recovers the results for the production of the massless quark derived in [28,29].

B. Twist-three polarized cross section

We now proceed to derive the single transverse-spin-dependent cross section by applying the same method developed in Refs. [7,17]. When both physically observed scales Q , $P_{h\perp} \gg \Lambda_{\text{QCD}}$, the spin-dependent cross section for D -meson production is expected to be factorized in terms of the twist-three transverse-spin-dependent trigluon correlation function [30],

$$\begin{aligned}d\Delta\sigma(s_\perp) &\propto \frac{1}{2S_{ep}} \int dz D(z) \int dx_1 dx_2 \mathcal{T}_G(x_1, x_2) i\epsilon^{\rho s_\perp n \bar{n}} \\ &\times \lim_{k_\perp \rightarrow 0} \frac{\partial}{\partial k_\perp^\rho} H(x_1, x_2, k_\perp),\end{aligned}\quad (26)$$

where $1/2S_{ep}$ is the flux factor and $\epsilon^{\rho s_\perp n \bar{n}} = \epsilon^{\rho\sigma\mu\nu} s_\perp \sigma n_\mu \bar{n}_\nu$,

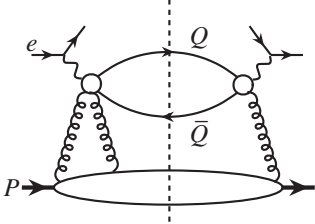


FIG. 2. A typical diagram that gives a nonvanishing contribution to the SSA.

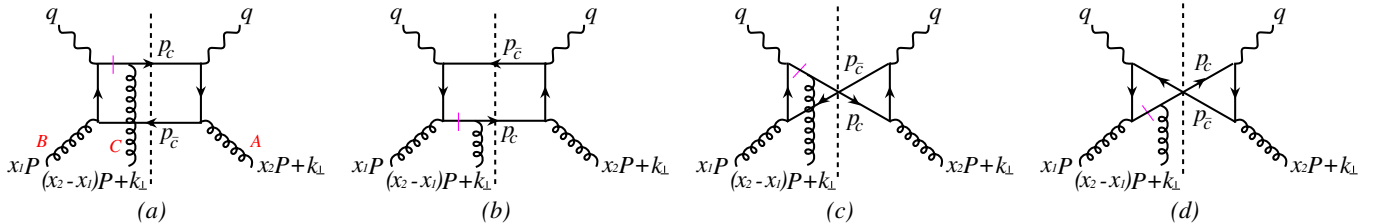


FIG. 3 (color online). Feynman diagrams that give the twist-three contribution to the spin-dependent cross section. The short bar indicates the propagator that produces the pole. The letters, A, B, and C represent the color of the initial-state gluons.

$$\begin{aligned}\mathcal{T}_G(x_1, x_2) &= \int \frac{P^+ dy_1^- dy_2^-}{2\pi} e^{ix_1 P^+ y_1^- + i(x_2 - x_1) P^+ y_2^-} \\ &\times d_{\alpha\beta} \langle P, s_\perp | A^\alpha(0) [\epsilon^{s_\perp \sigma n \bar{n}} F_\sigma^+(y_2^-)] \\ &\times A^\beta(y_1^-) | P, s_\perp \rangle,\end{aligned}\quad (27)$$

where $d_{\alpha\beta} = -g_{\alpha\beta} + \bar{n}_\alpha n_\beta + \bar{n}_\beta n_\alpha$. $\mathcal{T}_G(x_1, x_2)$ is related to the trigluon correlation function through $T_G(x, x) = x\mathcal{T}_G(x, x)$. Since $\mathcal{T}_G(x_1, x_2)$ is real, we need an imaginary part of the hard-scattering function $H(x_1, x_2, k_\perp)$ to contract with $i\epsilon^{\rho s_\perp n \bar{n}}$ in order to obtain a real $\Delta\sigma(s_\perp)$. This imaginary part comes from the interference between a real part of scattering amplitude with a single gluon initial state and an imaginary part of the partonic scattering amplitude with an extra gluon, see Fig. 2. Technically, the imaginary part, or the phase, “ i ” arises when the virtual momentum integral of the extra gluon is evaluated by the residue of an unpinched pole from a propagator in the amplitude with an extra gluon. Such a propagator is indicated by the one marked with a short bar in the diagrams in Fig. 3.

There are a total of eight partonic diagrams contributing to the twist-three polarized cross sections, $\Delta\sigma(s_\perp)$. Four of them are shown in Fig. 3, and the other four are obtained by attaching the extra gluon in the same way on the right side of the final-state cut. When the extra gluon is attached to the left side of the final-state cut, as shown in Fig. 3, the phase from the propagator marked by the bar arises effectively as

$$\begin{aligned}&\frac{1}{(p_c - (x_2 - x_1)P - k_\perp)^2 - m_c^2 + i\epsilon} \\ &= \frac{1}{2P \cdot p_c} \frac{1}{x_1 - x_2 + v_1 \cdot k_\perp + i\epsilon} + \mathcal{O}(k_\perp^2) \\ &\rightarrow \frac{-i\pi}{2P \cdot p_c} \delta(x_1 - x_2 + v_1 \cdot k_\perp),\end{aligned}\quad (28)$$

to fix the virtual loop momentum fraction $x_1 = x_2 - v_1 \cdot k_\perp$ with $v_1^\mu = -2p_c^\mu / 2P \cdot p_c$. On the other hand, the on-shell condition associated with the unobserved anticharm quark fixes the momentum fraction of the active initial-state gluon as

$$\delta(p_c^2 - m_c^2) = \delta((x_2 P + k_\perp + q - p_c)^2 - m_c^2) = \frac{1}{2P \cdot (q - p_c)} \delta(x_2 - x - v_2 \cdot k_\perp), \quad (29)$$

where terms at $\mathcal{O}(k_\perp^2)$ and higher are neglected and

$$x = -\frac{(q - p_c)^2 - m_c^2}{2P \cdot (q - p_c)}, \quad v_2^\mu = \frac{2p_c^\mu}{2P \cdot (q - p_c)}. \quad (30)$$

When the extra gluon is attached to the right-hand side of the cut, the phase arises as

$$\frac{1}{(p_c + (x_2 - x_1)P + k_\perp)^2 - m_c^2 - i\epsilon} = \frac{1}{2P \cdot p_c} \frac{1}{x_2 - x_1 - v_1 \cdot k_\perp - i\epsilon} + \mathcal{O}(k_\perp^2) \rightarrow \frac{i\pi}{2P \cdot p_c} \delta(x_2 - x_1 - v_1 \cdot k_\perp), \quad (31)$$

and the on-shell condition of the unobserved anticharm quark gives

$$\delta(p_c^2 - m_c^2) = \frac{1}{2P \cdot (q - p_c)} \delta(x_1 - x), \quad (32)$$

which has no $k_\perp$ -dependence.

Applying the so-called “master formula” in Ref. [17], we have from Eq. (26) the following general expression:

$$\begin{aligned} & \lim_{k_\perp \rightarrow 0} \frac{\partial}{\partial k_\perp^\rho} \int dx_1 \int dx_2 \mathcal{T}_G(x_1, x_2) [H_L(x_1, x_2, k_\perp) \delta(x_1 - x_2 + v_1 \cdot k_\perp) \delta(x_2 - x - v_2 \cdot k_\perp) \\ & \quad - H_R(x_1, x_2, k_\perp) \delta(x_2 - x_1 - v_1 \cdot k_\perp) \delta(x_1 - x)] \\ & = (v_2 - v_1)^\rho H_L(x, x, 0) \frac{d}{dx} \left(\frac{T_G(x, x)}{x} \right) + \frac{T_G(x, x)}{x} \lim_{k_\perp \rightarrow 0} \frac{\partial}{\partial k_\perp^\rho} [H_L(x + (v_2 - v_1) \cdot k_\perp, x + v_2 \cdot k_\perp, k_\perp) \\ & \quad - H_R(x, x + v_1 \cdot k_\perp, k_\perp)], \end{aligned} \quad (33)$$

where we have already used the facts that $H_L(x, x, 0) = H_R(x, x, 0)$ and $T_G(x, x) = x \mathcal{T}_G(x, x)$. The fact that Eq. (33) depends only on the diagonal part of the trigluon correlation function, $T_G(x_1, x_2)$, with $x_1 = x_2 = x$ is a consequence of the photon-gluon fusion subprocess at this order having only the so-called “soft-pole” contribution to the SSA [7,16]. Therefore, the measurement of the SSA in D -meson production in SIDIS is a direct measurement of the trigluon correlation function, $T_G(x, x)$.

In terms of $\hat{s}$, $\hat{t}$, $\hat{u}$ defined in the previous subsection, we have

$$v_1^\mu = \frac{2x}{\hat{u}} p_c^\mu, \quad v_2^\mu = -\frac{2x}{\hat{t}} p_c^\mu, \quad (v_2 - v_1)^\mu = -\frac{2x}{\hat{t}} \left(1 + \frac{\hat{t}}{\hat{u}} \right) p_c^\mu. \quad (34)$$

Using Eqs. (26) and (33), and adding the contributions from the eight diagrams together, we find the final expression for the fully differential single transverse-spin-dependent cross section:

$$\begin{aligned} \frac{d\Delta\sigma(s_\perp)}{dx_B dy dz_h dP_{h\perp}^2 d\phi} & = \sigma_0 \int_{x_{\min}}^1 dx \int \frac{dz}{z} D(z) \delta\left(\frac{P_{h\perp}^2}{z_h^2} - \frac{(1-\hat{x})(1-\hat{z})}{\hat{x}\hat{z}} Q^2 + \hat{z}^2 m_c^2\right) \left(\frac{1}{4}\right) \left[\epsilon^{P_{hs\perp} n \bar{n}} \left(\frac{\sqrt{4\pi\alpha_s}}{z\hat{t}} \right) \left(1 + \frac{\hat{t}}{\hat{u}} \right) \right] \\ & \quad \times \sum_{i=1}^4 \mathcal{A}_i \left[-x \frac{d}{dx} \left(\frac{T_G(x, x)}{x} \right) \hat{W}_i + \left(\frac{T_G(x, x)}{x} \right) \hat{N}_i \right], \end{aligned} \quad (35)$$

where $1/4$ is the color factor, $T_G(x, x)$ is the trigluon correlation function defined in Eq. (2), $\hat{W}_i$ are given in Eq. (24), and the hard parts for the nonderivative term $\hat{N}_i$ are given by

$$\begin{aligned}
\hat{N}_1 &= 4 \left[\frac{2m_c^2 - Q^2}{\hat{t}\hat{u}} + \frac{6\hat{x}^2}{Q^2} \right] \left[(\hat{s} - Q^2) - 2m_c^2 \left(\frac{\hat{u}}{\hat{t}} + \frac{\hat{t}}{\hat{u}} + 2 \right) \right], \\
\hat{N}_2 &= \frac{16\hat{x}^2}{Q^2} \left[(\hat{s} - Q^2) - 2m_c^2 \left(\frac{\hat{u}}{\hat{t}} + \frac{\hat{t}}{\hat{u}} + 2 \right) \right], \\
\hat{N}_3 &= \frac{2Q}{\hat{z}q_\perp} (\hat{u} - \hat{t}) \left[\left(\frac{4\hat{z}^2 q_\perp^2}{\hat{t}\hat{u}} - \frac{1}{Q^2 + \hat{s}} \right) \left(2m_c^2 \left(\frac{1}{\hat{t}} + \frac{1}{\hat{u}} \right) - \frac{Q^2 - \hat{s}}{Q^2 + \hat{s}} \right) - 2\hat{z}q_\perp^2 \right], \\
\hat{N}_4 &= 8 \left[2\hat{z}q_\perp^2 - \frac{\hat{t}\hat{u}}{Q^2 + \hat{s}} \right] \left[\frac{Q^2}{\hat{t}\hat{u}} + m_c^2 \left(\frac{1}{\hat{t}} + \frac{1}{\hat{u}} \right)^2 \right].
\end{aligned} \tag{36}$$

Equation (35) is our main result for the leading order twist-three $T_G(x, x)$ contribution to the fully differential polarized cross section $\Delta\sigma(s_\perp)$ of D -meson production in SIDIS. The single transverse-spin asymmetry for the D -meson production in SIDIS is obtained by substituting Eqs. (22) and (35) into Eq. (12).

Similar to the twist-three contributions to the SSAs generated by the fermionic quark-gluon correlation function $T_F(x, x)$, the gluonic twist-three contribution to the SSA of D -meson production in Eq. (35) has both the derivative and nonderivative terms, a unique feature of the twist-three contribution. It was found that the fermionic nonderivative and derivative terms can be combined into a simple form $T_F(x, x) - xT'_F(x, x)$ [17,23]. However, from Eqs. (24) and (36), it is clear that $\hat{W}_i \neq \hat{N}_i$. That is, our explicit calculation shows that such a simple combination does not hold for contributions from trigluon correlation function $T_G(x, x)$ and is not universal.

We close this section by a discussion on the calculation of the color factor $1/4$ in Eq. (35). The color factor in Eq. (35) depends on how colors of the three gluon fields in the matrix element of the trigluon correlation function in Eq. (2) are neutralized. If the color of the operator, $F^A(0)F^C(y_2^-)F^B(y_1^-)$, in Eq. (2) with the Lorentz indices suppressed, is neutralized by $(\mathcal{F}^C)_{AB} = -if^{CAB}$ with f^{CAB} the fully antisymmetric structure constant of the color SU(3) group, we refer to this trigluon correlation function as $T_G(x, x)$ as expressed in Eq. (2). The corresponding color factor for the partonic part in Eq. (35) is calculated by contracting the color indices of the Feynman diagrams in Fig. 3 with $\frac{i}{N(N^2-1)}f^{ABC}$, which gives the color factor $1/4$ in Eq. (35). On the other hand, if the color of the operator, $F^A(0)F^C(y_2^-)F^B(y_1^-)$, is neutralized by $(\mathcal{D}^C)_{AB} = d^{CAB}$, which is symmetric under the interchange of any two indices, we have a different trigluon correlation function and we refer to it as $\tilde{T}_G(x, x)$, which has the same expression as $T_G(x, x)$ except the difference in the contraction of the gluon color. The color factor for the corresponding partonic hard part is calculated by contracting the color indices of the same Feynman diagrams with $\frac{N}{(N^2-1)(N^2-4)}d^{ABC}$, which also gives the color factor $1/4$.

That is, there could be two trigluon correlation functions depending on how the colors of the three gluon fields are

neutralized [31]. We find that both correlation functions are gauge invariant after inserting the necessary gauge link in the adjoint representation between the gluon field strengths in the matrix element in Eq. (2), and they contribute to the SSAs with the same partonic hard parts, and potentially, different color factors [26]. Since the color factors are the same in our case, adding the potential contribution from $\tilde{T}_G(x, x)$ is to replace the $T_G(x, x)$ in Eq. (35) by $T_G(x, x) + \tilde{T}_G(x, x)$.

We noticed that all Feynman diagrams for producing a charm quark at this order in Fig. 3 have the same color structure, $\text{Tr}[T^A T^C T^B] = (d^{ACB} + if^{ACB})/4$, which leads to the overall color factor $1/4$ for the contributions from both $T_G(x, x)$ and $\tilde{T}_G(x, x)$. However, for the SSAs of producing a $\bar{D}$ meson, which is fragmented from an anti-charm quark, both the partonic part and the antisymmetric part of the color structure change sign, while the symmetric part of the color structure is unchanged. Therefore, the SSAs for $\bar{D}$ -meson production in the SIDIS at the leading order have the same functional form as that for the D -meson production except the sum of the trigluon correlation functions, $T_G(x, x) + \tilde{T}_G(x, x)$, is replaced by $T_G(x, x) - \tilde{T}_G(x, x)$. That is, by comparing the SSAs for producing D and $\bar{D}$ -mesons in SIDIS, we could gain valuable information on both trigluon correlation functions. However, the relation could be complicated in the D -meson production in hadronic collisions due to the additional color flow from the other colliding hadron [26].

We also notice that the correlation function $T_G(x, x)$ with the color neutralized by $(\mathcal{F}^C)_{AB} = -if^{CAB}$ could potentially be related to the spin-dependent TMD gluon distribution or the gluonic Sivers function [18], since the middle gluon field strength of the operator, $F^A(0)F^C(y_2^-)F^B(y_1^-)$, could be related to the gauge link in the adjoint representation that is needed to define the TMD gluon distribution. Without knowing the size and sign of either trigluon correlation functions, we will treat them as one combined trigluon correlation function, labeled by $T_G(x, x)$, in the following numerical estimates of the SSAs.

III. PHENOMENOLOGY

In this section we first evaluate the inclusive D -meson production rate at large $P_{h\perp}$ in SIDIS. We then propose a

simple model for trigluon correlation function $T_G(x, x)$ and estimate the size of SSA for the D -meson production in SIDIS.

The charm meson's transverse momentum $P_{h\perp}$ is chosen to be along the x -direction in the *hadron frame*, and therefore

$$\epsilon^{P_{h\perp} s_{\perp} n \bar{n}} = -P_{h\perp} \sin \phi_s. \quad (37)$$

The fully differential cross sections in Eqs. (22) and (35) can be decomposed in terms of the independent angular distributions as follows:

$$\begin{aligned} \frac{d\sigma}{dx_B dy dz_h dP_{h\perp}^2 d\phi} &= \sigma_0^U + \sigma_1^U \cos \phi + \sigma_2^U \cos 2\phi, \\ \frac{d\Delta\sigma}{dx_B dy dz_h dP_{h\perp}^2 d\phi} &= \sin \phi_s (\Delta\sigma_0 + \Delta\sigma_1 \cos \phi \\ &\quad + \Delta\sigma_2 \cos 2\phi). \end{aligned} \quad (38)$$

Before evaluating the SSA, we first estimate the D -meson production rate in the unpolarized SIDIS by using our LO formula in Eq. (22).

For the following numerical evaluations we use CTEQ6L parton distribution functions [32], and charm-to- D fragmentation functions from Ref. [33]. We choose

the factorization scale to be equal to the renormalization scale and set $\mu = \sqrt{Q^2 + m_c^2 + P_{h\perp}^2}$ with charm quark mass $m_c = 1.3$ GeV. In the following plots, we choose two sets of kinematic variables. The first one is $S_{ep} = 300$ GeV², $x_B = 0.01$, and $Q = 1$ GeV, which is close to the COMPASS kinematics. The other is $S_{ep} = 2500$ GeV², $x_B = 0.01$, and $Q = 4$ GeV, which is more relevant to the planned eRHIC experiment [22].

In Fig. 4, we show individual coefficients of the angular distribution, σ_0^U , σ_1^U , and σ_2^U , of the *fully differential* unpolarized cross section for D^0 production in Eq. (38) as a function of both z_h and $P_{h\perp}$ for the kinematics relevant to the COMPASS experiment. It is clear that the angular dependent pieces σ_1^U , $\sigma_2^U \ll \sigma_0^U$ and might be too small to be significant. Without worrying about the detection efficiency, the D -meson production at $P_{h\perp} \sim 1$ GeV could be measurable. Likewise, Fig. 5 shows the *fully differential* unpolarized cross section for D^0 production for eRHIC kinematics. With larger Q and $P_{h\perp}$, the production rate is smaller but may still have enough events with a high luminosity.

In order to obtain the numerical estimate for the SSAs of D -meson production, we have to model the unknown, but universal, trigluon correlation function $T_G(x, x)$. Similar to

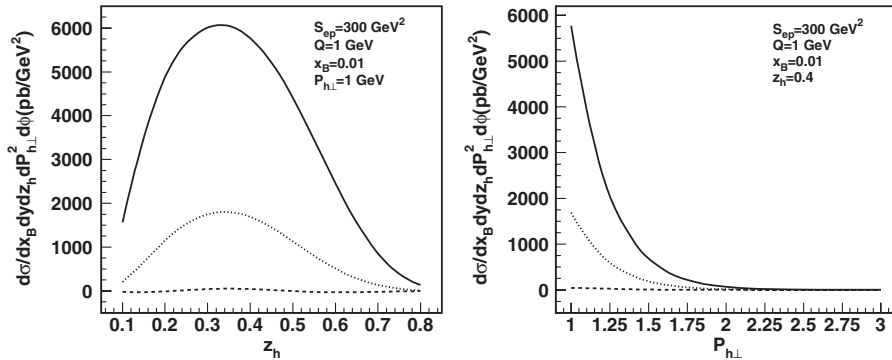


FIG. 4. The fully differential unpolarized cross section for D^0 production in SIDIS for COMPASS kinematics. The curves represent: σ_0^U (solid line), σ_1^U (dashed line), and σ_2^U (dotted line) in Eq. (38).

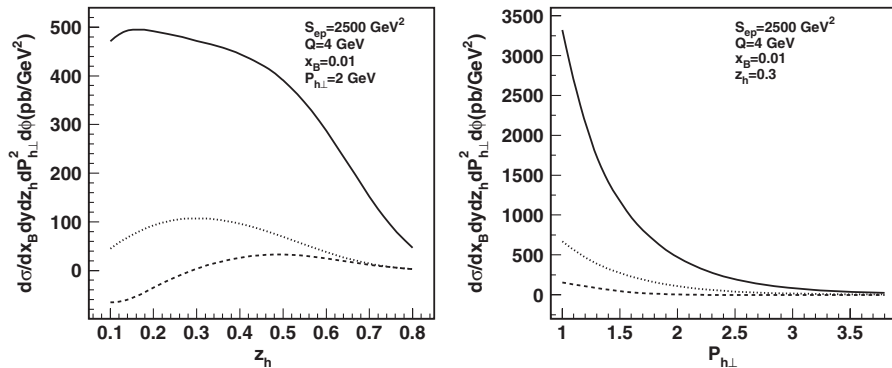


FIG. 5. The fully differential unpolarized cross section for D^0 production in SIDIS at the future eRHIC. The curves represent: σ_0^U (solid line), σ_1^U (dashed line), and σ_2^U (dotted line) in Eq. (38).

the ansatz for quark-gluon correlation function $T_F(x, x)$, which was originally introduced in [7] and found to be consistent with the latest experimental data [17], we model the trigluon correlation function $T_G(x, x)$ as

$$T_G(x, x) = \lambda_g G(x) \quad (39)$$

with $G(x)$ the normal unpolarized gluon distribution function. Because of its nonperturbative nature, $T_G(x, x)$ should be extracted from the experiments and the value and the sign of λ_g should be fixed by the data. For the following numerical estimate, we assume that λ_g has a positive sign and the same size as that for quark-gluon correlation function $T_F(x, x)$ [7], and we choose $\lambda_g = 0.07$ GeV.

In order to present the SSA and its angular dependence on the ϕ , the angle between the hadron plane and the lepton plane, we define the ϕ -integrated single-spin azimuthal asymmetries as

$$\langle \cos(n\phi) \rangle = \frac{1}{\sin\phi_s} \frac{\int_0^{2\pi} d\phi \cos(n\phi) \frac{d\Delta\sigma(s_\perp)}{dx_B dy dz_h dP_{h\perp}^2 d\phi}}{\int_0^{2\pi} d\phi \frac{d\sigma}{dx_B dy dz_h dP_{h\perp}^2 d\phi}}, \quad (40)$$

which gives

$$\langle 1 \rangle = \frac{\Delta\sigma_0}{\sigma_0^U}, \quad \langle \cos\phi \rangle = \frac{\Delta\sigma_1}{2\sigma_0^U}, \quad \langle \cos 2\phi \rangle = \frac{\Delta\sigma_2}{2\sigma_0^U}. \quad (41)$$

In Fig. 6 we plot the SSAs as a function of z_h (left) and $P_{h\perp}$ (right) for the COMPASS kinematics. The asymmetries, $\langle 1 \rangle$, $\langle \cos\phi \rangle$, and $\langle \cos 2\phi \rangle$, defined in Eq. (41), are shown by the solid, dot-dashed, and dotted curves, respectively. For a comparison between the size of the derivative and the nonderivative terms, we also show, by the dashed curves, the contribution to the SSA $\langle 1 \rangle$ from the derivative term only. It is clear that the derivative term dominates over the whole kinematic region. The asymmetries, $\langle \cos\phi \rangle$ and $\langle \cos 2\phi \rangle$, are too small to be observed experimentally. The SSA $\langle 1 \rangle$ is of the order of 10% and could be measurable at the COMPASS experiment.

Figure 6 indicates that the SSA hits a minimum at $z_h \sim 0.5$ and increases very fast when z_h becomes very large or very small. This is because the SSA, $\langle 1 \rangle \sim 1/(1 - x_{\min})$, due to the derivative of $T_G(x, x)$ [7]. From the definition of $x_{\min}$ in Eq. (23), the $z_h(1 - z_h)$ has a maximum at $z_h = 0.5$. Therefore, $x_{\min}$ increases, equivalently, the SSA increases when z_h becomes either smaller or larger than 0.5. When z_h is much further away from the central value 0.5, the $x_{\min}$ becomes so large that the perturbatively calculated asymmetry could increase sharply, which could signal a breakdown of the twist-three approximation and a need of higher power corrections. Nevertheless, the increase of the SSA when z_h is moving away from the central value 0.5 has the same physics origin as the observed increase of the SSA as a function of increasing x_F (or rapidity y) in the hadronic pion production [1,3], and it could be tested in the COMPASS experiment.

Figure 6 also indicates a monotonic increase of the SSA as a function of $P_{h\perp}$. Although we expect the SSA to fall when $P_{h\perp}$ increases, a natural behavior of the twist-three effect in QCD collinear factorization, the enhancement from the derivative of the $T_G(x, x)$ at large x wins over the suppression from large $P_{h\perp}$ due to the limited phase space at COMPASS kinematics. We will see below the decrease of the SSA as the increase of $P_{h\perp}$ is clearly seen at the eRHIC kinematics.

Similarly, we plot the SSAs for D^0 production for the eRHIC kinematics in Fig. 7. Because of the higher collision energy, the effective gluon momentum fraction x that dominates the SSAs is smaller, which leads to a smaller derivative of $T_G(x, x)$ and smaller SSAs. A similar feature has been seen in the SSA for hadronic pion production when we compare the data from the fixed-target experiments with that from RHIC experiments. The 5% SSA for D -meson production at eRHIC could be significant.

The slightly different shape of the SSA as a function of z_h is purely a consequence of the difference in effective range of the parton momentum fraction x . That is, the z_h -dependence of the SSA provides a good measurement of the x -dependence of the correlation function $T_G(x, x)$.

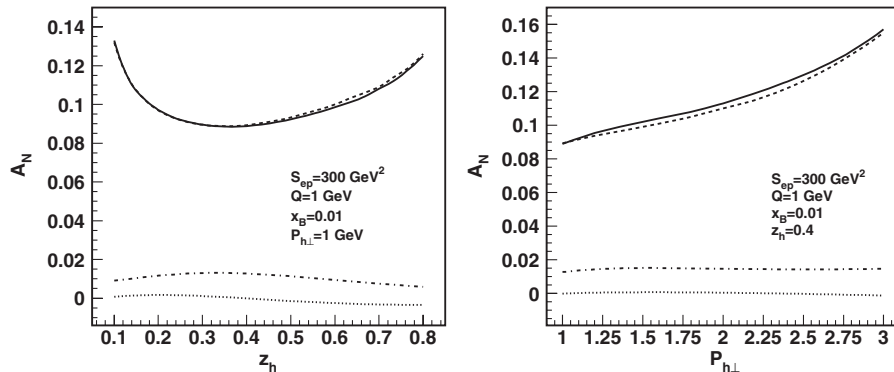


FIG. 6. Single transverse-spin-asymmetries defined in Eq. (41) for D^0 production in SIDIS for COMPASS kinematics. The curves are: solid- $\langle 1 \rangle$, dashed- $\langle 1 \rangle$ with derivative-term only, dot-dashed- $\langle \cos\phi \rangle$, and dotted- $\langle \cos 2\phi \rangle$.

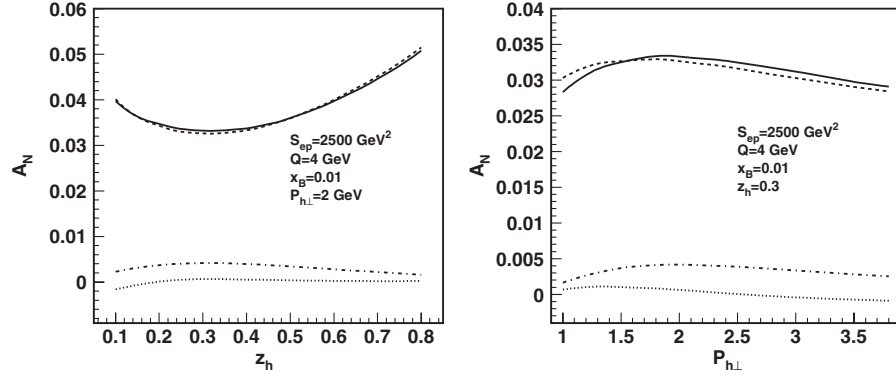


FIG. 7. Single transverse-spin-asymmetries defined in Eq. (41) for D^0 production in SIDIS for eRHIC kinematics. The curves are: solid- $\langle 1 \rangle$, dashed- $\langle 1 \rangle$ with derivative-term only, dot-dashed- $\langle \cos \phi \rangle$, and dotted- $\langle \cos 2\phi \rangle$.

On the other hand, the slow falloff of the SSA as a function $P_{h\perp}$ is natural due to the asymptotic $\lambda_g/P_{h\perp}$ behavior of the twist-three contribution when $P_{h\perp}$ increases. Of course, as discussed above, the $1/(1-x_{\min})$ dependence of the twist-three formalism compensates some of the $1/P_{h\perp}$ falloff due to the phase space limit on the parton momentum fraction x .

IV. CONCLUSIONS

In summary, we have studied the single transverse-spin asymmetry for D -meson production in SIDIS. In terms of the QCD collinear factorization approach, we calculated both derivative and nonderivative contributions to the SSAs. At large enough transverse momentum $P_{h\perp}$, the intrinsic charm contribution to the asymmetry might be neglected, and the SSA is directly proportional to the transverse-spin-dependent trigluon correlation function $T_G(x, x)$ (or $T_G(x, x) \pm \tilde{T}_G(x, x)$ if we include both color structures), which has not been studied experimentally. We pointed out that by comparing the SSAs for producing D and $\bar{D}$ mesons in SIDIS, we could gain valuable information on both trigluon correlation functions. With a simple model for the $T_G(x, x)$, we presented our estimates of the SSAs for the kinematics relevant for both COMPASS and future eRHIC experiments. From the inclusive production rate for the D -meson production and the estimated size of the SSAs, we argue that the SSAs of D -meson production in SIDIS could be a direct and clean probe of the unknown trigluon correlation function $T_G(x, x)$, which provides the important information on the spin dependence of the gluon's transverse motion inside a polarized hadron.

However, we stress that the SSAs shown in all figures are directly proportional to the value and the sign of the λ_g and our model for the twist-three trigluon correlation function $T_G(x, x)$. A different x -dependence from that of $G(x)$ could lead to a different derivative of $T_G(x, x)$ and a different prediction for the SSA. The actual sign and size of the SSA, and the function $T_G(x, x)$ should be determined by the experimental measurements, just like the PDFs. However, our calculation does predict the short-distance dynamics and the kinematic dependence of the SSAs, such as the increase of the SSA when z_h moves away from the central value 0.5.

Finally, we emphasize that the QCD collinear factorization approach to the SSAs allows us to calculate the SSAs of open charm production or other particle production in hadronic collisions. With the experimental extraction of the trigluon correlation function $T_G(x, x)$ as well as $\tilde{T}_G(x, x)$, and the existing and new knowledge of $T_F(x, x)$, we will be able to explore nonperturbative physics, in particular, the multiparton quantum correlations, beyond what we have learned from the parton distribution functions.

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