

Combining weak lensing tomography with halo clustering to probe dark energy

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(Received 19 June 2007; revised manuscript received 3 August 2007; published 18 October 2007)

Two methods of constraining the properties of dark energy are weak lensing tomography and cluster counting. Uncertainties in mass calibration of clusters can be reduced by using the properties of halo clustering (the clustering of clusters). However, within a single survey, weak lensing and halo clustering probe the same density fluctuations. We explore the question of whether this information can be used twice—once in weak lensing and then again in halo clustering to calibrate cluster masses—or whether the combined dark energy constraints are *weaker* than the sum of the individual constraints. For a survey like the Dark Energy Survey (DES), we find that the cosmic shearing of source galaxies at high redshifts is indeed highly correlated with halo clustering at lower redshifts. Surprisingly, this correlation does not degrade cosmological constraints for a DES-like survey, and in fact, constraints are marginally improved since the correlations themselves act as additional observables. This considerably simplifies the analysis for a DES-like survey: when weak lensing and halo clustering are treated as independent experiments, the combined dark energy constraints (cluster counts included) are accurate if not slightly conservative. Our findings mirror those of Takada and Bridle, who investigated correlations between the cosmic shear and cluster counts.

DOI: [10.1103/PhysRevD.76.083515](https://doi.org/10.1103/PhysRevD.76.083515)

PACS numbers: 98.80.Es, 95.30.Sf, 95.36.+x, 98.65.Cw

I. INTRODUCTION

Weak lensing tomography [1–6] and cluster counting [7–11] are complimentary methods of constraining dark energy parameters. Weak lensing tomography (WLT) measures how the images of galaxies at various redshifts are sheared by large scale structure, while cluster counting (CC) simply counts the number of massive clusters as a function of redshift. Both techniques exploit dark energy's geometric effects and its effects on large scale structure growth. The spatial clustering of clusters, or halo clustering (HC), provides free additional information that can break degeneracies in cluster surveys [11,12]. Although halo clustering is by far the weakest of the three probes, it becomes more valuable as systematic uncertainties (e.g. cluster mass calibration) degrade dark energy constraints. Future surveys will combine these techniques.

Since both the cosmic shear and the presence of clusters arise from fluctuations in the cosmic mass density field, we can expect their signals and error bars to be correlated. Intuitively, if a survey or combination of surveys observes a patch of sky with more clusters than average, we should expect the patch to have an above average cosmic shear signal. Hence, an analysis that treats lensing and clusters as independent observables will double-count information and may potentially overstate the bounds on cosmological parameters. On the other hand, the cross-correlation between lensing and clusters is an additional observable that can work to improve the parameter fit. Thus, combining lensing and clusters properly is important: treating them as independent could mean that we have artificially small

error bars or that we are throwing away valuable information—both disturbing possibilities!

Several authors have investigated WLT/CC correlations, finding that they arise primarily from the small scale power that individual halos contribute to the mass power spectrum. Approximating the halo number density as a smooth field related linearly to the cosmic mass density, Fang and Haiman [13] find negligible covariance between cosmic shear and total cluster counts. Takada and Bridle [14] further take into account the structure and discrete nature of halos; they find a correlation between cluster counts and the shear power spectrum on arcminute scales of about 10% (higher or lower depending on the minimum halo mass used).

We focus here on WLT/HC correlations: these should occur since shear maps and halos trace the same field of projected matter fluctuations (on scales larger than the average halo spacing). In other words, a local overdensity of mass tends to create a shear peak and an abundance of halos along the same line of sight. This correlation turns out to be significantly larger than the WLT/CC correlation, and it warrants a closer look at how halo clustering and weak lensing tomography should be combined. The WLT/HC correlation is most important in the regime where halo clustering is adding significant constraints to dark energy parameters; we will be working in that regime.

In this paper, we calculate the WLT/HC cross-correlation and determine its effect on a survey's ability to measure dark energy parameters. We restrict ourselves to large angular scales so that halo clustering is related to the mass overdensity by a linear bias and we can ignore the

effects of individual halos on the shear power spectra. We focus on projections for the Dark Energy Survey (DES) and find that including the full covariance does not degrade the projected constraints; in fact these tighten at the percent level.

II. WEAK LENSING AND HALO CLUSTERING

Divide a survey region into redshift bins with z_i denoting the minimum redshift of the i th bin. In this paper, we consider 10 bins of equal width up to $z = 1$. Let $\chi(z)$ denote the comoving distance to an object at redshift z so that $\chi_i \equiv \chi(z_i)$ (we will often use χ and z interchangeably). The weak lensing convergence is a good computational proxy for shear; in a given redshift bin, the convergence is a function of angular position on the sky and can be expressed as a weighted integral of the density along the line of sight:

$$\kappa_i(\boldsymbol{\theta}) = \int d\chi W_{(\text{Lens},i)}(\chi) \delta[\chi\boldsymbol{\theta}, \chi] \quad (1)$$

where the weighting function is

$$W_{(\text{Lens},i)}(\chi) = \frac{3\Omega_m H_0^2}{2\nu_i^{\text{gal}}} (1+z)\chi \int_{\max(\chi, \chi_i)}^{\chi_{i+1}} d\chi_s \frac{\chi_s - \chi}{\chi_s} \frac{d\nu^{\text{gal}}}{d\chi_s} \quad (2)$$

for $\chi \leq \chi_{i+1}$ and 0 otherwise [15]. Here ν_i^{gal} is the expected galaxy density in bin i which we parametrize as

$$\frac{d\nu^{\text{gal}}}{dz} = \nu_*^{\text{gal}} \frac{z^2}{2z_*^3} e^{-z/z_*}. \quad (3)$$

The distribution has $z_{\text{mode}} = 2z_*$ and $z_{\text{median}} = 2.67z_*$; for DES we use $z_* = 0.254$ and $\nu_*^{\text{gal}} = 15/\text{arcmin}^2$.

We approximate the local overdensity of halos in the i th redshift bin as another line-of-sight integral,

$$h_i(\boldsymbol{\theta}) \equiv \frac{\delta n_i(\boldsymbol{\theta})}{\bar{n}_i} = \int d\chi W_{(\text{Halo},i)}(\chi) \delta[\chi\boldsymbol{\theta}, \chi], \quad (4)$$

where $\bar{n}_i$ is the average halo density in bin i . We are counting all halos more massive than a threshold $M_{\text{min}} = 10^{14.2} M_\odot/h$, and we have assumed that the overdensity of halos of a given mass is proportional to δ (valid on large scales). The halo weighting function is

$$W_{(\text{Halo},i)} = \begin{cases} \chi^2 \bar{b}^M(\chi) & \chi_i < \chi < \chi_{i+1} \\ 0 & \text{otherwise} \end{cases} \quad (5)$$

where the *mass-averaged halo bias* $\bar{b}^M(\chi)$ depends on both redshift and our M_{min} :

$$\bar{b}^M(z) \equiv \frac{\int_{M_{\text{min}}}^{\infty} dM b(M, z) \frac{dn}{dM}(z)}{\int_{M_{\text{min}}}^{\infty} dM \frac{dn}{dM}(z)}. \quad (6)$$

We compute $\bar{b}^M(\chi)$ using the halo mass function

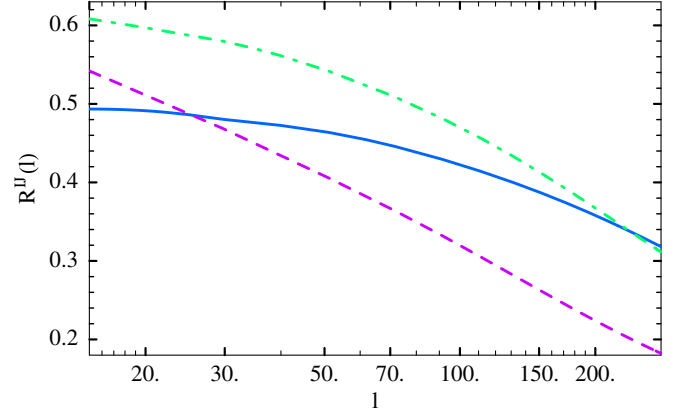


FIG. 1 (color online). Sample correlation coefficients; for a given lensing bin, the most strongly correlated halo bin is shown. All bins have width $\Delta z = 0.1$, and the average redshifts of the bins (Lens, Halo) are as follows: solid (0.95, 0.25); dot-dashed (0.65, 0.15); dashed (0.35, 0.05).

$dn/dM(z)$ of Jenkins *et al.* [16] and the halo bias model $b(M, z)$ of Sheth and Tormen [17].

Let u_I be one of our observables with the subscript denoting the data type and redshift bin, e.g. $u_I = \kappa_i$ for $I = (\text{Lens}, i)$. Then

$$u_I(\boldsymbol{\theta}) = \int d\chi W_I(\chi) \delta[\chi\boldsymbol{\theta}, \chi]. \quad (7)$$

Using the flat-sky approximation, decompose the observables u_I into 2D Fourier modes. Then we define

$$\langle u_I(\mathbf{l}) u_J(\mathbf{l}') \rangle = (2\pi)^2 \delta^2(\mathbf{l} - \mathbf{l}') C_{IJ}(\mathbf{l}) \quad (8)$$

where under the Limber approximation,

$$C_{IJ}(\mathbf{l}) = \int \frac{d\chi}{\chi^2} W_I(\chi) W_J(\chi) P_{IJ}\left(k = \frac{l}{\chi}; \chi\right). \quad (9)$$

The matter power spectrum $P_{IJ}(k)$ depends on the data types. We use a linear spectrum to compute halo-halo and halo-lens terms and a nonlinear spectrum to compute lens-lens terms.¹ Our matter power spectra are computed using the fitting formula of Eisenstein and Hu [18] and the nonlinear scaling of Smith *et al.* [19]. Note that the weighting functions W_I imply that

$$\langle h_i h_j \rangle = 0 \quad \text{for } i \neq j \quad (10)$$

$$\langle \kappa_i h_j \rangle = 0 \quad \text{for } i < j. \quad (11)$$

These follow from the Limber approximation and should be valid on scales $l > 10$.

To assess the degree to which lensing and halo clustering are correlated, it is useful to consider the correlation coef-

¹In the formalism we have chosen, the proper treatment of the halo-lens term is ambiguous, but our results are insensitive to our matter power spectrum choice.

ficient

$$R_{IJ} \equiv C_{IJ}(l)/\sqrt{C_{II}(l)C_{JJ}(l)}. \quad (12)$$

If lensing were completely uncorrelated with halo clustering, the correlation coefficients $R_{(\text{Lens},i)(\text{Halo},j)}$ would vanish; if they measured identical quantities, the coefficient would be unity. Figure 1 shows that on large scales, the lensing of high redshift background galaxies is highly correlated with clustering at intermediate redshifts. It is therefore possible that tighter constraints anticipated from including halo clustering are illusory: perhaps this information has already been used once—in weak lensing tomography—and cannot be double-counted. In that case, it would not be appropriate to blindly combine the constraints from (CC + HC) with those from WLT.

III. IMPACT ON DARK ENERGY PROJECTIONS

To assess the impact of the correlations depicted in Fig. 1, we compute the Fisher matrix for weak lensing tomography and cluster counting supplemented by halo clustering. We choose a standard set of cosmological parameters in a flat universe:

Parameter	$\Omega_m h^2$	$\Omega_b h^2$	Ω_{de}	w_0	w_a	n	δ_ζ^2
Fiducial	.14	.024	.73	-1	0	1	2.57×10^{-9}
Prior	1%	1%	—	—	—	1%	1%

where h is the Hubble constant in units of 100 km/sec/Mpc; Ω_{de} is the ratio of the dark energy density to the critical density (and similarly for the total matter and baryon densities); n is the spectral index of primordial scalar fluctuations, and δ_ζ is their amplitude² at $k = 0.002 \text{ Mpc}^{-1}$; the dark energy equation of state is taken to be $w = w_0 + w_a z/(1+z)$. We impose Gaussian priors expected from future CMB surveys. Additionally, we include (with no priors) the cluster mass calibration parameters A and n_A defined by Lima and Hu [12].

The Fisher matrix for our parameter set is, in the standard approximation, a sum of matrices for our three observables:

$$F^{\text{Total}} = F^{\text{CC}} + F^{\text{HC}} + F^{\text{WLT}}. \quad (13)$$

The full cluster Fisher matrix breaks nicely into a CC and HC term, as in the above equation [12]. However, we know from the results of § II (e.g. Fig. 1) and previous work ([13,14]) that lensing and clusters are not completely independent observables, and therefore, that the above approximation is flawed. To account for the correlation between halo clustering and cosmic shear, we generalize F^{Total} to

$$F^{\text{Total}} = F^{\text{CC}} + F^{\text{HC+WLT}} + \tilde{F}^{\text{WLT}} \quad (14)$$

²In practice, we vary $\ln \delta_\zeta^2$. The prior is defined at $k = 0.05 \text{ Mpc}^{-1}$.

where $\tilde{F}^{\text{WLT}}$ contains lensing information on scales smaller than can be probed with halo clustering.

The observed spectra $C^{\text{obs}}(l)$ are the sum of the *signals* given by Eq. (9) and Poisson noise (approximated as Gaussian):

$$C_{IJ}^{\text{obs}}(l) \equiv C_{IJ}(l) + \delta_{IJ} N_I \quad (15)$$

where δ_{IJ} is a Kronecker delta and

$$N_I = \begin{cases} \gamma_*^2 / \nu_i^{\text{gal}} & \text{for } I = (\text{Lens}, i) \\ 1/\bar{n}_i & \text{for } I = (\text{Halo}, i). \end{cases} \quad (16)$$

Here, γ_* is the RMS intrinsic shear of the source galaxies; for a DES-like survey, $\gamma_* \approx 0.25$ [20]. Let $\mathbf{C}_l$ be a matrix with elements $C_{IJ}^{\text{obs}}(l)$. Then for a set of parameters p_α ,

$$F_{\alpha\beta}^{\text{HC+WLT}} = f_{\text{sky}} \sum_l \frac{2l+1}{2} \text{Tr} \left[\frac{\partial \mathbf{C}_l}{\partial p_\alpha} \mathbf{C}_l^{-1} \frac{\partial \mathbf{C}_l}{\partial p_\beta} \mathbf{C}_l^{-1} \right], \quad (17)$$

where f_{sky} is the fraction of sky covered by the survey, and the trace sums over all redshift bins and data types.³ Similarly, we can write $\tilde{F}^{\text{WLT}}$ this way, summing only over the lensing part of $\mathbf{C}_l$. Note that $F^{\text{HC+WLT}}$ is a sum over l from l_{min} to l_{mid} , while $\tilde{F}^{\text{WLT}}$ is summed from $l_{\text{mid}} + 1$ to l_{max} . We choose $l_{\text{min}} = 20$ and $l_{\text{mid}} = 200$ so that the flat-sky approximation and our linear halo bias assumption are valid. As prescribed by Rudd *et al.* [21], we modestly set $l_{\text{max}} = 1000$ since baryonic effects on smaller scales are not completely understood.

If halo clustering were a completely independent observable from cosmic shear, then the matrices $\mathbf{C}_l$ would be block diagonal with no terms in the (HC, WLT) region. In that case, Eq. (14) reduces to Eq. (13). We compute F^{Total} for both cases, and since we are not investigating CC correlations, our cluster counting term is always given by

$$F_{\alpha\beta}^{\text{CC}} = \sum_i \frac{\partial m_i}{\partial p_\alpha} \frac{\partial m_i}{\partial p_\beta} \frac{1}{m_i}, \quad (18)$$

where m_i is the total number of clusters in the i th redshift bin with $M \geq M_{\text{min}}$. Note we have assumed that Poisson noise is the dominant source of scatter for cluster counting.

After computing the Fisher matrix, we use it to forecast 1σ errors for the dark energy parameters after marginalizing over all other parameters. We also compute the dark energy figure of merit (FoM), defined as the inverse area of the 95.4% error ellipse for (w_0, w_a) [22]. It turns out that the HC/WLT correlations have very little impact on the final parameter determination and even bring a slight improvement for DES. Numbers quantifying this small impact are given in Fig. 2 and Table I. We remind the reader

³We check that our numerical derivatives converge with decreasing step size. Ultimately, we use two-sided derivatives with steps that are 4% of the parameters or 0.04, 0.01, and 0.01 for w_a , A , and n_A respectively.

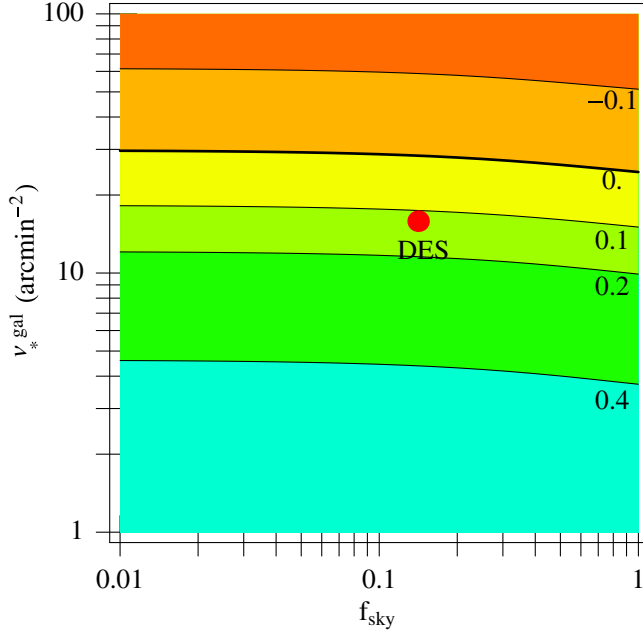


FIG. 2 (color online). Fractional change in the dark energy figure of merit once WLT + HC covariance is included, shown as a function of sky coverage and galaxy density. Positive values imply that neglecting the covariance results in an underestimation of the FoM (conservative parameter constraints).

that Fig. 2 varies two of several possible survey parameters: $M_{\min}$, z_* and γ_* are held fixed.

Our results show that cosmological constraints can be marginally improved by including the full HC/WLT covariance since the HC/WLT correlation itself acts as an additional observable. Whatever degradation occurs from acknowledging that lensing and clusters are correlated may be offset by our ability to predict and measure the correlation. We note that correlations do not necessarily imply error degradations in the first place if the observables are correlated in such a way that does not mimic their dependence on dark energy parameters. Equation (9) clearly shows that all differences between WL and HC enter through the weighting functions $W_I(\chi)$. Zhan and Knox [23] suggest that dark energy affects weak lensing primarily through its weighting function, and halo clustering may be similar, i.e. most of the information in halo clustering may come from its weighting function and perhaps from $\bar{b}^M(\chi)$ in particular.⁴

⁴Suggested by our anonymous referee

TABLE I. 1σ error forecasts and dark energy figure of merit for a DES-like survey ($f_{\text{sky}} = 0.12$, $\nu_*^{\text{gal}} = 15/\text{arcmin}^{-2}$). Constraints in the 2nd column combine CC + WLT only. Columns 3 and 4 show constraints from CC + WLT + HC, treating WLT + HC as correlated or as independent observables. The last column compares columns 3 and 4.

Parameter	No HC	WLT + HC Independent	WLT + HC Correlated	Difference
$\sigma(w_0)$	0.601	0.470	0.461	-2%
$\sigma(w_a)$	2.15	1.56	1.52	-2%
$\sigma(\Omega_{\text{de}})$	0.0387	0.0377	0.0362	-4%
FoM	0.485	1.01	1.15	+14%

Mirroring the results of Takada and Bridle [14], we conclude that a DES-like survey that treats halo clustering and weak lensing tomography as independent experiments will have accurate if not slightly conservative parameter constraints.

IV. CONCLUSIONS

The distribution of halos along a given line of sight probes the mass density along that line of sight in ways very similar to weak lensing. We have quantified this similarity with the cross-correlation coefficients shown in Fig. 1. However, for a survey like DES, projected constraints on dark energy parameters are not degraded by this cross-correlation (using the same information twice), and it is acceptable to simply add the constraints from weak lensing to those from clusters. Doing so actually results in slightly conservative parameter constraints.

ACKNOWLEDGMENTS

We thank Simon DeDeo, Sarah Hansen, Wayne Hu, Dragan Huterer, Doug Rudd, and Andrew Zentner for useful discussions. We thank our anonymous referee for constructive criticism of our manuscript. Thanks also to Antony Lewis, Zhaoming Ma and Eduardo Roza for assistance with code debugging. Our matter power spectra were calculated using code donated by Andrew Zentner. C. S. was supported in part by the Department of Energy and by the Kavli Institute for Cosmological Physics at the University of Chicago through NSF grant PHY-0114422, NSF grant PHY-0551142 and an endowment from the Kavli Foundation. S.D. is supported by the US Department of Energy.

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