

Measurement of $\mathcal{B}(D_s^+ \rightarrow l^+ \nu)$ and the decay constant $f_{D_s^+}$

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We examine $e^+e^- \rightarrow D_s^- D_s^{*+}$ and $D_s^{*-} D_s^+$ interactions at 4170 MeV using the CLEO-c detector in order to measure the decay constant $f_{D_s^+}$. We use the $D_s^+ \rightarrow \ell^+ \nu$ channel, where the ℓ^+ designates either a μ^+ or a τ^+ , when the $\tau^+ \rightarrow \pi^+ \bar{\nu}$. Analyzing both modes independently, we determine $\mathcal{B}(D_s^+ \rightarrow \mu^+ \nu) = (0.594 \pm 0.066 \pm 0.031)\%$, and $\mathcal{B}(D_s^+ \rightarrow \tau^+ \nu) = (8.0 \pm 1.3 \pm 0.4)\%$. We also analyze them simultaneously to find an effective value of $\mathcal{B}^{\text{eff}}(D_s^+ \rightarrow \mu^+ \nu) = (0.638 \pm 0.059 \pm 0.033)\%$ and extract $f_{D_s^+} = 274 \pm 13 \pm 7$ MeV. Combining with our previous determination of $\mathcal{B}(D^+ \rightarrow \mu^+ \nu)$, we also find the ratio $f_{D_s^+}/f_{D^+} = 1.23 \pm 0.11 \pm 0.04$. We compare to current theoretical estimates. Finally, we find $\mathcal{B}(D_s^+ \rightarrow e^+ \nu) < 1.3 \times 10^{-4}$ at 90% confidence level.

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I. INTRODUCTION

To extract precise information on the size of Cabibbo-Kobayashi-Maskawa (CKM) matrix elements from $B - \bar{B}$

mixing measurements, the “decay constants” for B_d and B_s mesons or their ratio, f_{B_d}/f_{B_s} , must be well-known [1]. These factors are related to the overlap of the heavy and

light quark wave functions at zero spatial separation. Indeed, the recent measurement of $B_s^0 - \bar{B}_s^0$ mixing by CDF [2] that can now be compared to the very well measured B^0 mixing [3] has pointed out the urgent need for precise values [4]. Decay constants have been calculated theoretically. The most promising of these calculations are based on lattice-gauge theory that includes light quark loops [5], often called “unquenched.” In order to ensure that these theories can adequately predict f_{B_s}/f_{B_d} it is useful to check the analogous ratio in charm decays $f_{D_s^+}/f_{D^+}$. We have previously measured f_{D^+} [6,7]. Here we present the most precise measurement to date of $f_{D_s^+}$ and the ratio $f_{D_s^+}/f_{D^+}$.

In the standard model (SM), the only way for a D_s meson to decay purely leptonically is via annihilation through a virtual W^+ , as depicted in Fig. 1. The decay rate is given by [8]

$$\Gamma(D_s^+ \rightarrow \ell^+ \nu) = \frac{G_F^2}{8\pi} f_{D_s^+}^2 m_\ell^2 M_{D_s^+} \left(1 - \frac{m_\ell^2}{M_{D_s^+}^2}\right)^2 |V_{cs}|^2, \quad (1)$$

where $M_{D_s^+}$ is the D_s^+ mass, m_ℓ is the mass of the charged final state lepton, G_F is the Fermi coupling constant, and $|V_{cs}|$ is a Cabibbo-Kobayashi-Maskawa matrix element with a value we take equal to 0.9738 [3].

In this paper we analyze both $D_s^+ \rightarrow \mu^+ \nu$ and $D_s^+ \rightarrow \tau^+ \nu$, $\tau^+ \rightarrow \pi^+ \bar{\nu}$. In both D_s decays the charged lepton must be produced with the wrong helicity because the D_s is a spin-0 particle, and the final state consists of a naturally left-handed spin-1/2 neutrino and a naturally right-handed spin-1/2 antilepton. Because the τ^+ has a mass close to that of the D_s^+ , the helicity suppression is broken with respect to the μ^+ decay, but there is an additional large phase space suppression.

New physics can affect the expected widths; any undiscovered charged bosons would interfere with the SM W^+ . These effects may be difficult to ascertain, since they would simply change the value of the decay constants. The ratio $f_{D_s^+}/f_{D^+}$ is much better predicted in the SM than the values individually, so deviations from the SM expectation are more easily seen. Any such discrepancies would point to beyond the SM charged bosons. For example, Akeroyd predicts that the presence of a charged Higgs boson would suppress this ratio significantly [9].

We can also measure the ratio of decay rates to different leptons, and the SM predictions then are fixed only by well-known masses. For example, for $\tau^+ \nu$ to $\mu^+ \nu$:

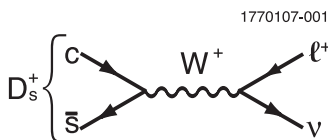


FIG. 1. The decay diagram for $D_s^+ \rightarrow \ell^+ \nu$.

$$R \equiv \frac{\Gamma(D_s^+ \rightarrow \tau^+ \nu)}{\Gamma(D_s^+ \rightarrow \mu^+ \nu)} = \frac{m_{\tau^+}^2 \left(1 - \frac{m_{\tau^+}^2}{M_{D_s^+}^2}\right)^2}{m_{\mu^+}^2 \left(1 - \frac{m_{\mu^+}^2}{M_{D_s^+}^2}\right)^2}. \quad (2)$$

Using measured masses [3], this expression yields a value of 9.72 with a negligibly small error. Any deviation in R from the value predicted by Eq. (2) would be a manifestation of physics beyond the SM. This could occur if any other charged intermediate boson existed that affected the decay rate differently than mass-squared. Then the couplings would be different for muons and τ 's. This would be a clear violation of lepton universality [10].

Previous measurements of $f_{D_s^+}$ have been hampered by a lack of statistical precision, and relatively large systematic errors [11–16]. One large systematic error source has been the lack of knowledge of the absolute branching fraction of the normalization channel, usually $D_s^+ \rightarrow \phi \pi^+$ [17]. The results we report here will not have this limitation.

II. EXPERIMENTAL METHOD

A. Selection of D_s candidates

The CLEO-c detector [18] is equipped to measure the momenta and directions of charged particles, identify them using specific ionization (dE/dx) and Cherenkov light (RICH) [19], detect photons, and determine their directions and energies.

In this study we use 314 pb^{-1} of data produced in e^+e^- collisions using the Cornell Electron Storage Ring (CESR) and recorded near a center-of-mass energy (E_{CM}) of 4.170 GeV. At this energy the e^+e^- annihilation cross section into $D_s^{*+} D_s^- + D_s^+ D_s^{*-}$ is approximately 1 nb, while the cross section for $D_s^+ D_s^-$ is about a factor of 20 smaller. In addition, D mesons are produced mostly as $D^* \bar{D}^*$, with a cross section of ~ 5 nb, and also in $D^* \bar{D} + D \bar{D}^*$ final states with a cross section of ~ 2 nb. The $D \bar{D}$ cross section is a relatively small ~ 0.2 nb [20]. There also appears to be $D \bar{D}^* \pi$ production. The underlying light quark “continuum” background is about 12 nb. The relatively large cross sections, relatively large branching fractions and sufficient luminosities allow us to fully reconstruct one D_s as a “tag,” and examine the properties of the other. In this paper we designate the tag as a D_s^- and examine the leptonic decays of the D_s^+ , though, in reality, we use both charges for tags and signals. Track requirements, particle identification, π^0 , η , and K_S^0 selection criteria are the same as those described in Ref. [6], except that we now require a minimum momentum of 700 MeV/ c for a track to be identified using the RICH.

We also use several resonances that decay via the strong interaction. Here we select intervals in invariant mass within ± 10 MeV of the known mass for $\eta' \rightarrow \pi^+ \pi^- \eta$, ± 10 MeV for $\phi \rightarrow K^+ K^-$, ± 100 MeV for $K^{*0} \rightarrow K^- \pi^+$, and ± 150 MeV for $\rho^- \rightarrow \pi^- \pi^0$.

We reconstruct tags from either directly produced D_s mesons or those that result from the decay of a D_s^* . The beam constrained mass, m_{BC} , is formed by using the beam energy to construct the D_s candidate mass via the formula

$$m_{\text{BC}} = \sqrt{E_{\text{beam}}^2 - \left(\sum_i \vec{p}_i\right)^2}, \quad (3)$$

where i runs over all the final state particles. If we ignore the photon from the $D_s^* \rightarrow \gamma D_s$ decay, and reconstruct the m_{BC} distribution, we obtain the distribution from Monte Carlo simulation shown in Fig. 2. The narrow peak occurs when the reconstructed D_s does not come from the D_s^* decay, but is directly produced.

Rather than selecting events based on only m_{BC} , we first select an interval that accepts most of the events, $2.015 < m_{\text{BC}} < 2.067$ GeV, and examine the invariant mass. Distributions from data for the 8 tag decay modes we use in this analysis are shown in Fig. 3. Note that the resolution in invariant mass is excellent, and the backgrounds not abysmally large, at least in these modes. To determine the number of D_s^- events we fit the invariant mass distributions to the sum of two Gaussians centered at the D_s^- mass, a function we refer to as “two-Gaussian.” The r.m.s. resolution (σ) is defined as

$$\sigma \equiv f_1 \sigma_1 + (1 - f_1) \sigma_2, \quad (4)$$

where σ_1 and σ_2 are the individual widths of each of the

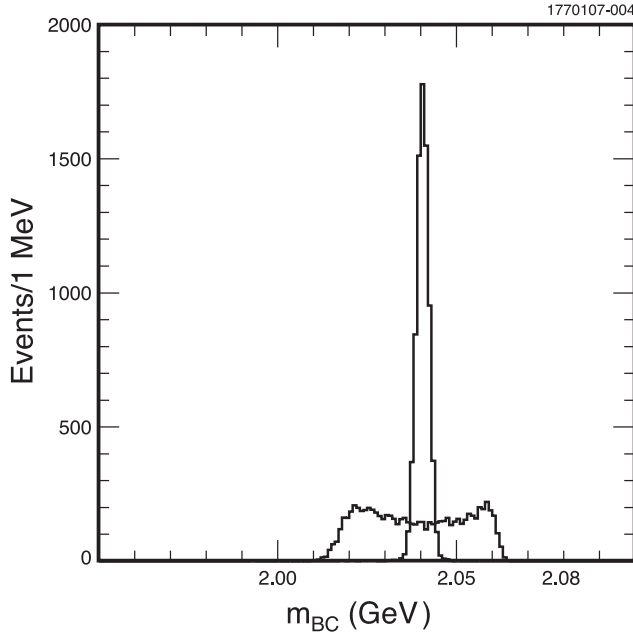


FIG. 2. The beam constrained mass m_{BC} from Monte Carlo simulation of $e^+e^- \rightarrow D_s^+ D_s^{*-}$, $D_s^\pm \rightarrow \phi \pi^\pm$ at an E_{CM} of 4170 MeV. The narrow peak is from the D_s^+ and the wider one from $D_s^{*-} \rightarrow \gamma D_s^-$. (The distributions are not centered at the D_s^+ or D_s^{*+} masses, because the reconstructed particles are assumed to have the energy of the beam.)

two Gaussians and f_1 is the fractional area of the first Gaussian. The number of tags in each mode is listed in Table I. We will later use sidebands of the signal peaks shown in Fig. 3 for part of the background estimate.

To select our sample of tag events, we require the invariant masses, shown in Fig. 3, to be within $\pm 2.5\sigma$ ($\pm 2\sigma$ for the $\eta\rho^-$ mode) of the known D_s^- mass. Then we look for an additional photon candidate in the event that satisfies our shower shape requirement. Regardless of whether or not the photon forms a D_s^* with the tag, for real $D_s^* D_s$ events, the missing mass-squared, MM^{*2} , recoiling against the photon and the D_s^- tag should peak at the D_s^+ mass-squared. We calculate

$$\text{MM}^{*2} = (E_{\text{CM}} - E_{D_s} - E_\gamma)^2 - (\vec{p}_{\text{CM}} - \vec{p}_{D_s} - \vec{p}_\gamma)^2, \quad (5)$$

where E_{CM} ($\vec{p}_{\text{CM}}$) is the center-of-mass energy (momentum), E_{D_s} ($\vec{p}_{D_s}$) is the energy (momentum) of the fully reconstructed D_s^- tag, and E_γ ($\vec{p}_\gamma$) is the energy (momentum) of the additional photon. In performing this calculation we use a kinematic fit that constrains the decay products of the D_s^- to the known D_s mass and conserves overall momentum and energy. All photon candidates in the event are used, except for those that are decay products of the D_s^- tag candidate.

The MM^{*2} distributions from the selected D_s^- event sample are shown in Fig. 4. We fit these distributions to determine the number of tag events. This procedure is enhanced by having information on the shape of the signal function. One possibility is to use the Monte Carlo simulation for this purpose, but that would introduce a relatively large systematic error. Instead, we use our relatively large sample of fully reconstructed $D_s D_s^*$ events, where we use the same decay modes listed in Table I; we find these events and then examine the signal shape in data when one D_s is ignored. The MM^{*2} distribution from this sample is shown in Fig. 5. The signal is fit to a Crystal Ball function [21,22]. The σ parameter, that represents the width of the distribution, is found to be 0.032 ± 0.002 GeV². We do expect this to vary somewhat depending on the final state, but we do not expect the parameters that fix the shape of the tail to change, since they depend mostly on beam radiation and the properties of photon detection.

We fit the MM^{*2} distributions for each mode using the Crystal Ball function with fixed tail parameters, but allowing σ to float, and a 5th order Chebyshev polynomial background. We find a total of $18\,645 \pm 426$ events within a $\pm 2.5\sigma$ interval defined by the fit to each mode. There is also a small enhancement of 4.8% on our ability to find tags in $\mu^+ \nu$ (or $\tau^+ \nu$, $\tau^+ \rightarrow \pi^+ \bar{\nu}$) events (tag bias) as compared with generic events, determined by Monte Carlo simulation, to which we assign a systematic error of 21% giving a correction of $(4.8 \pm 1.0)\%$. An overall systematic error of 5% on the number of tags is assigned by changing the fitting range, using 4th order and 6th order Chebyshev

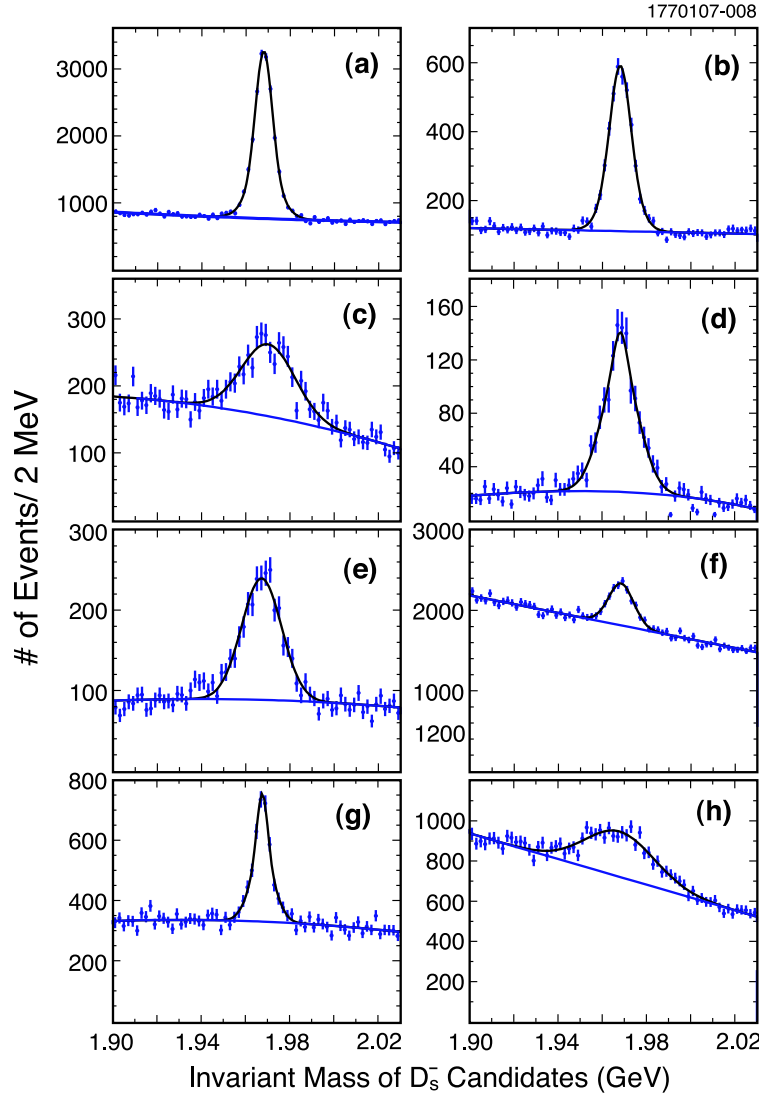


FIG. 3 (color online). Invariant mass of D_s^- candidates in the decay modes (a) $K^+ K^- \pi^-$, (b) $K_S^- K^-$, (c) $\eta \pi^-$, (d) $\eta' \pi^-$, (e) $\phi \rho^-$, (f) $\pi^+ \pi^- \pi^-$, (g) $K^{*-} K^{*0}$, and (h) $\eta \rho^-$, after requiring the total energy of the D_s^- candidate to be consistent with the beam energy. The curves are fits to two-Gaussian signal functions plus a polynomial background.

TABLE I. Tagging modes and numbers of signal and background events, within $\pm 2.5\sigma$ of the D_s^- mass for all modes, except $\eta \rho^-$ ($\pm 2\sigma$), determined from two-Gaussian fits to the invariant mass plots, and the number of photon tags in each mode, within $\pm 2.5\sigma$ of the D_s^- mass-squared determined from fits of the MM^{*2} distributions (see text) to a signal Crystal Ball function (see text) and a 5th order Chebyshev background polynomial, and the associated backgrounds.

Mode	Invariant Mass		MM^{*2}	
	Signal	Background	Signal	Background
$K^+ K^- \pi^-$	$13\,871 \pm 262$	10 850	8053 ± 211	13 538
$K_S^- K^-$	3122 ± 79	1609	1933 ± 88	2224
$\eta \pi^-; \eta \rightarrow \gamma \gamma$	1609 ± 112	4666	1024 ± 97	3967
$\eta' \pi^-; \eta' \rightarrow \pi^+ \pi^- \eta, \eta \rightarrow \gamma \gamma$	1196 ± 46	409	792 ± 69	1052
$\phi \rho^-; \phi \rightarrow K^+ K^-, \rho^- \rightarrow \pi^- \pi^0$	1678 ± 74	1898	1050 ± 113	3991
$\pi^+ \pi^- \pi^-$	3654 ± 199	25 208	2300 ± 187	15 723
$K^{*-} K^{*0}; K^{*-} \rightarrow K_S^0 \pi^-, K^{*0} \rightarrow K^+ \pi^-$	2030 ± 98	4878	1298 ± 130	5672
$\eta \rho^-; \eta \rightarrow \gamma \gamma, \rho^- \rightarrow \pi^- \pi^0$	4142 ± 281	20 784	2195 ± 225	17 353
Sum	$31\,302 \pm 472$	70 302	$18\,645 \pm 426$	63 520

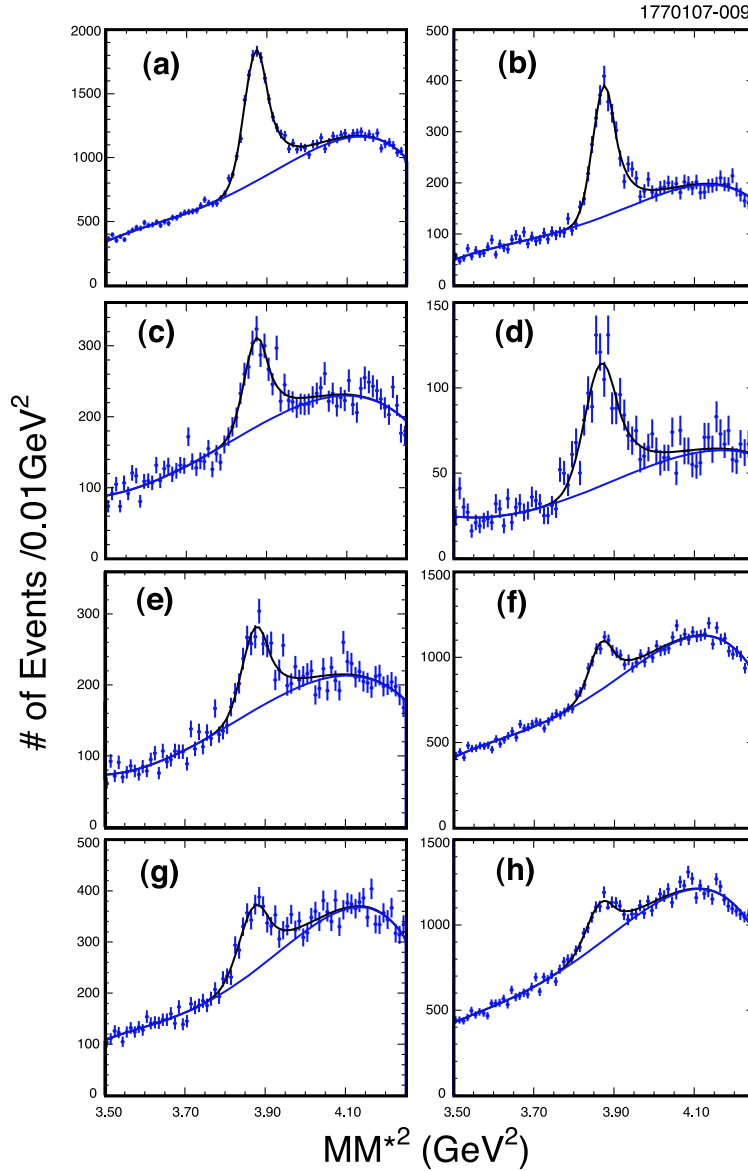


FIG. 4 (color online). The MM^{*2} distribution from events with a photon in addition to the D_s^- tag for the modes: (a) $K^+ K^- \pi^-$, (b) $K_S^0 K^-$, (c) $\eta \pi^-$, (d) $\eta' \pi^-$, (e) $\phi \rho^-$, (f) $\pi^+ \pi^- \pi^-$, (g) $K^{*-} K^{*0}$, and (h) $\eta \rho^-$. The curves are fits to the Crystal Ball function and a 5th order Chebyshev background function.

background polynomials, and allowing the parameters of the tail of the fitting function to float.

B. Signal reconstruction

We next describe the search for $D_s^+ \rightarrow \mu^+ \nu$. Candidate events are selected that contain only a single extra track with opposite sign of charge to the tag. The track must make an angle $> 35.9^\circ$ with respect to the beam line, and in addition we require that there not be any neutral cluster detected in the calorimeter with energy greater than 300 MeV. These cuts are highly effective in reducing backgrounds. The photon energy cut is especially useful to reject $D_s^+ \rightarrow \pi^+ \pi^0$, should this mode be significant, and $D_s^+ \rightarrow \eta \pi^+$.

Since we are searching for events where there is a single missing neutrino, the missing mass-squared, MM^2 , evaluated by taking into account the observed μ^+ , D_s^- , and γ should peak at zero; the MM^2 is computed as

$$MM^2 = (E_{\text{CM}} - E_{D_s} - E_\gamma - E_\mu)^2 - (\vec{p}_{\text{CM}} - \vec{p}_{D_s} - \vec{p}_\gamma - \vec{p}_\mu)^2, \quad (6)$$

where E_μ ($\vec{p}_\mu$) are the energy (momentum) of the candidate muon track and all other variables are the same as defined in Eq. (5).

We also make use of a set of kinematical constraints and fit each event to two hypotheses, one of which is that the D_s^- tag is the daughter of a D_s^{*-} and the other that the D_s^{*+}

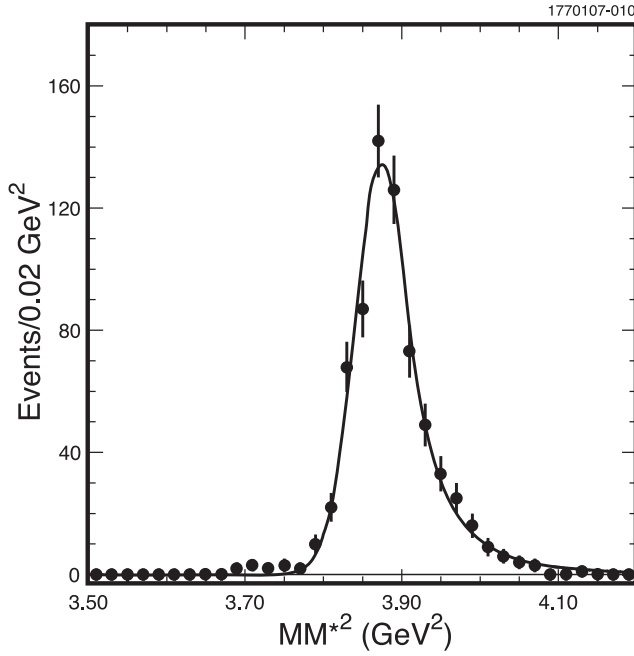


FIG. 5. The MM^2 distribution from a sample of fully reconstructed $D_s^- D_s^{*+}$ events where one D_s is ignored. The curve is a fit to the Crystal Ball function.

decays into γD_s^+ , with the D_s^+ subsequently decaying into $\mu^+ \nu$. The kinematical constraints, in the center-of-mass frame, are

$$\begin{aligned} \vec{p}_{D_s} + \vec{p}_{D_s^*} &= 0, & E_{\text{CM}} &= E_{D_s} + E_{D_s^*}, \\ E_{D_s^*} &= \frac{E_{\text{CM}}}{2} + \frac{M_{D_s^*}^2 - M_{D_s}^2}{2E_{\text{CM}}} \quad \text{or} \\ E_{D_s} &= \frac{E_{\text{CM}}}{2} - \frac{M_{D_s^*}^2 - M_{D_s}^2}{2E_{\text{CM}}}, \\ M_{D_s^*} - M_{D_s} &= 143.6 \text{ MeV}. \end{aligned} \quad (7)$$

In addition, we constrain the invariant mass of the D_s^- tag to the known D_s mass. This gives us a total of 7 constraints. The missing neutrino four-vector needs to be determined, so we are left with a three-constraint fit. We perform a standard iterative fit minimizing χ^2 . As we do not want to be subject to systematic uncertainties that depend on understanding the absolute scale of the errors, we do not make a χ^2 cut but simply choose the photon and the decay sequence in each event with the minimum χ^2 .

In this analysis, we consider three separate cases: (i) the track deposits <300 MeV in the calorimeter, characteristic of a noninteracting pion or a muon; (ii) the track deposits >300 MeV in the calorimeter, characteristic of an interacting pion, and is not consistent with being an electron; (iii) the track satisfies our electron selection criteria defined below. Then we separately study the MM^2 distributions for these three cases. The separation between muons and pions is not complete. Case (i) contains 99% of the muons but

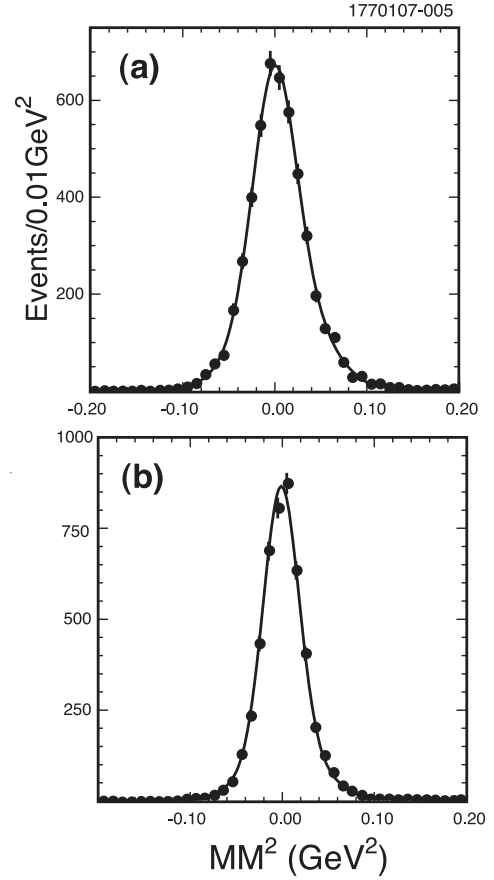


FIG. 6. The MM^2 resolution from Monte Carlo simulation for $D_s^+ \rightarrow \mu^+ \nu$ utilizing a $\phi \pi^-$ tag and a γ from either D_s^* decay, both before the kinematic fit (a) and after (b).

also 60% of the pions, while case (ii) includes 1% of the muons and 40% of the pions [7]. Case (iii) does not include any signal but is used later for background estimation. For cases (i) and (ii) we insist that the track not be identified as a kaon. For electron identification we require a match between the momentum measurement in the tracking system and the energy deposited in the CsI calorimeter and we also require that dE/dx and RICH information be consistent with expectations for an electron.

C. The expected MM^2 spectrum

For the $\mu^+ \nu$ final state the MM^2 distribution can be modeled as the sum of two Gaussians centered at zero [see Eq. (4)]. A Monte Carlo simulation of the MM^2 for the $\phi \pi^-$ subset of $K^+ K^- \pi^-$ tags is shown in Fig. 6 both before and after the fit. The fit changes the resolution from $\sigma = 0.032 \text{ GeV}^2$ to $\sigma = 0.025 \text{ GeV}^2$, a 22% improvement, without any loss of events.

We check the resolution using data. The mode $D_s^+ \rightarrow \bar{K}^0 K^+$ provides an excellent testing ground.¹ We search for

¹In this paper the notation $\bar{K}^0 K^+$ refers to the sum of $\bar{K}^0 K^+$ and $K^0 K^+$ final states.

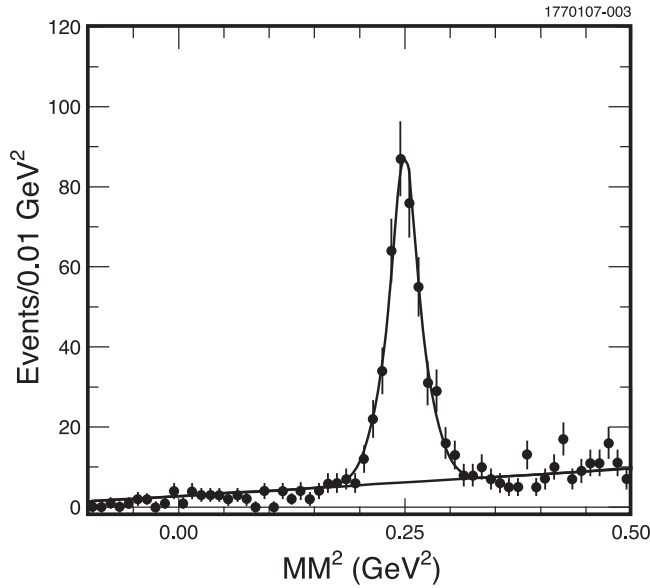


FIG. 7. The MM^2 distribution for events with an identified K^+ track. The kinematic fit has been applied. The curve is a fit to the sum of two Gaussians centered at the square of the K^0 mass and a linear background.

events with at least one additional track identified as a kaon using the RICH detector, in addition to a D_s^- tag. The MM^2 distribution is shown in Fig. 7. Fitting this distribution to a two-Gaussian shape gives a MM^2 resolution of 0.025 GeV^2 in agreement with Monte Carlo simulation.

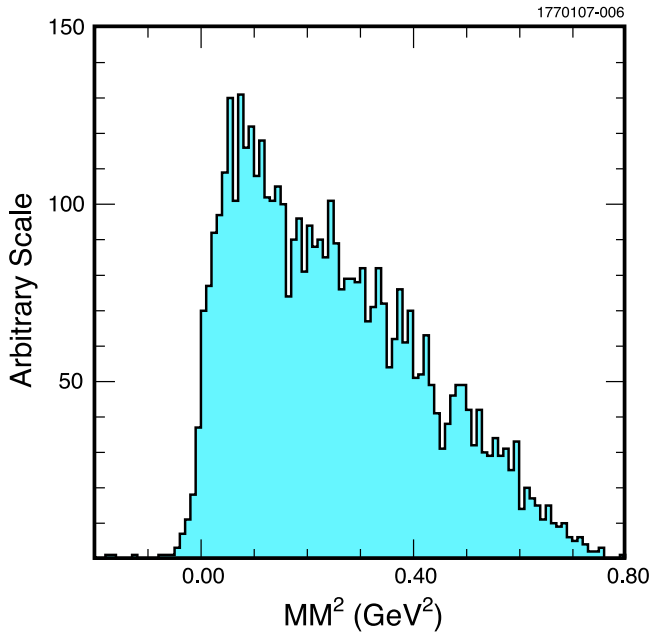


FIG. 8 (color online). The MM^2 distribution from Monte Carlo simulation for $D_s^+ \rightarrow \tau^+ \nu$, $\tau^+ \rightarrow \pi^+ \bar{\nu}$ at an E_{CM} of 4170 MeV.

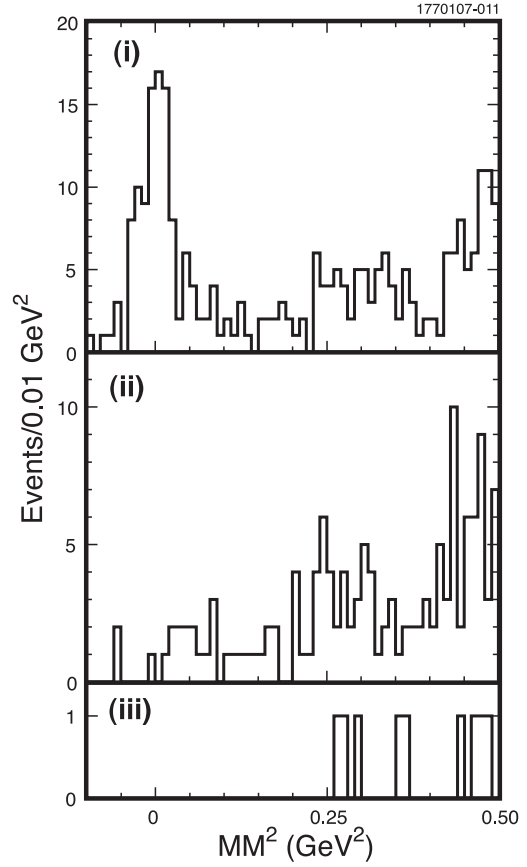


FIG. 9. The MM^2 distributions from data for events with a D_s^- reconstructed in a tag mode, an additional positively charged track and no neutral energy clusters above 300 MeV. Case (i): the single track deposits $<300 \text{ MeV}$ of energy in the calorimeter. The peak near zero is from $D_s^+ \rightarrow \mu^+ \nu$ events. Case (ii): track deposits $>300 \text{ MeV}$ in the crystal calorimeter but is not consistent with being an electron. Case (iii): the track is identified as an electron.

For the $\tau^+ \nu$, $\tau^+ \rightarrow \pi^+ \bar{\nu}$ final state a Monte Carlo simulation of the MM^2 spectrum is shown in Fig. 8. The extra missing neutrino results in a smeared distribution.

D. MM^2 spectra in data

The MM^2 distributions from data are shown in Fig. 9. The overall signal region we consider is $-0.05 < MM^2 < 0.20 \text{ GeV}^2$. The higher limit is imposed to exclude background from $\eta\pi^+$ and $K^0\pi^+$ final states. There is a clear peak in Fig. 9(i) due to $D_s^+ \rightarrow \mu^+ \nu$. Furthermore, the region between the $\mu^+ \nu$ peak and 0.20 GeV^2 has events that we will show are dominantly due to the $D_s^+ \rightarrow \tau^+ \nu$ decay. The events in Fig. 9(ii) below 0.20 GeV^2 are also dominantly due to $\tau^+ \nu$ decay.

The specific signal regions are defined as follows: for $\mu^+ \nu$, $-0.05 < MM^2 < 0.05 \text{ GeV}^2$, corresponding to $\pm 2\sigma$; for $\tau^+ \nu$, $\tau^+ \rightarrow \pi^+ \bar{\nu}$, in case (i) $0.05 < MM^2 < 0.20 \text{ GeV}^2$ and in case (ii) $-0.05 < MM^2 < 0.20 \text{ GeV}^2$. In these regions we find 92, 31, and 25 events, respectively.

TABLE II. Numbers of events in the signal region, and background events evaluated from sideband regions.

Case	Region (GeV ²)	Signal	Background
i	$-0.05 < \text{MM}^2 < 0.05$	92	3.5 ± 1.4
i	$0.05 < \text{MM}^2 < 0.20$	31	2.5 ± 1.1
ii	$-0.05 < \text{MM}^2 < 0.20$	25	3.0 ± 1.3
Sum	$-0.05 < \text{MM}^2 < 0.20$	148	9.0 ± 2.3

E. Background evaluations

We consider the background arising from two sources: one from real D_s^+ decays and the other from the background under the single-tag signal peaks. For the latter, we obtain the background from data. We define sidebands of the invariant mass signals shown in Fig. 2 starting at 4σ on the low and high sides of the invariant mass peaks for all modes. The intervals extend away from the signal peaks by approximately the same width used in selecting the signal, 5σ , so as to ensure that the number of background events in the sidebands accurately reflects the numbers under the signal peaks. Thus the amount of data corresponds to twice the number of background events under the signal peaks, except for the $\eta\pi^-$ and $\eta\rho^-$ modes, where the signal widths are so wide that we chose narrower sidebands only equaling the data. We analyze these events in exactly the same manner as those in the signal peak.²

The backgrounds are given here as the sum of two numbers, the first being the number from all modes, except $\eta\pi^-$ and $\eta\rho^-$, and the second being the number from these modes. For case (i) we find $2.5 + 1$ background in the $\mu^+\nu$ signal region and $2.5 + 0$ background in the $\tau^+\nu$ region. For case (ii) we find $2 + 1$ events. Our total sideband background summing over all of these cases is 9.0 ± 2.3 . The numbers of signal and background events due to false D_s^- tags as evaluated from sidebands are given in Table II.

This entire procedure was checked by performing the same study on a sample of Monte Carlo generated at an E_{CM} of 4170 MeV that includes known charm and continuum production cross sections. The Monte Carlo sample corresponds to an integrated luminosity that is 4 times larger than the data. We find the number of background events predicted directly by examining the decay generator of the simulation is 28 and the sideband method yields 22. These are slightly smaller than found in the data, but consistent within errors. We note that the Monte Carlo is far from perfect as many branching fractions are unknown and so are estimated.

The background from real D_s^+ decays is studied by identifying each possible source mode by mode. For the

²The D_s mass used in the fit is chosen to be the middle of the relevant sideband interval.

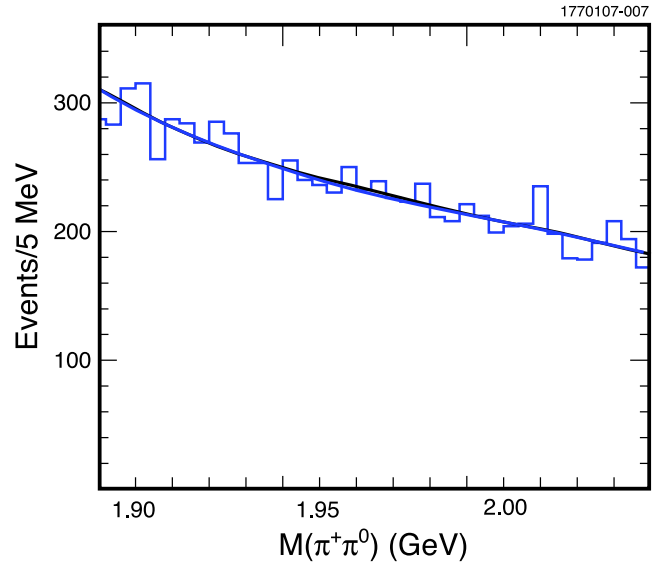


FIG. 10 (color online). The invariant $\pi^+\pi^0$ mass. The upper curve shows a fit using a background polynomial plus Gaussian signal functions, where the width of the Gaussian is fixed to a value determined by Monte Carlo simulation. The lower curve shows just the background polynomial.

$\mu^+\nu$ final state, the only possible background within the signal region is $D_s^+ \rightarrow \pi^+\pi^0$. This mode has not been studied previously. We show in Fig. 10 the $\pi^+\pi^0$ invariant mass spectrum from a 195 pb^{-1} subsample of our data. We do not see a signal and set an upper limit $< 1.1 \times 10^{-3}$ at 90% confidence level. Recall that any such events are also heavily suppressed by the extra photon energy cut of 300 MeV. There are also some $D_s^+ \rightarrow \tau^+\nu$, $\tau^+ \rightarrow \pi^+\bar{\nu}$ events that occur in the signal region. Using the SM expected ratio of decay rates from Eq. (2) we calculate a contribution of $7.4\pi^+\bar{\nu}\nu$ events that we will treat as part of the signal.

For the $\tau^+\nu$, $\tau^+ \rightarrow \pi^+\bar{\nu}$ final state the real D_s^+ backgrounds include, in addition to the $\pi^+\pi^0$ background discussed above, semileptonic decays, possible $\pi^+\pi^0\pi^0$ decays, and other τ^+ decays. Semileptonic decays involving muons are equal to those involving electrons shown in Fig. 9(iii). Since no electron events appear in the signal region, the background from muons is taken to be zero. The $\pi^+\pi^0\pi^0$ background is estimated by considering the $\pi^+\pi^+\pi^-$ final state whose measured branching fraction is $(1.02 \pm 0.12)\%$ [17]. This mode has large contributions from $f_0(980)\pi^+$ and other $\pi^+\pi^-$ resonant structures at higher mass [23]. The $\pi^+\pi^0\pi^0$ mode will also have these contributions, but the MM^2 opposite to the π^+ will be at large mass. The only component that can potentially cause background is the nonresonant component measured by FOCUS as $(17 \pm 4)\%$ [23]. This background has been evaluated by Monte Carlo simulation as have backgrounds from other τ^+ decays, and each is listed in Table III.

TABLE III. Backgrounds in the $D_s^+ \rightarrow \tau^+ \nu$, $\tau^+ \rightarrow \pi^+ \bar{\nu}$ sample for correctly reconstructed tags, case (i) for $0.05 < \text{MM}^2 < 0.20 \text{ GeV}^2$ and case (ii) for $-0.05 < \text{MM}^2 < 0.20 \text{ GeV}^2$.

Source	$\mathcal{B}$ (%)	# of events case (i)	# of events case (ii)	Sum
$D_s^+ \rightarrow X \mu^+ \nu$	8.2	$0_{-0}^{+1.8}$	0	$0_{-0}^{+1.8}$
$D_s^+ \rightarrow \pi^+ \pi^0 \pi^0$	1.0	0.03 ± 0.04	0.08 ± 0.03	0.11 ± 0.04
$D_s^+ \rightarrow \tau^+ \nu$	6.4			
$\tau^+ \rightarrow \pi^+ \pi^0 \bar{\nu}$	1.5	0.55 ± 0.22	0.64 ± 0.24	1.20 ± 0.33
$\tau^+ \rightarrow \mu^+ \bar{\nu} \nu$	1.0	0.37 ± 0.15	0	0.37 ± 0.15
Sum		$1.0_{-0}^{+1.8}$	0.7 ± 0.2	$1.7_{-0.4}^{+1.8}$

III. LEPTONIC BRANCHING FRACTIONS

The sum of MM^2 distributions for case (i) and case (ii), corresponding to the sum of $D_s^+ \rightarrow \mu^+ \nu$ and $D_s^+ \rightarrow \tau^+ \nu$, $\tau^+ \rightarrow \pi^+ \bar{\nu}$ candidates, is compared in Fig. 11 with the expected shape, assuming the SM value of R as given in Eq. (2) for the ratio of $\tau^+ \nu$ to $\mu^+ \nu$ rates. The curve is normalized to the total number of events below $\text{MM}^2 < 0.2 \text{ GeV}^2$. Besides the prominent $\mu^+ \nu$ peak and $\tau^+ \nu$; $\tau^+ \rightarrow \pi^+ \bar{\nu}$ shoulder, there is an enhancement between $0.25\text{--}0.35 \text{ GeV}^2$, due to $K^0 \pi^+$ and $\eta \pi^+$ final states, where the decay products other than the π^+ escape detection. The data are consistent with our expectation that the region $-0.05 < \text{MM}^2 < 0.2 \text{ GeV}^2$ contains mostly signal. Recall there are 148 total events only 10.7 of which we estimate are background, 9.0 from fake D_s^- tags, and 1.7 from real

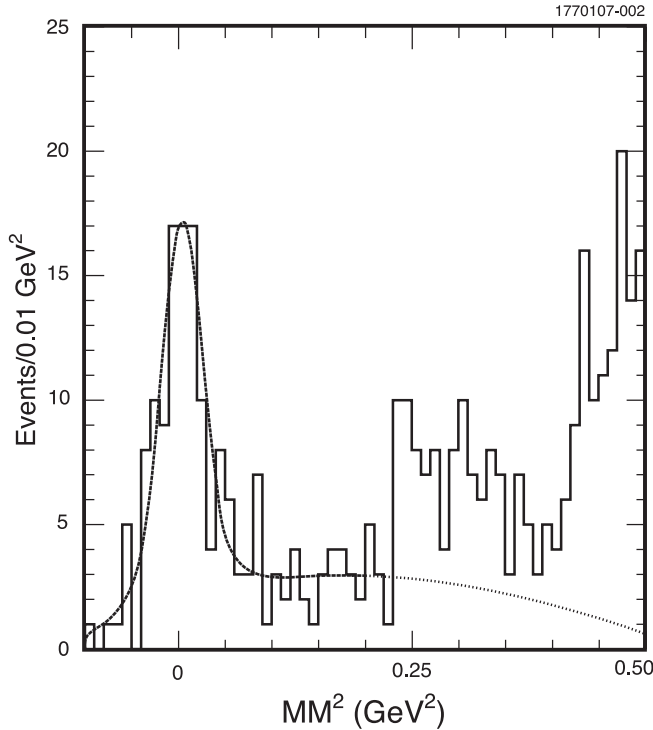


FIG. 11. The sum of case (i) and case (ii) MM^2 distributions (histogram) compared to the predicted shape (curve) for the sum of $D_s^+ \rightarrow \mu^+ \nu$ and $\tau^+ \nu$, $\tau^+ \rightarrow \pi^+ \bar{\nu}$. The curve is normalized to the total number of events below $\text{MM}^2 < 0.2 \text{ GeV}^2$.

tags and D_s^+ decays. Above 0.2 GeV^2 other, larger backgrounds enter.

The number of real $\mu^+ \nu$ events $N_{\mu\nu}$ is related to the number of events detected in the signal region N_{det} (92), the estimated background N_{bkgrd} (3.5), the number of tags, N_{tag} , and the branching fractions as

$$N_{\mu\nu} \equiv N_{\text{det}} - N_{\text{bkgrd}} = N_{\text{tag}} \cdot \epsilon' [\epsilon' \mathcal{B}(D_s^+ \rightarrow \mu^+ \nu) + \epsilon'' \mathcal{B}(D_s^+ \rightarrow \tau^+ \nu; \tau^+ \rightarrow \pi^+ \bar{\nu})], \quad (8)$$

where ϵ (80.1%) includes the efficiency for reconstructing the single charged track including final state radiation (77.8%), the $(98.3 \pm 0.2)\%$ efficiency of not having another unmatched cluster in the event with energy greater than 300 MeV, and for the fact that it is easier to find tags in $\mu^+ \nu$ events than in generic decays by 4.8%, as determined by Monte Carlo simulation. The efficiency labeled ϵ' (91.4%) is the product of the 99.0% muon efficiency for depositing less than 300 MeV in the calorimeter and 92.3% acceptance of the MM^2 cut of $|\text{MM}^2| < 0.05 \text{ GeV}^2$. The quantity ϵ'' (7.9%) is the fraction of $\tau^+ \nu$; $\tau^+ \rightarrow \pi^+ \bar{\nu}$ events contained in the $\mu^+ \nu$ signal window (13.2%) times the 60% acceptance for a pion to deposit less than 300 MeV in the calorimeter.

The two D_s^+ branching fractions in Eq. (8) are related as

$$\begin{aligned} \mathcal{B}(D_s^+ \rightarrow \tau^+ \nu; \tau^+ \rightarrow \pi^+ \bar{\nu}) &= R \cdot \mathcal{B}(\tau^+ \rightarrow \pi^+ \bar{\nu}) \\ &\times \mathcal{B}(D_s^+ \rightarrow \mu^+ \nu) \\ &= 1.059 \cdot \mathcal{B}(D_s^+ \rightarrow \mu^+ \nu), \end{aligned} \quad (9)$$

where we take the standard model ratio for R as given in Eq. (2) and $\mathcal{B}(\tau^+ \rightarrow \pi^+ \bar{\nu}) = (10.90 \pm 0.07)\%$ [3]. This allows us to solve Eq. (8). Since $N_{\text{det}} = 92$, $N_{\text{bkgrd}} = 3.5 \pm 1.4$, and $N_{\text{tag}} = 18\,645 \pm 426 \pm 1081$, we find

$$\mathcal{B}(D_s^+ \rightarrow \mu^+ \nu) = (0.594 \pm 0.066 \pm 0.031)\%. \quad (10)$$

We can also sum the $\mu^+ \nu$ and $\tau^+ \nu$ contributions, where we restrict ourselves to the MM^2 region below 0.20 GeV^2 and above -0.05 GeV^2 . Equation (8) still applies. The number of events in the signal region and the number of background events changes to 148 and $10.7 + 2.9 - 2.3$, respectively. The efficiency ϵ' becomes 96.2%, and ϵ'' increases to 45.2%. Using this method, we find an effective branching fraction of

$$\mathcal{B}^{\text{eff}}(D_s^+ \rightarrow \mu^+ \nu) = (0.638 \pm 0.059 \pm 0.033)\%. \quad (11)$$

The systematic errors on these branching fractions are given in Table IV. The error on track finding is determined from a detailed comparison of the simulation with double tag events where one track is ignored. “Minimum ionization” indicates the error due to the requirement that the charged track deposit no more than 300 MeV in the calorimeter; it is determined using two-body $D^0 \rightarrow K^- \pi^+$ decays (see Ref. [7]). The error on the photon veto efficiency, due to the 300 MeV/ c extra shower energy cut, is determined using Monte Carlo simulation. The Monte Carlo was cross-checked using a sample of fully reconstructed $D^+ D^-$ events and comparing the inefficiency due to additional photons with energy above 300 MeV/ c . These events have no real extra photons above 300 MeV/ c ; those that are present are due to interactions of the $D^\pm$ decay products in the detector material. The error on the number of tags of $\pm 5\%$ has been discussed earlier. In addition there is a small error of $\pm 0.6\%$ on the $\tau^+ \nu$ branching fraction due to the uncertainty on the τ^+ decay fraction to $\pi^+ \bar{\nu}$. Additional systematic errors arising from the background estimates are negligible. Note that the minimum ionization error does not apply to the summed branching fraction given in Eq. (11); in this case the total systematic error is 5.1%.

We also analyze the $\tau^+ \nu$ final state independently. We use different MM^2 regions for cases (i) and (ii) defined above. For case (i) we define the signal region to be the interval $0.05 < \text{MM}^2 < 0.20 \text{ GeV}^2$, while for case (ii) we define the signal region to be the interval $-0.05 < \text{MM}^2 < 0.20 \text{ GeV}^2$. Case (i) includes the $\mu^+ \nu$ signal, so we must exclude the region close to zero MM^2 , while for case (ii) we are specifically selecting pions so the signal region can be larger. The upper limit on MM^2 is chosen to avoid background from the tail of the $K^0 \pi^+$ peak. The fractions of the MM^2 range accepted are 32% and 45% for case (i) and (ii), respectively.

We find 31 signal and $3.5 + 1.7 - 1.1$ background events for case (i) and 25 signal and 5.1 ± 1.6 background events for case (ii). The branching fraction, averaging the two cases, is

$$\mathcal{B}(D_s^+ \rightarrow \tau^+ \nu) = (8.0 \pm 1.3 \pm 0.4)\%. \quad (12)$$

TABLE IV. Systematic errors on determination of the $D_s^+ \rightarrow \mu^+ \nu$ branching fraction.

Error Source	Size (%)
Track finding	0.7
Photon veto	1
Minimum ionization ^a	1
Number of tags	5
Total	5.2

^aNot applicable for summed rate.

Lepton universality in the standard model requires that the ratio R from Eq. (2) be equal to a value of 9.72. We measure

$$R \equiv \frac{\Gamma(D_s^+ \rightarrow \tau^+ \nu)}{\Gamma(D_s^+ \rightarrow \mu^+ \nu)} = 13.4 \pm 2.6 \pm 0.2. \quad (13)$$

Here the systematic error is dominated by the uncertainty on the minimum ionization cut that we use to separate the $\mu^+ \nu$ and $\tau^+ \nu$ regions at 300 MeV. We take this error as 2%, since a change here affects both the numerator and denominator. The ratio is consistent with the standard model prediction. Current results on D^+ leptonic decays also show no deviations [24]. The absence of any detected electrons opposite to our tags allows us to set an upper limit of

$$\mathcal{B}(D_s^+ \rightarrow e^+ \nu) < 1.3 \times 10^{-4} \quad (14)$$

at 90% confidence level; this is also consistent with standard model predictions and lepton universality.

IV. CHECKS OF THE METHOD

We perform an overall check of our procedures by measuring $\mathcal{B}(D_s^+ \rightarrow \bar{K}^0 K^+)$. For this measurement we compute the MM^2 [Eq. (6)] using events with an additional charged track but here identified as a kaon. These track candidates have momenta of approximately 1 GeV/ c ; here our RICH detector has a pion to kaon fake rate of 1.1% with a kaon detection efficiency of 88.5% [19]. For this study, we do not veto events with extra charged tracks, or neutral energy deposits $> 300 \text{ MeV}$, because of the presence of the K^0 .

The MM^2 distribution is shown in Fig. 7. The peak near 0.25 GeV^2 is due to the decay mode of interest. We fit this to a linear background from 0.02–0.50 GeV^2 plus a two-Gaussian signal function. The fit yields $375 \pm 23 \pm 18$ events. Events from the $\eta \pi^+$ mode where the π^+ fakes a K^+ are very rare and would not peak at the proper MM^2 . Since ηK^+ could in principle contribute a background in this region, we searched for this final state in a 195 pb^{-1} subsample of the data. Not finding any signal, we set an upper limit of $\mathcal{B}(D_s^+ \rightarrow \eta K^+) < 2.8 \times 10^{-3}$ at 90% confidence level, approximately a factor of 10 below our measurement. This final state would peak at a MM^2 of 0.30 GeV^2 and would cause an asymmetric tail on the high side of the peak. Since we see no evidence for an asymmetry in the $\bar{K}^0 K^+$ peak we ignore the ηK^+ final state from here on. In order to compute the branching fraction we must include the efficiency of detecting the kaon track 76.2%, including radiation [25], the particle identification efficiency of 88.5%, and take into account that it is easier to detect tags in events containing a $\bar{K}^0 K^+$ decay than in the average $D_s D_s^*$ event due to the track and photon multiplicities, which gives a 3% correction.³ These rates are

³The tag bias is less here than in the $\mu^+ \nu$ case because of the K^0 decays and interactions in the detector.

TABLE V. Systematic errors on determination of the $D_s^+ \rightarrow \bar{K}^0 K^+$ branching fraction.

Error Source	Size (%)
Signal shape	3
Background shape	2
Track finding	0.7
PID cut	1.0
Number of tags	5
Total	6.3

estimated by using Monte Carlo simulation. We determine

$$\mathcal{B}(D_s^+ \rightarrow \bar{K}^0 K^+) = (2.90 \pm 0.19 \pm 0.18)\%, \quad (15)$$

where the systematic errors are listed in Table V. We estimate the error from the signal shape by taking the change in the number of events when varying the signal width of the two-Gaussian function by $\pm 1\sigma$. The error on the background shapes is given by varying the shape of the background fit. The error on the particle identification efficiency is measured using two-body D^0 decays [19]. The other errors are the same as described in Table IV. Again, the largest component of the systematic error arises from the number of tag events (5%). In fact, to use this result as a check on our procedures, we need only consider the systematic errors that are different here than in the $\mu^+ \nu$ case. Those are due only to the signal and background shapes and the particle identification cut. Those systematic errors amount to 3.7% or ± 0.11 in the branching fraction.

To determine absolute branching fractions of charm mesons, CLEO-c uses a method where both particles are fully reconstructed (so called “double tags”) and the rates are normalized using events where only one particle is fully reconstructed. Our preliminary result using this

method for $\mathcal{B}(D_s^+ \rightarrow K_S K^+) = (1.50 \pm 0.09 \pm 0.05)\%$, which when doubled becomes $(3.00 \pm 0.19 \pm 0.10)\%$ [26]. This is in excellent agreement with the number in Eq. (15). These results are not independent.

We also performed the entire analysis on a Monte Carlo sample that corresponds to an integrated luminosity 4 times larger than the data sample. The input branching fraction in the Monte Carlo is 0.5% for $\mu^+ \nu$ and 6.57% for $\tau^+ \nu$, while our analysis measured $\mathcal{B}(D_s^+ \rightarrow \mu^+ \nu) = (0.514 \pm 0.027)\%$ for the case (i) $\mu^+ \nu$ signal and $(0.521 \pm 0.024)\%$ for $\mu^+ \nu$ and $\tau^+ \nu$ combined. We also find $(6.6 \pm 0.6)\%$ for the $\tau^+ \nu$ rate.

V. THE DECAY CONSTANT

Using our most precise value for $\mathcal{B}(D_s^+ \rightarrow \mu^+ \nu)$ from Eq. (11), that is derived using both our $\mu^+ \nu$ and $\tau^+ \nu$ samples, and Eq. (1) with a D_s lifetime of $(500 \pm 7) \times 10^{-15}\text{s}$ [3], we extract

$$f_{D_s^+} = 274 \pm 13 \pm 7 \text{ MeV}. \quad (16)$$

We combine with our previous result [6]

$$f_{D^+} = 222.6 \pm 16.7_{-3.4}^{+2.8} \text{ MeV} \quad (17)$$

and find a value for

$$\frac{f_{D_s^+}}{f_{D^+}} = 1.23 \pm 0.11 \pm 0.04, \quad (18)$$

where only a small part of the systematic error cancels in the ratio of our two measurements.

VI. CONCLUSIONS

Theoretical models that predict $f_{D_s^+}$ and the ratio $\frac{f_{D_s^+}}{f_{D^+}}$ are listed in Table VI. Our result for f_{D_s} is slightly higher than

TABLE VI. Theoretical predictions of $f_{D_s^+}$, f_{D^+} , and $f_{D_s^+}/f_{D^+}$. QL indicates quenched lattice calculations.

Model	$f_{D_s^+}$ (MeV)	f_{D^+} (MeV)	$f_{D_s^+}/f_{D^+}$
Lattice (HPQCD + UKQCD) [27]	241 ± 3	208 ± 4	1.162 ± 0.009
Lattice (FNAL + MILC + HPQCD) [28]	$249 \pm 3 \pm 16$	$201 \pm 3 \pm 17$	$1.24 \pm 0.01 \pm 0.07$
QL (QCDSF) [29]	$220 \pm 6 \pm 5 \pm 11$	$206 \pm 6 \pm 3 \pm 22$	$1.07 \pm 0.02 \pm 0.02$
QL (Taiwan) [30]	$266 \pm 10 \pm 18$	$235 \pm 8 \pm 14$	$1.13 \pm 0.03 \pm 0.05$
QL (UKQCD) [31]	$236 \pm 8_{-14}^{+17}$	$210 \pm 10_{-16}^{+17}$	$1.13 \pm 0.02_{-0.02}^{+0.04}$
QL [32]	$231 \pm 12_{-1}^{+6}$	$211 \pm 14_{-12}^{+2}$	1.10 ± 0.02
QCD Sum Rules [33]	205 ± 22	177 ± 21	$1.16 \pm 0.01 \pm 0.03$
QCD Sum Rules [34]	235 ± 24	203 ± 20	1.15 ± 0.04
Field Correlators [35]	210 ± 10	260 ± 10	1.24 ± 0.03
Quark Model [36]	268	234	1.15
Quark Model [37]	248 ± 27	230 ± 25	1.08 ± 0.01
LFQM (Linear) [38]	211	248	1.18
LFQM (HO) [38]	194	233	1.20
LF-QCD [39]	253	241	1.05
Potential Model [40]	241	238	1.01
Isospin Splittings [41]		262 ± 29	

TABLE VII. Our results for $\mathcal{B}(D_s^+ \rightarrow \mu^+ \nu)$, $\mathcal{B}(D_s^+ \rightarrow \tau^+ \nu)$, and $f_{D_s^+}$ compared with previous measurements. Results have been updated for the new value of the D_s lifetime [3]. ALEPH combines both measurements to derive a value for the decay constant.

Experiment	Mode	$\mathcal{B}$	$\mathcal{B}_{\phi\pi}$ (%)	$f_{D_s^+}$ (MeV)
CLEO-c	$\mu^+ \nu$	$(5.94 \pm 0.66 \pm 0.31) \cdot 10^{-3}$		$264 \pm 15 \pm 7$
CLEO-c	$\tau^+ \nu$	$(8.0 \pm 1.3 \pm 0.4) \cdot 10^{-2}$		$310 \pm 25 \pm 8$
CLEO-c	combined	$\dots$		$274 \pm 13 \pm 7$
CLEO [11]	$\mu^+ \nu$	$(6.2 \pm 0.8 \pm 1.3 \pm 1.6) \cdot 10^{-3}$	3.6 ± 0.9	$273 \pm 19 \pm 27 \pm 33$
BEATRICE [12]	$\mu^+ \nu$	$(8.3 \pm 2.3 \pm 0.6 \pm 2.1) \cdot 10^{-3}$	3.6 ± 0.9	$312 \pm 43 \pm 12 \pm 39$
ALEPH [13]	$\mu^+ \nu$	$(6.8 \pm 1.1 \pm 1.8) \cdot 10^{-3}$	3.6 ± 0.9	$282 \pm 19 \pm 40$
ALEPH [13]	$\tau^+ \nu$	$(5.8 \pm 0.8 \pm 1.8) \cdot 10^{-2}$		
L3 [14]	$\tau^+ \nu$	$(7.4 \pm 2.8 \pm 1.6 \pm 1.8) \cdot 10^{-2}$		$299 \pm 57 \pm 32 \pm 37$
OPAL [15]	$\tau^+ \nu$	$(7.0 \pm 2.1 \pm 2.0) \cdot 10^{-2}$		$283 \pm 44 \pm 41$
BaBar [16]	$\mu^+ \nu$	$(6.74 \pm 0.83 \pm 0.26 \pm 0.66) \cdot 10^{-3}$	4.71 ± 0.46	$283 \pm 17 \pm 7 \pm 14$

most theoretical expectations. We are consistent with lattice-gauge theory, and most other models, for the ratio of decay constants. There is no evidence at this level of precision for any suppression in the ratio due to the presence of a virtual charged Higgs [9].

By using a theoretical prediction for $f_{D_s^+}/f_{D^+}$ we can derive a value for the ratio of CKM elements $|V_{cd}/V_{cs}|$. Taking the value from Ref. [27] of 1.162 ± 0.009 , we find

$$|V_{cd}/V_{cs}| = 0.2171 \pm 0.021 \pm 0.0017, \quad (19)$$

where the first error is due the statistical and systematic errors of the experiment and the second is due to the stated error on the theoretical prediction. This value is expected to be almost equal to the ratio of the CKM elements $|V_{us}|/|V_{ud}|$.

We now compare with previous measurements. The branching fractions, modes, and derived values of $f_{D_s^+}$ are listed in Table VII. Our values are shown first. We are generally consistent with previous measurements, although ours are more precise.

Most measurements of $D_s^+ \rightarrow \ell^+ \nu$ are normalized with respect to $\mathcal{B}(D_s^+ \rightarrow \phi \pi^+) \equiv \mathcal{B}_{\phi\pi}$. An exception is the OPAL measurement which is normalized to the D_s fraction in Z^0 events that is derived from an overall fit to heavy

flavor data at LEP [42]. It still, however, relies on absolute branching fractions that are hidden by this procedure, and the estimated error on the normalization is somewhat smaller than that indicated by the error on $\mathcal{B}_{\phi\pi}$ available at the time of their publication. The L3 measurement is normalized taking the fraction of D_s mesons produced in c quark fragmentation as 0.11 ± 0.02 , and the ratio of D_s^*/D_s production of 0.65 ± 0.10 . The ALEPH results use $\mathcal{B}_{\phi\pi}$ for their $\mu^+ \nu$ results and a similar procedure as OPAL for their $\tau^+ \nu$ results. We note that the recent BaBar result uses a larger $\mathcal{B}_{\phi\pi}$ than the other results.

The CLEO-c determination of $f_{D_s^+}$ is the most accurate to date. It also does not rely on the independent determination of any normalization mode. (We note that a preliminary CLEO-c result using $D_s^+ \rightarrow \tau^+ \nu$, $\tau^+ \rightarrow e^+ \bar{\nu} \nu$ [43] is consistent with these results.)

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