

Evolution of large scale curvature fluctuations during the perturbative decay of the inflaton

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We study the evolution of cosmological fluctuations during and after inflation driven by a scalar field coupled to a perfect fluid through a friction term. During the slow-roll regime for the scalar field, the perfect fluid is also frozen and isocurvature perturbations are generated. After the end of inflation, during the decay of the inflaton, we find that a change in the observationally relevant large scale curvature fluctuations is possible.

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I. INTRODUCTION

The inflationary paradigm provides an explanation for the large scale curvature fluctuations seen in the pattern of anisotropies of the cosmic microwave background and of the large scale structure [1]. In an inflationary scenario driven by a single scalar field, large scale curvature perturbations are generated by the amplification of quantum fluctuations of the inflaton field during the accelerated expansion era and remain constant until their reentry in the Hubble radius in the absence of post-inflationary changes.

In the context of cold inflation, the release of entropy is left for the post-inflationary evolution, during which the inflaton decays into other intermediate fields or directly into the matter our present Universe is made of. The possibility of a change in the amplitude of large scale curvature perturbations during preheating has been widely investigated [2–8]. Such change is possible due to the coupling of isocurvature and curvature perturbations on large scales [9,10]. For additional scalar fields coupled to the inflaton, the possibility of a post-inflationary amplification of curvature perturbations is very model dependent, but possible and strongly related to the spectrum of the decay products generated during inflation.

Although a post-inflationary change of large scale curvature perturbations during reheating might be an unexpected twist for successful single field inflationary models, it may be useful for the single field models whose parameters clash with observations. Along this line, after it was

shown that curvature perturbations can indeed change during preheating, it was proposed that large scale curvature perturbations may be seeded by light scalar fields during reheating [11]. The proposal to seed curvature perturbations at reheating requires three components, according to [11]: the inflaton, the decay products (taken as a perfect fluid) coupled to the inflaton, and a light modulus on which the decay rate depends.

Although a lot of attention has been paid to systems of scalar fields, also interacting through the kinetic terms [12,13], here instead we analyze the evolution of cosmological perturbations in an inflationary scenario driven by a standard scalar field coupled to a perfect fluid (with equation of state $p_F = \omega_F \rho_F$) through a friction term Γ . This setting is motivated from the old theory of reheating [14–16] to describe the decay of the inflaton into matter after the accelerated stage, but we believe it is also interesting for other reasons. First, fluctuations of a scalar field with a nonvanishing potential are different from those of a perfect fluid. As a second point, it is conceivable to explore the consequences of a coupling for the inflaton not dictated by its interaction with another scalar field. Third, the interaction between the inflaton and the perfect fluid is invisible to gravity, in contrast with what happens to scalar fields where a term $g^2 \phi^2 \chi^2$ appears as part of the potential driving the inflationary stage. For these three reasons, duplication of the results of two-field inflationary models is not expected. This formalism is also suitable for investigating the prediction of warm inflation [17–19], in which the dissipative term Γ is much larger than the Hubble parameter during inflation, since the numerical value for the ratio Γ/H is left free in this paper.

We stress that the main goal of this paper is to study cosmological perturbations for an inflaton coupled to a

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perfect fluid *during and after* inflation. To our knowledge, in previous studies of cosmological perturbations in warm inflation the post-inflationary stage was not considered [18,19]. Apart from warm inflation, this setting has been applied only after inflation to describe the reheating stage with a decay rate computed by quantum field theory methods. However, the inflationary geometric amplification of zero-point fluctuations applies to any component present in the Lagrangian. If we think about the inflaton coupling to a perfect fluid as an effective way to describe its couplings to other degrees of freedom present in the Lagrangian, it is conceivable to consider the perfect fluid contribution also during inflation.

The outline of this paper is as follows. In Sec. II and III we present the governing equations for the background and scalar perturbations in the longitudinal gauge, respectively. In Sec. IV we present the evolution of scalar perturbations in the uniform curvature gauge (UCG), from which it is easy to obtain the coupled equations for the Mukhanov variable (the gauge-invariant scalar field fluctuation [20]) and the Lukash variable (the gauge-invariant fluid fluctuation [21]). The numerical analysis presented is based on this last set of equations. In Sec. V we conclude.

II. BACKGROUND EVOLUTION

In this section we present the background equations for the scalar field plus perfect fluid and several background quantities, such as the speed of sound for the various components. The equations of motion for the scalar field and the fluid we consider are the following:

$$\ddot{\phi} = -3H\dot{\phi} - \Gamma\dot{\phi} - V_{,\phi}, \quad (1)$$

$$\dot{\rho}_F = -3H(1 + \omega_F)\rho_F + \Gamma\dot{\phi}^2, \quad (2)$$

where $H \equiv \dot{a}/a$, w_F is the state parameter of the fluid—which we consider non-negative, and Γ is the friction coefficient—which can depend on any other quantity. Note that this coupling has a fully covariant description [19] and that other couplings of the inflaton to a perfect fluid may be considered [11]. The above equations correspond to

$$\dot{\rho}_i + 3H(1 + w_i)\rho_i = X_i, \quad (3)$$

where $i = \phi, F$ and $X_\phi = -X_F = -\Gamma\dot{\phi}^2$. The Einstein equations are

$$H^2 = \frac{8\pi G}{3} \left[\frac{\dot{\phi}^2}{2} + V(\phi) + \rho_F \right] \equiv \frac{8\pi G}{3} \rho_{\text{tot}}, \quad (4)$$

$$\begin{aligned} \dot{H} &= -4\pi G \left[\dot{\phi}^2 + \rho_F(1 + \omega_F) \right] \equiv -4\pi G(\rho_{\text{tot}} + p_{\text{tot}}) \\ &\equiv -4\pi G\rho_{\text{tot}}(1 + w_{\text{tot}}), \end{aligned} \quad (5)$$

where it is clear that the dissipative term Γ does not enter explicitly into the Einstein equations. The dissipative term

Γ slows down the inflaton in a slow-roll regime and damps it into the fluid during the oscillatory stage when the value of the Hubble rate H drops below Γ .

For future convenience we define the parameter ω_ϕ as

$$\omega_\phi = \frac{p_\phi}{\rho_\phi} = \frac{\dot{\phi}^2/2 - V(\phi)}{\dot{\phi}^2/2 + V(\phi)} \quad (6)$$

and the sound of speed associated with the scalar field as

$$c_\phi^2 \equiv \frac{\dot{p}_\phi}{\dot{\rho}_\phi} = 1 + \frac{2V_{,\phi}}{3H\dot{\phi}(1 + r)}, \quad (7)$$

where $r = \Gamma/(3H)$. This last formula is the generalization of Eq. (6) of [22] to the case $r \neq 0$. We also define the total speed of sound:

$$\begin{aligned} c_{\text{tot}}^2 &\equiv \frac{\dot{p}_\phi + \dot{p}_F}{\dot{\rho}_\phi + \dot{\rho}_F} = \omega_{\text{tot}} - \frac{\dot{w}_{\text{tot}}}{3H(1 + w_{\text{tot}})} \\ &= c_\phi^2 + \frac{p_F + \rho_F - r(p_\phi + \rho_\phi)}{p_{\text{tot}} + \rho_{\text{tot}}}(\omega_F - c_\phi^2). \end{aligned} \quad (8)$$

When the inflaton slow rolls, the following approximations hold:

$$\dot{\phi} \simeq -\frac{V_{,\phi}}{3H(1 + r)}, \quad (9)$$

$$\rho_F \simeq \frac{\Gamma\dot{\phi}^2}{3H(1 + \omega_F)} = \frac{r\dot{\phi}^2}{(1 + \omega_F)}. \quad (10)$$

It is clear that the perfect fluid also remains almost frozen because of the coupling to the inflaton, i.e., $\dot{\rho}_F \simeq 0$. The (cosmic) time derivative of the Hubble distance is

$$\begin{aligned} \epsilon &\equiv -\frac{\dot{H}}{H^2} = \epsilon_\phi + \epsilon_F = \frac{4\pi G\dot{\phi}^2}{H^2} + \frac{4\pi G\rho_F(1 + w_F)}{H^2} \\ &\simeq \epsilon_\phi(1 + r), \end{aligned} \quad (11)$$

where $\simeq$ denotes the validity during slow roll.

We note that this coupling of the inflaton ϕ to a perfect fluid is inequivalent to the coupling to another massless scalar field χ in a quartic way, $g^2\phi^2\chi^2$. If we study such a system,

$$\begin{aligned} H^2 &= \frac{8\pi G}{3} \left[\frac{\dot{\phi}^2}{2} + V(\phi) + \frac{\dot{\chi}^2}{2} + g^2 \frac{\phi^2\chi^2}{2} \right] \\ &\equiv \frac{8\pi G}{3} [\rho_\phi + \rho_\chi]. \end{aligned} \quad (12)$$

By considering $\rho_\chi = \dot{\chi}^2/2 + g^2\phi^2\chi^2/2$, the structure of the corresponding interacting term X_ϕ is different from the one in Eq. (2).

As a final comment in this section, it is useful to compare this setting with the $\Gamma = 0$ case, which is very similar to the single inflaton case since $\rho_F \propto a^{-3(1+w_F)}$ is rapidly washed out for $w_F > 0$. The dissipative term Γ slows down the inflaton during the slow-roll regime and damps its

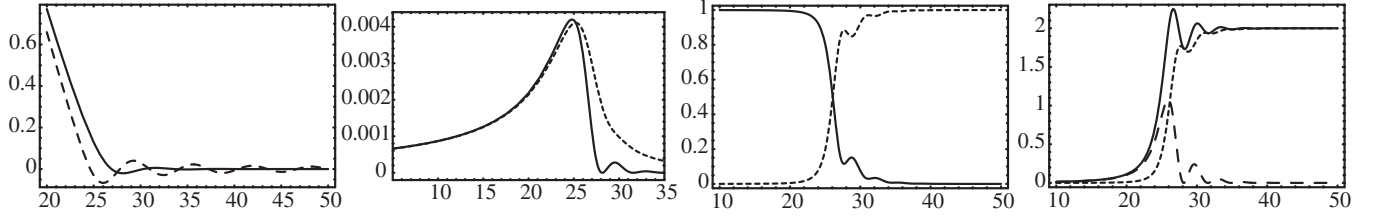


FIG. 1. Typical evolution of background quantities from inflation to radiation ($w_F = 1/3$, $\Gamma/m = 0.5$) for $V(\phi) = m^2\phi^2/2$ as function of an adimensional cosmic time mt . On the left panel we test validity of Eq. (10): we show the evolution of $\phi\sqrt{G}$ in the coupled (solid line) and uncoupled (dashed line) case. Note how Eq. (10) is a good approximation during inflation. In the second panel we show the time evolution of $\Gamma G \dot{\phi}^2 / (3Hm^2)$ (solid line) and $4\pi G \rho_F (1 + w_F) / m^2$ (dashed line). The time evolution of ρ_ϕ / ρ_{tot} (solid line) and ρ_F / ρ_{tot} (dashed line) is displayed in the third panel. In the right panel is shown the evolution of $-\dot{H}/H^2$ (solid line) and its contribution from the scalar field $4\pi G \dot{\phi}^2 / H^2$ (long-dashed line) and from radiation $4\pi G \rho_F (1 + w_F) / H^2$ (short-dashed line).

amplitude during the oscillatory stage. As an example, we show in the left panel of Fig. 1 the evolution of a massive inflaton— $V(\phi) = m^2\phi^2/2$ —with $\Gamma = 0.5m$. See the caption for the evolution of other relevant background quantities.

III. SCALAR PERTURBATIONS IN THE LONGITUDINAL GAUGE

The scalar perturbations around a flat Robertson-Walker metric are

$$ds^2 = -(1 + 2\alpha)dt^2 - a\beta_{,i}dtdx^i + a^2[\delta_{ij}(1 - 2\phi) + 2\gamma_{,ij}]dx^idx^j, \quad (13)$$

where the symbol $_{,i}$ denotes the derivative with respect to the spatial coordinates.

If we work in the longitudinal gauge, the perturbed metric is

$$ds^2 = -(1 + 2\Phi)dt^2 + a(t)^2(1 - 2\Phi)\delta_{ij}dx^idx^j, \quad (14)$$

where the two longitudinal perturbations are the same since neither the scalar field nor the perfect fluid have linear anisotropic terms in the pressure. The energy-momentum tensor for the fluid is

$$T_{\mu\nu} = (\rho + p)u_\mu u_\nu + pg_{\mu\nu}, \quad u_\mu u^\mu = -1, \quad (15)$$

and its perturbations are given by

$$\begin{aligned} \delta T_{F0}^0 &= -\delta\rho_F, & \delta T_{Fi}^0 &= \psi_{F,i} \\ \delta T_{Fj}^i &= \delta p_F \delta_j^i \equiv w_F \delta\rho_F \delta_j^i, \end{aligned} \quad (16)$$

where we have implicitly introduced the potential ψ_F for the spatial velocity of the fluid u_i .

The perturbed total energy and pressure are, respectively,

$$\delta\rho_{tot} = \dot{\phi}\delta\phi - \dot{\phi}^2\Phi + V_\phi\delta\phi + \delta\rho_F, \quad (17)$$

$$\delta p_{tot} = \dot{\phi}\delta\phi - \dot{\phi}^2\Phi - V_\phi\delta\phi + \omega_F\delta\rho_F. \quad (18)$$

From now on we shall work in the Fourier space, omitting

the index k . The Einstein constraints are

$$3H\dot{\Phi} + 3H^2\Phi + \frac{k^2}{a^2}\Phi = -4\pi G\delta\rho_{tot}, \quad (19)$$

$$H\Phi + \dot{\Phi} = 4\pi G(\dot{\phi}\delta\phi - \psi_F), \quad (20)$$

and the conservation of the energy-momentum tensor for the fluid leads to

$$\begin{aligned} \delta\dot{\rho}_F + 3H(1 + \omega_F)\delta\rho_F - \frac{k^2}{a^2}\psi_F - 3(1 + \omega_F)\rho_F\dot{\Phi} \\ = X_F\Phi + \delta X_F = 2\Gamma\dot{\phi}\delta\dot{\phi} - \Gamma\dot{\phi}^2\Phi + \delta\Gamma\dot{\phi}^2, \end{aligned} \quad (21)$$

$$\dot{\psi}_F + 3H\psi_F + \Phi(1 + \omega_F)\rho_F + \omega_F\delta\rho_F = -\Gamma\dot{\phi}\delta\phi, \quad (22)$$

with

$$\delta X_F = 2\Gamma\dot{\phi}\delta\dot{\phi} - 2\Gamma\dot{\phi}^2\Phi + \dot{\phi}^2\delta\Gamma = -\delta X_\phi. \quad (23)$$

The equation of motion for field fluctuations is

$$\begin{aligned} \delta\ddot{\phi} + (3H + \Gamma)\delta\dot{\phi} + \left[\frac{k^2}{a^2} + V_{\phi\phi}\right]\delta\phi \\ = 4\dot{\phi}\dot{\Phi} - 2V_\phi\Phi - \Gamma\dot{\phi}\Phi - \dot{\phi}\delta\Gamma. \end{aligned} \quad (24)$$

The curvature perturbation in the longitudinal gauge ζ is defined as

$$\zeta = \Phi - \frac{H}{\dot{H}}(\dot{\Phi} + H\Phi) = \Phi + \frac{2}{3}\frac{\dot{\Phi} + H\Phi}{H(1 + \omega_{tot})}. \quad (25)$$

Following [22], we define the entropy perturbation between the scalar field and the fluid,

$$\begin{aligned} S_{\phi F} &\equiv \frac{3H(1 + w_F)(1 + w_\phi)\rho_F}{(1 + w_{tot})\rho_{tot}} \left(\frac{\delta\rho_\phi}{\dot{\rho}_\phi} - \frac{\delta\rho_F}{\dot{\rho}_F} \right) \\ &= \frac{(1 + w_F)(1 + w_\phi)\rho_F}{(1 + w_{tot})\rho_{tot}} \left[\frac{\delta\rho_F}{(1 + w_F)\rho_F - r\dot{\phi}^2} \right. \\ &\quad \left. - \frac{\delta\rho_\phi}{(1 + r)\dot{\phi}^2} \right], \end{aligned} \quad (26)$$

and the intrinsic entropy perturbation of the scalar field,

$$S_\phi \equiv \frac{3H(1+w_\phi)c_\phi^2}{1-c_\phi^2} \left(\frac{\delta\rho_\phi}{\dot{\rho}_\phi} - \frac{\delta p_\phi}{\dot{p}_\phi} \right) = \frac{\delta p_\phi - c_\phi^2 \delta\rho_\phi}{\rho_\phi(1-c_\phi^2)(1+r)}. \quad (27)$$

We conclude this section with the equation of evolution for ζ ,

$$\dot{\zeta} = \frac{2}{3H(1+w_{\text{tot}})} \left[-c_{\text{tot}}^2 \frac{k^2}{a^2} \Phi + 4\pi G \delta p_{\text{nad}} \right], \quad (28)$$

where $\delta p_{\text{nad}} \equiv \delta p_{\text{tot}} - c_{\text{tot}}^2 \delta\rho_{\text{tot}}$ is the nonadiabatic pressure which depends on the intrinsic and relative entropy perturbations, as we can see making use of Eq. (8):

$$\frac{\delta p_{\text{nad}}}{\rho_\phi(1+r)} = (1-c_\phi^2)S_\phi + (\omega_F - c_\phi^2) \left(1 - r \frac{\dot{\phi}^2}{(1+w_F)\rho_F} \right) S_{F\phi}. \quad (29)$$

It is simple to see from Eqs. (28) and (29) that in the case of pure radiation ($\phi = \dot{\phi} = \delta\phi = 0$ and $\omega_F = c_s^2 = 1/3$) curvature perturbations are conserved on large scales.

Large scale curvature solution during slow roll

It is useful to give an analytic approximation for large scale cosmological perturbation during slow roll. By using Eqs. (9)–(11), the approximate solutions at leading order in the slow-roll parameters are

$$\begin{aligned} \Phi &\simeq \frac{4\pi G}{H} \dot{\phi}(1+r)\delta\phi, \\ \psi_F &\simeq -r\dot{\phi}\delta\phi, \\ \delta\rho_F &\simeq -\frac{H\dot{\phi}}{(1+r)(1+\omega_F)} \\ &\quad \times \left(2\frac{V_{\phi\phi}}{3H^2}r - \beta(1-r) - (3+r)r\epsilon \right) \delta\phi, \\ \delta\phi &\simeq -\frac{H}{1+r} \left(\frac{V_{\phi\phi}}{3H^2} + \beta - (2+r)\epsilon \right) \delta\phi, \\ \delta\phi &\propto \frac{\dot{\phi}}{H}, \end{aligned} \quad (30)$$

with $\beta = \ddot{\Gamma}/(3H^2)$. These solutions have been obtained neglecting the highest time derivatives and the terms $\propto k^2$ in Eqs. (20)–(22) and (24) and are valid if $\Gamma = \Gamma(\phi)$. It is interesting to use the above relations and write the leading term (in the slow-roll parameter) of curvature perturbations:

$$\zeta \simeq \frac{2}{3} \frac{\Phi}{1+\omega_{\text{tot}}} = \frac{\Phi}{\epsilon} \simeq \frac{H}{\dot{\phi}} \delta\phi, \quad (31)$$

which is formally just the single field expression; Γ is,

however, present in the dynamics for the inflaton, which differs from the single field case with the same $V(\phi)$ and $\Gamma = 0$. The expression differs from the multifield inflationary case in which all the scalar fields in slow roll contribute (with the same formal weight) to curvature perturbations. We end on noting that ζ is constant in time for the last expression of Eq. (30) at leading order in slow-roll parameters (i.e., $\dot{\zeta} \sim H\zeta\mathcal{O}(\epsilon)$).

IV. NUMERICAL EVOLUTION OF COSMOLOGICAL PERTURBATIONS

We will consider the numerical evolution of linear cosmological perturbations from inflation through reheating, following Ref. [5], which evolved the Mukhanov variables related to two scalar fields in a fully constrained evolution.

In this case we need to obtain the evolution of the Mukhanov variable associated to the scalar field coupled with the Lukash variable associated to the gradient velocity of the perfect fluid. Given a fully general metric perturbation as in Eq. (13), gauge-invariant variables for the scalar field and the fluid are defined as

$$\delta\phi^{\text{g.i.}} = \delta\phi + \frac{\dot{\phi}}{H}\psi, \quad \psi_F^{\text{g.i.}} = \psi_F - (\rho + p)\psi_{,i}. \quad (32)$$

In the UCG, where the spatial metric is unperturbed,

$$ds^2 = -(1+2\alpha)dt^2 - a\beta_{,i}dt dx^i + a^2\delta_{ij}dx^i dx^j, \quad (33)$$

the gauge-invariant variables coincide with the scalar field and fluid velocity potential fluctuations. It is therefore easier to obtain the evolution for these gauge-invariant variables in such a gauge. From here on we restrict ourselves to this gauge, and we shall write fluctuations of the scalar field and of the fluid, although different, in the same way as in the previous section.

The equations corresponding to (21), (22), and (24) are

$$\begin{aligned} \delta\dot{\rho}_F + 3H(1+\omega_F)\delta\rho_F - \frac{k^2}{a^2}\psi_F - (1+\omega_F)\rho_F \frac{k^2}{2a}\beta \\ = X_F\alpha + \delta X_F = 2\Gamma\dot{\phi}\delta\dot{\phi} - \Gamma\dot{\phi}^2\alpha + \delta\Gamma\dot{\phi}^2, \end{aligned} \quad (34)$$

$$\dot{\psi}_F + 3H\psi_F + \alpha(1+\omega_F)\rho_F + \omega_F\delta\rho_F = -\Gamma\dot{\phi}\delta\phi, \quad (35)$$

$$\begin{aligned} \delta\ddot{\phi} + (3H + \Gamma)\delta\dot{\phi} + \left[\frac{k^2}{a^2} + V_{\phi\phi} \right] \delta\phi \\ = \dot{\phi} \left(\dot{\alpha} + \frac{k^2}{2a}\beta \right) - 2V_{\phi}\alpha - \dot{\phi}\delta\Gamma - \alpha\Gamma\dot{\phi}. \end{aligned} \quad (36)$$

If $\delta\phi$ is used to quantize the linear scalar field fluctuation in the presence of gravity [20], the variable to quantize a fluid [21], as also explained in [23], is

$$Q_F \equiv \frac{\psi_F}{\sqrt{\rho_F(1+w_F)}}. \quad (37)$$

On restricting to the case in which Γ is constant in time, the coupled second order differential equations for $(Q_\phi, Q_F) = (\delta\phi, Q_F)$ are

$$\ddot{Q}_i + (3H\delta_{ij} + G_{ij})\dot{Q}_j + \Omega_{ij}Q_j = 0, \quad (38)$$

where G has the following components:

$$G_{\phi\phi} = \Gamma, \quad (39)$$

$$G_{\phi F} = \frac{4\pi G(w_F^2 - 1)\rho_F\dot{\phi}}{\sqrt{\rho_F(1 + w_F)}w_F H}, \quad (40)$$

$$G_{FF} = \frac{[-4\pi G(w_F^2 - 1)\rho_F + (2w_F + 1)H\Gamma]\dot{\phi}}{\sqrt{\rho_F(1 + w_F)}H}, \quad (41)$$

$$G_{F\phi} = \Gamma \frac{\dot{\phi}^2}{\rho_F}, \quad (42)$$

and where Ω is given by

$$\begin{aligned} \Omega_{\phi\phi} = & \frac{k^2}{a^2} + V_{\phi\phi} + 16\pi G \frac{\dot{\phi}V_\phi}{H} + 24\pi G \dot{\phi}^2 \\ & + 4\pi G \left(\frac{2w_F - 1}{w_F} \right) \frac{\dot{\phi}^2 \Gamma}{H} - 16\pi^2 G^2 \frac{\dot{\phi}^2}{H^2} \\ & \times \left(\rho_F \frac{(w_F + 1)^2}{w_F} + 2\dot{\phi}^2 \right), \end{aligned} \quad (43)$$

$$\begin{aligned} \Omega_{\phi F} = & \sqrt{\rho_F(1 + w_F)} \left(-8\pi G \frac{V_\phi}{H} - 6\pi G \frac{(1 + w_F)^2}{w_F} \dot{\phi} \right. \\ & + 16\pi^2 G^2 \frac{(1 + w_F)^2}{w_F} \rho_F \frac{\dot{\phi}}{H^2} + 32\pi^2 G^2 \frac{\dot{\phi}^3}{H^2} \\ & \left. + 2\pi G \frac{(w_F - 1)}{w_F} \frac{\Gamma}{H} \frac{\dot{\phi}^3}{\rho_F} - 4\pi G \Gamma \frac{\dot{\phi}}{H} \right), \end{aligned} \quad (44)$$

$$\begin{aligned} \Omega_{FF} = & \sqrt{\rho_F(1 + w_F)} \left(-4(1 + w_F)\pi G \frac{V_\phi}{H} \right. \\ & - 12\pi G(1 + w_F)\dot{\phi} - \frac{\Gamma V_\phi}{(1 + w_F)\rho_F} \\ & + \frac{(3w_F H - \Gamma)\Gamma\dot{\phi}}{(1 + w_F)\rho_F} + 32\pi^2 G^2(1 + w_F) \frac{\rho_F\dot{\phi}}{H^2} \\ & \left. + 16\pi^2 G^2(1 + w_F) \frac{\dot{\phi}^3}{H^2} + 4\pi G \frac{1}{(w_F + 1)} \frac{\Gamma\dot{\phi}^3}{H\rho_F} \right), \end{aligned} \quad (45)$$

$$\begin{aligned} \Omega_{FF} = & w_F \frac{k^2}{a^2} + \frac{9}{4} H^2(1 - w_F^2) \\ & + 6\pi G \rho_F(1 + w_F)(1 + 3w_F) \\ & - \frac{32\pi^2 G^2 \rho_F^2}{H^2} (1 + w_F)^2 - \Gamma \frac{V_\phi \dot{\phi}}{\rho_F} \\ & + 6\pi G \dot{\phi}^2(w_F - 1) - \frac{4\pi G \Gamma \dot{\phi}^2}{H} \\ & + \frac{\Gamma \dot{\phi}^2}{2\rho_F} (-2\Gamma + 3w_F H) - \frac{16\pi^2 G^2}{H^2} \rho_F \dot{\phi}^2(1 + w_F)^2 \\ & - \frac{\Gamma^2 \dot{\phi}^4}{4\rho_F^2}. \end{aligned} \quad (46)$$

Several cross-checks on the novel set of equations can be made:

- (1) For $\rho_F = \Gamma = 0$, the equation for Q_ϕ agrees with the Mukhanov equation for single field inflation [20].
- (2) When the two components are decoupled ($\Gamma = 0$) and the fluid is stiff matter ($w_F = 1$), the above equations agree with those for two scalar fields [5] when the second one, χ , is decoupled and massless (i.e., equivalent to stiff matter), reminding us that $Q_\chi = -Q_F$.

We now evolve the system in Eq. (38) starting from a wavelength well inside the Hubble radius $k \gg aH$ with initial conditions

$$Q_\phi \simeq \frac{e^{-ik\eta}}{a(2k)^{1/2}}, \quad Q_F \simeq \frac{e^{-iw_F^{1/2}k\eta}}{a(4k^2w_F)^{1/4}}. \quad (47)$$

The gauge-invariant curvature perturbation ζ in the UCG gauge is written as

$$\zeta = H \frac{\dot{\phi}Q_\phi - \sqrt{\rho_F(1 + w_F)}Q_F}{\dot{\phi}^2 + \rho_F(1 + w_F)} \equiv \zeta_\phi + \zeta_F. \quad (48)$$

The evolution of curvature perturbations is displayed in Fig. 2. Perfect fluid fluctuations are amplified similarly to inflaton fluctuations when $\Gamma \neq 0$, leading to a mixture of curvature and isocurvature fluctuations during slow roll. Isocurvature perturbations can then change the amplitude curvature perturbations during and after the inflaton decay.

The uncoupled case

As is clear from the numerics, the perfect fluid does not contribute to curvature perturbations in the uncoupled case $\Gamma = 0$. We have already observed that for $w_F > 0$, the homogeneous perfect fluid energy density is rapidly washed out for $\Gamma = 0$. If we neglect the terms which decay quasiexponentially in time, the equation for fluid perturbations becomes

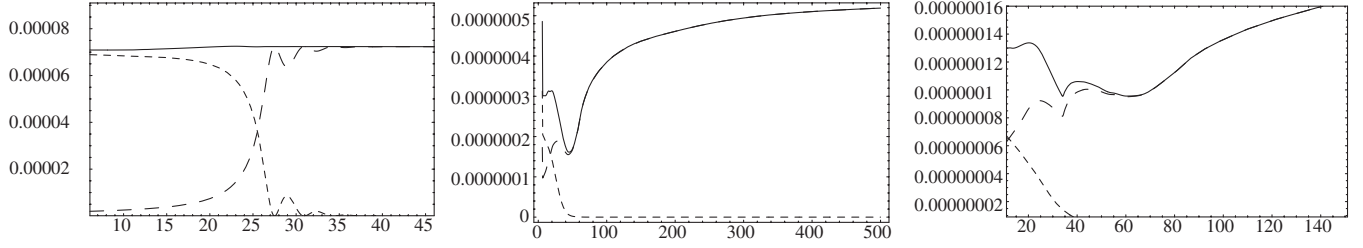


FIG. 2. Typical evolution of curvature perturbations (total as solid line; short-dashed line and long-dashed line for the field ζ_ϕ and fluid ζ_F contributions, respectively) from inflation to radiation as a function of an adimensional cosmic time mt . On the left panel, the evolution for $\Gamma/m = 0.5$, for which the curvature perturbations are given by the inflaton (solid and short-dashed lines are superimposed) and there is negligible post-inflationary change. On the middle and right panel, the evolution for $\Gamma/m = 6$ and $\Gamma/m = 8$.

$$\ddot{Q}_F + 3H\dot{Q}_F + \left[w_F \frac{k^2}{a^2} + \frac{9}{4} H^2 (1 - w_F^2) + 6\pi G \dot{\phi}^2 (w_F - 1) \right] Q_F \simeq 0. \quad (49)$$

For radiation, at leading order in a nearly de Sitter exponent, we have an effective mass $\simeq 2H^2$ which makes the fluid fluctuations similar to a massless conformally coupled fluid. The initial vacuum fluid fluctuations in Eq. (47) are almost left unchanged during the stretching to large scale, leading to a spectrum $P_F(k) = k^3 |Q_F|^2 / (2\pi^2) \sim k^2$, which is too blue to affect the nearly scale-invariant spectrum of inflaton fluctuations in the observable range.

V. DISCUSSIONS AND CONCLUSIONS

We have studied the coupling of the inflaton to a perfect fluid through a friction term Γ during and after inflation. Although this type of coupling has been used in the early days to describe the decay of the inflaton after the accelerated era, it has also been used in the regime $\Gamma \gg H$ for warm inflation.

By considering radiation as the perfect fluid and Γ constant in time for simplicity, we have shown how this coupling freezes the perfect fluid during the slow-roll evolution and large scale fluid fluctuations are amplified together with the scalar field ones, leading to a mixture of curvature and isocurvature fluctuations during the slow-roll regime. In the stage of the inflaton decay, large scale

curvature and isocurvature perturbations are not weakly coupled anymore, and there is a transfer of the latter leading to an amplitude for the former which is different from the one computed during slow roll. After the decay is completed, a purely adiabatic curvature perturbation is left. We believe that the qualitative behavior found here is generic and does not occur only for the simplest parameters studied in this paper—radiation as the perfect fluid and Γ constant in time. Note that a variation of large scale curvature perturbations was also found in the late decay of a massive curvaton [24].

Our results for coupling to a perfect fluid add to the changes of curvature perturbations during preheating found for other couplings to scalar fields, as studied in [5]. As already said, this coupling of the inflaton easily generates non-negligible isocurvature fluctuations during inflation, which is one of the key requirements in order to have change of curvature perturbations in the post-inflationary stage [5]. Since we see that the amount of isocurvature perturbations increases with Γ in the cases studied here, inflationary models with sizeable r have a larger variation of curvature perturbations in the post-inflationary era. As shown explicitly here, a successful conversion of isocurvature into curvature perturbations at reheating may not need a varying decay rate.

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