Helicity amplitude analysis of hyperon nonleptonic decays in J/ψ or $\psi(2S)$ decays

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We present formulae for the joint angular distributions for J/ψ or $\psi(2S)$ decays into $\Lambda\bar{\Lambda}$, $\Sigma^0\bar{\Sigma}^0$, $\Xi^-\bar{\Xi}^+$, and $\Omega^-\bar{\Omega}^+$, which allows the determination of the antihyperon decay parameters using helicity amplitude information. The prospects of measuring the antihyperon decay parameters in these decays are discussed. The estimation of the measurement sensitivity shows that J/ψ decays into $\Lambda\bar{\Lambda}$ and $\Xi^-\bar{\Xi}^+$ can be hopefully used to measure the $\bar{\Lambda}$ and $\bar{\Xi}^+$ decay parameters.

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I. INTRODUCTION

Hyperon nonleptonic decays have played an important role in understanding parity violation in particle physics. Since 1957, when the possibility of parity violation was proposed with the suggestion to search for it in hyperon nonleptonic decays [1], much experimental evidence has been established. The hyperon decay parameters, which characterize parity violation, have been measured in Λ, Σ, Ξ, and Ω nonleptonic decays by many experiments with high sensitivity [2]. However, in antihyperon nonleptonic decays, the uncertainties of the asymmetry parameters measured are still large, e.g. $\alpha(\bar{\Lambda} \to \bar{p}\pi^+) = -0.63 \pm 0.13$ [3] and $\alpha(\bar{\Omega}^+ \to \bar{\Lambda}K^+) = 0.017 \pm 0.077$ [4], and asymmetry parameters of $\bar{\Sigma}^-$ and $\bar{\Xi}^+$ have not been measured.

It has long been known that high precise measurements of hyperon and antihyperon decay parameters provide a test of CP symmetry [5]. In spin-$\frac{1}{2}$ hyperon nonleptonic decays, the angular distribution of the daughter baryon takes the form $\frac{dN}{d\Omega} = \frac{1}{4\pi}(1 + \alpha_Y \vec{P}_p \cdot \hat{p}_d)$, where $\vec{P}_p$ is the hyperon polarization, $\hat{p}_d$ is the daughter baryon momentum unit vector in its mother's rest frame, and α_Y is the hyperon asymmetry parameter. The observable of CP asymmetry is defined by $A_Y = \frac{\alpha_Y + \alpha_{\bar{Y}}}{\alpha_Y - \alpha_{\bar{Y}}}$. Under CP transformation, $\alpha_Y = -\alpha_{\bar{Y}}$ if CP is conserved. Hence any nonzero value of A_Y provides evidence for CP asymmetry. Experimentally, searches for CP asymmetry in Λ nonleptonic decays have been previously performed at $p\bar{p}$ colliders by the R608 [6] and PS185 [7] collaborations, and at a e^+e^- collider by DM2 Collaboration [3], but the precision of the measurements are limited by statistics. Recently, searches for direct CP violation in Ξ decays have been performed in the HyperCP experiment at Fermilab. With 117×10^6 Ξ^- and 41×10^6 $\bar{\Xi}^+$ events analyzed, the HyperCP collaboration reported that CP violation in charged $\Xi \to \Lambda\pi$ decays is consistent with standard model predictions [8]. For Ω decays, HyperCP collaboration measured the decay parameter in $\Omega^- \to \Lambda K^-$ to be $\alpha_\Omega = (2.07 \pm 0.51 \pm 0.81) \times 10^{-2}$ [9]. However, the uncertainty of the decay parameter measured in $\bar{\Omega}^+ \to \bar{\Lambda}K^+$ is still large, $\alpha_{\bar{\Omega}} = (1.7 \pm 7.7) \times 10^{-2}$, which was reported by the E756 collaboration [4]. In short, the CP violation test in hyperon nonleptonic decays needs more precise measurements of their decay parameters, especially in antihyperon nonleptonic decays.

J/ψ and $\psi(2S)$ decays at e^+e^- colliders provide an ideal laboratory to measure antihyperon decay parameters with clean signals [3]. Although J/ψ decay into a hyperon antihyperon pair has only about a 10^{-3} branching fraction, larger, higher statistics data samples are expected at CLEOc and the updated BEPCII. Since the hyperon and antihyperon are produced as a pair, this allows one to measure antihyperon asymmetry parameters with the same precision as the hyperon.

In Sec. II, we systematically present helicity amplitude formulae for hyperon nonleptonic decays in $e^+e^- \to J/\psi$ and $\psi(2S) \to Y\bar{Y}$. The formulae are calculated with the computer algebra system, MATHEMATICA, and the results are checked by using REDUCE. The codes are available from the authors. In Sec. III, we present an unbinned likelihood fit method to measure hyperon decay parameters, along with estimates of the expected measurement sensitivities. If $10^{10} J/\psi$ events can be accumulated with the updated BESIII/BEPCII detector, it will be possible to get more accurate antihyperon decay parameters.

II. HELICITY AMPLITUDES

The helicity amplitudes for sequential decays are constructed in the helicity frame defined by:

(1) In $e^+e^- \to J/\psi, \psi(2S) \to Y\bar{Y}$, the z-axis of the J/ψ or $\psi(2S)$ rest frame is along the hyperon outgoing direction, which changes from event to event. The e^+ beam is along the direction of the solid angle (θ, ϕ).

(2) For hyperon decays $Y_i \to BM$ (B: baryon, M: meson), the solid angle of the daughter particle $\Omega_i(\theta_i, \phi_i)$ is referred to the hyperon Y_i rest frame, where the z-axis is taken along the outgoing direc-

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tion of the hyperon Y_i in its mother particle rest frame. The helicity frame of antiparticle $\bar{Y}_i$ has a similar definition but is described by the solid angle $\bar{\Omega}_i(\bar{\theta}_i, \bar{\phi}_i)$. For example, Fig. 1 shows the definition of the helicity frame for the sequential decay $e^+e^- \to J/\psi$, $\psi(2S) \to \Lambda\bar{\Lambda} \to p\bar{p}\pi^+\pi^-$.

A. $e^+e^- \to J/\psi$, $\psi(2S) \to \Lambda\bar{\Lambda} \to p\bar{p}\pi^+\pi^-$

We denote, respectively, the helicity amplitudes for $J/\psi \to \Lambda\bar{\Lambda}$ and $\Lambda(\bar{\Lambda}) \to p\pi^-(\bar{p}\pi^+)$ with $A_{\lambda,\bar{\lambda}}$ and $B_{\lambda_p}(\bar{B}_{\lambda_{\bar{p}}})$, where λ, $\bar{\lambda}$, and $\lambda_p(\bar{\lambda}_p)$ are the helicity values for Λ, $\bar{\Lambda}$, and proton (antiproton). The joint helicity amplitude can be expressed by

$$|\mathcal{M}|^2 \propto \sum_{\lambda_1,\bar{\lambda}_1,\lambda_2,\bar{\lambda}_2,\lambda_p\bar{\lambda}_{\bar{p}}} \rho^{(\lambda_1-\bar{\lambda}_1,\lambda_2-\bar{\lambda}_2)}(\theta,\phi) A_{\lambda_1,\bar{\lambda}_1} A^*_{\lambda_2,\bar{\lambda}_2} \times B_{\lambda_p} B^*_{\lambda_p} \bar{B}_{\lambda_{\bar{p}}} \bar{B}^*_{\lambda_{\bar{p}}} D^{1/2*}_{\lambda_1,\lambda_p}(\Omega_1) D^{1/2}_{\lambda_2,\lambda_p}(\Omega_1) D^{1/2*}_{\bar{\lambda}_1,\lambda_{\bar{p}}}(\bar{\Omega}_1) \times D^{1/2}_{\bar{\lambda}_2,\bar{\lambda}_{\bar{p}}}(\bar{\Omega}_1), \tag{1}$$

where $D^J_{\lambda_1,\lambda_2}(\Omega_i) \equiv D^J_{\lambda_1,\lambda_2}(\phi_i,\theta_i,0)$, and $\rho^{(i,j)}(\theta,\phi) = \sum_{k=\pm 1} D^1_{i,k}(\Omega) D^{1*}_{j,k}(\Omega)$ is the density matrix for the J/ψ produced in e^+e^- annihilation. From parity invariance, one has $A_{-\lambda_1,-\lambda_2} = A_{\lambda_1,\lambda_2}$. The helicity amplitude for Λ decays into $p\pi$ is related to its asymmetry decay parameter by

$$\alpha_\Lambda = \frac{|B_{1/2}|^2 - |B_{-1/2}|^2}{|B_{1/2}|^2 + |B_{-1/2}|^2}, \qquad \alpha_{\bar{\Lambda}} = \frac{|\bar{B}_{1/2}|^2 - |\bar{B}_{-1/2}|^2}{|\bar{B}_{1/2}|^2 + |\bar{B}_{-1/2}|^2}. \tag{2}$$

If CP invariance in $\Lambda(\bar{\Lambda})$ decays into $p\pi^-(\bar{p}\pi^+)$ is assumed, then one has $\alpha_\Lambda = -\alpha_{\bar{\Lambda}}$. By integration over ϕ in Eq. (1), one obtains the angular distribution

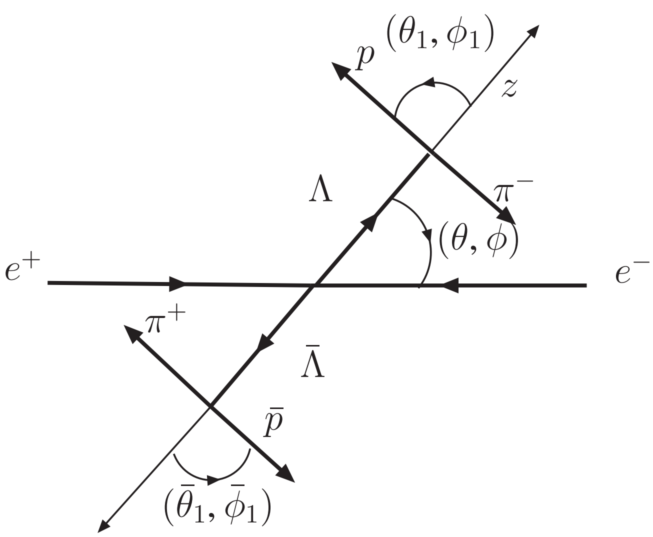

FIG. 1. Definition of the helicity frame for $e^+e^- \to J/\psi \to \Lambda\bar{\Lambda} \to p\bar{p}\pi^+\pi^-$.

$$\frac{d|\mathcal{M}|^2}{d(\cos\theta)d\Omega_1 d\bar{\Omega}_1} \propto 4|A_{1/2,1/2}|^2 \sin^2\theta \times [1 + \alpha_\Lambda \alpha_{\bar{\Lambda}}(\cos\theta_1 \cos\bar{\theta}_1 + \sin\theta_1 \sin\bar{\theta}_1 \cos(\phi_1 + \bar{\phi}_1))] - 2|A_{1/2,-1/2}|^2(1 + \cos^2\theta) \times (\alpha_\Lambda \alpha_{\bar{\Lambda}} \cos\theta_1 \cos\bar{\theta}_1 - 1). \tag{3}$$

A similar formula can be found in Ref. [3]. This formula is also applicable for the sequential decay $e^+e^- \to J/\psi$, $\psi(2S) \to \Sigma^+\bar{\Sigma}^-$, or $\Sigma^-\bar{\Sigma}^+$, with $\Sigma \to N\pi$, but the hyperon asymmetry parameter is replaced. If the solid angles Ω_1 and $\bar{\Omega}_1$ are integrated, then Eq. (3) is reduced to the well-known angle distribution for $J/\psi \to B_8\bar{B}_8$ (B_8: octet baryon), i.e.:

$$\frac{d|\mathcal{M}|^2}{d\cos\theta} \propto (1 + \alpha\cos^2\theta), \quad \text{with} \quad \alpha = \frac{|A_{1/2,-1/2}|^2 - 2|A_{1/2,1/2}|^2}{|A_{1/2,-1/2}|^2 + 2|A_{1/2,1/2}|^2}. \tag{4}$$

B. $e^+e^- \to J/\psi$, $\psi(2S) \to \Sigma^0\bar{\Sigma}^0 \to 2\gamma\Lambda\bar{\Lambda} \to 2\gamma\pi^+\pi^- p\bar{p}$

The helicity amplitude for $J/\psi \to \Sigma^0(\sigma)\bar{\Sigma}^0(\bar{\sigma})$ is denoted by $A_{\sigma,\bar{\sigma}}$; amplitudes for $\Sigma^0(\sigma) \to \gamma(r_1)\Lambda(\lambda)$ and $\Lambda(\lambda) \to p(\lambda_p)\pi^-$ are denoted by B_λ and C_{λ_p}, and their antiparticle counterparts by $\bar{B}_{\bar{\lambda}}$ and $\bar{C}_{\bar{\lambda}_p}$, respectively. The joint amplitude reads

$$|\mathcal{M}|^2 = \sum_{\substack{\sigma_1,\sigma_2,\bar{\sigma}_1,\bar{\sigma}_2,\\ \lambda_1,\lambda_2,\bar{\lambda}_1,\bar{\lambda}_2,\\ r_1,r_2,\lambda_p,\lambda_{\bar{p}}}} \rho^{(\sigma_1-\bar{\sigma}_1,\sigma_2-\bar{\sigma}_2)}(\theta,\phi) A_{\sigma_1,\bar{\sigma}_1} A^*_{\sigma_2,\bar{\sigma}_2} B_{r_1,\lambda_1} B^*_{r_1,\lambda_2} \times C_{\lambda_p} C^*_{\lambda_p} \bar{B}_{r_2,\bar{\lambda}_1} \bar{B}^*_{r_2,\bar{\lambda}_2} \bar{C}_{\lambda_{\bar{p}}} \bar{C}^*_{\lambda_{\bar{p}}} D^{1/2*}_{\sigma_1,r_1-\lambda_1}(\Omega_1) \times D^{1/2}_{\sigma_2,r_1-\lambda_2}(\Omega_1) D^{1/2*}_{\bar{\sigma}_1,r_2-\bar{\lambda}_1}(\bar{\Omega}_1) D^{1/2}_{\bar{\sigma}_2,r_2-\bar{\lambda}_2}(\bar{\Omega}_1) \times D^{1/2*}_{\lambda_1,\lambda_p}(\Omega_2) D^{1/2}_{\lambda_2,\lambda_p}(\Omega_2) D^{1/2}_{\bar{\lambda}_1,\lambda_{\bar{p}}}(\bar{\Omega}_2) D^{1/2}_{\bar{\lambda}_2,\lambda_{\bar{p}}}(\bar{\Omega}_2). \tag{5}$$

From consideration of parity invariance in $J/\psi \to \Sigma^0\bar{\Sigma}^0$ and $\Sigma^0 \to \gamma\Lambda$, one has the relations $A_{-\sigma_1-\sigma_2} = A_{\sigma_1\sigma_2}$, $B_{-r,-\lambda} = B_{r,\lambda}$, and $\bar{B}_{-r,-\lambda} = \bar{B}_{r,\lambda}$, respectively. One obtains the angular distribution by integrating the azimuthal angles ϕ, ϕ_1, and $\bar{\phi}_1$:

$$\frac{d|\mathcal{M}|^2}{d\cos\theta d\Omega_1 d\bar{\Omega}_1 d\cos\theta_2 d\cos\bar{\theta}_2} \propto |B_{1,1/2}|^2 |\bar{B}_{1,1/2}|^2 \{4\sin^2\theta[\cos\theta_2 \cos\bar{\theta}_2(\cos\theta_1 \cos\bar{\theta}_1 + \cos(\phi_1 + \bar{\phi}_1)\sin\theta_1 \sin\bar{\theta}_1)\alpha_\Lambda\alpha_{\bar{\Lambda}} + 1]|A_{1/2,1/2}|^2 - (\cos 2\theta + 3)(\cos\theta_1 \cos\theta_2 \cos\bar{\theta}_1 \cos\bar{\theta}_2 \alpha_\Lambda \alpha_{\bar{\Lambda}} - 1) \times |A_{1/2,-1/2}|^2\}. \tag{6}$$

Integrating over the polar angles θ_1 and $\bar{\theta}_1$, the $\alpha_\Lambda\alpha_{\bar{\Lambda}}$ terms vanish. One has

$$\frac{d|\mathcal{M}|^2}{d\cos\theta d\cos\theta_2 d\cos\bar{\theta}_2} \propto |B_{1,1/2}|^2|\bar{B}_{1,1/2}|^2(|A_{(1/2)-(1/2)}|^2 + 2|A_{(1/2)(1/2)}|^2)(1+\alpha\cos\theta), \quad (7)$$

where $\alpha = \frac{|A_{1/2,-1/2}|^2 - 2|A_{1/2,1/2}|^2}{|A_{1/2,-1/2}|^2 + 2|A_{1/2,1/2}|^2}$. As expected, the above equation shows that $d|\mathcal{M}|^2/d\cos\theta$ has the expected form for $J/\psi \to B_8\bar{B}_8$ angular distributions (see Eq. (4)).

C. $e^+e^- \to J/\psi, \psi(2S) \to \Xi^-\bar{\Xi}^+ \to \pi^+\pi^-\Lambda\bar{\Lambda} \to 2(\pi^+\pi^-)p\bar{p}$

The helicity amplitude for $J/\psi \to \Xi^-(\xi)\bar{\Xi}^+(\bar{\xi})$ is denoted by $A_{\xi,\bar{\xi}}$; amplitudes for $\Xi^-(\xi) \to \Lambda(\lambda)\pi^-$ and $\Lambda(\lambda) \to p(\lambda_p)\pi^-$ are denoted by B_λ and C_{λ_p}, and their antiparticle counterparts by $\bar{B}_{\bar{\lambda}}$ and $\bar{C}_{\bar{\lambda}_p}$, respectively. The joint amplitude is

$$|\mathcal{M}|^2 = \sum_{\substack{\xi_1,\xi_2,\bar{\xi}_1,\bar{\xi}_2,\\ \lambda_1,\lambda_2,\bar{\lambda}_1,\bar{\lambda}_2,\\ \lambda_p,\lambda_{\bar{p}}}} \rho^{(\xi_1-\bar{\xi}_1,\xi_2-\bar{\xi}_2)}(\theta,\phi)A_{\xi_1,\bar{\xi}_1}A^*_{\xi_2,\bar{\xi}_2}B_{\lambda_1}B^*_{\lambda_2} \times C_{\lambda_p}C^*_{\lambda_p}\bar{B}_{\bar{\lambda}_1}\bar{B}^*_{\bar{\lambda}_2}\bar{C}_{\lambda_{\bar{p}}}\bar{C}^*_{\lambda_{\bar{p}}}D^{1/2*}_{\xi_1,\lambda_1}(\Omega_1)D^{1/2}_{\xi_2,\lambda_2}(\Omega_1) \times D^{1/2*}_{\bar{\xi}_1,\bar{\lambda}_1}(\bar{\Omega}_1)D^{1/2}_{\bar{\xi}_2,\bar{\lambda}_2}(\bar{\Omega}_1)D^{1/2*}_{\lambda_1,\lambda_p}(\Omega_2)D^{1/2}_{\lambda_2,\lambda_p}(\Omega_2) \times D^{1/2*}_{\bar{\lambda}_1,\lambda_{\bar{p}}}(\bar{\Omega}_2)D^{1/2}_{\bar{\lambda}_2,\lambda_{\bar{p}}}(\bar{\Omega}_2). \quad (8)$$

Assuming parity invariance in $J/\psi \to \Xi^-\bar{\Xi}^+$, one has the relation $A_{-\xi_1-\xi_2} = A_{\xi_1\xi_2}$. The helicity amplitudes of Ξ decays can be related to their asymmetry parameters by

$$\alpha_{\Xi^+} = \frac{|B_{1/2}|^2 - |B_{-1/2}|^2}{|B_{1/2}|^2 + |B_{-1/2}|^2} \quad \text{and} \quad \alpha_{\bar{\Xi}^-} = \frac{|\bar{B}_{1/2}|^2 - |\bar{B}_{-1/2}|^2}{|\bar{B}_{1/2}|^2 + |\bar{B}_{-1/2}|^2}.$$

One obtains the angular distribution by integrating over all azimuthal angles:

$$\begin{aligned}\frac{d|\mathcal{M}|^2}{d\cos\theta d\cos\theta_1 d\cos\bar{\theta}_1 d\cos\theta_2 d\cos\bar{\theta}_2} &\propto (\cos2\theta+3)\{\cos\bar{\theta}_2\alpha_{\bar{\Lambda}}\alpha_{\bar{\Xi}} \\ &- \cos\theta_1\cos\bar{\theta}_1\alpha_\Xi(\cos\bar{\theta}_2\alpha_{\bar{\Lambda}}+\alpha_{\bar{\Xi}}) \\ &+ \cos\theta_2\alpha_\Lambda[\alpha_\Xi(\cos\bar{\theta}_2\alpha_{\bar{\Lambda}}\alpha_{\bar{\Xi}}+1) \\ &- \cos\theta_1\cos\bar{\theta}_1(\cos\bar{\theta}_2\alpha_{\bar{\Lambda}}+\alpha_{\bar{\Xi}})]+1\}|A_{1/2,-1/2}|^2 \\ &+ 4\sin^2\theta\{\cos\bar{\theta}_2\alpha_{\bar{\Lambda}}\alpha_{\bar{\Xi}} \\ &+ \cos\theta_1\cos\bar{\theta}_1\alpha_\Xi(\cos\bar{\theta}_2\alpha_{\bar{\Lambda}}+\alpha_{\bar{\Xi}}) \\ &+ \cos\theta_2\alpha_\Lambda[\cos\theta_1\cos\bar{\theta}_1(\cos\bar{\theta}_2\alpha_{\bar{\Lambda}}+\alpha_{\bar{\Xi}}) \\ &+ \alpha_\Xi(\cos\bar{\theta}_2\alpha_{\bar{\Lambda}}\alpha_{\bar{\Xi}}+1)]+1\}|A_{1/2,1/2}|^2. \end{aligned} \quad (9)$$

This formula is applicable to the sequential decay $e^+e^- \to J/\psi, \psi(2S) \to \Xi^0\bar{\Xi}^0 \to \Lambda\bar{\Lambda}\pi^0\pi^0 \to p\bar{p}\pi^+\pi^-\pi^0\pi^0$. Integrating over the solid angles θ_2 and $\bar{\theta}_2$, the joint angular distribution becomes

$$\begin{aligned}\frac{d|\mathcal{M}|^2}{d\cos\theta d\cos\theta_1 d\cos\bar{\theta}_1} &\propto 4|A_{1/2,1/2}|^2\sin^2\theta(\cos\theta_1\cos\bar{\theta}_1 \\ &\times \alpha_\Xi\alpha_{\bar{\Xi}}+1) - |A_{1/2,-1/2}|^2 \\ &\times (3+\cos2\theta)(\cos\theta_1\cos\bar{\theta}_1 \\ &\times \alpha_\Xi\alpha_{\bar{\Xi}}-1). \end{aligned} \quad (10)$$

The above equation reduces to the angular distribution Eq. (4) when the solid angles of the daughter particles are integrated over.

D. $e^+e^- \to \psi(2S) \to \Omega^-\bar{\Omega}^+ \to K^-K^+\Lambda\bar{\Lambda} \to K^+K^-\pi^+\pi^-p\bar{p}$

Though Ω^- has played a celebrated role in particle physics, the prediction of its $\frac{3}{2}$-spin has not yet been clearly verified. If we assume that Ω is spin-$\frac{3}{2}$, parity violation allows the final states in $\Omega^- \to \Lambda K^-$ to contain a mixture of P and D waves. Therefore, Ω^- asymmetry parameters can be defined similarly to those in Λ nonleptonic decays. The helicity amplitude for $\psi(2S) \to \Omega^-(\omega)\bar{\Omega}^+(\bar{\omega})$ is denoted by $A_{\omega,\bar{\omega}}$; amplitudes for $\Omega^-(\omega) \to \Lambda(\lambda)K^-$ and $\Lambda(\lambda) \to p(\lambda_p)\pi^-$ are denoted by B_λ and C_{λ_p}, and their antiparticle counterparts by $\bar{B}_{\bar{\lambda}}$ and $\bar{C}_{\bar{\lambda}_p}$, respectively. The joint amplitude reads

$$|\mathcal{M}|^2 = \sum_{\substack{\omega_1,\omega_2,\bar{\omega}_1,\bar{\omega}_2,\\ \lambda_1,\lambda_2,\bar{\lambda}_1,\bar{\lambda}_2,\\ \lambda_p,\lambda_{\bar{p}}}} \rho^{(\omega_1-\bar{\omega}_1,\omega_2-\bar{\omega}_2)}(\theta,\phi)A_{\omega_1,\bar{\omega}_1}A^*_{\omega_2,\bar{\omega}_2}B_{\lambda_1}B^*_{\lambda_2} \times C_{\lambda_p}C^*_{\lambda_p}\bar{B}_{\bar{\lambda}_1}\bar{B}^*_{\bar{\lambda}_2}\bar{C}_{\lambda_{\bar{p}}}\bar{C}^*_{\lambda_{\bar{p}}}D^{3/2*}_{\omega_1,\lambda_1}(\Omega_1)D^{3/2}_{\omega_2,\lambda_2}(\Omega_1) \times D^{3/2*}_{\bar{\omega}_1,\bar{\lambda}_1}(\bar{\Omega}_1)D^{3/2}_{\bar{\omega}_2,\bar{\lambda}_2}(\bar{\Omega}_1)D^{1/2*}_{\lambda_1,\lambda_p}(\Omega_2)D^{1/2}_{\lambda_2,\lambda_p}(\Omega_2) \times D^{1/2*}_{\bar{\lambda}_1,\lambda_{\bar{p}}}(\bar{\Omega}_2)D^{1/2}_{\bar{\lambda}_2,\lambda_{\bar{p}}}(\bar{\Omega}_2). \quad (11)$$

Assuming parity invariance in $\psi(2S) \to \Omega^-\bar{\Omega}^+$, one has the relation $A_{-\omega_1-\omega_2} = A_{\omega_1\omega_2}$. In the decay of an unpolarized Ω^- hyperon, the daughter Λ is produced with a longitudinal polarization [10], which leads to an asymmetry in $\Lambda \to p\pi^-$. The angular distribution of the proton is [11]

$$\frac{d|\mathcal{M}|^2}{d\cos\theta} \propto 1 + \alpha_\Omega\alpha_\Lambda\cos\theta_2. \quad (12)$$

Hence, helicity amplitudes of Ω decays can be related to their asymmetry parameters by

$$\alpha_{\Omega^-} = \frac{|B_{1/2}|^2 - |B_{-1/2}|^2}{|B_{1/2}|^2 + |B_{-1/2}|^2}$$

$$\text{and} \quad \alpha_{\bar{\Omega}^+} = \frac{|\bar{B}_{1/2}|^2 - |\bar{B}_{-1/2}|^2}{|\bar{B}_{1/2}|^2 + |\bar{B}_{-1/2}|^2}.$$

After integrating over all azimuthal angles and over the polar angles θ_1 and $\bar{\theta}_1$, one obtains

$$\frac{d|\mathcal{M}|^2}{d\cos\theta d\cos\theta_2 d\cos\bar{\theta}_2} \propto (1 + \alpha_\Lambda \alpha_\Omega \cos\theta_2)(1 + \alpha_{\bar{\Lambda}} \alpha_{\bar{\Omega}} \times \cos\bar{\theta}_2)[4(|A_{1/2,1/2}|^2 + |A_{3/2,3/2}|^2) \times \sin^2\theta + (|A_{1/2,3/2}|^2 + |A_{1/2,-1/2}|^2 + |A_{3/2,1/2}|^2)(3 + \cos 2\theta)]. \tag{13}$$

It can be seen that the above equation can be reduced to the angular distribution for the J/ψ, $\psi(2S) \to B_{10}\bar{B}_{10}$ (B_{10}: decuplet baryon) if the angles θ_2 and $\bar{\theta}_2$ are integrated over:

$$\frac{d|\mathcal{M}|^2}{d\cos\theta} \propto (1 + \alpha\cos^2\theta) \quad \text{with} \quad \alpha = \frac{|A_{1/2,-1/2}|^2 + |A_{1/2,3/2}|^2 + |A_{3/2,1/2}|^2 - 2(|A_{1/2,1/2}|^2 + |A_{3/2,3/2}|^2)}{|A_{1/2,-1/2}|^2 + |A_{1/2,3/2}|^2 + |A_{3/2,1/2}|^2 + 2(|A_{1/2,1/2}|^2 + |A_{3/2,3/2}|^2)}. \tag{14}$$

III. EXPERIMENTAL PROSPECTS

A. Experimental method

For polarized hyperon decays, their decay parameters can be measured by observing the angular distributions of the daughter baryons. This method was previously used by HyperCP, R608, and PS185 collaborations. However, the hyperon $Y(\bar{Y})$ produced in J/ψ, $\psi(2S) \to Y\bar{Y}$ is unpolarized. In this case, the hyperon decay parameter is a hidden parameter, which cannot be determined solely by the measurement of the angular distribution of the decay particle in the hyperon rest system. Because of the constraints of helicity conservation in $J/\psi \to Y\bar{Y}$, the helicities of the hyperon and antihyperon are correlated. The correlation between their decay particles can be used to measure decay parameters. For example, the DM2 collaboration measured $\alpha_\Lambda \alpha_{\bar{\Lambda}}$ in $J/\psi \to \Lambda\bar{\Lambda}$ by measuring the momentum correlation between the proton and antiproton [3], which takes the form $\alpha_\Lambda \alpha_{\bar{\Lambda}} \vec{a} \cdot \vec{b}$, where $\vec{a}$ and $\vec{b}$ are the unit-momentum vector for p and $\bar{p}$ in the Λ and $\bar{\Lambda}$ rest frames, respectively. In this work, we present an unbinned maximum likelihood method to fit the data sample to measure the hyperon decay parameters using helicity amplitude information.

If the probability to produce event i, characterized by the measurements p_i, is $\text{Prob}(p_i)$, then the joint probability density for observing the N events in the data sample with detection efficiency $\epsilon(p_i)$ is

$$\mathcal{L} = \prod_{i=1}^{N} \text{Prob}(p_i) = \prod_{i=1}^{N} \frac{|\mathcal{M}(p_i)|^2 \epsilon(p_i)}{\int dp_i |\mathcal{M}(p_i)|^2 \epsilon(p_i)}, \tag{15}$$

where $\mathcal{M}(p_i)$ is the amplitude of event i. As in partial wave analysis, to determine the parameters in question, one has to minimize the function defined by

$$\mathcal{S} = -\ln\mathcal{L} = -\sum_{i=1}^{N} \ln\left[\frac{|\mathcal{M}(p_i)|^2}{\int dp_i |\mathcal{M}(p_i)|^2 \epsilon(p_i)}\right], \tag{16}$$

where we neglect the term $\ln\epsilon(p_i)$ because it is independent of the hyperon decay parameters.

B. Sensitivity

The sensitivity of the measurement of parameter α is defined by

$$\delta(\alpha) = \frac{\sqrt{V(\alpha)}}{\alpha}, \tag{17}$$

where $V(\alpha)$ is the variance of parameter α. With the joint angular distribution, the evaluation of sensitivity is straightforward by using the maximum likelihood method as shown in [12].

For the measurement of $\alpha_\Lambda \alpha_{\bar{\Lambda}}$ in $J/\psi \to \Lambda\bar{\Lambda} \to \pi^+\pi^- p\bar{p}$ and $\alpha_\Xi \alpha_{\bar{\Xi}}$ in $J/\psi \to \Xi^-\bar{\Xi}^+ \to \pi^+\pi^-\Lambda\bar{\Lambda} \to 2(\pi^+\pi^-)p\bar{p}$, one has

$$\delta(\alpha_Y \alpha_{\bar{Y}}) \approx \frac{1}{\alpha_Y \alpha_{\bar{Y}}} \sqrt{\frac{9(d-2)^2(d+1)\sqrt{d^2-4}}{N\{48d^2 i \tanh^{-1}(\sqrt{\frac{2-d}{2+d}}) + (d+2)[(d-9)d+2]\sqrt{d^2-4}\}}} \quad \text{with} \quad d = \frac{2(1+\alpha)}{1-\alpha}, \tag{18}$$

where N is the number of observed events; α is the angular distribution parameter as defined in Eq. (4).

The decay $J/\psi \to \Sigma^0\bar{\Sigma}^0$ can also be used to measure $\alpha_\Lambda \alpha_{\bar{\Lambda}}$. The sensitivity can be estimated by

$$\delta(\alpha_\Lambda \alpha_{\bar{\Lambda}}) \approx \frac{1}{\alpha_\Lambda \alpha_{\bar{\Lambda}}} \sqrt{\frac{36(d-2)^2(d+1)\sqrt{d^2-4}}{N\pi^2\{48d^2 i\tanh^{-1}(\sqrt{\frac{2-d}{2+d}}) + (d+2)[(d-9)d+2]\sqrt{d^2-4}\}}} \quad \text{with} \quad d = \frac{2(1+\alpha)}{1-\alpha}, \tag{19}$$

where α is the angular distribution parameter for $J/\psi \to \Sigma^0\bar{\Sigma}^0$, and N is the number of observed events.

In $\psi(2S) \to \Omega^-\bar{\Omega}^+$, the sensitivity of $\alpha_{\bar{\Lambda}}\alpha_{\bar{\Omega}}$ can be estimated by

$$\delta(\alpha_{\bar{\Lambda}}\alpha_{\bar{\Omega}}) = \frac{1}{\alpha_{\bar{\Lambda}}\alpha_{\bar{\Omega}}}\sqrt{\frac{3}{N}}, \tag{20}$$

where N is the observed events. If the CP asymmetry observable is defined by

$$A_\Omega = \frac{\alpha_\Lambda\alpha_\Omega - \alpha_{\bar{\Lambda}}\alpha_{\bar{\Omega}}}{\alpha_\Lambda\alpha_\Omega + \alpha_{\bar{\Lambda}}\alpha_{\bar{\Omega}}}, \tag{21}$$

then the sensitivity of the measurement is given by

$$\delta A_\Omega = \frac{1}{\alpha_\Lambda\alpha_\Omega}\sqrt{\frac{3}{2N}}. \tag{22}$$

C. Prospects

$J/\psi \to \Lambda\bar{\Lambda}$ is an ideal place to measure the $\bar{\Lambda}$ decay parameter. Compared with $J/\psi \to \Sigma^0\bar{\Sigma}^0 \to \gamma\gamma\Lambda\bar{\Lambda}$, this decay is very clean and easily reconstructed with high efficiency by selecting four charged tracks, and the $\Lambda/\bar{\Lambda}$ decay length can be used to control the background. If the Λ decay parameter $\alpha_- = \alpha(\Lambda \to p\pi^-)$ is fixed at the world average value, $\alpha_- = 0.642 \pm 0.013$ [2], then the $\bar{\Lambda}$ decay parameter $\alpha_{\bar{\Lambda}}$ can be determined by a fit to the data with the joint angular distribution as shown in Eq. (3).

TABLE I. The $\Lambda\bar{\Lambda}$ reconstruction in $J/\psi \to \Lambda\bar{\Lambda}$ and the measurements of $\bar{\Lambda}$ asymmetry parameters in the product of $\alpha_\Lambda\bar{\alpha}_\Lambda$. In the table, $\alpha_-(\alpha_+)$ and $\alpha_0(\bar{\alpha}_0)$ are the decay parameters for $\Lambda(\bar{\Lambda})$ decays into $p\pi^-(\bar{p}\pi^+)$ and $n\pi^0(\bar{n}\pi^0)$, respectively.

No.	$\Lambda\bar{\Lambda}$ reconstruction	Measurement
I	$\Lambda \to p\pi^-$, $\bar{\Lambda} \to \bar{p}\pi^+$	$\alpha_-\alpha_+$
II	$\Lambda \to p\pi^-$, $\bar{\Lambda} \to \bar{n}\pi^0$	$\alpha_-\bar{\alpha}_0$
III	$\Lambda \to n\pi^0$, $\bar{\Lambda} \to \bar{p}\pi^+$	$\alpha_0\alpha_+$

TABLE II. Evaluation of sensitivity, where the efficiency quotes the BESII experimental information [13], and the measured $\alpha(\bar{\Lambda} \to p\pi^+)$ quotes -0.63 ± 0.13 [3].

Channel $J/\psi \to Y\bar{Y}$	Efficiency ϵ	Parameter α	Sensitivity $\lvert\delta(\alpha)\rvert$ measured	$10^{10} J/\psi$ sample
$\Lambda\bar{\Lambda}$	~20%	$\alpha(\bar{\Lambda} \to \bar{p}\pi^+)$	20%	$\sim 3 \times 10^{-3}$
$\Sigma^0\bar{\Sigma}^0$	~6%	$\alpha(\bar{\Lambda} \to \bar{p}\pi^+)$	20%	~2%
$\Xi^-\bar{\Xi}^+$	~1.4%	$\alpha(\bar{\Xi}^+ \to \bar{\Lambda}\pi^+)$	—	~1%

Table I lists measurements of $\alpha_{\bar{\Lambda}}$ parameters in terms of the $\Lambda\bar{\Lambda}$ reconstruction in different modes. For example, mode I measures $\alpha_+ = \alpha(\bar{\Lambda} \to \bar{p}\pi^+)$; mode II measures $\bar{\alpha}_0 = \alpha(\bar{\Lambda} \to \bar{n}\pi^0)$; then mode III measures $\alpha_0 = \alpha(\Lambda \to n\pi^0)$. With the 10^{10} J/ψ events collected, the sensitivity of α_+ is expected to be about 3×10^{-3} in $J/\psi \to \Lambda\bar{\Lambda}$ and about 2% in $J/\psi \to \Sigma^0\bar{\Sigma}^0 \to \gamma\gamma\Lambda\bar{\Lambda}$, as shown in Table II, which has higher precision than the current measured value, $\alpha(\bar{\Lambda} \to \bar{p}\pi^+) = -0.63 \pm 0.13$ [3].

The asymmetry parameter $\alpha(\bar{\Sigma}^- \to \bar{p}\pi^0)$ can be measured in $J/\psi \to \Sigma^+\bar{\Sigma}^- \to p\bar{p}\pi^0\pi^0$, but the parameter $\alpha(\bar{\Sigma}^+ \to \bar{n}\pi^+)$ is difficult to measure in $J/\psi \to \Sigma^-\bar{\Sigma}^+ \to n\bar{n}\pi^+\pi^-$, since two missing nucleons make the event reconstruction impossible. The sensitivity of the measurement of $\alpha(\bar{\Sigma}^- \to \bar{p}\pi^0)$ can be estimated with Eq. (18). However, experimental information on $J/\psi \to \Sigma^+\bar{\Sigma}^-$ are not available at the present time.

The asymmetry parameter $\alpha(\bar{\Xi}^+ \to \bar{\Lambda}\pi^+)$ can be measured in $J/\psi \to \Xi^-\bar{\Xi}^+ \to \pi^+\pi^-\Lambda\bar{\Lambda} \to 2(\pi^+\pi^-)p\bar{p}$. Experimentally, this channel is easily reconstructed by selecting six charged tracks with very low backgrounds. The parameter $\alpha(\bar{\Xi}^+ \to \bar{\Lambda}\pi^+)$ can be extracted from Eq. (10). The sensitivity of the $\alpha(\bar{\Xi}^+ \to \bar{\Lambda}\pi^+)$ measurement can be estimated by Eq. (18), and is about 1% if 10^{10} J/ψ events are collected, as shown in Table II.

The asymmetry parameter $\alpha_\Lambda\alpha_\Omega$, as well as its antiparticle counterpart $\alpha_{\bar{\Lambda}}\alpha_{\bar{\Omega}}$, can be extracted from the joint angular distribution (Eq. (13)) in $\psi(2S) \to \Omega^-\bar{\Omega}^+ \to K^-K^+\Lambda\bar{\Lambda} \to K^+K^-\pi^+\pi^-p\bar{p}$. The value of $\alpha_\Omega = 0.0175 \pm 0.0024$ [2] is quite small, so the A_Ω measurement needs a larger data sample. Unfortunately, this measurement is limited by the $\psi(2S)$ statistics. To date, no signal events are observed in $\psi(2S) \to \Omega^-\bar{\Omega}^+$ [14].

IV. SUMMARY

The formulae of the joint angular distributions for J/ψ or $\psi(2S)$ decays into $\Lambda\bar{\Lambda}$, $\Sigma^0\bar{\Sigma}^0$, $\Xi^-\bar{\Xi}^+$, and $\Omega^-\bar{\Omega}^+$ are presented. The prospects of measurement on antihyperon asymmetry parameters at a e^+e^- collider are discussed. The estimation of the measurement sensitivity shows that J/ψ decays into $\Lambda\bar{\Lambda}$ and $\Xi^-\bar{\Xi}^+$ can be hopefully used to measure the $\bar{\Lambda}$ and $\bar{\Xi}^+$ decay parameters. With a large data sample, e.g. 10^{10} J/ψ decays, the sensitivities of $\alpha(\bar{\Lambda} \to \bar{p}\pi^+)$ and $\alpha(\bar{\Xi}^+ \to \bar{\Lambda}\pi^+)$ measurements are about 3×10^{-3} and 1%, respectively. The sensitivity to look for CP asymmetry in the Ω^- decays is limited by the $\psi(2S)$ statistics.

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