Bound on minimal universal extra dimensions from $\bar{B} \to X_s \gamma$

Ulrich Haisch[1] and Andreas Weiler[2]

[1]*Institut für Theoretische Physik, Universität Zürich, CH-8057 Zürich, Switzerland*
[2]*Institute for High-Energy Phenomenology Newman Laboratory for Elementary-Particle Physics, Cornell University Ithaca, New York 14853, USA*

(Received 15 March 2007; published 23 August 2007)

We reexamine the constraints on universal extra dimensional models arising from the inclusive radiative $\bar{B} \to X_s \gamma$ decay. We take into account the leading order contributions due to the exchange of Kaluza-Klein modes as well as the available next-to-next-to-leading order corrections to the $\bar{B} \to X_s \gamma$ branching ratio in the standard model. For the case of one large flat universal extra dimension, we obtain a lower bound on the inverse compactification radius $1/R > 600$ GeV at 95% confidence level that is independent of the Higgs mass.

DOI: 10.1103/PhysRevD.76.034014 PACS numbers: 13.20.He, 11.25.Wx, 12.15.Lk, 12.60.−i

The branching ratio of the inclusive radiative $\bar{B}$-meson decay is known to provide stringent constraints on various nonstandard physics models at the electroweak scale, because it is accurately measured and its theoretical determination is rather precise.

The present experimental world average which includes the latest measurements by CLEO [1], Belle [2], and *BABAR* [3] is performed by the Heavy Flavor Averaging Group [4] and reads for a photon energy cut of $E_\gamma > E_0$ with $E_0 = 1.6$ GeV in the $\bar{B}$-meson rest frame

$$\mathcal{B}(\bar{B} \to X_s \gamma)_{\rm exp} = (3.55 \pm 0.24^{+0.09}_{-0.10} \pm 0.03) \times 10^{-4}. \tag{1}$$

Here the first error is a combined statistical and systematic one, while the second and third are systematic uncertainties due to the extrapolation from $E_0 = (1.8–2.0)$ GeV to the reference value and the subtraction of the $\bar{B} \to X_d \gamma$ event fraction, respectively.

After a joint effort [5–7], the first theoretical determination of the total $\bar{B} \to X_s \gamma$ branching ratio at next-to-next-to-leading order (NNLO) QCD has been presented recently in [6,8]. In [9] this fixed-order result has been supplemented with perturbative cut-related $\mathcal{O}(\alpha_s^2)$ corrections [10] and an estimate of enhanced $\Lambda_{\rm QCD}/m_b$ nonlocal power corrections using the vacuum insertion approximation [11]. For $E_0 = 1.6$ GeV the result of the improved standard model (SM) evaluation is given by [12]

$$\mathcal{B}(\bar{B} \to X_s \gamma)_{\rm SM} = (2.98 \pm 0.26) \times 10^{-4}, \tag{2}$$

where the uncertainties from higher-order perturbative effects ($^{+4}_{-6}\%$), hadronic power corrections ($\pm 5\%$), parametric dependencies ($\pm 4\%$), and the interpolation in the charm quark mass ($\pm 3\%$) have been added in quadrature to obtain the total error.

Compared with the experimental world average of Eq. (1), the new SM prediction of Eq. (2) is lower by around 1.4σ. Potential beyond SM contributions should now be preferably constructive, while models that lead to a suppression of the $b \to s\gamma$ amplitude are more severely constrained than in the past, where the theoretical determination used to be above the experimental one.

Among the scenarios of the latter category is the model of Appelquist, Cheng, and Dobrescu (ACD) [13] as emphasized in [14,15]. In the ACD framework the SM is extended from four-dimensional Minkowski space-time to five dimensions and the extra space dimension is compactified on the orbifold S^1/Z_2 in order to obtain chiral fermions in four dimensions. The five-dimensional fields can equivalently be described in a four-dimensional Lagrangian with heavy Kaluza-Klein (KK) states for every field that lives in the fifth dimension or bulk. In the ACD model all SM fields are promoted to the bulk. The orbifold compactification breaks KK number conservation, but preserves KK parity. This property implies that KK states can only be pair produced, that their virtual effect comes only from loops, and causes the lightest KK particle (LKP) to be stable, therefore providing a viable dark matter (DM) candidate [16] with promising prospects for direct and indirect detection. See [17] for a recent review on the DM and collider phenomenology of the ACD model.

The full Lagrangian of the ACD model includes both bulk and boundary terms. The bulk Lagrangian is determined by the SM parameters after an appropriate rescaling. The coefficients of the boundary terms, however, although volume suppressed, are free parameters and will get renormalized by bulk interactions. Flavor nonuniversal boundary terms would lead to unacceptably large flavor-changing neutral currents (FCNCs). In the following we will assume vanishing boundary terms at the cutoff scale and that the ultraviolet completion does not introduce additional sources of flavor and *CP* violation beyond the ones already present in the model. These additional assumptions define the minimal universal extra dimension (mUED) model which belongs to the class of constrained minimal-flavor-violating [18] scenarios. With this choice contributions from boundary terms are of higher order [19]

 © 2007 The American Physical Society

and one only has to consider the bulk Lagrangian in leading order (LO) calculations in the ACD model.

Since at LO the $b \to s\gamma$ amplitude turns out to be cutoff independent [15] the only additional parameter entering the mUED prediction of $\mathcal{B}(\bar{B} \to X_s\gamma)$ relative to the SM is the inverse of the compactification radius $1/R$. For a light Higgs mass of $m_h = 115$ GeV a careful analysis of oblique corrections [20] gives a lower bound of $1/R > 600$ GeV at 90% confidence level (CL), well above current collider limits of $1/R \gtrsim 300$ GeV [21]. With increasing Higgs mass the former constraint relaxes significantly leading to $1/R > 300$ GeV at 90% CL for $m_h = 500$ GeV [20]. Other constraints on $1/R$ that derive from the $Z \to b\bar{b}$ pseudo observables [22], the muon anomalous magnetic moment $(g-2)_\mu$ [23], and several FCNC processes [15,24,25] are with $1/R \gtrsim (200\text{–}250)$ GeV in general weaker.

Values of $1/R$ as low as 300 GeV would also lead to an exciting phenomenology in the next generation of colliders [17,21] and could be of interest in connection with DM searches [16]. Collider measurements alone do not place an upper bound on $1/R$. LKPs would overclose the Universe for $1/R \gtrsim 1.5$ TeV [16], providing motivation for considering weak-scale KK particles.

The purpose of this paper is to point out that combining the present experimental world average with the improved SM prediction of $\mathcal{B}(\bar{B} \to X_s\gamma)$ forces the compactification scale $1/R$ of the mUED model to lie above 600 GeV if errors are treated as Gaussian. This 95% CL exclusion bound is independent of the Higgs mass and therefore stronger than the constraint that follows from electroweak precision data. The possibility to derive such a powerful bound has already been anticipated in [15].

The mUED prediction of $\mathcal{B}(\bar{B} \to X_s\gamma)$ for $E_0 = 1.6$ GeV as a function of $1/R$ is displayed by the dark gray (red) band in Fig. 1. The light gray (yellow) and medium gray (green) band in the same figure shows the experimental and SM result as given in Eqs. (1) and (2), respectively. In all three cases, the middle line is the central value, while the widths of the bands indicate the uncertainties that one obtains by adding individual errors in quadrature. The strong suppression of $\mathcal{B}(\bar{B} \to X_s\gamma)$ in the mUED model with respect to the SM expectation [14,15] and the slow decoupling of KK modes is clearly seen in Fig. 1.

In our numerical analysis, matching of the mUED Wilson coefficients at the electroweak scale is complete up to the LO [15], while terms beyond that order include SM contributions only. The LO mUED matching correction to the $b \to s\gamma$ amplitude is found from the one-loop diagrams that can be seen in Fig. 2. The shown Feynman graphs have been calculated first in [15]. They contain apart from the ordinary SM fields, infinite towers of the KK modes corresponding to the W-boson, $W^\pm_{(k)}$, the pseudo Goldstone boson, $G^\pm_{(k)}$, the $SU(2)$ quark doublets, $\mathcal{Q}_{q(k)}$,

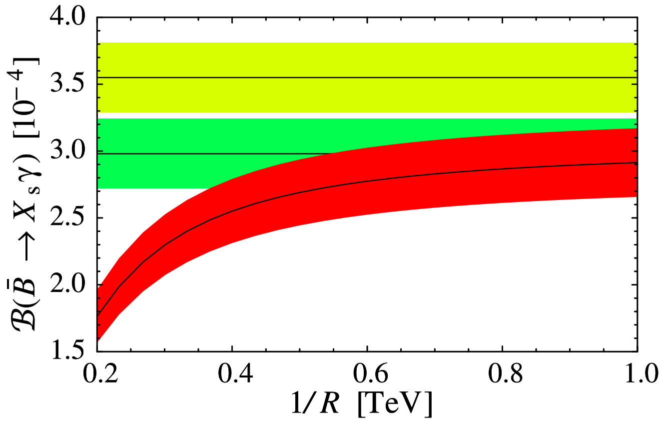

FIG. 1 (color online). $\mathcal{B}(\bar{B} \to X_s\gamma)$ for $E_0 = 1.6$ GeV as a function of $1/R$. The dark gray (red) band corresponds to the LO mUED result. The 68% CL range and central value of the experimental/SM result is indicated by the light/medium gray (yellow/green) band underlying the straight solid line. See text for details.

and singlets, $\mathcal{U}_{q(k)}$. Additionally, there appears a charged scalar, $a^\pm_{(k)}$, which has no counterpart in the SM. The uncertainty related to higher orders in the mUED model is estimated by varying the matching scale between 80 and 320 GeV. It does not exceed $^{+8}_{-9}\%$ for $1/R$ in the range of 0.2 and 1.5 TeV. Whether this provides a reliable estimate of the next-to-leading order (NLO) QCD corrections to $\mathcal{B}(\bar{B} \to X_s\gamma)$ in the mUED model can only be seen by performing a complete two-loop matching involving KK gluon corrections. Such a calculation seems worthwhile but it is beyond the scope of this paper.

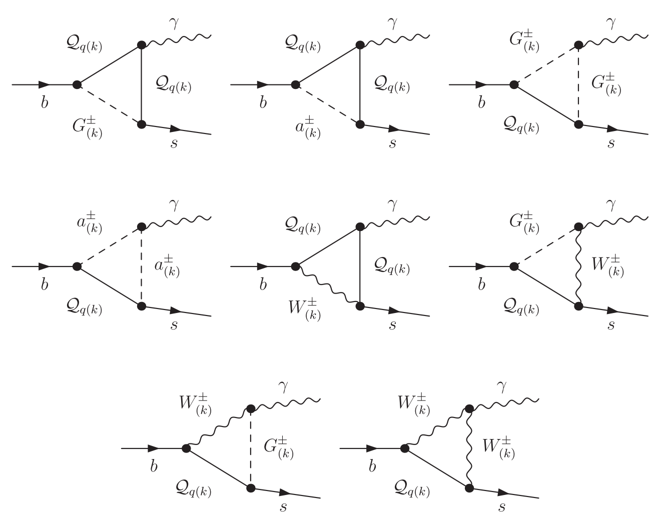

FIG. 2. One-loop corrections to the $b \to s\gamma$ amplitude in the mUED model involving infinite towers of the KK modes. Diagrams where the $SU(2)$ quark doublets $\mathcal{Q}_{q(k)}$ are replaced by the $SU(2)$ quark singlets $\mathcal{U}_{q(k)}$ are not shown. See text for details.

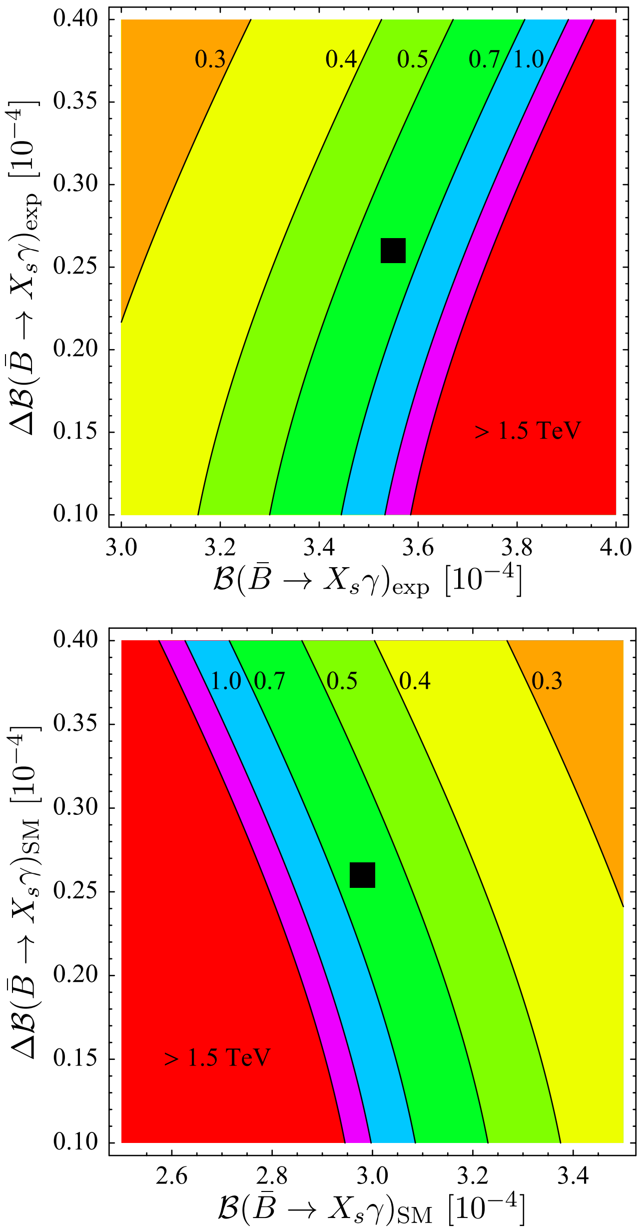

FIG. 3 (color online). The upper/lower panel displays the 95% CL limits on $1/R$ as a function of the experimental/SM central value (horizontal axis) and total error (vertical axis). The experimental/SM result from Eq. (1)/Eq. (2) is indicated by the black square. The contour lines represent values that lead to the same bound in TeV. See text for details.

Since the experimental result is at the present above the SM one and KK modes in the mUED model interfere destructively with the SM $b \to s\gamma$ amplitude, the lower bound on $1/R$ following from $\mathcal{B}(\bar{B} \to X_s\gamma)$ turns out to be stronger than what one can derive from any other currently available measurement. If all the uncertainties are treated as Gaussian and combined in quadrature, the 95% (99%) CL bound amounts to 600 (330) GeV. In contrast to the limit coming from electroweak precision measurements the latter exclusion is almost independent of the Higgs mass because genuine electroweak effects related to Higgs exchange enter $\mathcal{B}(\bar{B} \to X_s\gamma)$ first at the two-loop level. In the SM these corrections have been calculated [26] and amount to around -1.5% in the branching ratio. They are included in Eq. (2). Neglecting the corresponding two-loop Higgs effects in the mUED model calculation should therefore have practically no influence on the derived limits.

The upper (lower) contour plot in Fig. 3 shows the 95% CL bound of $1/R$ as a function of the experimental (SM) central value and error. The current experimental world average and SM prediction of Eqs. (1) and (2) are indicated by the black squares. These plots allow to monitor the effect of future improvements in both the measurements and the SM prediction. One should keep in mind, of course, that the derived bounds depend in a non-negligible way on the treatment of theoretical uncertainties. Furthermore, the found limits could be weakened by the NLO QCD matching corrections in the mUED model which remain unknown.

To conclude, we have pointed out that combining the present experimental with the improved standard model result for the branching ratio of the inclusive $\bar{B} \to X_s\gamma$ decay implies that the inverse compactification radius of the minimal universal extra dimension model has to satisfy $1/R > 600$ GeV at 95% confidence level if all the uncertainties are treated as Gaussian. This lower bound is independent from the Higgs mass and therefore stronger than the limits that can be derived from any other currently available measurement. This underscores the outstanding role of the inclusive radiative $\bar{B}$-meson decay in searches for new physics close to the electroweak scale.

We are grateful to Andrzej Buras for suggesting the topic, for his careful reading of the manuscript and his valuable comments. Enlightening discussions with Einan Gardi concerning the calculation of the differential $\bar{B} \to X_s\gamma$ decay rate using dressed gluon exponentiation [27] is acknowledged. We finally would like to thank Ignatios Antoniadis for bringing [28] to our attention and for his interest in our article. This work has been supported in part by the Schweizer Nationalfonds and the National Science Foundation under Grant No. PHY-0355005.

[1] S. Chen *et al.* (CLEO Collaboration), Phys. Rev. Lett. **87**, 251807 (2001).
[2] P. Koppenburg *et al.* (Belle Collaboration), Phys. Rev. Lett. **93**, 061803 (2004).
[3] B. Aubert *et al.* (*BABAR* Collaboration), Phys. Rev. Lett. **97**, 171803 (2006).
[4] E. Barbiero *et al.* (Heavy Flavor Averaging Group), arXiv:hep-ex/0603003.
[5] K. Bieri, C. Greub, and M. Steinhauser, Phys. Rev. D **67**, 114019 (2003); M. Misiak and M. Steinhauser, Nucl. Phys. **B683**, 277 (2004); M. Gorbahn and U. Haisch, Nucl. Phys. **B713**, 291 (2005); M. Gorbahn, U. Haisch, and M. Misiak, Phys. Rev. Lett. **95**, 102004 (2005); K. Melnikov and A. Mitov, Phys. Lett. B **620**, 69 (2005); I. Blokland *et al.*, Phys. Rev. D **72**, 033014 (2005); H. M. Asatrian *et al.*, Nucl. Phys. **B749**, 325 (2006); **B762**, 212 (2007).
[6] M. Misiak and M. Steinhauser, Nucl. Phys. **B764**, 62 (2007).
[7] M. Czakon, U. Haisch, and M. Misiak, J. High Energy Phys. 03 (2007) 008.
[8] M. Misiak *et al.*, Phys. Rev. Lett. **98**, 022002 (2007).
[9] T. Becher and M. Neubert, Phys. Rev. Lett. **98**, 022003 (2007).
[10] T. Becher and M. Neubert, Phys. Lett. B **633**, 739 (2006); **637**, 251 (2006).
[11] S. J. Lee, M. Neubert, and G. Paz, Phys. Rev. D **75**, 114005 (2007).
[12] The small NNLO corrections related to the four-loop $b \to sg$ mixing diagrams [7] and from the charm quark mass effects of the electromagnetic dipole operator contribution [29] is not included in the numerical result.
[13] T. Appelquist, H. C. Cheng, and B. A. Dobrescu, Phys. Rev. D **64**, 035002 (2001).
[14] K. Agashe, N. G. Deshpande, and G. H. Wu, Phys. Lett. B **514**, 309 (2001).
[15] A. J. Buras *et al.*, Nucl. Phys. **B678**, 455 (2004).
[16] G. Servant and T. M. P. Tait, Nucl. Phys. **B650**, 391 (2003); H. C. Cheng, J. L. Feng, and K. T. Matchev, Phys. Rev. Lett. **89**, 211301 (2002).
[17] D. Hooper and S. Profumo, arXiv:hep-ph/0701197.
[18] A. J. Buras *et al.*, Phys. Lett. B **500**, 161 (2001); G. D'Ambrosio *et al.*, Nucl. Phys. **B645**, 155 (2002); M. Blanke *et al.*, J. High Energy Phys. 10 (2006) 003.
[19] Boundary terms arise radiatively [30]. They affect the $b \to s\gamma$ amplitude first at the two-loop level. Since we perform a LO analysis of $\mathcal{B}(\bar{B} \to X_s\gamma)$ in the mUED model it is consistent to neglect these effects.
[20] I. Gogoladze and C. Macesanu, Phys. Rev. D **74**, 093012 (2006).
[21] C. Macesanu, C. D. McMullen, and S. Nandi, Phys. Rev. D **66**, 015009 (2002); Phys. Lett. B **546**, 253 (2002).
[22] J. F. Oliver, J. Papavassiliou, and A. Santamaria, Phys. Rev. D **67**, 056002 (2003).
[23] T. Appelquist and B. A. Dobrescu, Phys. Lett. B **516**, 85 (2001).
[24] A. J. Buras, M. Spranger, and A. Weiler, Nucl. Phys. **B660**, 225 (2003).
[25] P. Colangelo *et al.*, Phys. Rev. D **73**, 115006 (2006); **74**, 115006 (2006).
[26] P. Gambino and U. Haisch, J. High Energy Phys. 09 (2000) 001; 10 (2001) 020.
[27] J. R. Andersen and E. Gardi, J. High Energy Phys. 01 (2007) 029.
[28] I. Antoniadis, Phys. Lett. B **246**, 377 (1990).
[29] H. M. Asatrian *et al.*, Phys. Lett. B **647**, 173 (2007).
[30] H. Georgi, A. K. Grant, and G. Hailu, Phys. Lett. B **506**, 207 (2001); H. C. Cheng, K. T. Matchev, and M. Schmaltz, Phys. Rev. D **66**, 036005 (2002).