

Remark on the minimal seesaw model and leptogenesis with tribimaximal mixing

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We have studied a leptogenesis scenario in the framework of the minimal seesaw model with tribimaximal mixing. Usually, at least one of the elements in the Dirac mass matrix is fixed to be zero, for example, we denote it by $b_2 = 0$. We have pointed out that the absolute value of the CP asymmetry has several minimums and maximums with nonzero b_2 . Thus one can expect that more rich phenomena, such as an enhanced leptogenesis, are hidden in the $b_2 \neq 0$ space.

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I. INTRODUCTION

The recent neutrino oscillation experiments have provided us with robust evidence that neutrinos have tiny masses and their flavor mixing involves two large angles and one small angle [1]. A global analysis of current neutrino oscillation data yields $7.2 \times 10^{-5} \text{ eV}^2 \leq \Delta m_s^2 \leq 8.9 \times 10^{-5} \text{ eV}^2$ and $1.7 \times 10^{-3} \text{ eV}^2 \leq \Delta m_a^2 \leq 3.3 \times 10^{-3} \text{ eV}^2$ for the squared mass differences of solar and atmospheric neutrinos and $30^\circ \leq \theta_{12} \leq 38^\circ$, $36^\circ \leq \theta_{23} \leq 54^\circ$, and $0^\circ \leq \theta_{13} < 10^\circ$ for the flavor mixing angles at the 99% confidence level (the best-fit values are $\Delta m_s^2 = 8.0 \times 10^{-5} \text{ eV}^2$, $\Delta m_a^2 = 2.5 \times 10^{-3} \text{ eV}^2$, $\theta_{12} = 34^\circ$, $\theta_{23} = 45^\circ$, and $\theta_{13} = 0^\circ$) [2], where we define $\Delta m_s^2 = m_2^2 - m_1^2$ and $\Delta m_a^2 = |m_3^2 - m_2^2|$ with the neutrino mass eigenvalues m_1 , m_2 , and m_3 .

There are two important questions: (1) why are the two neutrino mixing angles large, almost maximal, and is the one angle small? and (2) why are the neutrino masses so tiny? Tribimaximal mixing is the one of the lepton mixing ansatz to answer the first question, which is characterized by the following mixing matrix [3]

$$U = \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}. \quad (1)$$

The texture of this matrix consists of $\sin\theta_{23} = 1/\sqrt{2}$, $\sin\theta_{12} = 1/\sqrt{3}$, and $\sin\theta_{13} = 0$. Thus, the tribimaximal mixing matrix could approximately describe the best-fit values of the current neutrino oscillation data: $\theta_{23} = 45^\circ$, $\theta_{12} = 34^\circ$, and $\theta_{13} = 0^\circ$. Recently, many theoretical efforts have been made to produce the tribimaximal mixing pattern arising from some relevant symmetries [4].

On the other hand, an attractive idea to answer the second question is given by the seesaw mechanism [5] which explains the smallness of the neutrino masses through the existence of right-handed Majorana neutrinos.

Among many realistic seesaw models existing in the literature, the minimal seesaw model contains only two right-handed Majorana neutrinos, thus it is the most economical one [6]. Also, the seesaw mechanism provides a very natural explanation of the baryon asymmetry in the Universe through the baryogenesis via leptogenesis scenarios [7]. The baryon asymmetry has been measured by the Wilkinson Microwave Anisotropy Probe (WMAP) as a baryon-photon ratio $\eta_B = (6.1_{-0.2}^{+0.3}) \times 10^{-10}$ [8].

The leptogenesis scenarios in the minimal seesaw model have been studied by many groups [9]. However, as mentioned below, if we try to analytically solve the Dirac neutrino mass matrix m_D in terms of the observed element of the light neutrino mass matrix, it is sometimes assumed that at least one of the elements in m_D should be fixed to a constant or we should assume an equality between two or more entries in the matrix m_D . For example, some groups have estimated the baryon asymmetry with the following Dirac mass matrix

$$m_D = \begin{pmatrix} A_1 & B_1 \\ A_2 & B_2 \\ A_3 & B_3 \end{pmatrix} = \begin{pmatrix} * & * \\ * & 0 \\ * & * \end{pmatrix}, \quad (2)$$

which is the so-called texture one-zero mass matrix. However, a natural question is: what happens if the B_2 element deviates from zero? This is our question in this paper.

II. MINIMAL SEESAW MODEL

We show the brief review of the minimal seesaw model. The minimal seesaw Lagrangian can be written as

$$L = -\bar{\ell}_L M_L \ell_R - \bar{\nu}_L m_D N_R + \frac{1}{2} \bar{N}^c M_R N_R + \text{H.c.}, \quad (3)$$

where $\nu_L = (\nu_e, \nu_\mu, \nu_\tau)^T$, $\ell_L = (e, \mu, \tau)^T$ and $N_R = (N_1, N_2)^T$ denote the left-handed (light) neutrinos, the left-handed charged leptons, and the right-handed (heavy) neutrinos, respectively. We have assumed that the mass matrices of both the heavy neutrinos $M_R = \text{diag}(M_1, M_2)$

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and the charged lepton $M_\ell = \text{diag}(m_e, m_\mu, m_\tau)$ are diagonal and real. The Dirac neutrino mass matrix m_D is a 3×2 rectangular matrix, which can be written by 6 parameters $(a_1, a_2, a_3, b_1, b_2, b_3)$ as [10]

$$m_D = \begin{pmatrix} \sqrt{M_1}a_1 & \sqrt{M_2}b_1 \\ \sqrt{M_1}a_2 & \sqrt{M_2}b_2 \\ \sqrt{M_1}a_3 & \sqrt{M_2}b_3 \end{pmatrix}. \quad (4)$$

Via the well-known seesaw mechanism, we can obtain the symmetric 3×3 Majorana mass matrix for light neutrinos as $M_\nu = -m_D M_R^{-1} m_D^T$ and which yields the following equation

$$M_\nu \equiv \begin{pmatrix} M_{11} & M_{12} & M_{13} \\ & M_{22} & M_{23} \\ & & M_{33} \end{pmatrix} = \begin{pmatrix} a_1^2 + b_1^2 & a_1 a_2 + b_1 b_2 & a_1 a_3 + b_1 b_3 \\ & a_2^2 + b_2^2 & a_2 a_3 + b_2 b_3 \\ & & a_3^2 + b_3^2 \end{pmatrix}. \quad (5)$$

This equation contains 6 parameters, however, we have only 5 independent conditions, the sixth condition is redundant because of $\det(M_\nu) = 0$ [6]. Therefore, if we choose one of the elements in the Dirac mass matrix $(a_1, a_2, a_3, b_1, b_2, b_3)$ as a constant term, we can analytically solve 5 parameters out of 6 of m_D in terms of the element of the light neutrino mass matrix. For $M_{11} \neq 0$, the solution in terms of a_1 or b_1 is

$$\begin{aligned} a_1 &= \sqrt{M_{11} - b_1^2} \quad \text{or} \quad b_1 = \sqrt{M_{11} - a_1^2}, & a_2 &= \frac{1}{M_{11}} [a_1 M_{12} - \sigma_2 b_1 \sqrt{M_{11} M_{22} - M_{12}^2}], \\ a_3 &= \frac{1}{M_{11}} [a_1 M_{13} - \sigma_3 b_1 \sqrt{M_{11} M_{33} - M_{13}^2}], & b_2 &= \frac{1}{M_{11}} [b_1 M_{12} + \sigma_2 a_1 \sqrt{M_{11} M_{22} - M_{12}^2}], \\ b_3 &= \frac{1}{M_{11}} [b_1 M_{13} + \sigma_3 a_1 \sqrt{M_{11} M_{33} - M_{13}^2}], & M_{23} &= \frac{1}{M_{11}} [M_{12} M_{13} + \sigma_2 \sigma_3 \sqrt{(M_{11} M_{22} - M_{12}^2)(M_{11} M_{33} - M_{13}^2)}], \end{aligned} \quad (6)$$

where $\sigma_{2,3} = \pm 1$ and $\text{sgn}(a_1) = +1$ [10,11]. The last equation is a consistency condition due to $\det(M_\nu) = 0$ and we can use it to fix the sign of $\sigma_2 \sigma_3$. The solution in terms of a_2 or b_2 is obtained under the exchanges $a_1 \leftrightarrow a_2, b_1 \leftrightarrow b_2, M_{11} \leftrightarrow M_{22}, M_{13} \leftrightarrow M_{23}, \sigma_1 \leftrightarrow \sigma_2$, for $M_{22} \neq 0$, and $\text{sgn}(a_2) = +1$ with $\sigma_{1,3} = \pm 1$. Similarly, under the transformations $a_1 \leftrightarrow a_3, b_1 \leftrightarrow b_3, M_{11} \leftrightarrow M_{33}, M_{12} \leftrightarrow M_{23}, \sigma_1 \leftrightarrow \sigma_3$ with $\sigma_{2,1} = \pm 1$, we obtain the solutions in terms of a_3 or b_3 for $M_{33} \neq 0$ and $\text{sgn}(a_3) = +1$.

The light neutrino mass matrix M_ν is also calculated to be $M_\nu = U^* M^{\text{diag}} U^\dagger$ where $M^{\text{diag}} = \text{diag}(m_1, m_2, m_3)$ is the diagonal neutrino mass matrix and U is the Maki-Nakagawa-Sakata mixing matrix which can be parametrized by

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \begin{pmatrix} e^{i\alpha_1/2} & 0 & 0 \\ 0 & e^{i\alpha_2/2} & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad (7)$$

where $c_{ij} = \cos\theta_{ij}$ and $s_{ij} = \sin\theta_{ij}$ ($i, j = 1, 2, 3$). There are two types of the leptonic CP violation phases: one is the Dirac phase δ and the other are the two Majorana phases α_1, α_2 if neutrinos are Majorana particle [12]. Now, we can reconstruct the Dirac mass matrix elements $(a_1, a_2, a_3, b_1, b_2, b_3)$ in terms of the low energy neutrino observables $m_1, m_2, m_3, \theta_{12}, \theta_{23}, \theta_{13}$ and CP violating phases $\delta, \alpha_1, \alpha_2$ with heavy neutrino masses M_1, M_2 .

Moreover, in the framework of the minimal seesaw model, the $\det(M_\nu) = 0$ condition holds if one of the neutrino mass eigenvalues (m_1, m_2, m_3) is exactly zero [6]. As a result, the experimental results of the square mass differences can be explained by the two types of mass hierarchies. One is called the normal mass hierarchy: $(m_1, m_2, m_3) = (0, \sqrt{\Delta m_s^2}, \sqrt{\Delta m_s^2 + \Delta m_a^2})$, and the other is

called the inverted mass hierarchy: $(m_1, m_2, m_3) = (\sqrt{\Delta m_a^2 - \Delta m_s^2}, \sqrt{\Delta m_a^2}, 0)$.

III. LEPTOGENESIS

We give an outline of the leptogenesis. The baryon-photon ratio η_B can be translated into the baryon asymmetry $Y_B = (n_B - n_{\bar{B}})/s$ as $\eta_B = 7.04 Y_B$, and in leptogenesis scenario, this baryon asymmetry produced via the sphaleron process is related to the lepton asymmetry Y_L [13]:

$$Y_B = -\frac{8N + 4m}{14N + 9m} Y_L, \quad (8)$$

where N is the number of generation of fermions and m is

the number of Higgs doublets. In the particle contents of the standard model, $N = 3$ and $m = 1$ thus $Y_B \simeq -0.549Y_L$.

The lepton asymmetry Y_L is parametrized by three term, the dilution factor d , the CP -asymmetry parameter ϵ , and the effective number of the relativistic degree of freedom g_*

$$Y_L = d \frac{\epsilon}{g_*}, \quad (9)$$

where, for instance, we have $g_* \simeq 106.75$ in the standard model. The dilution factor d describes the washout of the lepton asymmetry due to the various lepton number violating processes. We should solve the full Boltzmann equations to determine the magnitude of the dilution factor exactly; however, we can use good analytical approximation by Nielsen and Takanishi in Ref. [14]

$$d \sim \frac{1}{2.0\sqrt{K^2 + 9}} \quad (10)$$

for $0 \leq K \leq 10$ and

$$d \sim \frac{0.3}{K(\ln K)^{0.6}} \quad (11)$$

for $10 \leq K \leq 10^6$, where

$$K = \frac{M_{\text{pl}}}{1.66\sqrt{g_*}(8\pi v^2)} \frac{(m_D^\dagger m_D)_{11}}{M_1} \simeq \frac{1}{10^{-3} \text{ eV}} (|a_1|^2 + |a_2|^2 + |a_3|^2) \quad (12)$$

with the Planck mass $M_{\text{pl}} \simeq 1.22 \times 10^{19}$ GeV and the vacuum expectation value of the Higgs particle $v \simeq 174$ GeV.

The CP asymmetry ϵ has generated by the decay processes of the heavy Majorana neutrino:

$$\epsilon = \sum_{i=1,2} \epsilon_i = \sum_{i=1,2} \frac{\Gamma(N_i \rightarrow \ell H) - \Gamma(N_i \rightarrow \bar{\ell} H^*)}{\Gamma(N_i \rightarrow \ell H) + \Gamma(N_i \rightarrow \bar{\ell} H^*)}, \quad (13)$$

where H denotes the Higgs doublet in the standard model.

Now, we assume a hierarchical mass pattern of the heavy neutrinos $M_1 \ll M_2$. In this case, the interactions of N_1 can be in thermal equilibrium when N_2 decays and the asymmetry caused by the N_2 decay is washed out by the lepton number violating processes with N_1 . Thus, only the decays of N_1 are relevant for the generation of the final lepton asymmetry $\epsilon \simeq \epsilon_1$. In this case, the CP asymmetry parameter in the minimal seesaw model is calculated to be [9]

$$\begin{aligned} \epsilon &= \frac{1}{8\pi v^2} \frac{\text{Im}[(m_D^\dagger m_D)_{12}^2]}{(m_D^\dagger m_D)_{11}} f\left(\frac{M_2}{M_1}\right) \\ &= \frac{M_2}{8\pi v^2} \frac{\text{Im}[(a_1^* b_1 + a_2^* b_2 + a_3^* b_3)^2]}{|a_1|^2 + |a_2|^2 + |a_3|^2} f\left(\frac{M_2}{M_1}\right), \end{aligned} \quad (14)$$

where the function $f(x)$ is given by

$$f(x) = x \left[1 - (1 + x^2) \ln \left(\frac{1 + x^2}{x^2} \right) + \frac{1}{1 - x^2} \right]. \quad (15)$$

IV. b_2 DEPENDENCE TO η_B

As we have already addressed, the leptogenesis scenarios in the minimal seesaw model has been studied by many groups in the case that at least one of the elements in the Dirac neutrino mass matrix are fixed to zero, such as the $b_2 = 0$ case:

$$m_D = \begin{pmatrix} \sqrt{M_1} a_1 & \sqrt{M_2} b_1 \\ \sqrt{M_1} a_2 & 0 \\ \sqrt{M_1} a_3 & \sqrt{M_2} b_3 \end{pmatrix}. \quad (16)$$

Then, from Eq. (5), the low energy neutrino mass matrix M_ν can be written by five parameters (a_1, a_2, a_3, b_1, b_3) as

$$\begin{pmatrix} a_1^2 + b_1^2 & a_1 a_2 & a_1 a_3 + b_1 b_3 \\ & a_2^2 & a_2 a_3 \\ & & a_3^2 + b_3^2 \end{pmatrix} \quad (17)$$

and we can analytically explain these five parameters in terms of the five independent elements in the low energy neutrino mass matrix.

However, to solve the Dirac mass matrix we can take not only $b_2 = 0$ but also $b_2 = X$ where X is an any constant,

$$m_D = \begin{pmatrix} \sqrt{M_1} a_1 & \sqrt{M_2} b_1 \\ \sqrt{M_1} a_2 & \sqrt{M_2} X \\ \sqrt{M_1} a_3 & \sqrt{M_2} b_3 \end{pmatrix}. \quad (18)$$

There are six nonvanishing entries but there are only five independent parameters. Then, the low energy neutrino mass matrix can be written by five parameters (a_1, a_2, a_3, b_1, b_3) as

$$\begin{pmatrix} a_1^2 + b_1^2 & a_1 a_2 + b_1 X & a_1 a_3 + b_1 b_3 \\ & a_2^2 + X^2 & a_2 a_3 + X b_3 \\ & & a_3^2 + b_3^2 \end{pmatrix}. \quad (19)$$

This is same situation of the $b_2 = 0$ case and we can analytically obtain these five parameters in terms of the five independent elements in M_ν again. Thus, $b_2 = 0$ is just a special case of the $b_2 = X$, e.g., both the $b_2 = 0$ case and $b_2 \neq 0$ case such as $b_2 = 1, 2, 3, \dots$ are in the same ground for reconstruction of the Dirac mass matrix elements in terms of the low energy observables.

In this section, we ask the question: what happens if the b_2 element deviates from zero? To answer this question, we estimate the b_2 dependence to the baryon-photon ratio η_B by numerical calculations.

First, we choose the b_2 range. The solution of the elements in the Dirac mass matrix in the term of b_2 is obtained as $a_2 = \sqrt{M_{22} - b_2^2}$, etc. Thus, we take $b_2 = r\sqrt{M_{22}}$ with a parameter r and use the following parametrization:

$$\begin{aligned}
a_2 &= \sqrt{M_{22} - b_2^2}, & b_2 &= r\sqrt{M_{22}}, \\
a_1 &= \frac{1}{M_{22}}[a_2 M_{12} - \sigma_1 b_2 \sqrt{M_{22} M_{11} - M_{12}^2}], \\
a_3 &= \frac{1}{M_{22}}[a_2 M_{23} - \sigma_3 b_2 \sqrt{M_{22} M_{33} - M_{23}^2}], \\
b_1 &= \frac{1}{M_{22}}[b_2 M_{12} + \sigma_1 a_2 \sqrt{M_{22} M_{11} - M_{12}^2}], \\
b_3 &= \frac{1}{M_{22}}[b_2 M_{23} + \sigma_3 a_2 \sqrt{M_{22} M_{33} - M_{23}^2}], \\
M_{13} &= \frac{1}{M_{22}}[M_{12} M_{23} \\
&\quad + \sigma_1 \sigma_3 \sqrt{(M_{22} M_{11} - M_{12}^2)(M_{22} M_{33} - M_{23}^2)}].
\end{aligned} \tag{20}$$

The parameter r is to be varied in the range of $0 \leq r \leq 1$, where $r = 0$ means the $b_2 = 0$ case while $r = 1$ means the $a_2 = 0$ case.

For mixing angles, we assume the tribimaximal mixing pattern, e.g., $\sin\theta_{23} = 1/\sqrt{2}$, $\sin\theta_{12} = 1/\sqrt{3}$, and $\sin\theta_{13} = 0$ to be consistent with the observed properties of neutrinos. Thus, in our analysis, the Dirac CP phase δ will not be active. Also, we assume the light neutrinos have the normal hierarchical mass pattern with $\Delta m_s^2 = 7.0 \times 10^{-5} \text{ eV}^2$ and $\Delta m_a^2 = 2.5 \times 10^{-3} \text{ eV}^2$. In this case, only one Majorana phase α_2 in Eq. (7) is physically relevant. Finally, we assume the magnitude of the heavier neutrino mass $M_2 = 10M_1$ and use $g_* = 106.75$.

We show the numerical result in the figures. The main result in this study is shown in Figs. 5 and 7. Other figures are shown to explain the behavior of the main result or to check the consistency of our analysis.

Figure 1 shows the M_1 dependence to the baryon-photon ratio η_B in the case of $r = 0$, or equivalently $b_2 = 0$. This $r = 0$ case was first estimated by Chang, Kang, and Siyeon in Ref. [10]. Our result is consistent with Chang's solution and we have also confirmed that the baryon-photon ratio η_B increases with the mass of heavy neutrino M_1 . This M_1 dependence to the baryon-photon ratio η_B can be understood by the following fact. The function $f(x) \sim -3/(2x)$ for the large value of $x = M_2/M_1$, thus the CP asymmetry parameter ϵ , becomes

$$\epsilon \sim \frac{3}{16\pi v^2} M_1 \tilde{\epsilon}, \quad \tilde{\epsilon} \equiv \frac{\text{Im}[(a_1^* b_1 + a_2^* b_2 + a_3^* b_3)^2]}{|a_1|^2 + |a_2|^2 + |a_3|^2}. \tag{21}$$

Since $\tilde{\epsilon}$ is independent of M_1 , ϵ is nearly proportional to the mass of the heavy neutrino $\epsilon \propto M_1$.

Figure 2 shows the CP asymmetry ϵ as a function of r with $M_1 = 10^{12} \text{ GeV}$ and $\alpha_2 = 90^\circ$. The horizontal line shows $\epsilon = 0$. Note that in the small and large r region the CP asymmetry ϵ becomes negative. Thus, the absolute

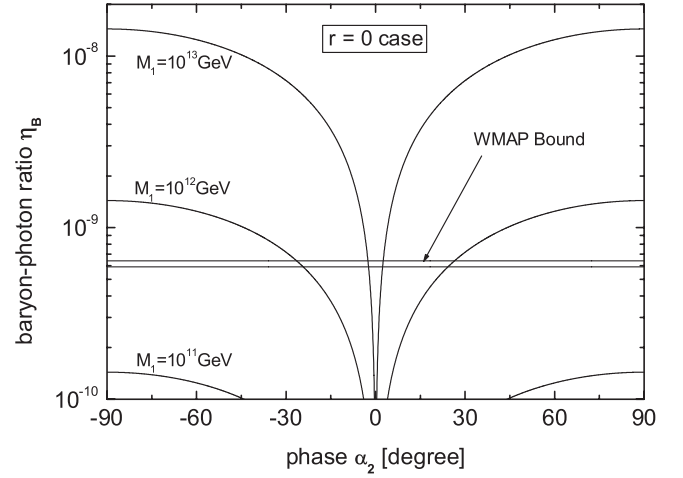


FIG. 1. The baryon-photon ratio η_B as a function of the Majorana phase α_2 in the $r = 0$ ($b_2 = 0$) case with various values of M_1 . The imposed horizontal lines show the upper and lower bound of η_B from the WMAP observation.

value of the CP asymmetry has several minimums and maximums as shown in Fig. 3. According to the relation of $\eta_B \propto Y_L \propto d|\epsilon|$, if the dilution factor d increases or decreases simply with r , we can expect that the baryon-photon ratio η_B also has several minimums and maximums.

Indeed, the dilution factor d decreases with r as shown in Fig. 4. The discontinuity at $r \sim 0.26$ arises as long as we use the Nielsen's approximations in Eq. (11) to estimate the dilution factor d . Also, it has been pointed out that in some cases the Nielsen's approximations could underestimate the suppression in the baryon asymmetry due to the washout effects [15]. However, the Nielsen's approximations are quite simple, so that we adopt their approximations to simplify our analysis.

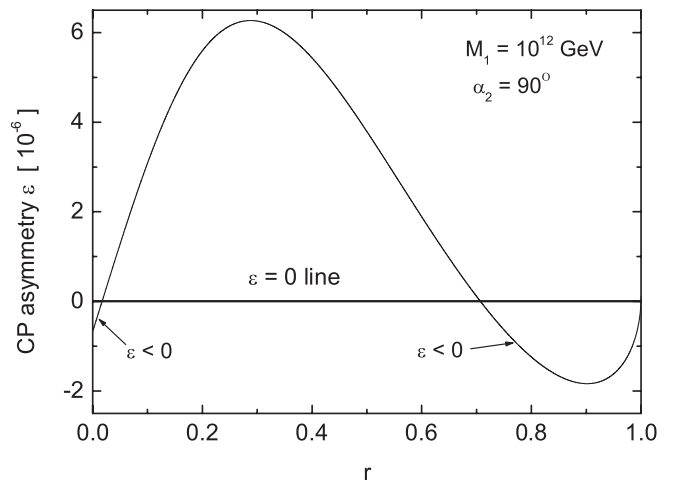
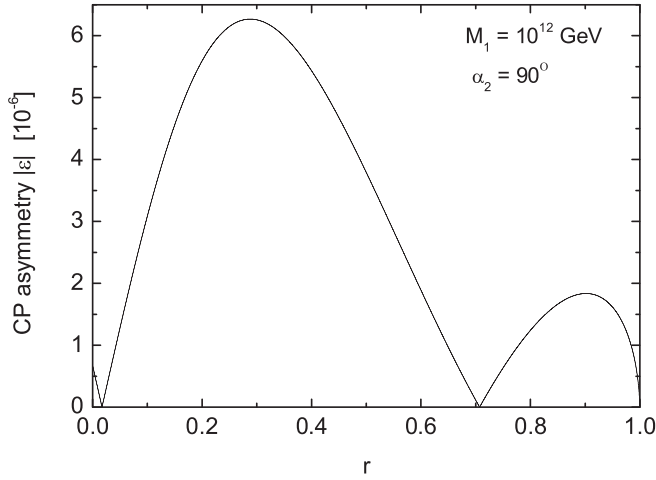
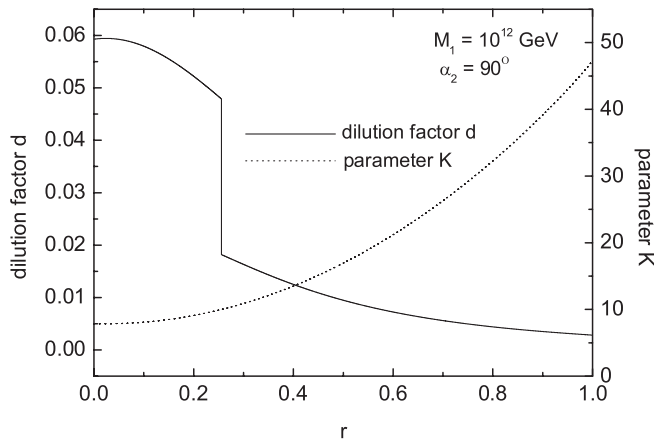
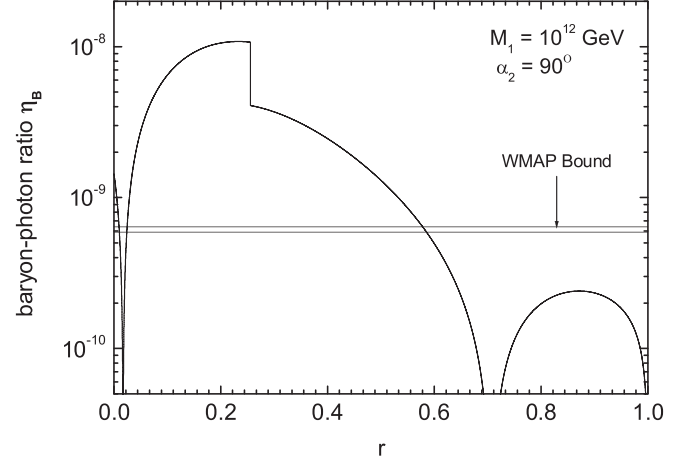


FIG. 2. CP asymmetry parameter ϵ as a function of r .

FIG. 3. Absolute value of the CP asymmetry ϵ .

As mentioned above, shown in Fig. 5 is the first main result in this paper. In this figure, we plot the baryon-photon ratio η_B as a function r with $M_1 = 10^{12}$ GeV and $\alpha_2 = 90^\circ$. The reasons of all the behavior of the curve are theoretically understood now. The minimums/maximums and the discontinuity at $r \sim 0.26$ in this figure are caused by the minimums/maximums of the CP asymmetry in Fig. 3 and by the discontinuity of the dilution factor in Fig. 4, respectively. From the figure, we see the baryon asymmetry significantly varies with r , equivalently with b_2 , e.g., the baryon asymmetry can be enhanced or suppressed at the particular points of r . It suggests that we can expect the presence of more rich phenomena, such as an enhanced leptogenesis [16], in the $b_2 \neq 0$ case.

To check the consistency of our numerical result, we have obtained the approximate formula of the CP asymmetry parameter in Eq. (21) up to the first order of r as follows:

FIG. 4. Dilution factor d as a function of r . The parameter K is shown in Eq. (12).FIG. 5. The baryon-photon ratio η_B as a function of r .

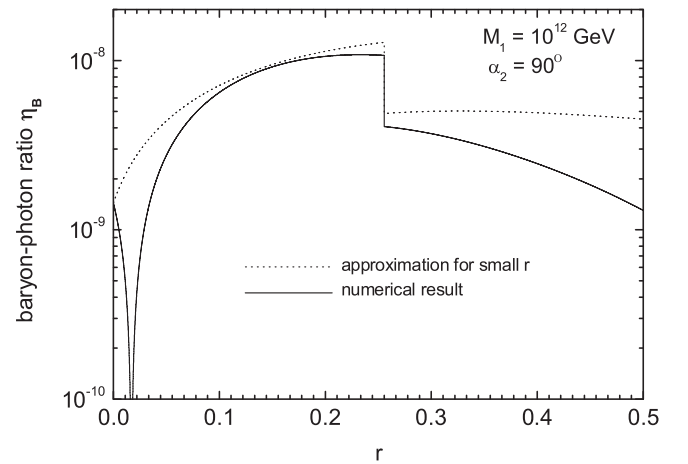
$$\tilde{\epsilon} = \frac{1}{|M_{22}|} \frac{\text{Im}[A^2] + 2rC \text{Im}[A]}{B - 2r \text{Re}[A]} + \mathcal{O}(r^2), \quad (22)$$

where

$$\begin{aligned} A &= \sigma_1 M_{12}^* Y + \sigma_3 M_{23}^* Z, \\ B &= |M_{12}|^2 + |M_{22}|^2 + |M_{23}|^2, \\ C &= B - |Y|^2 - |Z|^2, \\ Y &= \sqrt{M_{22} M_{11} - M_{12}^2}, \\ Z &= \sqrt{M_{22} M_{33} - M_{23}^2}. \end{aligned} \quad (23)$$

Figure 6 shows that the full numerical result is well explained by the approximate formula for small r without the pole at $r \sim 0.16$. It enhances the validity of our numerical result.

Up to now, we have fixed the value of the Majorana phase α_2 to 90° . Finally, we relax this fixing, however we still assume $M_1 = 10^{12}$ GeV. Shown in Fig. 7 is the second

FIG. 6. The approximation for small r .

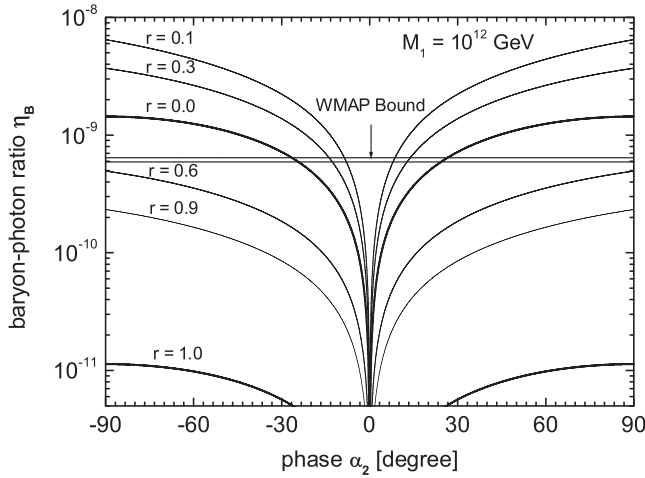


FIG. 7. The baryon-photon ratio η_B as a function of the Majorana phase α_2 with various values of r .

main result in this paper. In this figure, we show the baryon-photon ratio η_B as a function of the Majorana phase α_2 with various values of r . Recall that the Majorana phase α_2 is the only leptonic CP violation source in our analysis and the baryon-photon ratio η_B can be translated into the leptonic CP asymmetry. Thus, it is a natural consequence that the baryon-photon ratio tends to

vanish if the Majorana phase α_2 goes to zero. Also, Fig. 7 shows that the maximal Majorana CP phase $\alpha_2 = 90^\circ$ is one of the ingredients which is necessary to generate the maximal baryon asymmetry in the Universe.

V. SUMMARY

Summarizing our discussion, we have studied the generation of the baryon asymmetry in the Universe in the framework of the minimal seesaw model with tribimaximal mixing. Usually, at least one of the elements in the Dirac mass matrix m_D is fixed to be zero in order to reconstruct all elements in the m_D analytically. For example, the b_2 element is fixed to be zero. In this paper, we have taken nonzero b_2 . From numerical analysis, we have pointed out that the absolute value of the CP asymmetry ϵ has several minimums and maximums in the range of $0 \leq b_2 \leq \sqrt{M_{22}}$. Thus the baryon asymmetry $\eta_B \propto \epsilon$ significantly varies with b_2 and one can naturally expect that more rich phenomena, such as an enhanced leptogenesis, are hidden in the $b_2 \neq 0$ case.

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