

Scalar form factor of the nucleon and nucleon-scalar meson coupling constant in QCD

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(Received 12 December 2006; published 8 February 2007)

Scalar form factor of the nucleon is calculated in the framework of light-cone QCD sum rules, using the most general form of the baryon current. Using the result on scalar form factor of the nucleon, the nucleon-scalar σ and a_0 meson coupling constants are estimated. Our results on these couplings are in good agreement with the prediction of the external-field QCD sum rules method.

DOI: [10.1103/PhysRevD.75.045006](https://doi.org/10.1103/PhysRevD.75.045006)

PACS numbers: 11.55.Hx, 13.40.Gp, 14.20.Dh

I. INTRODUCTION

In spite of the great success of QCD as a theory of strong interactions, the quark structure of scalar mesons is still an open question and widely discussed in literature (see [1] for a review). There are many scenarios for the scalar meson structures like two-quark $\bar{q}q$, four-quark $\bar{q}q\bar{q}q$ states, meson-meson bound states and even a scalar glueball. It is very likely that the scalar meson structure is not made of such simple structures, but are superpositions of these contents. Therefore different scenarios can give quite different predictions on the production and decays of scalar mesons which can be tested in experiments. One efficient way to make progress about the nature scalar bosons is to study their role in the various two-baryon reactions (NN , ΛN , ΣN , $\Lambda\Lambda$, $\Sigma\Sigma$).

Our aim in this paper is to calculate the scalar form factor of the proton in the framework of light-cone sum rules (LCSR) [2,3] using the most general form of the proton current and then relate it to the nucleon-scalar σ or a_0 mesons coupling constants. This method is an alternative approach to the traditional sum rule [4]. LCSR is based on the operator product expansion (OPE) near the light cone, which is an expansion of the time ordered product over the twist rather than the dimension of the operators. Main contribution of this approach comes from the operators having lower twist. The main ingredients of the LCSR are the wave functions of hadrons which define the matrix elements of nonlocal operators between the vacuum and one hadron states.

This method applied to study the nucleon electromagnetic form factors [5], and the weak $\Lambda_b \rightarrow p\ell\bar{\nu}_\ell$ decay [6]. The higher twist wave functions of baryons have been obtained recently in [7].

The plan of this article is as follows: In Sec. II we derive the sum rule for the proton form factor due to the scalar current. Section III is devoted to the numerical analysis and discussion.

II. SUM RULES FOR THE PROTON FORM FACTOR DUE TO THE SCALAR CURRENT

In the present section we derive sum rules for the scalar form factor of the proton using the most general form of nucleon interpolating current. For this aim we start with the following correlation function:

$$\Pi(p, q) = i \int d^4x e^{iqx} \langle 0 | T \{ \eta(0) J(x) \} | N(p) \rangle, \quad (1)$$

where $J(x) = \bar{u}u + (-1)^I \bar{d}d$ is the interpolating current for the scalar σ ($I = 0$) and a_0 ($I = 1$) mesons, respectively, and η is a suitable interpolating current with nucleon quantum number. Few words about the interpolating current of the nucleon are in order. There exist the following currents with the isotopic spin 1/2, without derivative terms (see [8]):

$$\eta_1(x) = 2\varepsilon^{abc} \sum_{\ell=1}^2 (u^{Ta}(x) C A_1^\ell d^b(x)) A_2^\ell u^c(x), \quad (2)$$

$$\eta_2(x) = \frac{2}{3} \varepsilon^{abc} [(u^{Ta}(x) C \not{z} u^b(x)) \gamma_5 \not{z} d^c(x) - (u^{Ta}(x) C \not{z} d^b(x)) \gamma_5 \not{z} u^c(x)], \quad (3)$$

where $A_1^1 = I$, $A_2^1 = \gamma_5$, $A_1^2 = \gamma_5$, $A_2^2 = \beta$, and in C is the charge conjugation operator, a, b, c are the color indices, and z in Eq. (3) is a lightlike vector with $z^2 = 0$. In Eq. (2), $\beta = -1$ corresponds to the Ioffe current [9]. Note that in [5] $\eta_2(x)$ is modified as

$$\eta_3(x) = \varepsilon^{abc} [(u^{Ta}(x) C \not{z} u^b(x)) \gamma_5 \not{z} d^c(x)].$$

The nucleon-scalar form factor which is obtained using the current $\eta_3(x)$ in the light-cone QCD sum rule approach is studied in [10]. However, the current $\eta_3(x)$ couples also to the states with isospin 3/2, and these contributions should be eliminated from the final result, which is not done in [10]. Moreover, terms that are proportional to $\not{z}\not{z}$ and $\not{z}\not{z}$ in the coordinate space which give contribution to the considered structure in [11], are also neglected. In the present work η_1 is chosen as the general form of the interpolating current.

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Let us firstly calculate the hadronic part of the correlator function. By inserting the complete set of states between the currents in Eq. (1) with quantum numbers of the corresponding nucleon, we get the following hadronic representation of the correlator

$$\Pi(p, q) = \frac{\langle 0 | \eta | N(p') \rangle \langle N(p') | J | N(p) \rangle}{m_N^2 - p'^2} + \sum_h \frac{\langle 0 | \eta | h(p') \rangle \langle h(p') | J | N(p) \rangle}{m_h^2 - p'^2}, \quad (4)$$

where $p' = p - q$, and q is the momentum carried by the scalar current. The second term in Eq. (4) takes into account higher states and continuum contributions and hadrons h form a complete set of baryons having the same quantum number as the ground state baryon B (here B represents p and n). The matrix elements in Eq. (4) are defined as

$$\langle 0 | \eta | N(p') \rangle = \lambda_N u_N(p'), \quad (5)$$

$$\langle N(p') | J | N(p) \rangle = g(Q^2) \bar{u}(p') u(p). \quad (6)$$

Using Eqs. (4)–(6) and performing summation over nucleon spin, for the phenomenological part of the correlator we obtain

$$\Pi(p, q) = \frac{\lambda_N g(Q^2) (\not{p}' + m_N) u(p)}{m_N^2 - (p - q)^2} + \dots, \quad (7)$$

where $\dots$ represents contributions from the higher states and the continuum. It follows from Eq. (7) that the corre-

lator function contains three different structures, namely, structures $\sim \not{p}$, $\sim \not{q}$ and structure proportional to the unit operator I . In further analysis we will choose the structure $\sim \not{p}$.

Here we would like to make the following remark: The nucleon-scalar meson S coupling constant g_{NNS} can be related to the scalar form factor $g(Q^2)$, and for this aim we define the following coupling constant

$$\langle N(p') | S | N(p) \rangle \equiv g_{NNS} \bar{u}(p') u(p). \quad (8)$$

It follows then from Eq. (1) that

$$\Pi(p, q) = \frac{\langle 0 | J | S \rangle \langle N(p') | S | N(p) \rangle \langle 0 | \eta | N(p') \rangle}{(m_N^2 - p'^2)(m_S^2 - q^2)}. \quad (9)$$

The matrix element of the scalar current J between the vacuum and the scalar meson state is defined as

$$\langle 0 | J | S \rangle = \lambda_S. \quad (10)$$

Using Eqs. (8)–(10) and comparing with Eq. (7), we get the following relation between nucleon-scalar form factor and g_{NNS}

$$g(Q^2) = \frac{g_{NNS} \lambda_S}{m_S^2 + Q^2}. \quad (11)$$

At large Euclidean momenta $p'^2 = (p - q)^2$ and $q^2 = -Q^2$, the QCD part of the correlator can be calculated from the explicit form of interpolating currents $J(x)$ and $\eta(0)$, from which we obtain

$$\begin{aligned} \Pi = & \frac{1}{2} \int d^4x e^{iqx} \sum_{\ell=1}^2 \{ (CA_1^\ell)_{\alpha\gamma} [A_2^\ell S(-x)]_{\rho\beta} 4\epsilon^{abc} \langle 0 | T \{ u_\alpha^a(0) u_\beta^b(x) d_\gamma^c(0) \} | N \rangle \\ & + (A_2^\ell)_{\rho\alpha} [(CA_1^\ell)^T S(-x)]_{\gamma\beta} 4\epsilon^{abc} \langle 0 | T \{ u_\alpha^a(0) u_\beta^b(x) d_\gamma^c(0) \} | N \rangle + (A_2^\ell)_{\rho\beta} [CA_1^\ell S(-x)]_{\alpha\gamma} 4\epsilon^{abc} \langle 0 | T \{ u_\alpha^a(0) u_\beta^b(0) \} d_\gamma^c(x) | N \rangle \}, \end{aligned} \quad (12)$$

where $S(x)$ is the light quark propagator. It follows from Eq. (12) that for calculation of the correlator from QCD side we need to know the explicit form light quark propagator and the nucleon distribution amplitudes (DAs). The full propagator of the massless quark is

$$\begin{aligned} S(x) = & \frac{i \not{x}}{2\pi^2 x^4} - \frac{\langle q\bar{q} \rangle}{12} \left(1 + \frac{m_0^2 x^2}{16} \right) \\ & - i g_s \int_0^1 dv \left[\frac{\not{x}}{16\pi^2 x^4} G_{\mu\nu} \sigma^{\mu\nu} \right. \\ & \left. - v x^\mu G_{\mu\nu} \gamma^\nu \frac{i}{4\pi^2 x^2} \right]. \end{aligned} \quad (13)$$

The contribution $\sim G_{\mu\nu}$ can give rise to four and five particle nucleon DAs. These DAs are usually small [12,13] and can be neglected.

The second term in Eq. (13) does not give any contribution to the sum rule after the Borel transformation. So, only the first term is relevant for our calculations.

As has already been noted, in order to calculate the nonperturbative contributions to the theoretical part of the correlator function one needs to know the matrix element of $\epsilon^{abc} \langle 0 | u_\alpha^a(a_1 x) u_\beta^b(a_2 x) d_\gamma^c(a_3 x) | N(p) \rangle$ between the proton and vacuum states. This matrix element of the nonlocal operator is parametrized in terms of the nucleon DAs [7,13–17]. Their explicit forms are given in the Appendix A.

Taking into account the three valence quark light-cone DAs up to twist-6 and performing integration over x in the coordinate space, we finally obtain the following expression for the correlator function:

$$\begin{aligned}
\Pi = & \frac{1}{4} m_N \int_0^1 dt_2 \int_0^{1-t_2} dt_1 \frac{\not{q} - \not{p} t_2}{(q - p t_2)^2} \{2(1 + \beta) \tilde{V}_2(t_i) + 2[3 \mp 1 + (3 \pm 1)\beta] V_3(t_i) \pm 2(1 + \beta) P_1(t_i) \\
& - 2[2 \pm 1 - (2 \mp 1)\beta] S_1(t_i) + 2(1 + \beta) \tilde{\mathcal{A}}_2(t_i) + 2[3 \pm 1 + (3 \mp 1)\beta] \mathcal{A}_3(t_i) + [2 \mp 1 - (2 \pm 1)\beta] \tilde{\mathcal{T}}_2(t_i) \\
& + 12(1 - \beta) T_7(t_i) + [6 \mp 1 - (6 \pm 1)\beta] \tilde{\mathcal{T}}_4(t_i)\} \\
& - \frac{m_N^3}{2} \int_0^1 d\tau \int_1^\tau d\lambda \int_1^\lambda dt_2 \int_0^{1-t_2} dt_1 \frac{\not{q} - \not{p} \tau}{(q - p \tau)^4} \{\pm(1 - \beta) \tilde{V}_6(t_i) \mp (1 - \beta) \tilde{\mathcal{A}}_6(t_i) + 3[2 \mp 1 - (2 \pm 1)\beta] \tilde{\mathcal{T}}_8(t_i)\} \\
& - \frac{m_N}{2} \int_0^1 d\lambda \int_1^\lambda dt_2 \int_0^{1-t_2} dt_1 \frac{\not{p}}{(q - p \lambda)^2} \{[2 \mp 1 + (2 \pm 1)\beta] \tilde{V}_2(t_i) + [2 \pm 1 + (2 \mp 1)\beta] \tilde{\mathcal{A}}_2(t_i) \\
& + [2 \mp 1 - (2 \pm 1)\beta] \tilde{\mathcal{T}}_2(t_i) \pm 2(1 + \beta) \tilde{\mathcal{T}}_4(t_i)\}
\end{aligned} \tag{14}$$

where any function $F(t_i)$ means $F(t_i) = F(t_1, t_2, 1 - t_1, t_2)$, and functions with $\sim$ are defined as

$$\begin{aligned}
\tilde{V}_2(t_i) &= V_1(t_i) - V_2(t_i) - V_3(t_i), \\
\tilde{\mathcal{A}}_2(t_i) &= -A_1(t_i) + A_2(t_i) - A_3(t_i), \\
\tilde{\mathcal{A}}_6(t_i) &= A_1(t_i) - A_2(t_i) + A_3(t_i) + A_4(t_i) - A_5(t_i) \\
&\quad + A_6(t_i), \\
\tilde{\mathcal{T}}_2(t_i) &= T_1(t_i) + T_2(t_i) - 2T_3(t_i), \\
\tilde{\mathcal{T}}_4(t_i) &= T_1(t_i) - T_2(t_i) - 2T_7(t_i), \\
\tilde{\mathcal{T}}_6(t_i) &= 2T_2(t_i) - 2T_3(t_i) - 2T_4(t_i) + 2T_5(t_i) + 2T_7(t_i) \\
&\quad + 2T_8(t_i), \\
\tilde{\mathcal{T}}_8(t_i) &= -T_1(t_i) + T_2(t_i) + T_5(t_i) - T_6(t_i) + 2T_7(t_i) \\
&\quad + 2T_8(t_i).
\end{aligned}$$

Explicit expressions of $A_\alpha(t_i)$, $T_\alpha(t_i)$ and $V_\alpha(t_i)$ can be found in [17]. Note also that in deriving Eq. (14) we omit the terms proportional with the unit operators which do not give contribution to the structure $\sim \not{p}$. Upper (lower) sign in Eq. (14) corresponds to the scalar form factor due to the scalar current with the quantum numbers of σ (a_0).

In order to suppress the continuum and higher state contributions we need to apply Borel transformation to the phenomenological and theoretical parts of the correlation function with respect to the variable $(q - p)^2$.

For the theoretical part the Borel transformation and the continuum subtraction can be done by using the following substitution rules

$$\begin{aligned}
\int dx \frac{\rho(x)}{(q - xp)^2} &\rightarrow - \int dx \frac{\rho(x)}{x} e^{-s/M_B^2}, \\
\int dx \frac{\rho(x)}{(q - xp)^4} &\rightarrow \frac{1}{M_B^2} \int dx \frac{\rho(x)}{x^2} e^{-s/M_B^2} \\
&\quad + \frac{\rho(x_0)}{Q^2 + x_0^2 M_B^2} e^{-s_0/M_B^2},
\end{aligned} \tag{15}$$

where

$$\begin{aligned}
s &= (1 - x)m_N^2 + \frac{1 - x}{x} Q^2, \\
x_0 &= \frac{\sqrt{(Q^2 + s_0 - m_N^2)^2 + 4m_N^2 Q^2} - (Q^2 + s_0 - m_N^2)}{2m_N^2},
\end{aligned}$$

and $Q^2 = -q^2$. Finally, we get the following sum rule for the scalar form factor.

$$\begin{aligned}
g(Q^2) = & \frac{e^{m_N^2/M_B^2}}{\lambda_N} \left[\frac{1}{4} m_N \int_{x_0}^1 dt_2 \int_0^{1-t_2} dt_1 e^{-s/M_B^2} \{2(1 + \beta) \tilde{V}_2(t_i) + 2[3 \mp 1 + (3 \pm 1)\beta] V_3(t_i) \pm 2(1 + \beta) P_1(t_i) \right. \\
& - 2[2 \pm 1 - (2 \mp 1)\beta] S_1(t_i) + 2(1 + \beta) \tilde{\mathcal{A}}_2(t_i) + 2[3 \pm 1 + (3 \mp 1)\beta] \mathcal{A}_3(t_i) + [2 \mp 1 - (2 \pm 1)\beta] \tilde{\mathcal{T}}_2(t_i) \\
& + 12(1 - \beta) T_7(t_i) + [6 \mp 1 - (6 \pm 1)\beta] \tilde{\mathcal{T}}_4(t_i)\} \\
& + \frac{m_N^3}{2} \left(\int_{x_0}^1 d\tau \int_1^\tau d\lambda \int_1^\lambda dt_2 \int_0^{1-t_2} dt_1 \frac{1}{\tau M_B^2} e^{-s/M_B^2} \{\pm(1 - \beta) \tilde{V}_6(t_i) \mp (1 - \beta) \tilde{\mathcal{A}}_6(t_i) \right. \\
& + 3[2 \mp 1 - (2 \pm 1)\beta] \tilde{\mathcal{T}}_8(t_i)\} + \frac{x_0}{Q^2 + x_0^2 M_B^2} \int_1^{x_0} d\lambda \int_1^\lambda dt_2 \int_0^{1-t_2} dt_1 \frac{1}{\tau M_B^2} e^{-s/M_B^2} \{\pm(1 - \beta) \tilde{V}_6(t_i) \\
& \mp (1 - \beta) \tilde{\mathcal{A}}_6(t_i) + 3[2 \mp 1 - (2 \pm 1)\beta] \tilde{\mathcal{T}}_8(t_i)\} \Big) + \frac{1}{2} m_N \int_{x_0}^1 \frac{dt_2}{t_2} \int_0^{1-t_2} dt_1 e^{-s/M_B^2} \{[2 \mp 1 + (2 \pm 1)\beta] \tilde{V}_2(t_i) \\
& + [2 \pm 1 + (2 \mp 1)\beta] \tilde{\mathcal{A}}_2(t_i) + [2 \mp 1 - (2 \pm 1)\beta] \tilde{\mathcal{T}}_2(t_i) \pm 2(1 + \beta) \tilde{\mathcal{T}}_4(t_i)\} \Big]
\end{aligned} \tag{16}$$

It follows from Eq. (16) that for determining the scalar form factor, the residue λ_N of the nucleon needs to be known. The residue λ_N is obtained from the mass sum rules for the baryons (see for example [18])

$$\lambda_N^2 e^{-m_N^2/M_B^2} = \frac{M^6}{256\pi^4} E_2(x)(5 + 2\beta + \beta^2) + \frac{\langle \bar{u}u \rangle}{6} [-6(1 - \beta^2)\langle \bar{d}d \rangle + (-1 + \beta^2)\langle \bar{u}u \rangle] - \frac{m_0^2}{24M_B^2} \langle \bar{u}u \rangle [-12(1 - \beta^2)\langle \bar{d}d \rangle + (-1 + \beta^2)\langle \bar{u}u \rangle], \quad (17)$$

where $x = s_0/M_B^2$, and

$$E_n(x) = 1 - e^x \sum_{k=0}^n \frac{x^k}{k!},$$

which describes the subtraction of higher states and continuum contributions.

III. NUMERICAL ANALYSIS

In this section we present the numerical analysis for the scalar form factor $g(Q^2)$. It follows from Eq. (16) that the main ingredients of LCSR are the DAs, which involve nonperturbative parameters f_N , λ_1 , λ_2 , f_1^u , f_1^d , f_2^d , A_1^u , V_1^d (see Appendix B).

In the numerical calculations we use three different sets of the DAs (for more details, see [5,17]).

- (1) QCD sum rules based DAs,
- (2) Asymptotic DAs,
- (3) A model for the nucleon DAs. The parameters in this case are chosen in such a way that, the form of DAs describes very well the existing experimental data on nucleon form factors.

The values of the above-mentioned nonperturbative parameters for these cases at $\mu = 1$ GeV scale are given in [5,17]. Our numerical calculations show that the values of the scalar form factors of nucleon due to the scalar current in QCD based DAs case are larger compared to that of the other two scenarios, and, for customary reasons we present our numerical results only for this case.

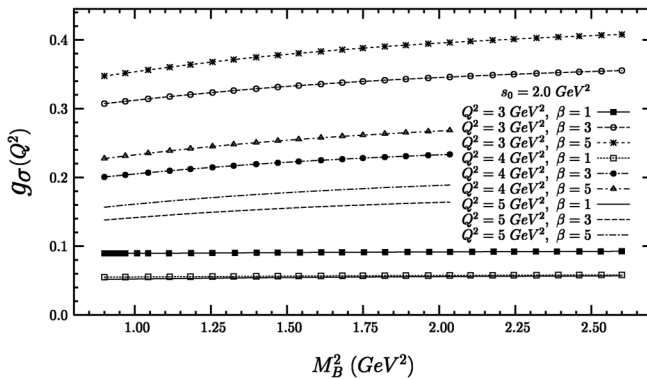


FIG. 1. The dependence of the scalar form factor of nucleon due to the scalar current with quantum numbers of σ meson, on Borel parameter M_B^2 at $s_0 = 2$ GeV² for three different values of Q^2 and β : $Q^2 = 3$ GeV², $Q^2 = 4$ GeV², $Q^2 = 5$ GeV² and $\beta = 1$, $\beta = 3$, $\beta = 5$.

In the problem at hand we have three auxiliary parameters, namely, Borel mass square M_B^2 , continuum threshold s_0 and β in the interpolating current of nucleon. Obviously, any physically measurable must be independent of the auxiliary parameters. So, in the first hand we have to find the appropriate regions of these parameters, in where the form factor is independent of them. The mass sum rules analysis shows that optimal value for the continuum thresholds is in the region $s_0(2 \div 2.5)$ GeV².

For this aim we consider the following three-step procedure. In the first step we try to find the working region of M_B^2 where scalar form factor is independent of M_B^2 at fixed values of s_0 and β . In Figs. 1–4 (Figs. 5–8) we present the dependence of the scalar form factor induced by $I = 0(I = 1)$ scalar current on M_B^2 at different fixed values of Q^2 and β at $s_0 = 2.0$ GeV² and $s_0 = 2.5$ GeV², respectively.

From these figures we see that, in both cases, the scalar form factor is practically independent of M_B^2 at different values of Q^2 , β and s_0 . To decide on an upper bound for M_B^2 we require the contribution of continuum be less than the contribution of continuum-subtracted sum rules and the lower bound can be determined by requiring that the contribution of the highest power of $1/M_B^2$ to be less than, say, 30% of the higher powers of M_B^2 . As a result of our analysis we find that the region $1 \text{ GeV}^2 \leq M_B^2 \leq 2.5 \text{ GeV}^2$ satisfies both conditions simultaneously.

The parameter β is an auxiliary parameter, and hence, the physical quantities should be independent of it. For this reason we need to find the working region of β where the scalar form factor is independent of it. In determining this region of β two conditions should be satisfied: Eq. (17) and its logarithmic derivative with respect to $1/M_B^2$ should be

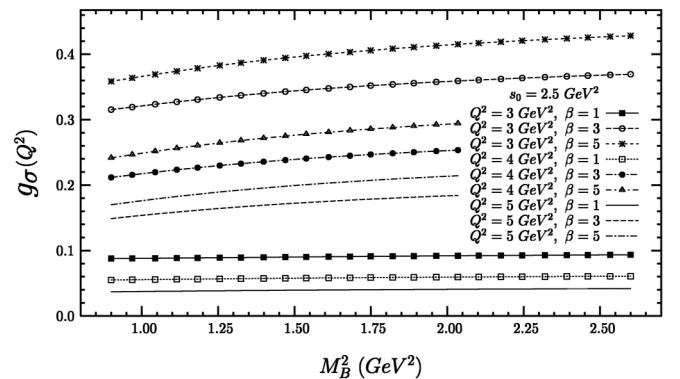


FIG. 2. The same as in Fig. 1, but at $s_0 = 2.5$ GeV².

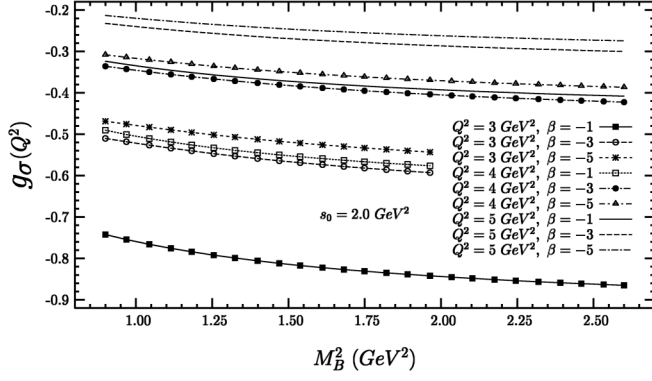


FIG. 3. The same as in Fig. 1, but for values of β : $\beta = -1, -3, -5$.

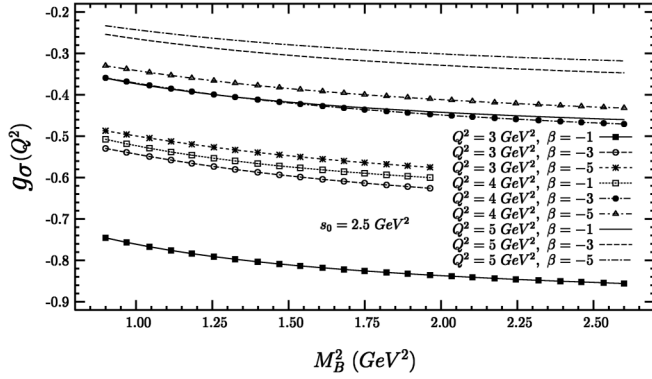


FIG. 4. The same as in Fig. 2, but for values of β : $\beta = -1, \beta = -3, \beta = -5$.

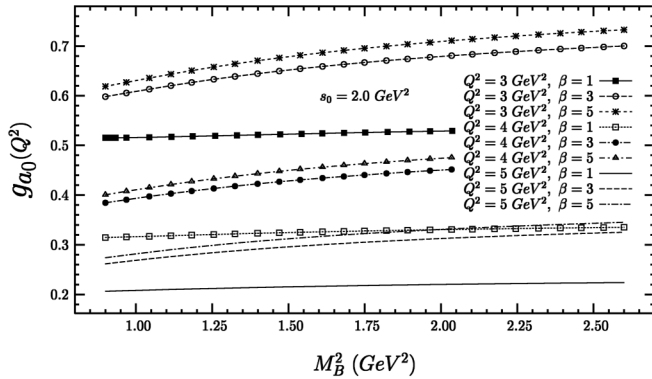


FIG. 5. The same as in Fig. 1, but for the scalar form factor induced by the scalar current with quantum number of a_0 meson.

positive and independent of β . From an analysis of Eq. (17) it is found that $-0.5 < \beta < 0.5$ is unphysical [18]. It is further shown in [18] that the working region of β where the second condition is satisfied is obtained for $-0.6 < \cos\theta < 0.3$, where $\tan\theta = \beta$, which yields $\beta < -1.3$ and $\beta > 3.3$.

We perform calculations depicting the dependence of the scalar form factors on $\cos\theta$ at fixed values of s_0 ($s_0 =$

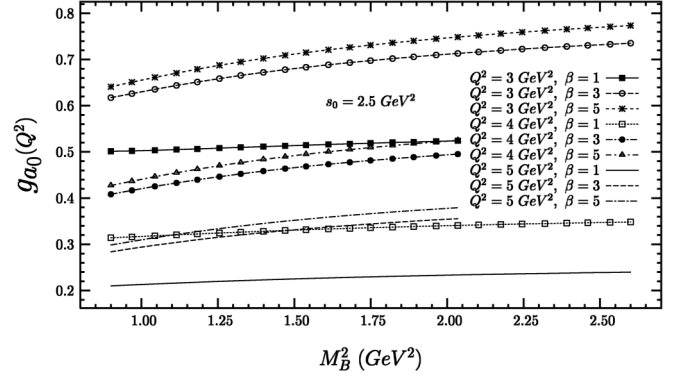


FIG. 6. The same as in Fig. 5, but at $s_0 = 2.5 \text{ GeV}^2$.

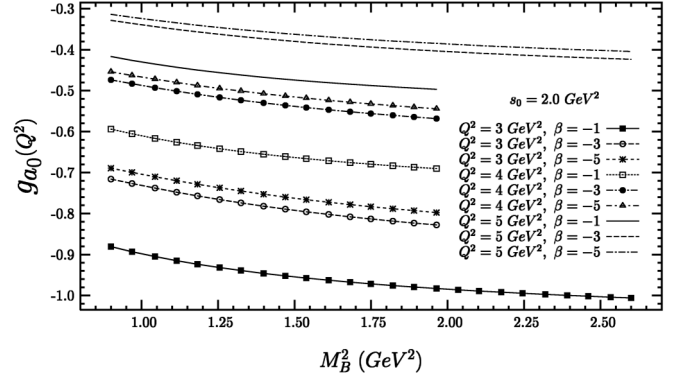


FIG. 7. The same as in Fig. 5, but for values of β : $\beta = -1, \beta = -3, \beta = -5$.

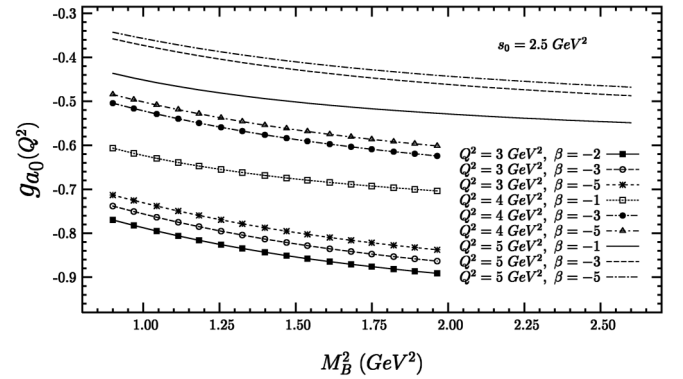


FIG. 8. The same as in Fig. 7, but at $s_0 = 2.5 \text{ GeV}^2$.

2.0 GeV^2 and $s_0 = 2.5 \text{ GeV}^2$), M_B^2 ($M_B^2 = 1.0 \text{ GeV}^2$ and $M_B^2 = 1.5 \text{ GeV}^2$) at three fixed values of Q^2 ($Q^2 = 3.0 \text{ GeV}^2$, $Q^2 = 4.0 \text{ GeV}^2$ and $Q^2 = 5 \text{ GeV}^2$); and confirmed that in the regions when $\beta < -1.3$ and $\beta > 3.3$, the form factor is practically independent of β . So, the common region where mass sum and form factor are independent of β is the region $-0.5 < \cos\theta < 0.3$, which corresponds to $\beta < -1.3$ and $\beta > 3.3$ and which we will use in further discussions. Note that, the Monte Carlo analysis that is performed in [19] predicts the optimal value

of β to be $\beta = -1.2$. Our lower bound is slightly different compared to this value.

We perform the OPE in the light cone with large Euclidean Q^2 and $(p - q)^2$ momenta, where the scalar form factor $g(Q^2)$ can reliably be determined at the range $Q^2 > 2 \text{ GeV}^2$. In the region $Q^2 < 2 \text{ GeV}^2$ OPE becomes questionable and for this reason in determining $g(Q^2)$ we restrict ourselves to the region $Q^2 > 2 \text{ GeV}^2$.

In Figs. 9–12 we present the dependence of $g(Q^2)$ induced by $I = 0(I = 1)$ current on Q^2 at fixed values of β which lies in the above-mentioned region of β ; at $M_B^2 = 1 \text{ GeV}^2$ and $s_0 = 2 \text{ GeV}^2$, $s_0 = 2.5 \text{ GeV}^2$, respectively, for the central values all parameters. We observe that the value of $g(Q^2)$ seems to be insensitive to the choice of s_0 . We further see from these figures that for positive (negative) values β , the sign of the form factor is positive (negative). Therefore determination of the sign of $g(Q^2)$ can give valuable information about parameter β .

Having the obtained the Q^2 dependence of the form factor, we can determine g_{NNS} . It follows from Eq. (10) that

$$g_{NNS} = \frac{(m_S^2 + Q^2)g(Q^2)}{\lambda_S}.$$

The residues λ_σ and λ_{a_0} of the scalar currents are determined from two-point QCD sum rules analysis in [20,21], which is predicted to have the value $\lambda_\sigma = 0.2$ and $\lambda_{a_0} = 0.3$. Using the results of Figs. 3 and 4, for the scalar form factor $g_{NN\sigma}$ we get

$$|g_{NN\sigma}| = \begin{cases} 6 \pm 1 & \text{for } \beta = 5, \\ 8 \pm 2 & \text{for } \beta = -5, \beta = -3, \end{cases}$$

$$|g_{NNa_0}| = \begin{cases} 5.5 \pm 1.5 & \text{for } \beta = 5, \\ 7 \pm 1 & \text{for } \beta = -5, \beta = -3. \end{cases}$$

For completeness we present the results for g_{NNS} coupling for the Ioffe current case, despite that $\beta = -1$ lies

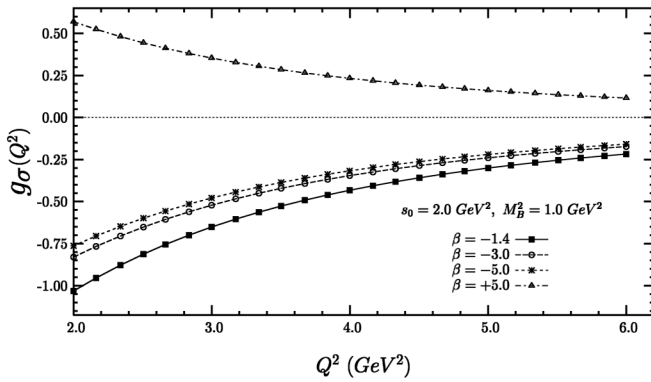


FIG. 9. The dependence of the scalar form factor induced by the scalar current with σ meson quantum numbers, on Q^2 at $M_B^2 = 1 \text{ GeV}^2$ and $s_0 = 2 \text{ GeV}^2$, at fixed values of $\beta = -1.4$, $\beta = -3$, $\beta = -5$, $\beta = 5$.

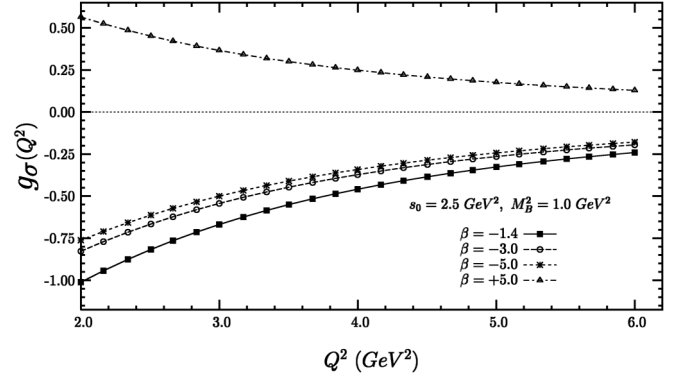


FIG. 10. The same as in Fig. 9, but at $s_0 = 2.5 \text{ GeV}^2$.

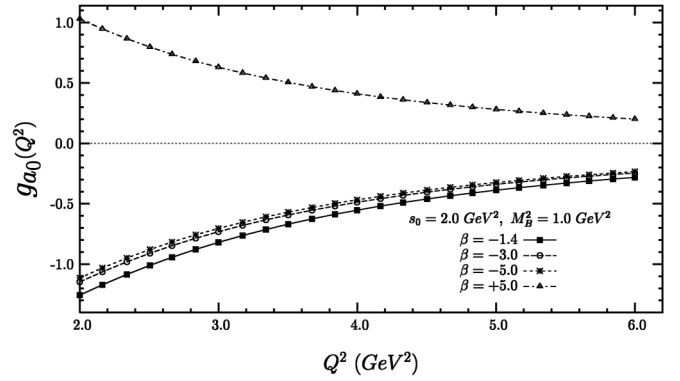


FIG. 11. The dependence of the scalar form factor induced by the scalar current with a_0 meson quantum numbers, on Q^2 at $M_B^2 = 1 \text{ GeV}^2$ and $s_0 = 2 \text{ GeV}^2$, at fixed values of $\beta = -1.4$, $\beta = -3$, $\beta = -5$, $\beta = 5$.

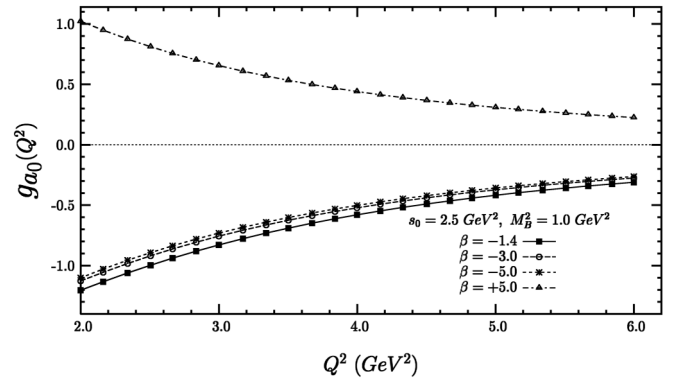


FIG. 12. The same as in Fig. 11, but at $s_0 = 2.5 \text{ GeV}^2$.

outside the working region of β . Our results are

$$|g_{NN\sigma}| = 12 \pm 2, \quad |g_{NNa_0}| = 11 \pm 2.$$

Note that this coupling constant in framework of the external-field QCD sum rules method using the Ioffe current is evaluated in [22], and it is obtained that $g_{NN\sigma} = 14.4 \pm 3.7$. This same coupling constant is also shown to

have the value $g_{NN\sigma} = 16.9$ [1] from a fit to the NN scattering data.

A comparison of our result for $\beta = -1$ shows that, within the error limits, there is a good agreement between our result and the above-mentioned one.

In conclusion, we have calculated the nucleon-scalar form factor in the framework of light-cone QCD sum rules, using the most general form of the baryon current. Using the obtained result on scalar form factor, we estimate $g_{NN\sigma}$. Our result in regard to the coupling constant $g_{NN\sigma}$ is in a good agreement with the prediction of the external-field QCD sum rules method for the Ioffe current. Measurement of the nucleon-scalar meson coupling constant can give valuable information about the quark structure of scalar meson as well as about the value of parameter β .

ACKNOWLEDGMENTS

One of the authors (T. M. A) is grateful to TÜBİTAK for partially support of this work under the project No. 105T131.

APPENDIX A: NUCLEON DISTRIBUTION AMPLITUDES

In this appendix we present the three-quark distribution DAs from twist-3 to twist-6 (for more details, see [7,17]).

The matrix element of the nonlocal operator $\epsilon^{abc}\langle 0|u_a^a(a_1x)u_b^b(a_2x)d_\gamma^c(a_3x)|N(p)\rangle$ between the proton and vacuum states is parametrized in terms of the nucleon DAs as follows:

$$\begin{aligned}
4\epsilon^{abc}\langle 0|u_a^a(a_1x)u_b^b(a_2x)d_\gamma^c(a_3x)|P\rangle = & S_1 m_N C_{\alpha\beta}(\gamma_5 N)_\gamma + S_2 m_N^2 C_{\alpha\beta}(\not{x}\gamma_5 N)_\gamma + P_1 m_N(\gamma_5 C)_{\alpha\beta} N_\gamma \\
& + P_2 m_N^2(\gamma_5 C)_{\alpha\beta}(\not{x}N)_\gamma + \left(\mathcal{V}_1 + \frac{x^2 m_N^2}{4} \mathcal{V}_1^M\right)(\not{P}C)_{\alpha\beta}(\gamma_5 N)_\gamma \\
& + \mathcal{V}_2 m_N(\not{P}C)_{\alpha\beta}(\not{x}\gamma_5 N)_\gamma + \mathcal{V}_3 m_N(\gamma_\mu C)_{\alpha\beta}(\gamma^\mu \gamma_5 N)_\gamma + \mathcal{V}_4 m_N^2(\not{x}C)_{\alpha\beta}(\gamma_5 N)_\gamma \\
& + \mathcal{V}_5 m_N^2(\gamma_\mu C)_{\alpha\beta}(i\sigma^{\mu\nu}x_\nu \gamma_5 N)_\gamma + \mathcal{V}_6 m_N^3(\not{x}C)_{\alpha\beta}(\not{x}\gamma_5 N)_\gamma \\
& + \left(\mathcal{A}_1 + \frac{x^2 m_N^2}{4} \mathcal{A}_1^M\right)(\not{P}\gamma_5 C)_{\alpha\beta} N_\gamma + \mathcal{A}_2 m_N(\not{P}\gamma_5 C)_{\alpha\beta}(\not{x}N)_\gamma \\
& + \mathcal{A}_3 m_N(\gamma_\mu \gamma_5 C)_{\alpha\beta}(\gamma^\mu N)_\gamma + \mathcal{A}_4 m_N^2(\not{x}\gamma_5 C)_{\alpha\beta} N_\gamma \\
& + \mathcal{A}_5 m_N^2(\gamma_\mu \gamma_5 C)_{\alpha\beta}(i\sigma^{\mu\nu}x_\nu N)_\gamma + \mathcal{A}_6 m_N^3(\not{x}\gamma_5 C)_{\alpha\beta}(\not{x}N)_\gamma \\
& + \left(\mathcal{T}_1 + \frac{x^2 m_N^2}{4} \mathcal{T}_1^M\right)(P^\nu i\sigma_{\mu\nu} C)_{\alpha\beta}(\gamma^\mu \gamma_5 N)_\gamma + \mathcal{T}_2 m_N(x^\mu P^\nu i\sigma_{\mu\nu} C)_{\alpha\beta}(\gamma_5 N)_\gamma \\
& + \mathcal{T}_3 m_N(\sigma_{\mu\nu} C)_{\alpha\beta}(\sigma^{\mu\nu} \gamma_5 N)_\gamma + \mathcal{T}_4 m_N(P^\nu \sigma_{\mu\nu} C)_{\alpha\beta}(\sigma^{\mu\rho} x_\rho \gamma_5 N)_\gamma \\
& + \mathcal{T}_5 m_N^2(x^\nu i\sigma_{\mu\nu} C)_{\alpha\beta}(\gamma^\mu \gamma_5 N)_\gamma + \mathcal{T}_6 m_N^2(x^\mu P^\nu i\sigma_{\mu\nu} C)_{\alpha\beta}(\not{x}\gamma_5 N)_\gamma \\
& + \mathcal{T}_7 m_N^2(\sigma_{\mu\nu} C)_{\alpha\beta}(\sigma^{\mu\nu} \not{x}\gamma_5 N)_\gamma + \mathcal{T}_8 m_N^3(x^\nu \sigma_{\mu\nu} C)_{\alpha\beta}(\sigma^{\mu\rho} x_\rho \gamma_5 N)_\gamma. \tag{A1}
\end{aligned}$$

The calligraphic DAs do not have definite twist and can be related to the one with definite twist as:

$$\begin{aligned}
S_1 &= S_1, & (2P \cdot x)S_2 &= S_1 - S_2, \\
P_1 &= P_1, & (2P \cdot x)P_2 &= P_2 - P_1, \\
\mathcal{V}_1 &= V_1, & (2P \cdot x)\mathcal{V}_2 &= V_1 - V_2 - V_3, \\
2\mathcal{V}_3 &= V_3, & (4P \cdot x)\mathcal{V}_4 &= -2V_1 + V_3 + V_4 + 2V_5, \\
(4P \cdot x)\mathcal{V}_5 &= V_4 - V_3, & (2P \cdot x)^2\mathcal{V}_6 &= -V_1 + V_2 + V_3 + V_4 + V_5 - V_6, \\
\mathcal{A}_1 &= A_1, & (2P \cdot x)\mathcal{A}_2 &= -A_1 + A_2 - A_3, \\
2\mathcal{A}_3 &= A_3, & (4P \cdot x)\mathcal{A}_4 &= -2A_1 - A_3 - A_4 + 2A_5, \\
(4P \cdot x)\mathcal{A}_5 &= A_3 - A_4, & (2P \cdot x)^2\mathcal{A}_6 &= A_1 - A_2 + A_3 + A_4 - A_5 + A_6, \\
\mathcal{T}_1 &= T_1, & (2P \cdot x)\mathcal{T}_2 &= T_1 + T_2 - 2T_3, \\
2\mathcal{T}_3 &= T_7, & (2P \cdot x)\mathcal{T}_4 &= T_1 - T_2 - 2T_7, \\
(2P \cdot x)\mathcal{T}_5 &= -T_1 + T_5 + 2T_8, & (2P \cdot x)^2\mathcal{T}_6 &= 2T_2 - 2T_3 - 2T_4 + 2T_5 + 2T_7 + 2T_8, \\
(4P \cdot x)\mathcal{T}_7 &= T_7 - T_8, & (2P \cdot x)^2\mathcal{T}_8 &= -T_1 + T_2 + T_5 - T_6 + 2T_7 + 2T_8.
\end{aligned}$$

These DAs can be written as

$$F(a_i p \cdot x) = \int dx_1 dx_2 dx_3 \delta(x_1 + x_2 + x_3 - 1) e^{-ip \cdot x \sum_i x_i a_i} F(x_i). \quad (\text{A2})$$

These light-cone DAs are scale dependent and can be expanded next-to-leading spin accuracy with the conformal operators whose explicit expressions are,

$$\begin{aligned}
V_1(x_i, \mu) &= 120x_1x_2x_3[\phi_3^0(\mu) + \phi_3^+(\mu)(1 - 3x_3)], \\
V_2(x_i, \mu) &= 24x_1x_2[\phi_4^0(\mu) + \phi_4^+(\mu)(1 - 5x_3)], \\
V_3(x_i, \mu) &= 12x_3\{\psi_4^0(\mu)(1 - x_3) + \psi_4^-(\mu)[x_1^2 + x_2^2 - x_3(1 - x_3)] + \psi_4^+(\mu)(1 - x_3 - 10x_1x_2)\}, \\
V_4(x_i, \mu) &= 3\{\psi_5^0(\mu)(1 - x_3) + \psi_5^-(\mu)[2x_1x_2 - x_3(1 - x_3)] + \psi_5^+(\mu)[1 - x_3 - 2(x_1^2 + x_2^2)]\}, \\
V_5(x_i, \mu) &= 6x_3[\phi_5^0(\mu) + \phi_5^+(\mu)(1 - 2x_3)], \\
V_6(x_i, \mu) &= 2[\phi_6^0(\mu) + \phi_6^+(\mu)(1 - 3x_3)], \\
A_1(x_i, \mu) &= 120x_1x_2x_3\phi_3^-(\mu)(x_2 - x_1), \\
A_2(x_i, \mu) &= 24x_1x_2\phi_4^-(\mu)(x_2 - x_1), \\
A_3(x_i, \mu) &= 12x_3(x_2 - x_1)\{\psi_4^0(\mu) + \psi_4^+(\mu) + \psi_4^-(\mu)(1 - 2x_3)\}, \\
A_4(x_i, \mu) &= 3(x_2 - x_1)\{-\psi_5^0(\mu) + \psi_5^-(\mu)x_3 + \psi_5^+(\mu)(1 - 2x_3)\}, \\
A_5(x_i, \mu) &= 6x_3(x_2 - x_1)\phi_5^-(\mu), \\
A_6(x_i, \mu) &= 2(x_2 - x_1)\phi_6^-(\mu), \\
T_1(x_i, \mu) &= 120x_1x_2x_3\left[\phi_3^0(\mu) + \frac{1}{2}(\phi_3^- - \phi_3^+)(\mu)(1 - 3x_3)\right], \\
T_2(x_i, \mu) &= 24x_1x_2[\xi_4^0(\mu) + \xi_4^+(\mu)(1 - 5x_3)], \\
T_3(x_i, \mu) &= 6x_3\{(\xi_4^0 + \phi_4^0 + \psi_4^0)(\mu)(1 - x_3) + (\xi_4^- + \phi_4^- - \psi_4^-)(\mu)[x_1^2 + x_2^2 - x_3(1 - x_3)] \\
&\quad + (\xi_4^+ + \phi_4^+ + \psi_4^+)(\mu)(1 - x_3 - 10x_1x_2)\}, \\
T_4(x_i, \mu) &= \frac{3}{2}\{(\xi_5^0 + \phi_5^0 + \psi_5^0)(\mu)(1 - x_3) + (\xi_5^- + \phi_5^- - \psi_5^-)(\mu)[2x_1x_2 - x_3(1 - x_3)] \\
&\quad + (\xi_5^+ + \phi_5^+ + \psi_5^+)(\mu)[1 - x_3 - 2(x_1^2 + x_2^2)]\}, \\
T_5(x_i, \mu) &= 6x_3[\xi_5^0(\mu) + \xi_5^+(\mu)(1 - 2x_3)], \\
T_6(x_i, \mu) &= 2\left[\phi_6^0(\mu) + \frac{1}{2}(\phi_6^- - \phi_6^+)(\mu)(1 - 3x_3)\right], \\
T_7(x_i, \mu) &= 6x_3\{(-\xi_4^0 + \phi_4^0 + \psi_4^0)(\mu)(1 - x_3) + (-\xi_4^- + \phi_4^- - \psi_4^-)(\mu)[x_1^2 + x_2^2 - x_3(1 - x_3)] \\
&\quad + (-\xi_4^+ + \phi_4^+ + \psi_4^+)(\mu)(1 - x_3 - 10x_1x_2)\}, \\
T_8(x_i, \mu) &= \frac{3}{2}\{(-\xi_5^0 + \phi_5^0 + \psi_5^0)(\mu)(1 - x_3) + (-\xi_5^- + \phi_5^- - \psi_5^-)(\mu)[2x_1x_2 - x_3(1 - x_3)] \\
&\quad + (-\xi_5^+ + \phi_5^+ + \psi_5^+)(\mu)[1 - x_3 - 2(x_1^2 + x_2^2)]\}, \\
S_1(x_i, \mu) &= 6x_3(x_2 - x_1)[(\xi_4^0 + \phi_4^0 + \psi_4^0 + \xi_4^+ + \phi_4^+ + \psi_4^+)(\mu) + (\xi_4^- + \phi_4^- - \psi_4^-)(\mu)(1 - 2x_3)], \\
S_2(x_i, \mu) &= \frac{3}{2}(x_2 - x_1)[-(\psi_5^0 + \phi_5^0 + \xi_5^0)(\mu) + (\xi_5^- + \phi_5^- - \psi_5^-)(\mu)x_3 + (\xi_5^+ + \phi_5^+ + \psi_5^+)(\mu)(1 - 2x_3)], \\
P_1(x_i, \mu) &= 6x_3(x_2 - x_1)[(\xi_4^0 - \phi_4^0 - \psi_4^0 + \xi_4^+ - \phi_4^+ - \psi_4^+)(\mu) + (\xi_4^- - \phi_4^- + \psi_4^-)(\mu)(1 - 2x_3)], \\
P_2(x_i, \mu) &= \frac{3}{2}(x_2 - x_1)[(\psi_5^0 + \phi_5^0 - \xi_5^0)(\mu) + (\xi_5^- - \phi_5^- + \psi_5^-)(\mu)x_3 + (\xi_5^+ - \phi_5^+ - \psi_5^+)(\mu)(1 - 2x_3)],
\end{aligned} \quad (\text{A3})$$

where V_1, A_1 and T_1 are leading twist-3; $S_1, P_1, V_2, V_3, A_2, A_3, T_2, T_3$ and T_7 are twist-4; $S_2, P_2, V_4, V_5, A_4, A_5, T_4, T_5$ and T_8 are twist-5; and V_6, A_6 and T_6 are twist-6 DAs.

APPENDIX B: NON-PERTURBATIVE PARAMETERS

The coefficients $\phi_3^0, \phi_6^0, \phi_4^0, \phi_5^0, \xi_4^0, \xi_5^0, \psi_4^0, \psi_5^0, \phi_3^-, \phi_3^+, \phi_4^-, \phi_4^+, \psi_4^-, \psi_4^+, \xi_4^-, \xi_4^+, \phi_5^-, \phi_5^+, \psi_5^-, \psi_5^+, \xi_5^-, \xi_5^+, \phi_6^-, \phi_6^+$ in the expansions of the DAs can be expressed in terms of eight nonperturbative parameters $f_N, \lambda_1, \lambda_2, f_1^u, f_1^d, f_2^d, A_1^u, V_1^d$ as follows (see [17]):

For the leading conformal spin,

$$\begin{aligned} \phi_3^0 &= \phi_6^0 = f_N, & \phi_4^0 &= \phi_5^0 = \frac{1}{2}(f_N + \lambda_1), \\ \xi_4^0 &= \xi_5^0 = \frac{1}{6}\lambda_2, & \psi_4^0 &= \psi_5^0 = \frac{1}{2}(f_N - \lambda_1), \end{aligned} \quad (\text{B1})$$

for the next-to-leading spin, for twist-3:

$$\phi_3^- = \frac{21}{2}f_N A_1^u, \quad \phi_3^+ = \frac{7}{2}f_N(1 - 3V_1^d), \quad (\text{B2})$$

for twist-4:

$$\begin{aligned} \phi_4^+ &= \frac{1}{4}[f_N(3 - 10V_1^d) + \lambda_1(3 - 10f_1^d)], \\ \phi_4^- &= -\frac{5}{4}[f_N(1 - 2A_1^u) - \lambda_1(1 - 2f_1^d - 4f_1^u)], \\ \psi_4^+ &= -\frac{1}{4}[f_N(2 + 5A_1^u - 5V_1^d) - \lambda_1(2 - 5f_1^d - 5f_1^u)], \\ \psi_4^- &= \frac{5}{4}[f_N(2 - A_1^u - 3V_1^d) - \lambda_1(2 - 7f_1^d + f_1^u)], \\ \xi_4^+ &= \frac{1}{16}\lambda_2(4 - 15f_2^d), \\ \xi_4^- &= \frac{5}{16}\lambda_2(4 - 15f_2^d), \end{aligned} \quad (\text{B3})$$

for twist-5:

$$\begin{aligned} \phi_5^+ &= -\frac{5}{6}[f_N(3 + 4V_1^d) - \lambda_1(1 - 4f_1^d)], \\ \phi_5^- &= -\frac{5}{3}[f_N(1 - 2A_1^u) - \lambda_1(f_1^d - f_1^u)], \\ \psi_5^+ &= -\frac{5}{6}[f_N(5 + 2A_1^u - 2V_1^d) - \lambda_1(1 - 2f_1^d - 2f_1^u)], \\ \psi_5^- &= \frac{5}{3}[f_N(2 - A_1^u - 3V_1^d) + \lambda_1(f_1^d - f_1^u)], \\ \xi_5^+ &= \frac{5}{36}\lambda_2(2 - 9f_2^d), \\ \xi_5^- &= -\frac{5}{4}\lambda_2 f_2^d, \end{aligned} \quad (\text{B4})$$

and for twist-6:

$$\begin{aligned} \phi_6^+ &= \frac{1}{2}[f_N(1 - 4V_1^d) - \lambda_1(1 - 2f_1^d)], \\ \phi_6^- &= \frac{1}{2}[f_N(1 + 4A_1^u) + \lambda_1(1 - 4f_1^d - 2f_1^u)]. \end{aligned} \quad (\text{B5})$$

The values of the nonperturbative parameters $f_N, \lambda_1, \lambda_2, f_1^u, f_1^d, f_2^d, A_1^u, V_1^d$ can be found in [5,17].

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