

Quark lepton complementarity and renormalization group effects

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We consider a scenario for the quark-lepton complementarity relations between mixing angles in which the bimaximal mixing follows from the neutrino mass matrix. According to this scenario in the lowest order the angle θ_{12} is $\sim 1\sigma$ ($1.5^\circ - 2^\circ$) above the best fit point coinciding practically with the tribimaximal mixing prediction. Realization of this scenario in the context of the seesaw type-I mechanism with leptonic Dirac mass matrices approximately equal to the quark mass matrices is studied. We calculate the renormalization group corrections to θ_{12} as well as to θ_{13} in the standard model (SM) and minimal supersymmetric standard model (MSSM). We find that in a large part of the parameter space corrections $\Delta\theta_{12}$ are small or negligible. In the MSSM version of the scenario, the correction $\Delta\theta_{12}$ is in general positive. Small negative corrections appear in the case of an inverted mass hierarchy and opposite CP parities of ν_1 and ν_2 when leading contributions to θ_{12} running are strongly suppressed. The corrections are negative in the SM version in a large part of the parameter space for values of the relative CP phase of ν_1 and ν_2 : $\varphi > \pi/2$.

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I. INTRODUCTION

Implications of the observed pattern of neutrino mass and mixing (with two large angles) for fundamental physics are still an open question. This pattern has not yet led to a better understanding of the origins of the neutrino mass as well as fermion masses and mixing in general. In contrast, it made the situation more complicated and more intriguing [1]. In this connection, any hint from data and any empirical relation should be taken seriously and analyzed in detail.

In fact, one feature has been realized recently that (if not accidental) may lead to a substantially different approach to the underlying physics. Namely, the sums of the mixing angles of quarks and leptons for the 1–2 and 2–3 generations agree with 45° within 1σ . In other words, the quark and lepton mixings sum up to maximal mixing [2–4]:

$$\theta_{12} + \theta_C \approx \frac{\pi}{4}, \quad \theta_{23} + \theta_{cb} \approx \frac{\pi}{4}. \quad (1)$$

Here θ_C is the Cabibbo angle, $\theta_{cb} \equiv \arcsin V_{cb}$, and V_{cb} is the element of the Cabibbo-Kobayashi-Maskawa (CKM) matrix. According to Eqs. (1) which are called the quark-lepton complementarity (QLC) relations, the quark and lepton mixings are complementary to the maximal mixing. (A possibility that the lepton mixing responsible for the solar neutrino conversion equals maximal mixing minus θ_C was first mentioned in [5], and corrections to the

bimaximal mixing [6] from the CKM-type rotations were discussed in [7].)

For various reasons it is difficult to expect exact equalities (1). However certain correlations clearly show up:

- (i) The 2–3 leptonic mixing is close to maximal because the 2–3 quark mixing, V_{cb} , is very small.
- (ii) The 1–2 leptonic mixing deviates from maximal substantially because the 1–2 quark mixing (i.e., the Cabibbo angle) is relatively large.

If not accidental coincidence, the QLC relations imply [3,4,8]

- (i) a kind of quark-lepton symmetry or quark-lepton unification which propagates the information about mixing from the quark sector to the lepton sector;
- (ii) existence of some additional structure which produces maximal or bimaximal mixing.

Even within this context, one expects some deviations from exact quark-lepton complementarity due to

- (i) broken quark-lepton symmetry,
- (ii) renormalization group (RG) effects.

There are a number of attempts to reproduce the QLC relations on the basis of already existing ideas about fermion mass matrices [3,8–10]. Usually they lead to too small deviations of θ_{12} from $\pi/4$, and therefore require further corrections or deviations from the bimaximal mixing or from the Cabibbo mixing already in the lowest order. So, in the majority of the models proposed so far, an approximate QLC relation appears as a result of an interplay of different independent factors or as a sum of several independent contributions. In these cases the QLC relation seems to be accidental. There are few attempts to construct

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a consistent gauge model which explains the QLC relations. The simplest possibility is the $SU(2)_L \times SU(2)_R \times SU(4)_C$ model that implements the quark-lepton symmetry in the most straightforward way [8,10]. Phenomenology of schemes with QLC relations has been extensively studied [4,9,11,12].

The relation (1) is realized at some high energy scale, M_F , of flavor physics and quark-lepton unification. Therefore one should take into account the renormalization group effects on the QLC relations when confronting them with the low energy data. In fact, it was marked in [4] that in the minimal supersymmetric standard model (MSSM) the corrections are typically positive but negative $\Delta\theta_{12}$ can be obtained from the RG effects in the presence of nonzero 1–3 mixing. Also threshold corrections due to some new scale of physics, such as the low scale supersymmetry, can produce a negative shift of θ_{12} , thus enhancing its deviation from $\pi/4$ [13].

The Cabibbo mixing can be transmitted to the lepton sector in a more complicated way (than via the quark-lepton symmetry). In fact, $\sin\theta_C$ may turn out to be a generic parameter of the theory of fermion masses—the “quantum” of flavor physics. Therefore it may appear in various places: mass ratios, mixing angles. One can consider the Cabibbo angle as an expansion parameter for mixing matrices [4,7,14,15].

In this paper we study in detail the RG effects in the QLC scenario where the bimaximal mixing is generated by the neutrino mass matrix. We calculate corrections to the angles both in the standard model (SM) and MSSM. We analyze the dependence of the corrections on various parameters and obtain bounds on the parameters from consistency condition with QLC. In particular, we find regions where the corrections are negative. The paper is organized as follows. In Sec. II we formulate the scenario and comment on parametrization dependence of the QLC relations as well as confront the relations with experimental data. In Sec. III we consider realization of the scenario in the seesaw type I mechanism. The RG effects in MSSM and SM are described in Secs. IV and V correspondingly. We consider the RG effects on 1–3 mixing and dependence of the effects on the scale of new physics in Sec. VI. Conclusions are formulated in Sec. VII.

II. UPDATE ON QLC

A. A scenario

A general scheme for the QLC relations is that

$$\text{“lepton mixing} = \text{bimaximal mixing} - \text{CKM”}, \quad (2)$$

where the bimaximal mixing matrix is [6]

$$U_{bm} = U_{23}^m U_{12}^m = \frac{1}{2} \begin{pmatrix} \sqrt{2} & \sqrt{2} & 0 \\ -1 & 1 & \sqrt{2} \\ 1 & -1 & \sqrt{2} \end{pmatrix}. \quad (3)$$

Here U_{ij}^m is the maximal mixing ($\pi/4$) rotation in the ij -plane.

We assume that the bimaximal mixing is generated by the neutrino mass matrix. That is, the same mechanism which is responsible for the smallness of neutrino mass leads also to the large lepton mixing, and it is the seesaw mechanism [16] that plays the role of additional structure that generates the bimaximal mixing. Therefore

$$U_{\text{PMNS}} = U_l^\dagger U_\nu = V_{\text{CKM}}^\dagger \Gamma_\alpha U_{bm}, \quad (4)$$

where $\Gamma_\alpha \equiv \text{diag}(e^{i\alpha_1}, e^{i\alpha_2}, e^{i\alpha_3})$ is the phase matrix that can appear, in general, at diagonalization of the charged lepton or neutrino Dirac mass matrices.

Similarity of the Dirac mass matrices in the lepton and quark sectors, related to the quark-lepton symmetry, is the origin of the CKM rotations in the lepton sector. Here, there are two possibilities:

- (i) In a certain (“symmetry”) basis, where the theory of flavor is formulated, the neutrino mass matrix is of the bimaximal form. So $U_\nu = U_{bm}$, and the charged lepton mass matrix is diagonalized by the CKM rotation:

$$U_l = V_{\text{CKM}}. \quad (5)$$

The problem here is that the masses of charged leptons and down quarks are different: in particular, $m_e/m_\mu = 0.0047$, whereas $m_d/m_s = 0.04\text{--}0.06$, and also $m_\mu \neq m_s$ at the grand unified theory (GUT) scale. Since $m_l \neq m_d$, the equality (5) implies a particular structure of the mass matrices in which mixing weakly depends on eigenvalues.

- (ii) In the symmetry basis both the bimaximal and CKM mixings come from the neutrino mass matrix, and the charged lepton mass matrix is diagonal. That is, the symmetry basis coincides with the flavor basis. In this case, the Dirac mass matrix of neutrinos is the origin of the CKM rotation, whereas the Majorana mass matrix of the right-handed (RH) neutrinos is responsible for the bimaximal mixing. Since the eigenvalues of the Dirac neutrino mass matrix are unknown, we can assume an exact equality of the mass matrices,

$$m_u = m_\nu^D, \quad (6)$$

as a consequence of the quark-lepton symmetry. The equality (6) propagates the CKM mixing from the quark to the lepton sector precisely. In this case, however, the Gatto-Sartori-Tonin relation between the Cabibbo angle and the ratio of down quark masses [17] turns out to be accidental. Furthermore, one needs to explain why in the symmetry basis both the charged lepton and down quark mass matrices are diagonal simultaneously in spite of difference of eigenvalues.

These two cases have different theoretical implications, however the phenomenological consequences and the RG effects are the same.

In the scenario under consideration, the relation (1) is not realized precisely even for zero phases α_i since the U_{12}^{CKM} rotation matrix should be permuted with U_{23}^m in (4) to reduce the mixing matrix to the standard parametrization form [4]. From (4) we obtain the following expressions for the leptonic mixing angles:

$$\begin{aligned} U_{e2} &\equiv \cos\theta_{13} \sin\theta_{12} \\ &= \sin(\pi/4 - \theta_C) \\ &\quad + 0.5 \sin\theta_C [\sqrt{2} - 1 - V_{cb} \cos(\alpha_3 - \alpha_1)] \\ &\quad + 0.5 V_{ub} \cos(\alpha_3 - \alpha_1 - \delta_q) \end{aligned} \quad (7)$$

where δ_q is the quark CP -violating phase. This expression differs from the one derived in [4] by a factor $\cos\theta_{13}$ as well as by the last term, that turn out to be relevant at the level of accuracy we will consider here. The 1–3 mixing is large in this scenario [4,11,12]:

$$\begin{aligned} \sin\theta_{13} &= -\frac{\sin\theta_C}{\sqrt{2}} (1 - V_{cb} \cos\alpha_3) - \frac{V_{ub}}{\sqrt{2}} \cos(\alpha_3 - \delta_q) \\ &\approx -\frac{\sin\theta_C}{\sqrt{2}} \end{aligned} \quad (8)$$

and, hence, the Dirac CP phase δ is close to 180° . So, for the 1–2 mixing we find from (7) and (8)

$$\sin\theta_{12} \approx U_{e2} (1 + \frac{1}{4} \sin^2\theta_C), \quad (9)$$

and U_{e2} is given in (7). The expression for the 2–3 mixing reads

$$\begin{aligned} U_{\mu 3} &= \cos\theta_{13} \sin\theta_{12} \\ &= \cos\theta_C \left[\sin(\pi/4 - \theta_C) + \frac{V_{cb}}{\sqrt{2}} (1 - \cos\alpha_3) \right]. \end{aligned} \quad (10)$$

The RG effect on V_{CKM} is negligible.

B. QLC and parametrization

The QLC relations are essentially parametrization independent. They can be expressed in terms of physical quantities (compare with [18]). Indeed, the moduli of elements of the mixing matrix, $U_{\alpha i}$, are physical quantities immediately related to observables and, consequently, parametrization independent. In the standard parametrization

$$|U_{e2}| = |\cos\theta_{13} \sin\theta_{12}|, \quad |U_{e3}| = |\sin\theta_{13}|, \quad (11)$$

and therefore

$$|\sin\theta_{12}| = \frac{1}{\sqrt{1 - |U_{e3}|^2}} |U_{e2}|. \quad (12)$$

Notice that the presence of 1–3 mixing produces some ambiguity in formulation of the QLC relations. One can write the relations in terms of angles in the standard

parametrization or in terms of matrix elements:

$$\arcsin(V_{us}) + \arcsin(U_{e2}) = \pi/4. \quad (13)$$

Both forms coincide in the limit $U_{e3} \rightarrow 0$.

C. Experimental status

In Fig. 1 we show results of determination of θ_{12} by different groups. The SNO collaboration did analysis of the data in terms of 2ν mixing [19], whereas in [20,21] complete 3ν analyses have been performed. Furthermore, in [21] a nonzero best fit value of 1–3 mixing has been obtained. Results of different analyses are in very good agreement:

$$\begin{aligned} \theta_{12} &= (33.8 \pm 2.2)^\circ (\theta_{13} = 0), \\ \theta_{12} &= (34.2 \pm 1.5)^\circ (\theta_{13} = 7^\circ). \end{aligned} \quad (14)$$

Notice that the determination of θ_{12} follows mainly from analysis of the solar neutrino data. In this analysis θ_{12} and θ_{13} correlate. In particular, the CC/NC ratio that gives the most important restriction on mixing is determined by $P \sim \cos^4\theta_{13} \sin^2\theta_{12}$. The best fit values (14) are along with the trajectory $P = \text{constant}$.

In Fig. 1 we show the range of QLC values of θ_{12} obtained from Eqs. (7) and (9) by varying $(\alpha_3 - \alpha_1)$. This variation gives

$$\theta_{12}^{\text{QLC}} = 35.65^\circ - 36.22^\circ, \quad (15)$$

or

$$\sin^2\theta_{12}^{\text{QLC}} = 0.340 - 0.349. \quad (16)$$

The smallest value of θ_{12} corresponds to $\alpha_3 - \alpha_1 = -24^\circ$.

For the 1–3 mixing we obtain

$$\sin^2\theta_{13} \approx \frac{1}{2} \sin^2\theta_C = 0.024, \quad (17)$$

which is at the upper 1σ edge from the analysis [21].

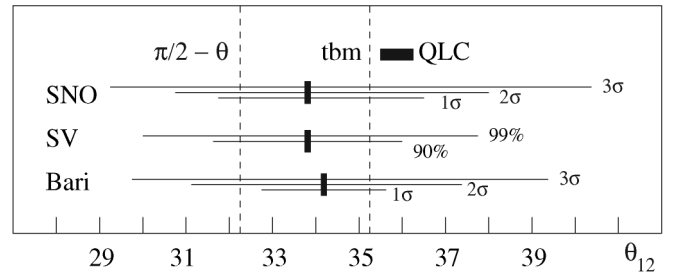


FIG. 1. The 1–2 mixing angle: experimental results and predictions. Shown are the best fit values and allowed regions for θ_{12} from analyses of different groups SNO [19], Strumia-Vissani [20], and Bari [21]. The vertical dashed lines correspond to value $(\pi/4 - \theta_C)$ and tribimaximal mixing. The horizontal bar shows values of QLC predictions without RG corrections.

The sums of angles equal

$$\theta_{12} + \theta_C = 46.7^\circ \pm 2.4^\circ \quad (1\sigma) \quad (18)$$

$$\theta_{23} + \theta_{cb} = (43.9^{+5.1}_{-3.6})^\circ \quad (1\sigma). \quad (19)$$

The QLC prediction is slightly larger than the experimental best fit point:

$$\theta_{12}^{\text{QLC}} - \theta_{12}^{\text{bf}} = 1.5^\circ - 2.0^\circ. \quad (20)$$

The difference is well within 1σ of experimental measurements. The exact complementarity value, $45^\circ - \theta_C$, is $1.8^\circ - 2.0^\circ$ below the best fit value. To disentangle these possibilities, one needs to measure the $1-2$ angle with accuracy better than 1 degree: $\Delta\theta_{12} < 1^\circ$, that is translated into

$$\frac{\Delta \sin^2 \theta_{12}}{\sin^2 \theta_{12}} = \frac{2}{\tan \theta_{12}} \Delta \theta_{12} \sim 5\% (\Delta \theta_{12}/1^\circ), \quad (21)$$

or

$$\frac{\Delta \sin^2 2\theta_{12}}{\sin^2 2\theta_{12}} = \frac{4}{\tan 2\theta_{12}} \Delta \theta_{12} \sim 2.7\% (\Delta \theta_{12}/1^\circ). \quad (22)$$

Forthcoming results from SNO phase-III (He) will improve determination of the CC/NC ratio, and consequently, θ_{12} . Future low energy solar neutrino experiments aimed at measurements of the pp -neutrino flux will have a $(1-2)\%$ sensitivity to $\sin^2 2\theta_{12}$ provided that degeneracy with $1-3$ mixing is resolved. Similar sensitivity could be achieved in dedicated reactor neutrino experiments with a large baseline [22].

D. QLC and tribimaximal mixing

The QLC prediction (16) is practically indistinguishable from the tribimaximal mixing [23] prediction $\sin^2 \theta_{12} = 1/3$. So, it turns out that almost the same values of $1-2$ mixing are obtained from two different and independent combinations of matrices:

$$U_{23}^m U_{12}(\arcsin(1/\sqrt{3})) \quad \text{and} \quad U_{12}(\theta_C) U_{23}^m U_{12}^m. \quad (23)$$

There are two possible interpretations of this fact.

The coincidence is accidental, which means that one of the two approaches (QLC or tribimaximal mixing) does not correspond to reality. To some extent that can be tested by measuring the $1-3$ mixing. In the QLC-scenario, one obtains (17), whereas the tribimaximal mixing implies $\sin^2 \theta_{13} = 0$ unless some corrections are introduced.

The coincidence is not accidental, and therefore it implies a nontrivial expression for the Cabibbo angle. Indeed, from the equality $\sin \theta_{12}^{\text{QLC}} = \sin \theta_{12}^{\text{tbm}}$ we obtain

$$\sin \theta_C = \frac{2}{3\sqrt{3}} \left(\sqrt{\frac{5}{2}} - 1 \right). \quad (24)$$

III. SEESAW AND QLC

The seesaw mechanism [16] that provides a natural explanation of smallness of neutrino mass can also be the origin of the difference of the quark and lepton mixings, and, in particular, the origin of bimaximal mixing. In the context of seesaw type-I this implies a particular structure of the RH neutrino mass matrix.

We assume that the type-I seesaw gives the dominant contribution to neutrino masses since it can provide the closest relation between the quark and lepton mass matrices, as required by the QLC. The relevant terms of the Lagrangian are

$$- \mathcal{L} = l^c Y_e L H_d + N^T Y_\nu L H_u + \frac{1}{2} N^T M_R N + \text{H.c.}, \quad (25)$$

where $L \equiv (\nu, l)$ is the leptonic doublet, $N \equiv (\nu_R)^c$, H_d and H_u are two different Higgs doublets in the MSSM and $H_u = i\tau_2 H_d$ in SM; Y_e and Y_ν are the charged lepton and neutrino Yukawa coupling matrices. We consider M_R as a bare mass matrix of the RH neutrinos formed already at the GUT or even higher scale. It can be generated by some new interactions at higher scales.

Decoupling of N leads to the low energy effective $D = 5$ operator

$$\nu^T Y_\nu^T M_R^{-1} Y_\nu \nu H_u H_u. \quad (26)$$

After the electroweak symmetry breaking, this operator generates the mass term for light neutrinos, $\nu^T m_\nu \nu$ with

$$m_\nu = -m_D^T M_R^{-1} m_D, \quad (27)$$

where $m_D = Y_\nu \langle H_u \rangle$.

Let us consider the basis where the neutrino Dirac mass matrix is diagonal:

$$Y_\nu = Y_\nu^{\text{diag}} \equiv \text{diag}(y_1, y_2, y_3). \quad (28)$$

Then the light neutrino mass matrix equals

$$m_\nu = -m_D^{\text{diag}} M_R^{-1} m_D^{\text{diag}}, \quad (29)$$

and $m_D^{\text{diag}} \equiv Y_\nu^{\text{diag}} \langle H_u \rangle$.

According to our assumption, the matrix (29) should generate the bimaximal rotation:

$$m_\nu = m_{bm}, \quad (30)$$

where, in general,

$$m_{bm} = \Gamma_\delta U_{bm} \Gamma_{\varphi/2} m_\nu^{\text{diag}} \Gamma_{\varphi/2} U_{bm}^T \Gamma_\delta. \quad (31)$$

Here

$$\Gamma_\delta \equiv \text{diag}(e^{i\delta_1}, e^{i\delta_2}, e^{i\delta_3}), \quad (32)$$

is the phase matrix,

$$m_\nu^{\text{diag}} \equiv \text{diag}(m_1, m_2, m_3) \quad (33)$$

is the diagonal matrix of the light neutrinos, and

$$\Gamma_\varphi \equiv \text{diag}(e^{i\varphi_1/2}, e^{i\varphi_2/2}, 1), \quad (34)$$

with φ_i being the Majorana phases of light neutrinos.

According to our assumption, the CKM rotation follows from diagonalization of the charged lepton matrix

$$V_{\text{CKM}}^\dagger Y_e^\dagger Y_e V_{\text{CKM}} = \text{diag}(y_e^2, y_\mu^2, y_\tau^2), \quad (35)$$

and we will parametrize it as

$$V_l = \Gamma_\phi V_{\text{CKM}}(\theta_q, \delta_q). \quad (36)$$

Here the diagonal matrix of the phase factors on the RH side has been absorbed in the charged lepton field redefinition; V_{CKM} is the CKM matrix in the standard parametrization, θ_q and δ_q are the quark (CKM) mixing angles, and

$$\Gamma_\phi \equiv \text{diag}(e^{i\phi_1}, e^{i\phi_2}, e^{i\phi_3}). \quad (37)$$

Thus, in general, there are three matrices of phases, Γ_δ , Γ_φ , and Γ_ϕ , relevant for relations between the mixing angles. Finally, from (31) and (36) we obtain

$$U_{\text{PMNS}} = V_{\text{CKM}}^\dagger(\theta_q, \delta_q) \Gamma(\delta_l - \phi_l) U_{bm}, \quad (38)$$

and therefore in (4) $\alpha_j = (\delta_j - \phi_j)$.

The neutrino mass matrix in the flavor basis equals

$$m_\nu^f = V_{\text{CKM}}^T m_{bm} V_{\text{CKM}}. \quad (39)$$

From (29) and (31) we find an expression for the RH neutrino mass matrix:

$$M_R = \Gamma_\delta m_D^{\text{diag}} U_{bm} \Gamma_{\varphi/2} (m_\nu^{\text{diag}})^{-1} \Gamma_{\varphi/2} U_{bm}^T m_D^{\text{diag}} \Gamma_\delta. \quad (40)$$

Omitting the phase factor $\Gamma(\delta_i)$ (that can be absorbed in the definition of M_R) and including the CP phases φ_i into masses of light neutrinos $\Gamma_{\varphi/2} (m_\nu^{\text{diag}})^{-1} \Gamma_{\varphi/2} = (m_{\nu,\varphi}^{\text{diag}})^{-1}$, we obtain

$$M_R = m_D^{\text{diag}} U_{bm} (m_{\nu,\varphi}^{\text{diag}})^{-1} U_{bm}^T m_D^{\text{diag}}. \quad (41)$$

Explicitly

$$M_R = \frac{1}{4} m_D^{\text{diag}} \begin{pmatrix} 2A & -\sqrt{2}B & -\sqrt{2}B \\ \dots & C+A & C-A \\ \dots & \dots & C+A \end{pmatrix} m_D^{\text{diag}}, \quad (42)$$

where

$$A \equiv \frac{1}{m_1} + \frac{1}{m_2}, \quad B \equiv \frac{1}{m_2} - \frac{1}{m_1}, \quad C \equiv \frac{2}{m_3}, \quad (43)$$

(with phases φ_i included). We can parametrize m_D^{diag} as

$$m_D^{\text{diag}} = m_t \text{diag}(\epsilon'^2, \epsilon, 1), \quad (44)$$

with m_t being the mass of top quark and $\epsilon' \approx \epsilon \sim 3 \times 10^{-3}$. Using smallness of ϵ 's it is easy to estimate the mass eigenvalues:

$$M_3 \approx \frac{m_t^2}{4} (A + C), \quad M_2 \approx m_t^2 \epsilon^2 \frac{AC}{A + C}, \quad (45)$$

$$M_1 \approx m_t^2 \epsilon'^4 \frac{A^2 - B^2}{2A}.$$

Furthermore, the 1–2 and 2–3 mixing angles are of the order ϵ , whereas 1–3 mixing is of the order ϵ^2 .

In the case of normal mass hierarchy, $m_1 \ll m_2 \ll m_3$, Eqs. (45) lead to

$$M_3 \approx \frac{m_t^2}{4m_1}, \quad M_2 \approx \frac{2m_t^2 \epsilon^2}{m_3}, \quad M_1 \approx \frac{2m_t^2 \epsilon'^4}{m_2}, \quad (46)$$

in agreement with results of [24]. Notice a permutation character of these expressions: the masses of RH neutrinos 1, 2, 3 are determined by light masses 2, 3, 1. With $m_1 \rightarrow 0$, apparently, $M_3 \rightarrow \infty$. For $\epsilon' = \epsilon \sim 3 \times 10^{-3}$ and $m_1 = 10^{-3}$ eV values of masses equal

$$M_3 = 9 \times 10^{15} \text{ GeV}, \quad M_2 = 1 \times 10^{10} \text{ GeV}, \quad (47)$$

$$M_1 = 5 \times 10^5 \text{ GeV}.$$

So, masses have a “quadratic” hierarchy.

In the case of inverted mass hierarchy, $m_3 \ll m_1 \approx m_2 \equiv m_A$, and the same CP phases of ν_1 and ν_2 , we obtain from (45)

$$M_3 \approx \frac{m_t^2}{4m_3}, \quad M_2 \approx \frac{2m_t^2 \epsilon^2}{m_A}, \quad M_1 \approx \frac{2m_t^2 \epsilon'^4}{m_A}, \quad (48)$$

where $m_A \equiv \sqrt{|\Delta m_{31}^2|}$. This leads again to a strong mass hierarchy. Notice that now the mass of the lightest RH neutrino is determined by the atmospheric mass scale. Thus, apart from special regions in the parameter space that correspond to level crossings (see Sec. V) the QLC implies generically a very strong (quadratic) mass hierarchy of the RH neutrinos and very small mixing: $\Theta_{ij} \sim \epsilon$. As we will see, this determines substantially the size of the RG effects.

Let us introduce the unitary matrix, U_R , which diagonalizes the right-handed neutrino mass matrix

$$U_R^T M_R U_R = M_R^{\text{diag}} \equiv \text{diag}(M_1, M_2, M_3), \quad (49)$$

and the mixing matrix can be parametrized as

$$U_R = \Gamma_\Delta V_{\text{CKM}}(\Theta_{ij}, \Delta) \Gamma_{\xi/2}, \quad (50)$$

where Θ_{ij} and Δ are the angles and CP -phase of the RH neutrino mixing matrix.

In what follows we will not elaborate further on the origin of particular structures of M_R (42), just noticing that it can be related to the double (cascade) seesaw mechanism [25] with the “screening” of Dirac structure [26,27].

IV. RG EFFECTS: GENERAL CONSIDERATION AND THE MSSM CASE

A. General consideration

The quark-lepton symmetry implied by the QLC relation means that physics responsible for this relations should be realized at some scale M_F which is at the quark-lepton unification scale, M_{GUT} , or even higher scales. An alternative possibility would be the quark-lepton relation due to the Pati-Salam symmetry [28] broken below the GUT scale. Consequently, there are three different regions of RG running:

- (i) below the seesaw scales, $\mu < M_1$, where M_1 is the lightest RH neutrino mass. In this region all three neutrinos decouple and the $D = 5$ operator (26) is formed;
- (ii) between the seesaw scales, $M_1 < \mu < M_3$, where M_3 is the heaviest RH neutrino mass;
- (iii) above the seesaw scales $M_3 < \mu < M_F$. If $M_F > M_{\text{GUT}}$ new features of running can appear above M_{GUT} .

The RG equation for the neutrino mass matrix is given by [29–31]

$$16\pi^2 \dot{m}_\nu = P^T m_\nu + m_\nu P + \kappa m_\nu, \quad (51)$$

where $\dot{m}_\nu \equiv \mu dm_\nu/d\mu$, μ is the renormalization scale, and κm_ν includes the gauge interaction terms that can influence the flavor structure in the SM case (see below);

$$P \equiv C_e Y_e^\dagger Y_e + C_\nu Y_\nu^\dagger Y_\nu, \quad (52)$$

$C_e = -3/2$, $C_\nu = 1/2$ in the SM, and $C_e = C_\nu = 1/2$ in the MSSM. From the evolution equation (51) for the mass matrix, we can obtain the equations [32] for observables (masses and mixings) [33–36]. Above and below the seesaw scales the gauge interactions produce a flavor universal effect and all contributions (from all RH neutrinos) to the neutrino mass matrix have the same renormalization group equation.

In the limit of vanishing 1–3 mixing, the evolution of θ_{12} is described approximately by

$$32\pi^2 \dot{\theta}_{12} = Q_{12} [\sin 2\theta_{12} (P_{11} - c_{23}^2 P_{22} - s_{23}^2 P_{33} + s_{23} \text{Re} P_{23}) + 2 \cos 2\theta_{12} (c_{23} \text{Re} P_{21} - s_{23} \text{Re} P_{31})] + 4S_{12} (c_{23} \text{Im} P_{21} - s_{23} \text{Im} P_{31}), \quad (53)$$

where $s_{23} \equiv \sin \theta_{23}$, $c_{23} \equiv \cos \theta_{23}$, etc.,

$$Q_{ij} \equiv \frac{|m_i e^{i\varphi_i} + m_j e^{i\varphi_j}|^2}{\Delta m_{ji}^2} \quad (54)$$

and

$$S_{12} \equiv \frac{m_1 m_2 \sin(\varphi_1 - \varphi_2)}{\Delta m_{21}^2}. \quad (55)$$

Above the seesaw scale, one needs to consider renormalization of couplings of the full Lagrangian (25). The evolution of the effective operator which gives masses to neutrinos after the electroweak symmetry breaking is determined by evolution of the neutrino Yukawa couplings Y_ν and the mass terms of right-handed neutrinos.

Below the seesaw scales, running is dominated by P_{33} in the flavor basis which results in an increase of θ_{12} in MSSM and a slight decrease in the SM due to different signs of C_e :

$$32\pi^2 \dot{\theta}_{12} \approx -Q_{12} \sin 2\theta_{12} s_{23}^2 P_{33}. \quad (56)$$

Above the seesaw scales, the leading contribution is again given by P_{33} , and the next-to-leading contribution comes from P_{32} . This yields an increase of θ_{12} when running to low scales both in the MSSM and in SM. Explicitly, the corresponding evolution equation can be written as

$$32\pi^2 \dot{\theta}_{12} = -Q_{12} C_\nu \sin 2\theta_{12} \sin \theta_{23} [\sin \theta_{23} - V_{cb} \cos \theta_{23} \cos(\phi_2 - \phi_3)], \quad (57)$$

and the phases ϕ_i are determined in Eqs. (36) and (37).

The effect of running between the seesaw scales (about 10 orders of magnitude in μ) is more complicated. In this range the Yukawa coupling matrix has two terms (contributions):

- (1) $D = 5$ effective operators for the light neutrinos formed after decoupling of one or two RH neutrinos,
- (2) Dirac-type couplings and mass terms for uncoupled RH neutrinos given by the Lagrangian (25).

These terms are renormalized differently. In particular, for the terms of the second type the neutrino Yukawa couplings are important. The difference, however, cancels (between the seesaw scales) in the case of MSSM in which only the wave function renormalization takes place due to the nonrenormalization theorem. In contrast, in the SM due to vertex corrections to the $D = 5$ operators, the difference does not cancel, and, as we will see, produces a significant effect. So, in the MSSM, the RG equations are the same for both contributions and Eq. (53) is valid. In the SM they are not equal and Eq. (53) cannot be applied.

After the heaviest right-handed neutrino is integrated out, the right-handed neutrino mixing at the threshold influences running of θ_{12} . In the second order of $\sin \theta_C$, the expression for $\dot{\theta}_{12}$ reads

$$32\pi^2 \dot{\theta}_{12} = -\frac{1}{4} Q_{12} C_\nu (s_{23} - V_{cb} c_{23} \cos(\phi_2 - \phi_3)) \times (3 - 2 \cos 2\Theta_{23} \cos^2 \Theta_{13} - \cos 2\Theta_{13}) \times \sin 2\theta_{12} s_{23}, \quad (58)$$

where Θ_{ij} are the right-handed neutrino mixing angles at the scale at which the heaviest neutrino is integrated out. The unitary rotation of the right-handed neutrino fields is

done at the threshold of the heaviest right-handed neutrino, and the exact definitions of the angles are given in Eqs. (49) and (50).

B. RG evolution and scales of flavor physics

We have performed running from the M_F scale down to the electroweak scale and calculated $\Delta\theta_{12} \equiv \theta_{12}(M_Z) - \theta_{12}(M_F)$. For that we solved numerically a complete set of the RG equations including subleading effects due to the nonzero 1–3 mixing. In most of our calculations we take for definiteness $M_F = M_{\text{GUT}} = 2 \times 10^{16}$ GeV. We consider separately dependence of the results on M_F . Notice that the renormalization of θ_{12} in the bimaximal scheme has been studied in [37].

The following free parameters determine the RG effects substantially: the absolute scale of light masses m_1 , the CP (Majorana) phases of light neutrinos, φ_i , and the phases α_i . We studied dependence of the RG effects on these parameters. For each set of parameters we have calculated the RH mass matrix and running effects. The angles are fixed by the QLC relation at M_F , and the mass squared differences are adjusted to lie in the experimentally allowed region at the electroweak scale. For the neutrino Yukawa couplings we take $y_1:y_2:y_3 = \epsilon^2:\epsilon:1$, ($\epsilon = \epsilon'$) and $\epsilon = 3 \times 10^{-3}$.

We consider RG evolution in the MSSM with a unique SUSY threshold 1 TeV. The RG effects depend on the absolute mass scale, m_1 , $\tan\beta$, and the relative phase between the first and second mass eigenstates $\varphi \equiv \varphi_2 - \varphi_1$. Dependence on other parameters (e.g., other phases) is rather weak. Still we will use explicitly the phase φ_2 keeping everywhere $\varphi_1 = 0$.

In what follows we will describe results of our numerical calculations. We give an interpretation of the results using approximate formulas presented in Secs. III and IVA.

C. RG effect in MSSM with normal mass hierarchy

In Fig. 2 we show some examples of the scale dependence of θ_{12} for various values of parameters. With the increase of m_1 , two factors enhance the RG effects:

- (1) The largest mass M_3 decreases according to (46). Correspondingly, the region above the seesaw scale, $M_3 - M_{\text{GUT}}$, increases.
- (2) Corrections to the mass matrix elements are proportional to values of elements: $\Delta m_{\alpha\beta} \propto m_{\alpha\beta}$ and since with increase of m_1 the masses, $m_{\alpha\beta}$, generically increase, the corrections increase correspondingly.

For relatively small $\tan\beta \sim (3-10)$, the dominant contribution follows from the region above the seesaw scales due to large $(Y_\nu)_{33}$. Evolution below M_3 is mainly due to the Yukawa couplings Y_e which are relatively small. The effect increases fast with m_1 :

$$\Delta\theta_{12} \propto Q_{12} \log(M_{\text{GUT}}/M_3). \quad (59)$$

Notice that $M_3 \propto 1/m_1$. Therefore for $m_1 \sim 10^{-3}$ eV the running of θ_{12} is mainly related to an increase of the region above the seesaw scale. For $m_1 > 10^{-2}$ eV, the spectrum of light neutrinos becomes degenerate and $\Delta\theta_{12} \propto Q_{12} \propto m_1^2$ [Fig. 2(a)]. For large $\tan\beta$ and small m_1 , the dominant contribution to $\Delta\theta_{12}$ comes from the region below M_3 where $\Delta\theta_{12} \propto \tan^2\beta$ [see Fig. 2(b)].

A combined dependence of corrections, $\Delta\theta_{12}$, on m_1 and $\tan\beta$ is presented in Fig. 3 where we show contours of constant $\Delta\theta_{12}$ in the $(m_1 - \tan\beta)$ plane. The change of behavior of contours at $m_1 = 8 \times 10^{-4}$ eV is a consequence of our boundary condition: At $m_1 < 8 \times 10^{-4}$ eV we have $M_3 > M_{\text{GUT}}$, and therefore the region above the seesaw scale disappears.

In Fig. 4 we show the correction $\Delta\theta_{12}$ as functions of m_1 for different values of φ_2 . The dependence of $\Delta\theta_{12}$ on φ_2 ,

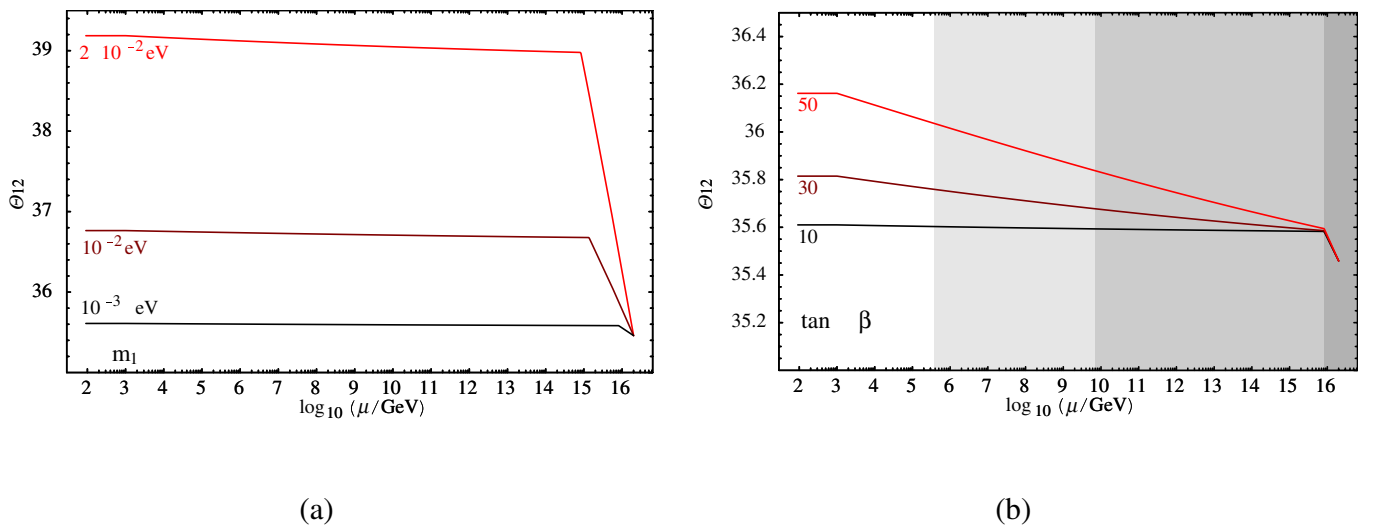


FIG. 2 (color online). Examples of running of θ_{12} in the case of MSSM and normal mass hierarchy. The dependence of θ_{12} on μ (a) for different values of m_1 , and $\tan\beta = 10$; (b) on $\tan\beta$ for $m_1 = 10^{-3}$ eV. All the CP -phases are taken to be zero.

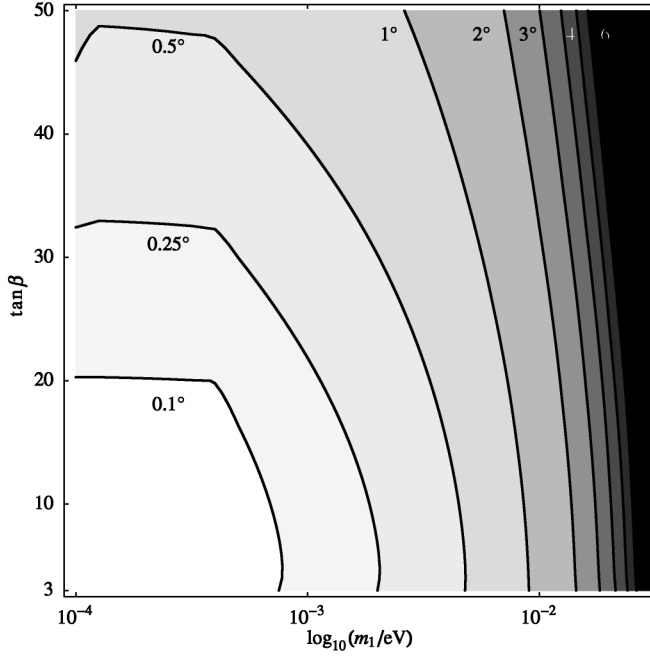


FIG. 3. Contours of constant RG corrections, $\Delta\theta_{12}$, in the $\tan\beta - m_1$ plane in the case of MSSM and normal mass hierarchy. All the CP -phases are taken to be zero.

given essentially by the factor Q_{12} , is weak for the hierarchical spectrum, $m_1 \ll 8 \times 10^{-3}$ eV, and very strong for the degenerate spectrum: $\Delta\theta_{12} \propto (1 + \cos\varphi)$. The corrections are strongly suppressed for the opposite CP -parities, $\varphi_2 = 180^\circ$ (Fig. 4), which agrees with the results of previous studies of corrections in the degenerate case [38–41].

Corrections $\Delta\theta_{12}$ are positive. This fact is essentially a consequence of strong hierarchy of the Yukawa couplings Y_ν and Y_e . The evolution is given approximately by Eq. (53), where $P_{33} \propto 1/2(|(Y_e)_{33}|^2 + |(Y_\nu)_{33}|^2) > 0$. The

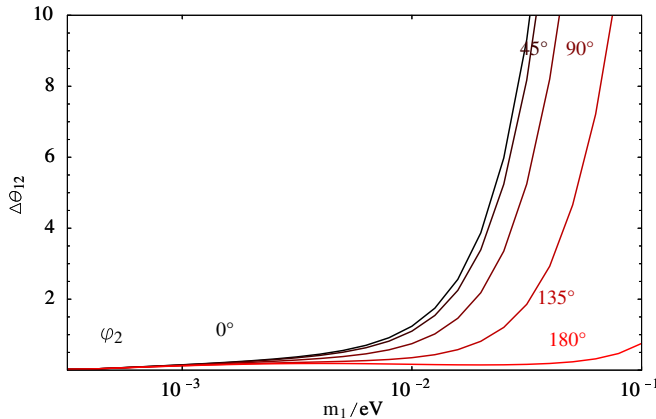


FIG. 4 (color online). The dependence of the RG correction, $\Delta\theta_{12}$ (in degrees), on m_1 for different values of φ_2 (figures at the curves) in the MSSM with the normal mass hierarchy. The lines correspond to $\tan\beta = 10$ and $\varphi_1 = 0$.

off-diagonal couplings P_{ij} are much smaller. Since $Q_{12} > 0$, we obtain $\dot{\theta}_{12} < 0$, that is, the angle θ_{12} increases with decrease of μ .

The condition that the QLC prediction for θ_{12} is within 1σ of the best fit experimental value requires $\Delta\theta_{12} < 0.5^\circ - 1^\circ$. This, in turn, leads to bounds on parameters of neutrino spectrum and $\tan\beta$. In particular, according to Fig. 4 the degenerate neutrino spectrum is excluded for the same CP parities ($\varphi_2 = 0$). In the case of large $\tan\beta$, it requires a strongly hierarchical spectrum: $m_1 < 10^{-3}$ eV that eliminates the running region above the seesaw scale. However, a degenerate spectrum is allowed for $\varphi \sim 180^\circ$.

Taking the 2σ upper bound $\Delta\theta_{12} < 2^\circ$, we find that the quasidegenerate spectrum with $m_1 \sim 10^{-2}$ eV is allowed even for the same parities. For normal mass hierarchy with $m_1 < 10^{-3}$ eV and $\tan\beta \sim (3-10)$, the running effect is negligible: $\Delta\theta_{12} < 0.1^\circ$.

D. MSSM and inverted mass hierarchy

In the case of inverted mass hierarchy, the states ν_1 and ν_2 associated to 1–2 mixing are strongly degenerate. Therefore, the RG effects are similar to those in the normal hierarchy case for $m_1 = m_A \sim 5 \times 10^{-2}$ eV. So, the corrections, $\Delta\theta_{12}$, are enhanced by the factor

$$\frac{(\Delta\theta_{12})^{\text{IH}}}{(\Delta\theta_{12})^{\text{NH}}} = \frac{(m_2^{\text{IH}})^2}{(m_2^{\text{NH}})^2}, \quad (60)$$

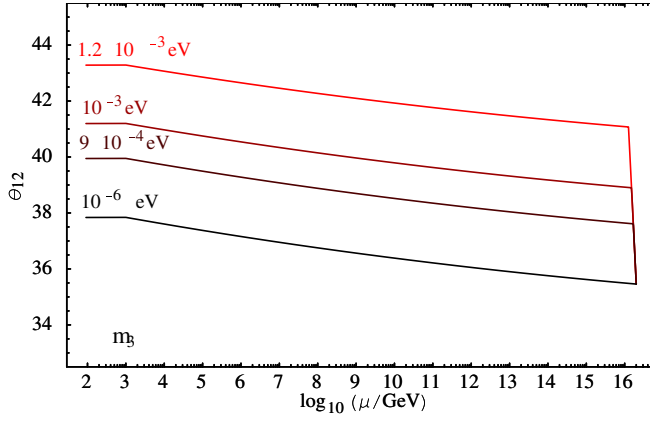
where subscripts NH and IH stand for normal and inverted mass hierarchy. This factor equals

$$\frac{\Delta m_{13}^2}{\Delta m_{21}^2} \quad \text{or} \quad \frac{(m_1^{\text{IH}})^2}{(m_1^{\text{NH}})^2} \quad (61)$$

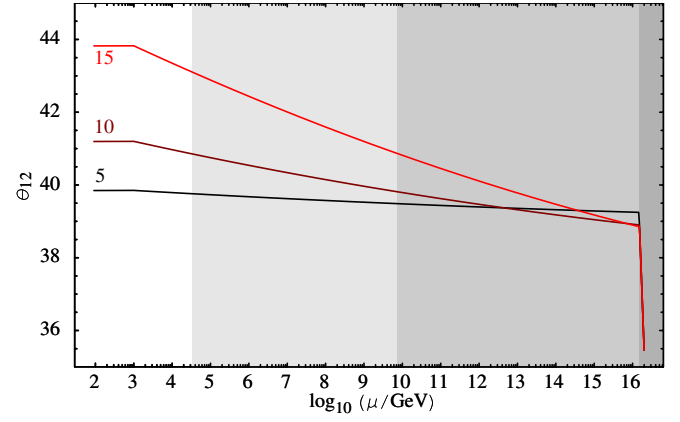
for the strong normal hierarchy and normal ordering ($m_1 \approx m_2$) correspondingly.

In Fig. 5 we show examples of running of θ_{12} for different values of masses and phases. Dependences of θ_{12} are well described by Q_{12} , as in the case of normal mass hierarchy. Notice that now the heaviest RH neutrino mass is determined by m_3 , and two others by m_A . With an increase of m_3 (now the lightest neutrino mass) [Fig. 5(a)], the range above the seesaw scales, where the evolution of θ_{12} is most strong, increases. The change of θ_{12} below M_3 is slower being of the same size for different values of m_3 (until $m_3 \ll m_A$). In this range the evolution is essentially due to Y_e couplings, so that $\Delta\theta_{12} \propto \tan^2\beta$ [Fig. 5(b)]. The correction can be strongly suppressed for the opposite CP -parities of ν_1 and ν_2 : $\Delta\theta_{12} \propto (1 + \cos\varphi)$.

As in the case of normal hierarchy (ordering), in a large part of the parameter space the correction is positive, $\Delta\theta_{12} > 0$, due to the dominant effect of P_{33} . For $\varphi_2 = 0$, consistency of the QLC prediction with data taken as $\Delta\theta_{12} < 2^\circ$, implies $\tan\beta < 10$ and $m_3 < 8 \times 10^{-4}$ eV. For $\varphi_2 \sim \pi$ corrections can be strongly suppressed, so that a larger region of the parameter space becomes al-



(a)



(b)

FIG. 5 (color online). Examples of running of θ_{12} in the case of MSSM and inverted mass hierarchy. The dependence of θ_{12} on μ (a) for different values of m_1 , and $\tan\beta = 10$, (b) on $\tan\beta$ for $m_1 = 10^{-3}$ eV. The value $\varphi_2 = 0$ is taken.

lowed. The corrections become negative for $\varphi_2 = \pi$ (see Figs. 6 and 7) when the leading RG effects are strongly suppressed and the running is mainly due to a subleading effect related to nonzero 1–3 mixing. This possibility has been mentioned in [4]. The sign of contribution due to nonzero θ_{13} to the RG running of θ_{12} due to nonzero θ_{13} depends on the parameter (masses, phases) region.

In general, for nonzero θ_{13} , the contribution to $\dot{\theta}_{12}$ is given by [34]

$$\begin{aligned} \frac{C_\nu \theta_{13}}{32\pi^2} \sin 2\theta_{23} \left[(Q_{12} \cos 2\theta_{12} + Q_{13} s_{12}^2 + Q_{23} c_{12}^2) \cos \delta \right. \\ \left. + 2 \left(\frac{m_1 m_2}{\Delta m_{21}^2} \sin(\varphi_1 - \varphi_2) + \frac{m_1 m_3}{\Delta m_{31}^2} \sin \varphi_1 s_{12}^2 \right. \right. \\ \left. \left. + \frac{m_2 m_3}{\Delta m_{32}^2} \sin \varphi_2 c_{12}^2 \right) \sin \delta \right]. \end{aligned} \quad (62)$$

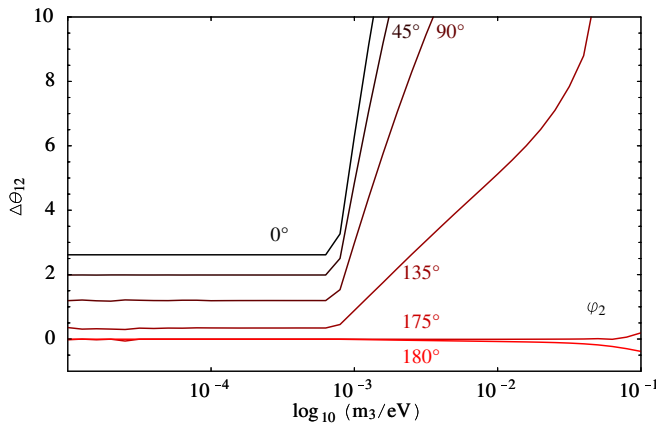


FIG. 6 (color online). The dependence of the RG correction $\Delta\theta_{12}$ on m_1 for different values of φ_2 (figures at the curves) in the case of MSSM, the inverted mass hierarchy, and $\tan\beta = 10$.

According to this equation for $\varphi_2 = 180^\circ$, $\varphi_1 = 0^\circ$, and $\delta = 180^\circ$, the dominant contribution is determined by the combination $-[(m_3 + m_1)/(m_3 - m_1)] \sin^2 \theta_{12} \sin 2\theta_{23}$, which is positive in the inverted hierarchy case, and therefore θ_{12} decreases from high to low energies.

V. RG EFFECTS IN THE STANDARD MODEL

In the standard model the evolution of θ_{12} is more complicated. As we have already mentioned, apart from

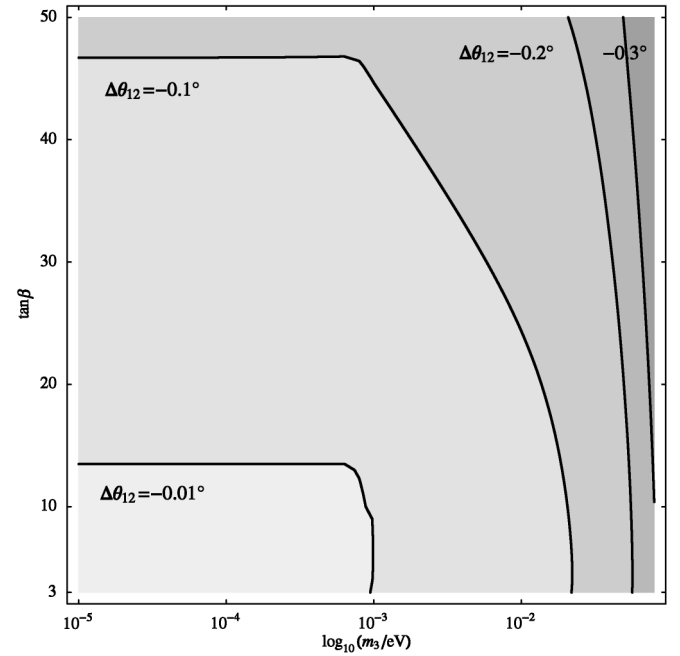
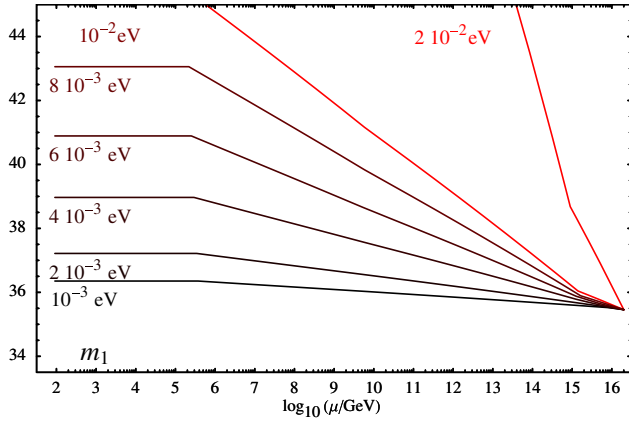
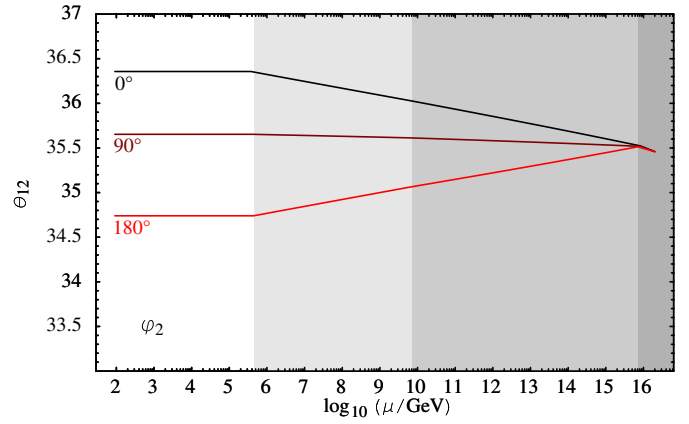


FIG. 7. Contours of constant RG corrections, $\Delta\theta_{12}$, in the $\tan\beta - m_1$ plane in the case of MSSM, inverted mass hierarchy, and $\varphi = \varphi_2 = \pi$.



(a)



(b)

FIG. 8 (color online). Examples of running of θ_{12} in the case of SM and normal mass hierarchy. The dependence of θ_{12} on μ (a) for different values of m_1 , and $\varphi_2 = 0$, (b) on φ_2 for $m_1 = 10^{-3}$ eV.

the Yukawa coupling contributions described by Eq. (51) there are additional vertex diagrams that cancel in the SUSY case [42]. Furthermore, the vertex diagrams with the gauge bosons become important: their contribution to running between the seesaw scales influences the flavor structure of mass matrix and therefore changes θ_{12} [34–36]. The point is that individual RH neutrinos, N_i , have the flavor dependent couplings with the left-handed components of neutrinos. Therefore the gauge boson corrections to the corresponding couplings will influence the flavor structure. Above the seesaw scales (where all RH neutrinos are operative) and below the seesaw scales (where all RH neutrinos decouple), flavor universality of the gauge interaction corrections is restored. There is no simple analytic formula for the θ_{12} renormalization in the SM.

In Fig. 8 we show examples of the θ_{12} scale evolution. Above the seesaw scales the running is due to the Yukawa interactions, Y_ν , and the effect is well described by the analytic results (53). Below the seesaw scales, $\mu < M_1$, the evolution is negligible: it is related to Y_e couplings that are small in the SM. The main effect is collected between the seesaw scales. As we mentioned above, it is mainly due to the gauge vertex corrections: After decoupling of N_3 the $D = 5$ operator

$$\frac{1}{M_3} (Y_\nu)_{ii} (U_R)_{i3} (U_R)_{3j} (Y_\nu)_{jj} L_i^T L_j H H \quad (63)$$

is formed. The vertex diagram corrections to this operator due to the gauge (and also Yukawa) interactions produce running. Notice that, for other RH neutrinos that do not decouple, the corresponding couplings produce box diagrams with propagators of the RH neutrinos. Those diagrams are finite and do not lead to logarithmic corrections. The gauge interaction effect dominates since N_3 with the

largest Yukawa coupling is decoupled and Y_e are small. The corrections increase with m_1 .

The most interesting dependence of $\Delta\theta_{12}$ is the one on the CP -violation phase φ_2 [Fig. 8(b)]. The corrections are positive, $\Delta\theta_{12} > 0$, for $\varphi_2 = 0$. They are strongly suppressed for $\varphi_2 \sim \pi/2$, in contrast to the SUSY case where suppression is realized for $\varphi \sim \pi$. The corrections are negative for $\varphi_2 > \pi/2$. The angle of zero corrections, $\varphi_2(0)$, depends on m_1 and in general deviates from $\pi/2$.

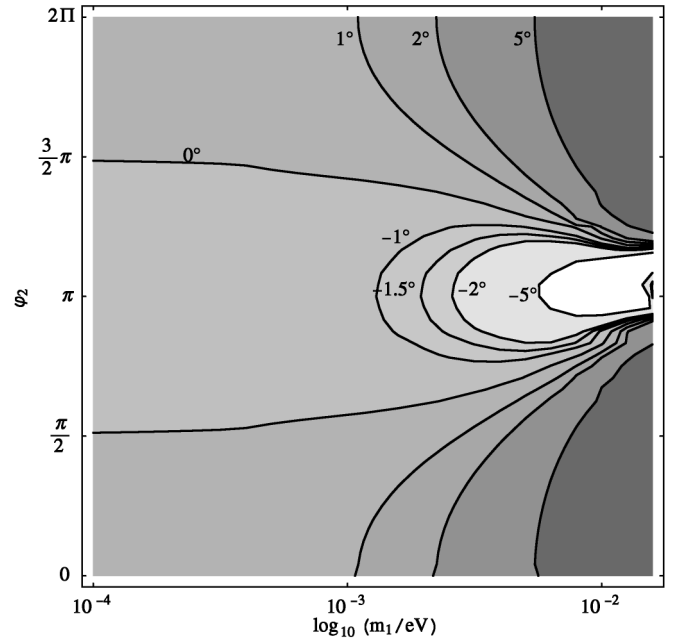


FIG. 9. Contours of constant RG corrections to θ_{12} (figures at the curves) in the $\varphi_2 - m_1$ plane in the case of SM and normal mass hierarchy.

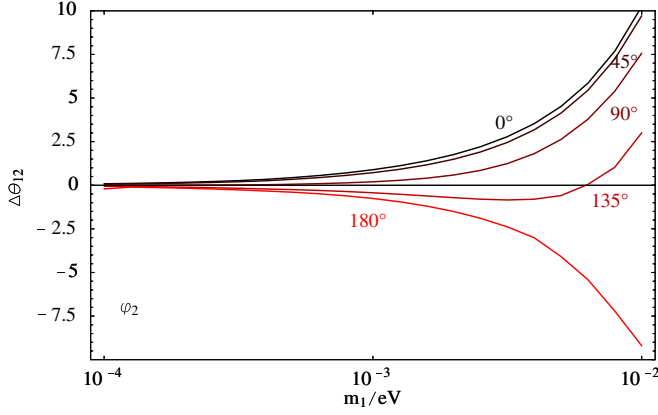


FIG. 10 (color online). The dependence of the RG correction $\Delta\theta_{12}$ on m_1 for different values of φ_2 (figures at the curves) in the SM with the normal mass hierarchy.

The deviation is due to the Yukawa interaction effects that produce the positive shift for strong Yukawa coupling hierarchy as we discussed before. The shift occurs both above and between the seesaw scales [see Fig. 8(b)].

In Fig. 9 we show contours of constant corrections in the $m_1 - \varphi_2$ plane and, in Fig. 10, an explicit dependence of $\Delta\theta_{12}$ on m_1 for different values of φ_2 . The line $\Delta\theta_{12} = 0$, is close to $\varphi_2 = \pi/2, 3\pi/2$ for $m_1 \rightarrow 0$, and it approaches π with increase of m_1 when spectrum becomes strongly degenerate. The pattern is nearly symmetric with respect to $\varphi = \pi$ for small m_1 , the asymmetry appears for $m_1 > 3 \times 10^{-3}$ eV.

The line $\Delta\theta_{12} = 2^\circ$ restricts the region consistent with the QLC relation. Along the contours $\Delta\theta_{12} = -1.5^\circ$, the best fit experimental value for θ_{12} can be reproduced. This corresponds to $m_1 > 2 \times 10^{-3}$ eV and $\varphi_2 \sim 5\pi/6 - 7\pi/6$. Large negative corrections appear in the region $m_1 > 5 \times 10^{-3}$ eV and $\varphi_2 \sim \pi$.

VI. RENORMALIZATION OF 13 MIXING. LEVEL CROSSING. EVOLUTION ABOVE M_{GUT}

A. Renormalization of 13 mixing

In the scenario discussed in this paper, the 1–3 mixing is nonzero and relatively large at the boundary (17). Notice that θ_{13} (i) interferes with 1–2 mixing in the QLC relation as we discussed before; (ii) produces subleading effects in renormalization of θ_{12} ; (iii) can provide further bounds on the considered scenario if RG corrections are positive and large.

The dominant contribution to the renormalization of θ_{13} is given by [34–36]

$$64\pi^2 \dot{\theta}_{13} = C_\nu \sin 2\theta_{12} \sin 2\theta_{23} (\mathcal{A}_{13} - \mathcal{A}_{23}), \quad (64)$$

where

$$\mathcal{A}_{i3} \equiv \frac{1}{\Delta m_{3i}^2} [(m_i^2 + m_3^2) \cos \delta + 2m_i m_3 \cos(\delta - \varphi_i)]. \quad (65)$$

In our case $\sin 2\theta_{12} > 0$, $\sin 2\theta_{23} > 0$, $\delta \approx 180^\circ$, and for vanishing Majorana CP phases, $\varphi_i = 0$, the dominant contribution can be approximated to

$$64\pi^2 \dot{\theta}_{13} = C_\nu \sin 2\theta_{12} \sin 2\theta_{23} (\mathcal{Q}_{23} - \mathcal{Q}_{13}), \quad (66)$$

and the last factor in (66): $(\mathcal{Q}_{23} - \mathcal{Q}_{13}) = (\mathcal{A}_{13} - \mathcal{A}_{23})$ is negative, irrespective of the mass hierarchy. Consequently θ_{13} increases when running to low energies. For nonvanishing phases φ_i this factor can be positive, thus leading to a decrease of θ_{13} when μ decreases.

In the case of strong mass hierarchy, Eq. (64) gives

$$64\pi^2 \dot{\theta}_{13} = -2 \sin 2\theta_{12} \sin 2\theta_{23} \cos(\delta - \varphi_2) \sqrt{\frac{\Delta m_{21}^2}{\Delta m_{31}^2}}. \quad (67)$$

The running is suppressed by a small mass ratio. Therefore only a small RG effect on 1–3 mixing appears for the hierarchical (normal as well as inverted) case. For instance, we find that, for the parameter sets used in Fig. 2 (MSSM), the correction $\Delta\theta_{13}$ is always smaller than 0.2° . In the SM, it is smaller than 0.3° .

For the degenerate spectrum, there can be a larger effect which strongly depends on the CP -phases. From (64) we find

$$64\pi^2 \dot{\theta}_{13} \approx 2 \sin 2\theta_{12} \sin 2\theta_{23} \frac{m_1^2}{\Delta m_{31}^2} \times [\cos(\delta - \varphi_1) - \cos(\delta - \varphi_2)]. \quad (68)$$

Notice that for zero CP phases the cancellation occurs again. In the MSSM for $m_1 = 0.03$ eV and $\tan \beta = 50$, we find $\Delta\theta_{13} \sim 0.5^\circ$. In contrast, for $\delta = \varphi_1 = \pi$ and $\varphi_2 = 0$, the two terms in (68) sum up and we obtain positive running: $64\pi^2 \dot{\theta}_{13} \approx 4 \sin 2\theta_{12} \sin 2\theta_{23}$. Consequently, θ_{13} becomes smaller at low energies.

B. Level crossing points

As we have established in Sec. III the spectrum of the right-handed Majorana neutrinos is generically hierarchical. However, there are the level crossing points, where two of the RH neutrino masses become equal [24]. The case of degeneracy of two lighter RH neutrino states, $M_1 \approx M_2$, is of special interest from the point of view of generation of the baryon asymmetry in the Universe. In this case the resonance leptogenesis becomes possible which produces large enough asymmetry in spite of smallness of the masses and, consequently, a large wash out effect.

From (45) we find

$$M_1 = \frac{2m_t^2 \epsilon'^4}{m_1 + m_2}, \quad M_2 = \frac{2m_t^2 \epsilon'^2(m_1 + m_2)}{(m_1 + m_2)m_3 + 2m_1 m_2}. \quad (69)$$

(Here the Majorana phases are included in m_i .) It is easy to see that due to smallness of ϵ the condition $M_1 \approx M_2$ can be satisfied only in the case of strong mass degeneracy $|m_1| \approx |m_2| \approx m_0$ when

$$m_1 + m_2 = \frac{\Delta m_{21}^2}{2m_0} \approx 0. \quad (70)$$

Then from the condition $M_1 \approx M_2$ we find

$$m_0 = \sqrt{\frac{\Delta m_{21}^2}{2\sqrt{2}\epsilon}} \sim 0.1 \text{ eV}. \quad (71)$$

In this special case the mass

$$M_1 \approx M_2 \frac{4m_t^2 \epsilon'^4 m_0}{\Delta m_{21}^2} = M_1^{\text{NH}} \frac{2m_0}{\sqrt{\Delta m_{21}^2}} \quad (72)$$

is enhanced by factor $2m_0/\sqrt{\Delta m_{21}^2} \sim 20$ and the third mass is much smaller than in the hierarchical case:

$$M_3 \approx \frac{m_t^2}{2m_3}, \quad (73)$$

that is, smaller by factor $m_1^{\text{NH}}/m_3 < 10^{-3}$.

The level crossing condition (70) implies the opposite CP -violating phases; it coincides with the condition of strong suppression of the RG effects. It also implies small-

ness of the 11-element of m_{bm} matrix. The condition for level crossing differs from that in [24] since here we require the neutrino Dirac matrix to be diagonal in the basis where the mass matrix of light neutrinos has exactly bimaximal form. If instead we use a generic matrix with nonmaximal 1–2 mixing, the level crossing condition can be realized for the hierarchical spectrum [24].

In Fig. 11 we show scale evolution of the mixing angles for parameters that correspond to the level crossing point $M_1 = M_2$. In this point $M_1 = M_2 = 8 \times 10^6 \text{ GeV}$, $M_3 = 8 \times 10^{13} \text{ GeV}$, $\varphi_1 = 0$, $\varphi_2 = \pi$, $m_1 = 0.13 \text{ eV}$. The angle θ_{12} evolves very weakly due to cancellation $Q_{12} = S_{12} \approx 0$ related to (70). In contrast, the 1–3 mixing evolves substantially above thresholds: $\Delta\theta_{13} = 7^\circ$. The 2–3 mixing shows relatively weak evolution, that, however, can influence the second QLC relation.

We find that in this crossing point the solar mass squared difference becomes large even if it is very small at the boundary. So, the 1–2 split has the radiative origin. The 1–3 split decreases by factor ~ 2 .

C. Evolution above the GUT scale

For $M_F > M_{\text{GUT}}$, one should perform running also above the GUT scale. Restoration of the GUT symmetry and unification of the gauge couplings does not prevent from different running of the Yukawa couplings, and therefore, from change of mixing angles. Renormalization of mixing angles would stop after possible unification of the Yukawa couplings which can be related, e.g., to restoration at M_F a non-Abelian flavor symmetry. An alternative is the boundary at the string or Planck scale, where the Yukawa couplings are formed and their properties are determined immediately by some symmetry or/and string selection rules.

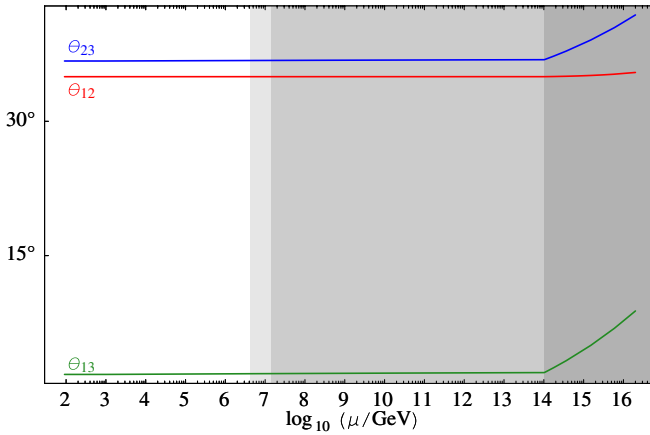


FIG. 11 (color online). Examples of running of mixing angles in the case of $M_1 \approx M_2$ in MSSM and normal mass ordering. We show the dependence of θ_{12} , θ_{13} , θ_{23} on μ for $\tan\beta = 10$, $\varphi_1 = 0$, $\varphi_2 = \pi$, and $m_1 = 0.13 \text{ eV}$.

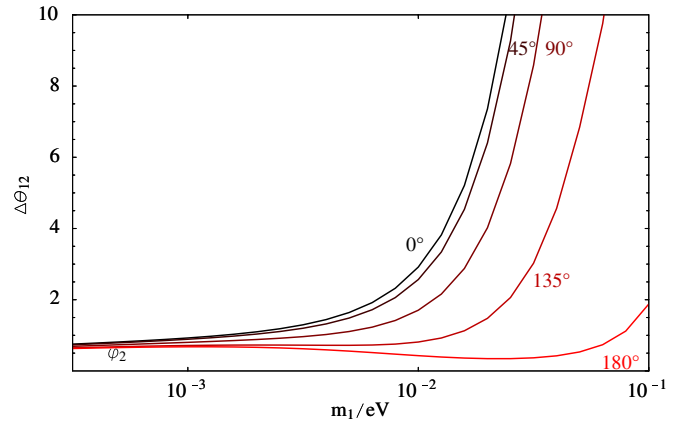


FIG. 12 (color online). The dependence of the RG correction $\Delta\theta_{12}$ on m_1 for different values of φ_2 (figures at the curves) in the MSSM with the normal mass hierarchy and $\tan\beta = 10$. The boundary condition is at M_{Pl} .

For illustration we performed the running in the MSSM up to the Planck scale (ignoring possible GUT effects). In Fig. 12 we show the dependence of $\Delta\theta_{12}$ on m_1 for the same (QLC) initial conditions at the Planck scale: $M_F = M_{\text{Pl}} = 1.2 \times 10^{19}$ GeV. The RG effect becomes much larger. In particular, the contribution from the region above the seesaw scale due to large Yukawa coupling Y_ν increases substantially. It is enhanced in comparison with the case of running up to M_{GUT} by the factor

$$\frac{\log(M_{\text{Pl}}/M_3)}{\log(M_{\text{GUT}}/M_3)} \quad (74)$$

that can be as large as 3–5 in some cases. Still for $\varphi_2 = \pi$ or for small m_1 the RG effects are suppressed and can be consistent with the QLC relations.

Similar RG effects are expected in the SU(5) model with the RH singlet neutrinos. In fact, no new diagrams with large Y_ν appear. The effect of the charged lepton couplings Y_e is enhanced by factor 4 above M_{GUT} due to the loop diagrams with down quarks (squarks) and $H^{1/3}$ charged Higgs bosons (Higgsinos).

The flavor-diagonal parts of the RG equations do influence the angles only indirectly through the change of the mass eigenvalues. Thus, the main effect of these interactions is due to the evolution of Δm_{12}^2 .

VII. CONCLUSION

Experimental results on the 1–2 and 2–3 mixing in the quark and lepton sectors show certain correlations that can be interpreted as the quark-lepton complementarity. We considered the QLC scenario in which the bimaximal mixing follows from diagonalization of the neutrino mass matrix. In the lowest order of the perturbation theory, the value of angle θ_{12} predicted in this scenario is about $\sim 1\sigma$ larger than the best fit experimental value. It coincides practically with the value given by the tribimaximal mixing. We commented on implications of this equality as well as on perspectives of future tests of the QLC relations.

In this paper, we assumed that the QLC relations are not accidental coincidences, but a consequence of the quark-lepton symmetry and additional structure in the theory that produces the bimaximal mixing. Here the accidental coincidences would mean that values of mixing angles are the result of interplay of two or more independent contributions. In this connection, we proposed a realization of the QLC scenario that provides the closest relation between quarks and leptons. It is based on

- (i) the seesaw type-I mechanism that generates the bimaximal mixing due to specific structure of the RH neutrino mass matrix;
- (ii) approximate equalities of the Dirac mass matrices: $m_u \approx m_D$, $m_l \approx m_d$ that follow from the approximate quark-lepton symmetry or unification. A cer-

tain small violation of equalities of these matrices produces difference of mass hierarchies but does not affect substantially the mixing.

The only additional (and, in fact, unavoidable) factor that can affect the QLC relations is RG corrections.

One of the consequences of the proposed scenario is a very strong hierarchy of the RH neutrino masses, apart from several particular level crossing points. The latter are realized for strongly degenerate spectrum of light neutrinos, and particular values of the CP -violating phases. This determines substantially the RG effects.

We performed a systematic study of the RG effects in the SM and MSSM. We find that, in the MSSM, the RG corrections to θ_{12} are generically positive due to a dominant effect of the Yukawa coupling Y_{33} . So, these corrections worsen agreement of the predicted θ_{12} with data.

In the MSSM, small negative corrections $|\Delta\theta_{12}| < 0.5^\circ$ can appear for the opposite CP parities of ν_1 and ν_2 and inverted mass hierarchy, in which case the main terms in the RG equations are strongly suppressed and running is due to the subleading effects related to the nonzero 1–3 mixing. The correction increases with m_1 and strongly depends on the relative Majorana phase. For $\varphi_2 = 0$, the consistency of the QLC prediction for θ_{12} with data implies strong mass hierarchy of light neutrinos and small $\tan\beta$. For $\varphi_2 = \pi$ corrections are suppressed and even the degenerate spectrum becomes allowed. For the inverted mass hierarchy the corrections are generically enhanced by larger values of masses of ν_1 and ν_2 .

The situation is qualitatively different in the SM. Here important contributions follow from the vertex corrections to the $D = 5$ operator in the range between the seesaw scales. The Yukawa couplings (especially for small m_1) give a subleading contribution. The RG corrections are negative in the interval $\varphi_2 = \pi/2 - 3\pi/2$ for small m_1 and the range of negative corrections becomes narrower, $\varphi_2 = (0.9-1.2)\pi$, for the degenerate neutrinos ν_1 and ν_2 .

Corrections depend substantially on the boundary scale M_F . The value $\Delta\theta_{12}$ can be enhanced by factor 2–5 if M_F increases from M_{GUT} to M_{Pl} .

For the hierarchical mass spectrum renormalization of the 1–3 mixings is, in general, small: $\Delta\theta_{13} \sim 0.2^\circ-0.3^\circ$. The correction can be large, $\Delta\theta_{13} \sim \theta_{13}$, for the degenerate spectrum. The sign of correction depends on values of CP -violating phases.

In conclusion, in a large part of the parameter space, especially for the strong mass hierarchy and opposite CP phases of ν_1 and ν_2 , the RG corrections to the QLC relation are small. The corrections are positive in the MSSM apart from a small region of parameter space that corresponds to the degenerate spectrum of light neutrinos and their opposite CP parities. The corrections are negative in the SM for $\varphi_2 > \pi/2$. In the considered QLC scenario, the RG corrections allow one to reproduce the best fit experimental value of θ_{12} exactly.

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