

Synchrotron and inverse Compton constraints on Lorentz violations for electrons

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We present a method for constraining Lorentz violation in the electron sector, based on observations of the photons emitted by high-energy astrophysical sources. The most important Lorentz-violating operators at the relevant energies are parametrized by a tensor $c^{\nu\mu}$ with nine independent components. If c is nonvanishing, then there may be either a maximum electron velocity less than the speed of light or a maximum energy for subluminal electrons; both these quantities will generally depend on the direction of an electron's motion. From synchrotron radiation, we may infer a lower bound on the maximum velocity, and from inverse Compton emission, a lower bound on the maximum subluminal energy. With observational data for both these types of emission from multiple celestial sources, we may then place bounds on all nine of the coefficients that make up c . The most stringent bound, on a certain combination of the coefficients, is at the 6×10^{-20} level, and bounds on the coefficients individually range from the 7×10^{-15} level to the 2×10^{-17} level. For most of the coefficients, these are the most precise bounds available, and with newly available data, we can already improve over previous bounds obtained by the same methods.

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I. INTRODUCTION

Currently, there is a great deal of interest in the possibility that Lorentz and *CPT* symmetries might not be exact in nature. If violations of these very basic symmetries are discovered, they would be of tremendous importance. They would be very important clues about the nature of physics at the Planck scale. Many candidate theories of quantum gravity suggest the possibility of Lorentz symmetry breaking in certain regimes. For example, Lorentz violation could arise spontaneously in string theory [1,2] or elsewhere [3]. There could also be Lorentz-violating physics in loop quantum gravity [4,5] and noncommutative geometry [6,7] theories, or Lorentz violation through spacetime-varying couplings [8], or anomalous breaking of Lorentz and *CPT* symmetries [9] in certain spacetimes.

Ultimately, the correctness of Lorentz symmetry must be verified experimentally. To date, there have been many high-precision experimental tests of Lorentz invariance. These have included studies of matter-antimatter asymmetries for trapped charged particles [10–13] and bound state systems [14,15], determinations of muon properties [16,17], analyses of the behavior of spin-polarized matter [18,19], frequency standard comparisons [20–23], Michelson-Morley experiments with cryogenic resonators [24–26], Doppler effect measurements [27,28], measurements of neutral meson oscillations [29–34], polarization measurements on the light from distant galaxies [35–38], and others.

There are many systems and reaction processes that could potentially be used to set further bounds on Lorentz violation. This work focuses on experimental lim-

its based on observations of synchrotron and inverse Compton (IC) radiation from ultrarelativistic electrons in astrophysical sources. Some of these limits have already appeared [39,40], but with new experimental data available, even better bounds are now possible.

In order to evaluate the results of sensitive Lorentz tests, it has been useful to develop a local effective field theory that parametrizes all possible Lorentz violations. The most general such theory is the standard model extension (SME) [41,42]. The SME includes, in addition to all Lorentz-violating operators that can be written down using standard model fields, all possible Lorentz-violating terms in the gravitational sector as well [43]. Although many theories describing new physics suggest the possibility of Lorentz violation, none of them are understood well enough to make firm predictions. The utility of the SME is its generality. Working within the SME, we can place bounds on the coefficients that parameterize any Lorentz violation, and these bounds do not depend on the ultimate mechanism underlying the Lorentz violation.

Of course, the number of parameters in the most general version of the SME is infinite. Practically, it is usually more useful to restrict attention to a finite subset of these coefficients. The most commonly considered subset is the minimal SME. This includes operators which are superficially renormalizable (that is, of dimension three or four) and invariant under the standard model's $SU(3)_c \times SU(2)_L \times U(1)_Y$ gauge group. The one-loop renormalizability of a further subset of the minimal SME—the minimal QED extension—has been verified explicitly [44]. Moreover, radiative corrections to this particular theory can be extremely interesting. Such interesting effects as photon splitting [45], ambiguous radiative corrections [46–49], and radiatively induced photon masses [50] are pos-

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sible. The stability of the minimal SME operators, as well as their causality properties, have also been looked at in detail [51].

Many of the coefficients in the minimal SME have been tightly constrained, but many others have not. For example, the neutrino sector is barely constrained at all. This work presents the best bounds on several coefficients in an important sector of the theory.

One of the radiation processes considered here—synchrotron radiation—has already been the subject of a number of analyses that included Lorentz violations. Several of these have focused on Lorentz violation through changes to particle dispersion relations. This follows the popular approach of Myers and Pospelov [52]. Taking a preferred direction v^μ in spacetime, one may add an operator proportional to $i\phi^*(v^\mu\partial_\mu)^3\phi$ to the Lagrange density for a scalar particle. If v^μ has a time component only, this will add a term proportional to E^3 to the usual relativistic energy-momentum relation $E^2 = \vec{p}^2 + m^2$. Since the statement that v^μ is purely timelike is not Lorentz-invariant, this condition must be taken to hold in a particular preferred frame, which is typically assumed to be the rest frame of the cosmic microwave background. In this framework, the electromagnetic field is incorporated through the usual minimal coupling procedure. In the presence of this kind of Lorentz violation, the motion of a charged particle in a constant magnetic field is modified, but the projection of the trajectory onto the plane perpendicular to $\vec{B}$ remains circular, and the particle's speed remains constant. The radiation in the far field can be determined (including information about polarization) and circumstances that could enhance observable effects have been identified [53,54].

Stringent bounds on Lorentz violations with modified dispersion relations have been obtained from data from the Crab nebula [55–57]. These modifications can lead to maximum particle velocities that are less than the speed of light, but the Crab nebula shows evidence of synchrotron emission from electrons with Lorentz factors of $\gamma = (1 - \tilde{v}^2)^{-1/2} \sim 3 \times 10^9$. For electrons with the conventional dispersion relation, this corresponds to energies of 1500 TeV. The existence of electrons with velocities this large can be used to constrain the dispersion relation models. If the coefficient of the Lorentz-violating operator in the Lagrangian has a particular sign, the data show that it must be at least 7 orders of magnitude smaller than $\mathcal{O}(E/M_P)$ Planck-level suppression. This method, of placing bounds on the size of the Lorentz violation based on the inferred velocities of astrophysical electrons, is the same one we shall use here. However, we shall apply the arguments to more important superficially renormalizable operators.

If spacetime is noncommutative, this will result in Lorentz noninvariant physics [7]. Synchrotron processes have also been examined within this framework. Because

of the noncommutativity of the coordinates, there automatically are modifications to all sectors of the theory. The electron sector, the free photon sector, and the minimal coupling between the two are all affected. The discussion in [58] covers the particular case in which a magnetic field and the Lorentz-violating noncommutativity parameter are aligned, so that the orbits of charged particles in the plane perpendicular to $\vec{B}$ are still given by circles. It is possible to work out the far fields within this model at leading order in the noncommutativity. However, there are a number of potential difficulties associated with the interpretation; these include acausality and problems with quantization.

In Sec. II, we shall introduce the SME terms that are likely to make the largest contributions to observable high-energy astrophysical processes. We shall explain why all other terms should probably contribute negligibly in comparison, but we shall also discuss how the same experimental data we shall utilize could nonetheless be used to constrain other Lorentz-violating models. Our bounds on Lorentz violation are related to the high-energy behavior of electrons' velocities. The Lorentz-violating modifications of the velocity are worked out in Sec. III. There are two important effects. The maximum electron speed might be less than one; or it might be greater than one, so that there is a maximum energy for subluminal electrons. In Sec. IV, we discuss the necessary details of the astrophysical synchrotron and inverse Compton processes, and in Sec. V, we describe how parameters such as the maximum electron speed can be inferred from observed spectral data. Then in Sec. VI, we look at the data that is actually available and use it to place bounds on the Lorentz-violating coefficients. Finally, we discuss our conclusions and the prospects for further improvement in Sec. VII.

II. LORENTZ-VIOLATING QED

A. The c term

Both modified dispersion relation and noncommutative spacetime theories involve Lorentz-violating operators that are nonrenormalizable by power counting. On the other hand, there is a unique spin-independent, superficially renormalizable SME coupling that is consistent with the gauge invariance of the standard model and which grows in relative importance at high energies. This is a CPT -even two-index tensor $c^{\nu\mu}$, and this is the term which we will seek to constrain.

The Lagrange density we shall consider is

$$\mathcal{L} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \bar{\psi}[\Gamma^\mu(i\partial_\mu - eA_\mu) - M]\psi, \quad (1)$$

where $\Gamma^\mu = \gamma^\mu + c^{\nu\mu}\gamma_\nu$ and $M = m$, so that

$$\mathcal{L} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \bar{\psi}[(\gamma^\mu + c^{\nu\mu}\gamma_\nu)(i\partial_\mu - eA_\mu) - m]\psi. \quad (2)$$

Here, ψ is the electron field, and c contains nine parameters that contribute to Lorentz-violating physics at leading

order. The expression (2) is not the full Lagrange density for the electron and photon sectors of the minimal SME. More generally, we could have

$$\Gamma^\mu = \gamma^\mu + c^{\nu\mu}\gamma_\nu - d^{\nu\mu}\gamma_\nu\gamma_5 + e^\mu + if^\mu\gamma_5 + \frac{1}{2}g^{\lambda\nu\mu}\sigma_{\lambda\nu} \quad (3)$$

and

$$M = m + \not{a} - \not{b}\gamma_5 + \frac{1}{2}H^{\mu\nu}\sigma_{\mu\nu} + im_5\gamma_5, \quad (4)$$

as well as terms $-\frac{1}{4}k_F^{\mu\nu\rho\sigma}F_{\mu\nu}F_{\rho\sigma}$ and $\frac{1}{2}k_{AF}^\mu\epsilon_{\mu\nu\rho\sigma}F^{\nu\rho}A^\sigma$ in the electromagnetic sector. However, the c coefficients are the only sources of Lorentz violation that we shall need to consider. As we shall discuss shortly, all other Lorentz-violating terms can either be absorbed into c or will have their contributions suppressed.

The trace $c^\mu{}_\mu$ only affects the overall normalization of the electron field, and the antisymmetric part of c has no effects at first order, because at that order, it is equivalent to a change in the representation of the Dirac matrices.

We shall choose a c that is not symmetric, but rather one with $c^{\nu 0} = 0$. This can be accomplished using a field redefinition. We do this because it will simplify our calculations. The utility of this choice comes from the fact that, despite the Lorentz violation, the electromagnetic field is coupled conventionally to the velocity via a $v^\mu A_\mu$ term. Because $c^{\nu 0} = 0$, the electrostatic potential $\Phi = A^0$ is coupled, as usual, to the charge density $e\psi^\dagger\psi$, and the vector potential $\vec{A}$ couples to $e\psi^\dagger\vec{x}\psi$. (Although the operator $e\psi^\dagger\vec{x}\psi$ technically contains *Zitterbewegung*, it will be perfectly valid to ignore this effect.) So all the usual results for the electromagnetic field of a moving pointlike charge continue to hold, once the charge's motion is prescribed. Determining this motion is also relatively simple, because the equation of motion for the particle is the unmodified Lorentz force law.

Moreover, the canonical quantization of the fermion field requires some care when certain c coefficients are nonvanishing. If $c^{\nu 0}$ were nonzero, then $\mathcal{L}$ would contain nonstandard time derivative terms. In that case, a matrix transformation $\psi' = R\psi$ would be required, to ensure that $\Gamma^0 = \gamma^0$. An explicit power-series expression for the required R is given in [59]. For simplicity, we shall assume that any such necessary transformation has already been performed and $c^{\nu 0} = 0$. However, this will require us to consider the canonical quantization in a single frame only. We may not boost the theory into another frame, because doing so would reintroduce the problematic time derivatives. The precise frame in which we shall take $c^{\nu 0} = 0$ is the standard sun-centered celestial equatorial coordinate frame that is used in the study of Lorentz violation [60].

The c coefficients for protons are generally more tightly constrained than those for electrons. The proton's c values can be measured by precision atomic clock comparison experiments. Unfortunately, the hyperfine transitions used

in these measurements are not sensitive to c -type Lorentz violation in the electron sector. However, other atomic transitions at much higher energies may be sensitive to the electron c terms; future laboratory experiments could use measurements of those transitions to constrain many of the same terms that are being discussed here.

B. Terms to be neglected

There are other superficially renormalizable couplings contained in the minimal SME, but as already noted, the c couplings are most natural in this context, and they should make the largest contributions to the effects we plan to study. Nonrenormalizable operators—which are outside the minimal SME—we naturally expect to be suppressed relative to renormalizable ones by powers of a large momentum scale. When considering synchrotron and IC radiation, one is primarily interested in particles with very high energies. Lorentz-violating coefficients that modify the kinetic part of the Lagrangian will grow in relative importance at high energies, as the components of the momentum become large, so it is natural to consider only such kinetic modifications; thus we may neglect a , b , and H , whose effects do not increase with energy. There remain in Γ^μ only two sets of Lorentz-violating that are consistent with the actual standard model's chiral gauge couplings—the $c^{\nu\mu}$ terms and also the set of $d^{\nu\mu}$ terms, which have the same form as the c interactions, except for the addition of a γ_5 .

However, we do not consider the d interactions here, because their effects are expected to be small. Contributions from d should average out, because there should be no net polarization of the electrons in high-energy sources. In fact, for an electron undergoing circular cyclotron motion, with the spin oriented in the plane of the orbit, the spin rotates by $2\pi\gamma\frac{g-2}{2}$ radians with each orbital revolution. For $\gamma \gg \alpha^{-1}$, the spin will rotate many times during one orbital period, and any effects proportional to the helicity will be diminished by the resultant averaging. Moreover, mixing between the standard kinetic term and the c term (which have the same basic Dirac structure) also causes the effects of c also grow in importance relative to those of d at high energies [51]. If the scale of the dimensionless coefficients c and d is $\mathcal{O}(m/M_P)$, where M_P is some large momentum scale, then the effects of c grow to be large at momenta $|\vec{p}| \sim \sqrt{mM_P}$, while those of d grow more slowly, becoming large only when $|\vec{p}| \sim M_P$. [If there is physical Lorentz violation, then M_P may represent the Planck scale. However, for the present purposes, we may take M_P to be effectively defined by the size of c ; M_P is whatever scale is needed so that c will be $\mathcal{O}(m/M_P)$.]

Modifications of the kinetic Lagrangian that are not invariant under the standard model's $SU(2)_L$ gauge symmetry can also exist; these are e , f , and g . However, these can only appear as part of electroweak symmetry breaking, as vacuum expectation values of higher dimensional (i.e.,

nonrenormalizable) operators. These operators should therefore be further suppressed, and we shall also neglect them.

Finally, we may also neglect any Lorentz violation in the photon sector. Modifications of the free electromagnetic Lagrangian will generally change the speed of photon propagation. Most possible Lorentz-violating terms in the free electromagnetic sector (all the components of k_{AF} and half those of k_F) give rise to photon birefringence, because photons with right- and left-handed circular polarizations travel at different speeds. This birefringence has been searched for and not seen. The limits on the relevant forms of Lorentz violation are very strong, and we may safely neglect them. The purely electromagnetic terms that do not cause birefringence can be accounted for by adding

$$\mathcal{L}_F = -\frac{1}{4}(k_F)^\alpha{}_{\mu\alpha\nu}(F^{\rho\mu}F_\rho{}^\nu + F^{\mu\rho}F^\nu{}_\rho). \quad (5)$$

to $\mathcal{L}$. However, a coordinate transformation $x^\mu \rightarrow x^\mu - \frac{1}{2}(k_F)^\alpha{}_{\mu\alpha\nu}x^\nu$ will eliminate all the Lorentz violation from the photon sector at leading order [61,62]. This transformation shifts the Lorentz-violating physics into the charged matter sector, where it manifests itself exactly as a $c^{\nu\mu}$ term. So we see that consideration of c captures all the possible sources of Lorentz violation in a synchrotron process that are not significantly further suppressed. However, it is important to note that the transformation that eliminates k_F is frame-dependent, and the new coordinates need not even be rectangular relative to the original ones; so by choosing to consider only this form of Lorentz violation, we are restricting ourselves to working in a very particular and special coordinate system.

C. Alternative formulations

1. Alternative form for c

In many applications, it is more convenient to consider a $c^{\nu\mu}$ which is both traceless and symmetric. We shall refer to the equivalent c tensor with these properties as c_{TS} , and all the bounds given here can be easily translated into bounds on the coefficients of c_{TS} .

The trace $c^\mu{}_\mu$ can actually be eliminated from the theory to all orders by a field redefinition. $g^{\nu\mu} + c^{\nu\mu}$ is a bilinear form that connects γ_ν and p_μ in the action. However, its trace can be fixed to four by inserting a new set of fermion fields $\psi' = \sqrt{1 + \frac{1}{4}c^\mu{}_\mu}\psi$. (This field redefinition must be accompanied by a change in the value of the mass parameter m ; however, this change is unimportant for the ultrarelativistic phenomena considered here.) At leading order, the redefinition changes $c^{\nu\mu}$ to $c^{\nu\mu} - \frac{1}{4}c^\alpha{}_\alpha g^{\nu\mu}$. Symmetrizing this expression is trivial to accomplish, so the final traceless symmetric c_{TS} equivalent (at leading order) to our c is

$$c_{TS}^{\nu\mu} = \frac{1}{2}(c^{\nu\mu} + c^{\mu\nu}) - \frac{1}{4}c^\alpha{}_\alpha g^{\nu\mu}. \quad (6)$$

In fact, clock comparison experiments, which set the best bounds on the c coefficients for protons and neutrons, have only bounded such combinations as $c_{(XY)}$ [where the parentheses denote the symmetrized expression $c_{(XY)} = c_{XY} + c_{YX}$], c_{0X} , $c_{XX} - c_{YY}$, and $c_Q = c_{XX} + c_{YY} - 2c_{ZZ}$. (The coordinate system in which X , Y , and Z are defined is explained in Sec. VI.) None of these combinations of coefficients is actually sensitive to whether or not c is traceless. These experiments only constrain eight of the nine physical c coefficients per species. Clock comparison bounds on the remaining nucleon coefficients—the $(c_{TS})_{00}$ [or equivalently, the $(c_{TS})_{jj}$] — have not been calculated, because they would be suppressed by two powers of the Earth's revolution speed, $v_\oplus \sim 10^{-4}$. Moreover, the bounds on the nucleons' c_{0j} terms are already worse than the bounds on the other coefficients by one power of $v_\oplus$. In general, for each time index on an element of c , the observable effects (which come from violations of boost invariance) are suppressed by one power of the velocities involved. For laboratory experiments, where the largest speed available is $v_\oplus$, this is a major impediment. However, when the bounds are based on observations of ultrarelativistic electrons, this fact presents no problem at all, because the particles' speeds are all very close to 1. Therefore, we can find bounds on all the physically meaningful electron c coefficients at the same level of accuracy; any differences in the bounds are due solely to the differing the quality of the data available for sources in various directions.

2. Other theories

We shall be bounding the c coefficients for electrons by looking at the structure of the theory near the electrons' maximum velocities. Obviously, without any Lorentz violation, this maximum velocity is one. However, a c term will generally result in a change to the maximum speed. If $c_{jk} \propto \delta_{jk}$, the electrons' limiting speed is the same in all directions; the free electron theory looks just like ordinary special relativity, but with a different value for the speed of light. As we will see, a c_{0j} term results in a similar distortion of the energy-momentum relation, with the added wrinkle that the maximum speed is direction-dependent. This angular dependence is dipolar; for electrons travelling in a direction $\hat{e}$, the maximum speed is $1 - c_{0j}\hat{e}_j$. An electron moving the opposite direction will have a different maximum speed, and in the plane normal to the three-vector defined by the c_{0j} , the top speed is one. The traceless part of c_{jk} has a similar effect, except that the deformation of the maximum speed has a quadrupole pattern. (We shall derive all these facts in Sec. III.)

So c leads to three kinds of deviations in the maximum electron speed, which have three of the most obvious possible forms. There are other possibilities too, however. Direction-dependent changes in the speed of light with

octopole or higher multipole characteristics are not possible with just a c term, but they might occur in more general theories. Modifications of Lorentz invariance which are only important above some large scale M , such as would result from a deformed energy-momentum relation $E^2 = m^2 + \vec{p}^2 + |\vec{p}|^3/M$ are also possible. However, regardless of the structure of a specific theory, if the electrons' maximum speed is different from one, bounds of the sort discussed here will be available, based on the very same observations. As already mentioned, for the theory with the $|\vec{p}|^3/M$ modification, some of these bounds have already been worked out [55,56]. We shall not pursue the analyses of such alternative theories any further, since the effective field theory operators that give rise to these kinds of Lorentz violations are generally exotic and nonrenormalizable, making them less important than c . However, we should keep in mind that bounds similar to the ones here are possible in these nonrenormalizable theories as well.

III. ELECTRON VELOCITIES

A. Derivation of $\vec{v}$

Because the free electromagnetic sector and the coupling to charged matter in (2) are completely conventional, standard effects in electrodynamics may be used as sensitive probes of the Lorentz-violating electron sector. In particular, we may place strong bounds on c by looking at the relationships between energy, momentum, and velocity in the c -modified theory. At ultrarelativistic energies, the bounds on c that can be derived from observing the emissions of an electron with Lorentz factor γ go as γ^{-2} . This strong dependence on γ is a consequence of the rapid growth in the importance of c with energy. If c is $\mathcal{O}(m/M_P)$, its effects will become important at scale $E \sim \sqrt{mM_P}$, and the Lorentz factor at this scale is $\gamma \sim \sqrt{M_P/m}$. So if no effects of Lorentz violation are observed up to some Lorentz factor γ , this constrains c to be smaller than $\mathcal{O}(\gamma^{-2})$. (On the other hand, any bounds on the d , e , f , and g coefficients would scale as γ^{-1} , at best.)

We must understand the effects of Lorentz violation in the free electron sector in order to place constraints on c . So we shall for the moment neglect the electromagnetic coupling and just look at how the free electrons behave. In particular, we shall look at the structure of the electron velocity. This means doing single-particle relativistic quantum mechanics, starting from a modified Dirac equation [63]. We shall see that the conventional relations between energy, momentum, and velocity— $E = \gamma m$ and $\vec{\pi} = \gamma m \vec{v}$ —no longer hold because of the Lorentz violation.

It turns out however, that even in the presence of any of c , the velocity may be found exactly. With the Lorentz violation, the single-particle Hamiltonian becomes

$$H = \alpha_j \pi_j - c_{lj} \alpha_l \pi_j - c_{0j} \pi_j + \beta m. \quad (7)$$

As usual, the Dirac matrices are $\alpha_j = \gamma^0 \gamma^j$ and $\beta = \gamma^0$. $\vec{\pi}$ is the mechanical momentum, which coincides with the canonical momentum in the free case; however when we reintroduce the electromagnetic coupling, the velocity will depend on $\vec{\pi} = \vec{p} - e\vec{A}$, rather than the gauge-noninvariant canonical momentum $\vec{p}$. The Hamiltonian H generates the Heisenberg equation of motion for the position operator x_k ,

$$\dot{x}_k = i[H, x_k] = \alpha_k - c_{lk} \alpha_l - c_{0k}; \quad (8)$$

so in the presence of the Lorentz violation, $\vec{a}$ is no longer the velocity operator, although the velocity is still an affine function of the α_j .

To find the velocity, we must solve the Heisenberg equation of motion for α_k . This equation is

$$\dot{\alpha}_k = i[-2\alpha_k(H + c_{0j}\pi_j) + 2\pi_k - 2c_{kj}\pi_j], \quad (9)$$

and it has an exact solution analogous to the Lorentz-invariant one—

$$\alpha_k(t) = (\pi_k - c_{kj}\pi_j)(H + c_{0j}\pi_j)^{-1} + [\alpha_k(0) - (\pi_k - c_{kj}\pi_j)(H + c_{0j}\pi_j)^{-1}]e^{-2i(H+c_{0j}\pi_j)t}. \quad (10)$$

H and $\vec{\pi}$ are constants of the motion. The second term on the right-hand side of (10) is matrix-valued and oscillatory. It is a *Zitterbewegung* term, completely analogous to the one found in the Lorentz-invariant case. The *Zitterbewegung* arises from interference between positive- and negative-frequency plane waves, and it is solely responsible for the fact that the components of the velocity do not commute with one-another. The *Zitterbewegung* is purely quantum-mechanical in origin, and it may be neglected. The *Zitterbewegung* motion can be eliminated entirely if the electron wave packet contains no negative-frequency components, which is possible if the particle is spread out spatially, with a position uncertainty larger than the Compton wavelength $1/m$. Moreover, to the extent that we want to consider the radiation from electrons with well-defined velocities, we must drop the *Zitterbewegung*, because it prevents us from resolving more than one component of the velocity at a time.

The *Zitterbewegung*-free contribution to the velocity is therefore

$$v_k = \frac{1}{E + c_{0j}\pi_j} (\pi_k - c_{kj}\pi_j - c_{jk}\pi_j + c_{jk}c_{jl}\pi_l) - c_{0k}. \quad (11)$$

The Hamiltonian has been replaced by its eigenvalue E . E is the energy corresponding to the momentum $\vec{\pi}$, $E = \sqrt{m^2 + (\pi_k - c_{kj}\pi_j)(\pi_k - c_{kl}\pi_l) - c_{0j}\pi_j}$. The group velocity derived from E is the same as the *Zitterbewegung*-free $\vec{v}$; however, the algebraic method of deriving the velocity is more general.

To first order, only the symmetric part of c_{kj} contributes to $\vec{v}$, as expected. The antisymmetric part corresponds at this order merely to a change in the representation of the γ_j Dirac matrices; such a change can have no physical consequences. [The fact that (11) depends asymmetrically on c_{0j} and not c_{j0} is a consequence of the fact that our calculational methods have already made use of the fact that $c^{\nu 0} = 0$.]

We can now see why the effects of c become large at the scale $\sqrt{mM_p}$. According to (11), the velocity might become superluminal when $|\vec{\pi}|/E \approx 1 - |c|$, where $|c|$ is a characteristic size for the Lorentz-violating coefficients. This gives us an estimate of the maximum value of γ that can be achieved before new physics must come into play if some form of causality is to be preserved. For ultrarelativistic particles, $\gamma \approx [2(1 - |\vec{\pi}|/E)]^{-1/2}$, and this diverges at an energy scale $E_{\max} \sim m/\sqrt{|c|} \sim \sqrt{mM_p}$. Above this scale, the description of the Lorentz violation through an effective field theory containing only $c^{\nu\mu}$ terms will generally break down, and higher dimension operators should become important.

B. Maximum velocity

We can exploit the Lorentz violation in the relationship between momentum and velocity to place bounds on c . There are two crucial and complementary effects. The first effect is that the maximum electron speed in a given direction $\hat{e}$ is generally different from one. The value of this new maximum velocity can be easily calculated to first order in c . We simply expand $v_j \hat{e}_j$ to $\mathcal{O}(c)$, finding

$$v_j \hat{e}_j = \frac{1}{\sqrt{m^2 + \pi_j \pi_j}} \left(\pi_j \hat{e}_j + c_{jk} \frac{\pi_j \pi_k}{m^2 + \pi_l \pi_l} \pi_l \hat{e}_l - c_{jk} \pi_j \hat{e}_k - c_{jk} \pi_k \hat{e}_j \right) - c_{0j} \hat{e}_j. \quad (12)$$

At large momenta, we may neglect the mass m . Then in terms of the unit vector $\hat{\pi}$ in the momentum direction, $v_j \hat{e}_j$ becomes

$$v_j \hat{e}_j = \hat{\pi}_j \hat{e}_j + c_{jk} \hat{\pi}_j \hat{\pi}_k \hat{\pi}_l \hat{e}_l - c_{jk} \hat{\pi}_j \hat{e}_k - c_{jk} \hat{\pi}_k \hat{e}_j - c_{0j} \hat{e}_j. \quad (13)$$

The speed will be maximum when the momentum direction is $\hat{\pi} = \hat{e} + \mathcal{O}(c)$. In the terms that already contain c , the c -dependent corrections to this direction may be neglected; we may therefore replace $\hat{\pi}$ with $\hat{e}$ in the $\mathcal{O}(c)$ terms and then maximize the resulting expression, which is just $\hat{\pi}_j \hat{e}_j - c_{jk} \hat{e}_j \hat{e}_k - c_{0j} \hat{e}_j$. This is clearly still maximized by setting $\hat{\pi} = \hat{e}$, as in the Lorentz-invariant case. So the limiting value of $v_j \hat{e}_j$ is

$$(v_j \hat{e}_j)_{\max} = 1 - c_{jk} \hat{e}_j \hat{e}_k - c_{0j} \hat{e}_j. \quad (14)$$

If this is less than one, it can have readily observable consequences.

C. Maximum subluminal energy

The complementary effect is that there may be a maximum energy available to electrons with subluminal velocities. This will also generally depend on the direction of a particle's motion. To determine this energy, we again use (12), setting $v_j \hat{e}_j = 1$. However, in this case the mass cannot be neglected; instead, we expand to leading order in $m^2/\pi_j \pi_j$. This gives

$$1 + \frac{m^2}{2\pi_j \pi_j} = \hat{\pi}_j \hat{e}_j + c_{jk} \hat{\pi}_j \hat{\pi}_k \hat{\pi}_l \hat{e}_l \left(1 - \frac{m^2}{\pi_j \pi_j} \right) - c_{jk} \hat{\pi}_j \hat{e}_k - c_{jk} \hat{\pi}_k \hat{e}_j - c_{0j} \hat{e}_j. \quad (15)$$

We know from the previous calculation that the velocity in the $\hat{e}$ -direction will reach the speed of light most easily (that is, with the smallest momentum) when $\hat{\pi}$ and $\hat{e}$ are aligned. Any corrections to this alignment are at least second order in c . So we may replace $\hat{\pi}$ with $\hat{e}$ everywhere. Doing this, we have

$$\frac{m^2}{2\pi_j \pi_j} (1 + 2c_{jk} \hat{e}_j \hat{e}_k) = -c_{jk} \hat{e}_j \hat{e}_k - c_{0j} \hat{e}_j. \quad (16)$$

Up to corrections of $\mathcal{O}(c)$ or $\mathcal{O}(m^2/\pi_j \pi_j)$, $\pi_j \pi_j/m^2$ is simply E^2/m^2 . The corrections just mentioned, as well as the c -dependent terms on the left-hand side of (16), may be neglected. They are small corrections to the expression

$$\frac{E}{m} = \frac{1}{\sqrt{-2c_{jk} \hat{e}_j \hat{e}_k - 2c_{0j} \hat{e}_j}}. \quad (17)$$

The maximum subluminal energy E is proportional to the inverse square root of c , which is not surprising, since for vanishing c , E must be infinite. As is obvious from (17), such a maximum value for E need not always exist. In fact, according to (14), this maximal subluminal energy does not exist precisely when the maximum speed in the relevant direction is less than or equal to one, and this is exactly what one would expect.

D. Relationship to bounds on c

The two conditions we have found, that $v_j \hat{e}_j$ cannot exceed $1 - c_{jk} \hat{e}_j \hat{e}_k - c_{0j} \hat{e}_j$ and that E/m cannot exceed $(-2c_{jk} \hat{e}_j \hat{e}_k - 2c_{0j} \hat{e}_j)^{-1/2}$ without an electron becoming superluminal, look very different, although they do both depend on c through the expression $c_{jk} \hat{e}_j \hat{e}_k + c_{0j} \hat{e}_j$. However, these two conditions can actually be cast in very similar forms, when they are related to experimental observations. If we separately observe the existence of electrons with Lorentz factors up to some value $\gamma_{\max} = (1 - v_{\max}^2)^{-1/2}$ moving in the $\hat{e}$ -direction and subluminal electrons with energies up to $E_{\max}$ traveling in the same direction, then this restricts $c_{jk} \hat{e}_j \hat{e}_k + c_{0j} \hat{e}_j$ to lie in the

range

$$-\frac{1}{2(E_{\max}/m)^2} < c_{jk}\hat{e}_j\hat{e}_k + c_{0j}\hat{e}_j < \frac{1}{2\gamma_{\max}^2}. \quad (18)$$

Naturally, in the absence of Lorentz violation, $\gamma = E/m$. By measuring $\gamma_{\max}$ and $E_{\max}$, we are constraining the electrons' energy-momentum relation. The speed of light is fixed to be one by the conventional electromagnetic sector. This provides a basis for comparison when we search for Lorentz violations in other sectors. If Lorentz symmetry is exact, the maximum possible γ and E are both infinite. Yet this is not generally the case with Lorentz violation, and the Lorentz-violating c terms distort the behaviors of the Lorentz factor and the energy differently. So it is not surprising that the bounds derived from $\gamma_{\max}$ and $E_{\max}$ are different, and it is quite convenient that they are actually so complementary.

IV. HIGH-ENERGY RADIATION PROCESSES

A. Synchrotron process

The classical synchrotron process involves electrons revolving helically around lines of magnetic flux. These accelerated particles emit radiation over a broad spectrum of frequencies, up to a characteristic cutoff. We shall review the crucial features of this phenomenon, emphasizing those characteristics that will be important for our study of Lorentz violation. An excellent source that discusses the importance of various effects in determining astrophysical synchrotron spectra is [64], although the basic material can be found in many treatments.

We have previously presented a much more detailed account (to all orders in c), of the synchrotron process in the presence of Lorentz violation [39]. However, most of the detailed results arising from this treatment turn out to be unimportant for our attempt to set bounds on c . The predominant effects are the ones discussed in Sec. III, which were derived only from the electrons' energy-momentum relation. There are some other interesting qualitative changes in the behavior of the system that we shall mention; however, we shall not treat them in great detail.

Thus far, we have used rationalized units for the electromagnetic field and charge. However, for explicit calculations involving electrodynamics with moving sources, it is easier and more conventional to use Gaussian units. We shall use these unrationalized units in Secs. IV and V, so that the formulas will look more familiar.

1. Synchrotron motion

The basic phenomenon of synchrotron motion is well known. Charged particles (in this case electrons) moving at high speeds are accelerated perpendicular to the direction of their motion by a basically homogeneous magnetic field. Their trajectories curve around the magnetic field lines.

This confines the electrons in the plane perpendicular to $\vec{B}$, although they still move freely along the direction of the field. This picture changes only slightly when a Lorentz-violating c term is added.

At the energies we are interested in, all quantum effects can be neglected. This applies to both the electrons' motions and, as we shall see, to the radiation emissions. Quantum effects could enter through Landau level quantization or spin-dependent effects, or through the *Zitterbewegung*. The spin-orbit and Landau level effects make the spectrum discrete, but the difference between adjacent energy levels is miniscule compared to the energies involved. Since the quantum numbers are very large, the classical treatment is an excellent approximation. Similarly, although the velocity that the three-vector potential couples to is, in principle, the velocity $e\psi^\dagger \vec{x} \psi$ including the *Zitterbewegung*, we know empirically that *Zitterbewegung* is unimportant in the emission of astrophysical synchrotron radiation. The changes to the *Zitterbewegung* due to the Lorentz violation are minor, and result in no qualitative differences that would suddenly make this quantum interference effect important. So we may treat the electromagnetic field as if it were coupled simply to the group velocity $\vec{v}$.

The classical result is that an electron's motion perpendicular to $\vec{B}$ is circular, with angular revolution frequency $\omega_B = |e\vec{B}|/\gamma m$. The radius of gyration is $\rho = |\vec{p} \times \vec{B}|/|e|\vec{B}^2 \approx E \sin\theta/|e\vec{B}|$, where θ is the angle between the magnetic field and the direction of the electron's motion. The frequency is only slightly modified by the Lorentz-violating c . Depending on which formula is used, the radius ρ may be more significantly modified; however, this effect turns out not to be important. Lorentz violation will generally also shift the periodic part of the electron's motion out of the plane normal to $\vec{B}$; the circular path in the plane becomes an elliptical one tilted out of the plane, and the electron's velocity depends on time, varying with an angular frequency $2\omega_B$. The deviations in the shape and orientation of the orbit are $\mathcal{O}(c)$ and can be neglected. Moreover, although changes to the velocity of the emitting electrons are exactly what we want to measure, we cannot observe the periodic time variations in $\vec{v}$ directly, because of the way the synchrotron radiation is emitted.

2. Synchrotron radiation

As an electron revolves in the magnetic field, it emits radiation, which, because of the particle's ultrarelativistic velocity, is beamed into a narrow pencil of angles around the instantaneous direction of the velocity. The characteristic angular spread of the emission is $\mathcal{O}(\gamma^{-1})$, and this width is generally neglected; instead, we treat all the radiation as if it were emitted precisely along the tangent vector to the trajectory. The intensity of the radiation caused by any accelerations parallel to the velocity is

smaller than the synchrotron radiation by a factor of $\mathcal{O}(\gamma^{-2})$, provided the accelerations are comparable. Moreover, this radiation is beamed into the same narrow pencil of angles as the synchrotron emission, so it has no angular properties to distinguish it, and it has no meaningful effect on the observable spectrum.

The frequency spectrum is discrete, all the power being radiated in harmonics of the fundamental frequency $\nu_0 = \frac{\omega_B}{2\pi} \csc^2 \theta$. The emitted power is spread over all harmonics less than the critical frequency

$$\nu_c = \frac{3}{4\pi} \gamma^3 \omega_B \sin \theta = \frac{3}{4\pi m} \gamma^2 |e\vec{B}| \sin \theta. \quad (19)$$

Most of the power is emitted close to this frequency, and above ν_c , the radiated power falls off very rapidly. The rate of energy loss of an electron, found by summing the emission over all frequencies, is

$$\frac{dE}{dt} = -\frac{2e^4}{3m^2} \gamma^2 \vec{B}^2 \sin^2 \theta. \quad (20)$$

The ν_c in (19) represents the critical frequency for the emissions of a single electron. If a source contains significant numbers of electrons with velocities up to some maximum Lorentz factor $\gamma_{\max}$, then the observed cutoff in the spectrum will be at the cutoff frequency for the most energetic electrons— $\nu_c = \frac{3}{4\pi m} \gamma_{\max}^2 |e\vec{B}| \sin \theta$. (Detailed calculations for a truncated power law spectrum are given in [64].) This result for ν_c is what will allow us to infer $\gamma_{\max}$ from observations of spectra.

Quantum effects are unimportant in the emission part of the synchrotron process, just as they are in determining the electrons' trajectories. The leading order quantum corrections to the standard synchrotron formulas are negligible if $2\pi\nu_c \ll E$, or equivalently if $\gamma_{\max} \ll m^2/|e\vec{B}|$ [65]. Inserting the electron mass and charge, this is $\gamma \ll (3 \times 10^{13})/|\vec{B}|$ if the magnetic field is measured in Gauss. Since typical magnetic field strengths in synchrotron sources are fractions of mG, the range of γ values over which no quantum modifications are necessary is extremely large. In particular, the classical treatment will apply up to well beyond the scale of any observed $\gamma_{\max}$.

B. Inverse Compton process

The highest-energy photons that are emitted by astrophysical sources arise in IC processes. Low-energy photons (which often come from synchrotron emission) scatter off ultrarelativistic electrons. An electron may transfer a substantial fraction of its own energy to a photon during such a collision, resulting in the emission of photons whose energies may range almost up to the scale of the highest electron energies.

The details of IC scattering can be worked out by taking the Klein-Nishina formula and the usual kinematics of Compton scattering (starting from a frame in which the

electron is at rest) and transforming into a frame where the electron is moving with a speed $v \approx 1$. The details of the cross section will be unimportant in this instance. What matters is the transformation of the kinematics, which we shall now examine. This subject is covered in more detail in [64], although the emphasis there is not on precisely the same limit.

Collisions of photons with ultrarelativistic electrons are practically all head-on when viewed in the electron's frame. To see this, we note that the Lorentz transformation law for $\cos \psi$, where ψ is the angle between some given direction (which we shall take to be the direction of the photon's motion) and the boost direction (which is the direction of the electron's motion, since we are boosting the electron from rest into an ultrarelativistic frame) is

$$\cos \psi_i^e = \frac{\cos \psi_i^o - v}{1 - v \cos \psi_i^o}, \quad (21)$$

where the superscripts denote quantities taken in the rest frame of the observer (*o*) or of the electron (*e*), and the subscripts denote that these are initial values, applying prior to the scattering. As $v \rightarrow 1$, $\cos \psi^e \rightarrow -1$, so $\psi_i^e \approx \pi$. Overtaking collisions are extremely rare, occurring only in a miniscule range of observer frame solid angles.

We are interested in collisions where the fractional change in the photon energy is substantial. In such collisions, the photon's scattering angle θ^e in the electron's rest frame can never be small. When we boost into the observer's frame, all vectors that are not aligned almost perfectly antiparallel to the boost direction lie, after the boost, within a small pencil of angles about the boost direction. The fact that θ^e is not small just means that the outgoing photon is not moving nearly antiparallel to the electron's initial direction; therefore, it rebounds back with a very large observer frame scattering angle, $\theta^o \approx \pi$. So the emitted photon is propagating in essentially the same direction as the initial electron.

This beaming can be understood more simply if we make some approximations. The photon's initial energy is small, so for illustrative purposes, we may neglect it entirely. Then the kinematics of the process are the same as if the incoming photon did not exist; instead, it looks as if the electron has simply emitted a photon. The electron behaves almost like a massless particle at these energies, so the process looks almost like the splitting of one massless particle into two. The only way that energy and momentum conservation can be satisfied during such a process is if all three momenta are aligned. When the electron's mass and the photon's initial energy are taken into account, small deviations from perfect collinearity are allowed, but these are unimportant. Just as in the synchrotron case, the vast majority of the radiation is beamed into an extremely narrow pencil of angles around the direction of the electron's velocity.

In the electron's rest frame, the usual relationship between the initial and final photon energies ϵ_i and ϵ_f is

$$\epsilon_f^e = \frac{\epsilon_i^e}{1 + \frac{\epsilon_i^e}{m}(1 - \cos\theta^e)}. \quad (22)$$

Transformed into the observers frame, this is (in the $v \rightarrow 1$ limit),

$$\epsilon_f^o = \epsilon_i^o \gamma^2 \frac{(1 - \cos\psi_i^o)(1 + \cos\theta^e \cos\psi_i^e)}{1 + \gamma \frac{\epsilon_i^o}{m}(1 - \cos\psi_i^o)(1 - \cos\theta^o)}. \quad (23)$$

The Lorentz factor γ is that of the electron. Depending on the value of $\gamma\epsilon_i^o/m$, the final photon energy may be $\mathcal{O}(\gamma)$ to $\mathcal{O}(\gamma^2)$ larger than its initial energy. If $\gamma\epsilon_i^o/m \approx 0.1$, for example, then the maximum energy that can be carried off by the photon is $(\epsilon_f^o)_{\max} \approx 0.4\gamma m$. So the IC process allows some electrons to transfer sizable fractions of their energy to photons, and observed IC photon energies range up to ~ 100 TeV.

All these calculations have been done in the absence of Lorentz violation. With Lorentz violation, the two most important results continue to hold. The radiation is strongly beamed along the direction of the velocity, and the highest-energy IC photons can carry off significant fractions of the electrons' energies. Of course, with Lorentz violation, we must be careful to distinguish between quantities such as γ and E/m , but this is not a serious complication.

V. ANALYSIS OF SPECTRAL DATA

The raw data from which our bounds must be derived are the spectra of high-energy photon sources. These spectral profiles provide information about the emitting electrons' energy and velocity distributions. The particular quantities we want to extract are $\gamma_{\max}$ and $E_{\max}$, so that we may use (18) to bound c . Lower limits on these two quantities can be extracted in a fairly robust fashion, although for $\gamma_{\max}$, the analysis can be a bit tricky.

We shall review here how the values of $\gamma_{\max}$ and $E_{\max}$ are determined. The synchrotron part of the spectrum tells us about $\gamma_{\max}$. The highest-energy photons, which arise from IC scattering, give a lower bound on $E_{\max}$. The bounds are arrived at in entirely different ways, and they can be derived completely independently even for a single source.

A. Extracting $\gamma_{\max}$

As it turns out, the analysis required to find $\gamma_{\max}$ is fairly involved. From (19), we can infer that $\gamma_{\max} \propto \sqrt{\nu_c/|\vec{B}|}$. The constant of proportionality depends on the sine of the pitch angle; however, the $\gamma_{\max}$ inferred from assuming $\sin\theta = 1$ is always less than the true $\gamma_{\max}$. The cutoff at ν_c is an obvious feature of synchrotron spectra, and a lower bound on this cutoff frequency can easily be obtained from any spectrum that clearly has a synchrotron origin.

However, the tricky part is calculating the strength of the magnetic field.

The magnetic field is generally taken to be the minimum energy field which can generate the low-frequency part of the synchrotron spectrum. This can be estimated fairly accurately by the following calculation. For illustrative purposes, we shall neglect most numerical constants, but these would of course be retained in a more detailed calculation. More details are given for the case of a truncated power law electron spectrum in [64], and for a broken power law in [66]. Since only the relatively low-energy part of the spectrum is needed to calculate the magnetic field strength, we may also ignore the Lorentz violation.

Let $N(E)$ be the energy distribution of the electrons. The total synchrotron luminosity of these electrons between the energies E_1 and E_2 is

$$L = \int_{E_1}^{E_2} dE \frac{2e^4}{3m^2} \gamma^2 \vec{B}^2 \sin^2\theta N(E) \quad (24)$$

$$\propto \frac{e^4}{m^4} \vec{B}^2 \int_{E_1}^{E_2} dE E^2 N(E). \quad (25)$$

The total energy of the electrons is

$$E_e = \int_{E_1}^{E_2} dE E N(E). \quad (26)$$

Eliminating the electron distribution between these two and neglecting the dependence on the lower limit of integration E_1 gives

$$E_e \propto \frac{m^4}{e^4} \frac{L}{\vec{B}^2} E_2^{-1}. \quad (27)$$

Neglecting the dependence on E_1 is not necessarily a good approximation. In fact, it is possible for the integrals to be dominated by the energy range near E_1 . In that case, the dependence of E_e would be on E_1^{-1} instead of E_2^{-1} , but the results of our calculation would be qualitatively unchanged. Of course, a great deal more care would be required if we actually wanted to extract information about a real source, and the full behavior of the integrals over the entire range of energies would need to be taken into account.

We may replace the energy dependence of (27) with a frequency dependence. Since a given electron emits most of its energy around the critical frequency ν_c , we may replace the upper limit on the energy E_2 with an upper limit on frequency, and this upper limit ν_2 is precisely the critical frequency for an electron of energy E_2 . So E_e finally depends on

$$E_e \propto \frac{m^{11/2}}{e^{7/2}} L |\vec{B}|^{-3/2} \nu_2^{-1/2}. \quad (28)$$

We must also determine the energy that is contained in other particles and in the magnetic field. We assume that the energy in positively charged particles is proportional to the energy contained in the electrons. The constant of proportionality k can potentially range from $k \approx 1$ if the positive particles are positrons, to $k \approx 2000$ if the particles are protons accelerated to the same speeds as electrons. However, although k may cover a wide range, the final results will depend quite weakly on its value. A change in k by a factor of 100 will change $\gamma_{\max}$ by less than a factor of 2.

The total energy in particles is proportional to $(1 + k)L|\vec{B}|^{-3/2}$, and the energy in the magnetic field is easy to calculate. It is simply proportional to $\vec{B}^2 V$, where V is the volume of the region containing the field. So the total energy is

$$E_{\text{tot}} = C \left(\frac{m^{11/2}}{e^{7/2} \nu_2^{1/2}} \right) (1 + k) L |\vec{B}|^{-3/2} + \left(\frac{1}{8\pi} \right) \vec{B}^2 V, \quad (29)$$

where C is a numerical constant we have neglected. We may either minimize this as a function of the magnetic field, or choose $|\vec{B}|$ according to equipartition, so that the energy in the magnetic field is equal to the total energy of the particles. Numerically, the difference between these two methods is negligible, and the dependence of the field strength on the other quantities is

$$|\vec{B}| \propto \frac{m^{11/7}}{e} (1 + k)^{2/7} \nu_2^{-1/7} V^{-2/7} L^{2/7}. \quad (30)$$

This depends very weakly on most of the parameters that must be fitted from the observed spectrum. Therefore, the value of the field can be determined quite robustly. By considering only a limited range of frequencies, up to some ν_2 , the relevant parameters in (30) can be determined just from the lower-energy part of the spectrum, making the inferred value of $|\vec{B}|$ independent of the value of ν_c with which it must be combined to give the final value of $\gamma_{\max}$. Ultimately, $\gamma_{\max}$ depends on at most the seventh root of any fit parameter (except ν_c) that might be in error, and so its value is quite robust. The limited impact of any possible errors is discussed at length in [66].

The above argument relies on the assumption that a source has had enough time to reach a near equilibrium state, and this assumption may not be valid for real astrophysical sources. However, because we are using sources that are believed to be relatively well understood and because the dependence of the inferred magnetic field on the observational parameters is so weak, the calculated field strengths should be reliable enough for our purposes.

B. Extracting $E_{\max}$

Placing a lower bound on $E_{\max}$ is much simpler. Models of a source's structure can yield information about the energies of the electrons doing the emitting. In general,

the models typically require maximum electron energies that are several times larger than the highest observed photon energies, because IC scattering events do not transfer all of the high-energy electrons' energies to the photons. However, to get a more robust bound, we may take $E_{\max}$ to be the highest actually observed photon energy; this conservative estimate usually differs from a model-derived bound by less than an order of magnitude. The only input we require from a model is that the source's γ -ray emission is well described by the IC process.

Choosing $E_{\max}$ in this way ensures that Lorentz-violating distortions of the energy-momentum relation at higher than observed energies are not a problem. If the electrons' energy-momentum relation became significantly Lorentz-violating at the same scale as the highest particle energies, then the model results might be inaccurate, because they assume an unmodified electron dispersion relation up to arbitrarily large energies. However, the maximum observed photon energy is an absolute lower bound on the electrons' highest energies, independent of whether or not there is Lorentz violation.

As already stated, the importance of the models is that they can tell us whether the high-energy end of a source's emission spectrum fits with the hypothesis that IC scattering is the source of the radiation. There certainly are astrophysical sources whose spectra are not understood; such sources absolutely cannot be used to derive bounds on c . The quantity $E_{\max}$ is the maximum energy of subluminal electrons in a source. We can only infer that the electron involved in a particular IC event is moving more slowly than light if we know that all the electrons in a source have subluminal speeds. The signature of superluminal electrons would be a radiation spectrum that does not fit any known mechanism. Faster-than-light motion would definitely represent new physics, so we cannot predict with any assurance what the radiation from electrons with speeds greater than one would look like. However, we do expect on very general grounds that they should radiate energy extremely quickly. For charged particles with superluminal speeds, the rate of synchrotron emission diverges if the radiation reaction force is neglected. There will also be vacuum Cerenkov radiation. It is conceivable that the poorly understood spectra of some extremely high-energy sources may actually be evidence of superluminal electron motion, and it is precisely this reason that only well understood IC sources can be used to place bounds on $E_{\max}$.

VI. EXPERIMENTAL RESULTS AND CONSTRAINTS ON c

There are many sources for which measurements of $\gamma_{\max}$ and $E_{\max}$ have already been made. The sources with the largest observed values of these quantities will give the best bounds on the Lorentz-violating coefficients, and we have gleaned the data necessary for setting optimal bounds on c from the existing observational literature. There are a

number of model-derived values of $\gamma_{\max}$ in the literature, and we have located seven that are numerically large enough to contribute usefully to the bounds on c . Unfortunately, even for many well-studied sources, there are no reliable published values of $\gamma_{\max}$, and it might be a worthwhile future undertaking to determine $\gamma_{\max}$ for more of these sources.

Because the evaluation of $E_{\max}$ is more straightforward, we have found more useful $E_{\max}$ values than $\gamma_{\max}$ values. Many of the best measurements of IC radiation presently available come from the H.E.S.S. telescope in Namibia. It is the excellent sensitivity of this device that makes many the limits on the IC side possible. However, the sky coverage of this device is limited, which is a drawback.

Bounds on SME coefficients are generally given in a sun-centered celestial equatorial coordinate frame [60]. Right ascension and declination constitute a system of polar coordinates for this same reference frame. However, for parameterizing a quantity such as c , it is necessary to introduce Cartesian coordinates. The origin of the coordinates is at the center of the sun. The Z -axis points along the direction of the Earth's rotation, and the X -axis points toward the vernal equinox point on the celestial sphere. (That is, the X -direction is the direction from the Earth to the sun at the occurrence of the vernal equinox, so at the time of this equinox, the Earth lies along the negative X -axis.) The Y -direction is chosen according to the right hand rule. The Earth's orbit is inclined by approximately 23° from the XY -plane. Although it is unimportant here, the origin of time ($T = 0$) is conventionally taken to be at the vernal equinox in the year 2000.

Since both synchrotron and inverse Compton radiation are strongly beamed along the direction of an electron's motion, when we observe this radiation from a given source, we are observing emissions from electrons moving in the source-to-Earth direction at ultrarelativistic speeds. This gives the correct $\hat{e}$ to appear in (18). In terms of the right ascension α and declination δ , the components of $\hat{e}$ are $\hat{e}_X = -\cos\delta \cos\alpha$, $\hat{e}_Y = -\cos\delta \sin\alpha$, and $\hat{e}_Z = -\sin\delta$; the minus signs come from the fact that $\hat{e}$ is the direction for the source to the Earth, not vice versa. It is important to note that the velocity of the source as a whole will not enter into our calculations in any way; we are looking at the velocities of individual electrons, and the bulk motion of the source is irrelevant. Of course however, for distant extragalactic sources, cosmological red shifts would need to be taken into account.

There are nine components of c that can be bounded with the astrophysical data—the three c_{0j} and the six-component symmetric part of c_{jk} . Each of the inequalities derived from (18) generally couples all nine of the coefficients in a nontrivial way. These bounds may be fairly awkward. However, the coupled bounds may be translated into bounds on the separate coefficients by means of linear programming. The linear program produces absolute

bounds on each coefficient; these values are the largest and smallest that a given coefficient can be under any circumstances. Of course, there are also additional correlations, as it is not generally possible for several of the coefficients to take on their extreme values simultaneously.

To constrain the nine Lorentz-violating coefficients to lie within a bounded region of the parameter space requires at least ten inequalities. If these inequalities are all derived from measurements of $\gamma_{\max}$ or $E_{\max}$, then at least nine sources must be used. Together, the $\gamma_{\max}$ and $E_{\max}$ bounds for a single source constrain the allowed parameters to the region between two parallel hyperplanes; nine such hyperplane pairs would be needed to produce a completely bounded region.

Table I lists the data for 14 sources for which good measurements are available. We have added several more sources to the list that was used in [40]. These include sources for which good data have only very recently been released, as well as additional extragalactic sources, such as the blazars Markarian 421 and Markarian 501. The additional sources allow for significant improvements in the bounds on some of the c coefficients. While the TeV γ -rays from the two blazars, as well as from the galactic sources G 347.3-0.5 and PSR B1259-63, are believed to be dominated by IC radiation, the hadronic process $\pi^0 \rightarrow 2\gamma$ has not been conclusively ruled out as a major contributor. Leaving these sources out of our analysis would result in a modest weakening of the bounds on c .

In addition to the astrophysical bounds, we have included some other comparable bounds in the linear program. This improves the resolution for the individual coefficients significantly, because the inequalities derived just from the astrophysical bounds do not complement one-another optimally. If only the astrophysical bounds are considered, the nine-dimensional polytope region within which the c coefficients must lie turns out to be fairly elongated. Individual $c^{\nu\mu}$ coefficients are allowed to take on values significantly larger than the typical $\gamma_{\max}^{-2}$ values, because of possible large cancellations with other coefficients. However, since bounds derived from laboratory experiments have completely different forms, they are excellent complements to the astrophysical constraints and improve the situation markedly.

The additional bounds that we use come from optical resonator tests. These tests are usually used to place bounds on the parameters of the SME photon sector. However, the same experiments may be used to place bounds on the electron c coefficients [86]. The key realization is that electronic Lorentz violations will modify the structure of a crystalline resonator, and this effect can be worked out systematically, provided Lorentz violations for nucleons can be safely neglected. These experiments then yield measurements of $c_{(XY)}$, $c_{(XZ)}$, $c_{(YZ)}$, and $c_{XX} - c_{YY}$. The measured values of these coefficients in [86] are at roughly the 10^{-15} level, with standard errors of the same magni-

TABLE I. Parameters for the astrophysical sources that we shall use to constrain c . References are given for each value of $\gamma_{\max}$ or $E_{\max}$. The four $E_{\max}/m$ values marked with asterisks denote the four sources for which the IC origin of the observed γ -rays is merely strongly favored, rather than completely assured.

Emission source	$\hat{e}_X$	$\hat{e}_Y$	$\hat{e}_Z$	$\gamma_{\max}$	$E_{\max}/m$
3C 273	0.99	0.13	-0.04	3×10^7 [67]	2×10^5 [67]
Centaurus A	0.68	0.27	0.68	2×10^8 [68]	...
Crab nebula	-0.10	-0.92	-0.37	3×10^9 [69]	2×10^8 [69,70]
G 0.9 + 0.1	0.05	0.88	0.47	...	10^7 [71]
G 12.82 - 0.02	-0.06	0.95	0.29	...	5×10^7 [72]
G 18.0 - 0.7	-0.11	0.97	0.24	...	7×10^7 [73,74]
G 347.3 - 0.5	0.16	0.75	0.64	3×10^7 [75]	$2 \times 10^{7*}$ [76]
MSH 15 - 52	0.34	0.38	0.86	...	8×10^7 [77]
Mkn 421	0.76	-0.19	-0.62	...	$3 \times 10^{7*}$ [78,79]
Mkn 501	0.22	0.74	-0.64	...	$4 \times 10^{7*}$ [80]
PSR B1259 - 63	0.42	0.12	0.90	...	$6 \times 10^{6*}$ [81]
RCW 86	0.35	0.30	0.89	10^8 [82]	...
SNR 1006 AD	0.52	0.53	0.67	2×10^7 [83]	7×10^6 [83]
Vela SNR	0.44	-0.55	0.71	3×10^8 [84]	1.3×10^8 [85]

tude. {Note that [86] uses a different convention for symmetrizing the c_{jk} coefficients, so that the quoted values of $c_{(jk)}$ in that paper are smaller by a factor of 2.} There is no compelling evidence from these measurements that any of the coefficients are nonzero. Therefore, we shall use these results to derive bounds that are symmetric about zero. If the measured value and error for a particular $c_{(jk)}$ is $\alpha \pm \beta$, we shall treat this as a bound $|c_{(jk)}| < |\alpha| + 2\beta$, which is valid at the 2σ level or better. The resulting bounds on $|c_{(XY)}|$ and $|c_{(XZ)}|$ are about 3×10^{-15} ; for $|c_{(YZ)}|$ and $|c_{XX} - c_{YY}|$, they are slightly more stringent, at better than a 2.5×10^{-15} level.

The output of the linear program is given in Table II. Since these bounds represent the absolute maximum and minimum values that are possible for each coefficient, they are not always as tight numerically as the raw bounds. We observe that the cryogenic resonator bounds are still the

TABLE II. Independent bounds on the components of c . The astrophysical data do not improve on the results of [86] for $c_{(XY)}$ and $c_{(XZ)}$. The lower limit on $c_{(YZ)}$ is also controlled solely by the data from [86]; however, the upper limit has been improved by adding the astrophysical information.

$c_{\mu\nu}$	Maximum	Minimum
c_{XX}	5×10^{-15}	-3×10^{-15}
c_{YY}	2.5×10^{-15}	-7×10^{-16}
c_{ZZ}	2.5×10^{-15}	-1.6×10^{-15}
$c_{(XY)}$	3×10^{-15}	-3×10^{-15}
$c_{(YZ)}$	3×10^{-15}	-3×10^{-15}
$c_{(YZ)}$	1.8×10^{-15}	-2.5×10^{-15}
c_{0X}	4×10^{-15}	-7×10^{-15}
c_{0Y}	1.5×10^{-15}	-5×10^{-16}
c_{0Z}	2×10^{-17}	-4×10^{-17}

best for $|c_{(XY)}|$ and $|c_{(XZ)}|$; the lower limit on $c_{(YZ)}$ also comes solely from [86]. However, the bounds on almost all the other coefficients are comparable to or better than the resonator bounds, and there are some significant improvements over the results presented in [40] as well. The bounds on c_{0Z} are especially strong, while in laboratory experiments, boost invariance violation coefficients such as c_{0j} are typically harder to constrain. This shows the advantage of deriving bounds from emissions by relativistic sources.

VII. CONCLUSION

The various bounds in Table II are generally similar in their orders of magnitude, and this observation is actually quite interesting. The methods we have used could not actually detect a nonzero $c^{\nu\mu}$ directly. Instead, the signature of Lorentz violation in high-energy astrophysical sources would be emission spectra that could not be modeled by conventional radiation mechanisms. If the calculated bounds on certain Lorentz-violating coefficients were significantly weaker than others, that would be an indication that those coefficients might actually be nonzero. However, we see no indications that any particular components of c are more likely to be nonzero than others.

The bounds that can be derived by combining the astrophysical and resonator data are in some cases many orders of magnitude better than the bounds on the same coefficients that could be extracted by other methods. Doppler shift measurements can only constrain the c_{0j} coefficients to be less than about 10^{-2} . So the improvements here are by more than 12 orders of magnitude (fourteen orders for c_{0Z}). Moreover, with better sky coverage and more sensitive detectors, even greater precision ought to be possible.

However, there are some fundamental limits to the how accurate these astrophysical measurements can be. There are different ways to search for Lorentz violation. Laboratory tests generally involve extremely high-precision measurements. One may then compare the results of these measurements when they are made in different reference frames. The rotation and revolution of the earth naturally provide a selections of observation frames with different orientations and velocities. Some experiments also utilize measurement apparatuses that rotate in the laboratory or beams of particles moving with substantial velocities in the lab frame. In all these kinds of scenarios, experimental accuracy is the most important limitation to setting tight bounds. Questions relating to metrology and long-term experimental stability can be very important, and the experiments can be extremely demanding technically. However, as the accuracy with which measurements can be made improves, the bounds on Lorentz violation will always improve accordingly. Bounds on Lorentz violations can usually be smaller than the errors involved in the absolute measurement of a quantity, because Lorentz violations possess characteristic signatures (such as side-real variations) whose existence can often be very strongly excluded.

Astrophysical tests of Lorentz symmetry are rather different. The astrophysical bounds on k_F and k_{AF} come from searches for photon birefringence. The data sets involved are noisy; the sources are irregular, and the detectors are not always especially precise. However, extremely tight bounds may be derived by taking advantage of cosmological distances. A tiny effect on the propagation of photons will be magnified by an incredibly long line of sight to the source. It is this line of sight that is the most important limitation on the precision of these bounds, and so these bounds cannot generally be improved by systematic improvements the way laboratory bounds can.

The bounds derived from astrophysical synchrotron and IC sources are similar. In this case, it is not very large

astrophysical distances that we are utilizing, but very high astrophysical energies. At these energies, miniscule Lorentz-violating terms can lead to obvious changes in the observed spectra. However, again these bounds are not subject to perpetual improvement, however much the systematics of our measurements improve. Better sky coverage and more accurate detectors surely will allow us to improve these bounds somewhat, but there is an ultimate limit that we cannot pass beyond. This limit is set by the actual maximum energies of electrons in these sources. Above some energy, there will simply exist too few electrons for us to observe their radiation, no matter how sensitive our detectors become. So other laboratory-based methods of constraining the $c^{\nu\mu}$ coefficients, perhaps relying on precision measurements of atomic transitions at optical frequencies, may ultimately produce more accurate results than these astronomical methods.

However, at present, the astrophysical bounds we have derived are the best currently available for most of the $c^{\nu\mu}$ coefficients. The bounds on all the coefficients are at about the 10^{-15} level or a little better. These bounds are comparable to other bounds, obtained by similar means but assuming unbroken rotation invariance, on the difference between the maximum electron speed and the speed of light [87]; however, the present bounds are much more general. As better experimental data become available, the bounds will continue to improve, and these results are further clear demonstrations of the usefulness of astrophysical data in constraining violations of Lorentz invariance.

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