Unitarity violation of the CKM matrix in a nonuniversal gauge interaction model

Kang Young Lee*

Department of Physics, Korea Advanced Institute of Science and Technology, Daejeon 305-701, Korea

(Received 4 January 2005; revised manuscript received 31 March 2005; published 21 June 2005)

We explore the unitarity violation of the Cabibbo-Kobayashi-Maskawa (CKM) matrix in the model where the third generation fermions are subjected to the separate $SU(2)_L$ gauge interaction. With the recent LEP and SLAC linear collider (SLC) data at Z-pole and low-energy neutral current interaction data, the analysis on the parameter space of the model is updated, and the unitary violation is predicted under the constraint.

DOI: 10.1103/PhysRevD.71.115008 PACS numbers: 12.60.Cn, 13.25.Es

I. INTRODUCTION

The Cabibbo-Kobayashi-Maskawa (CKM) matrix describes quark mixing for the charged current interaction. Each element of the CKM matrix is not determined theoretically but just parametrized by three angles and one phase due to the unitarity nature of the CKM matrix in the framework of the standard model (SM). The unitarity of the CKM matrix is a universal feature of the flavour physics and also holds in many new physics scenarios like the minimal supersymmetric standard model (MSSM) as well as in the SM. Recently the precision test on the CKM matrix becomes possible and the unitarity check on the first row of the CKM matrix has been reported [1]. We parametrize the unitarity relation for the first row of the matrix as

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1 - \Delta, \tag{1}$$

and Δ should be zero in the SM. Experimentally each element has been measured through various weak interaction processes. From the recent nuclear β-decay data, the first element is determined such that $|V_{ud}| = 0.9740 \pm 0.0005$. Combined with $|V_{us}| = 0.2196 \pm 0.0023$ from kaon decays and $|V_{ub}| = 0.0036 \pm 0.0009$ from B decays, it leads to $\Delta = 0.0031 \pm 0.0014$, which indicates a violation of the unitarity by 2.2σ standard deviation [2]. However, recent experimental results on $|V_{us}|$ supports the unitarity again [3] and this issue is still controversial.

If the violation of the unitarity relation in the CKM matrix turns out to be true, it is a clear evidence of the new physics beyond the SM. It suggests an interesting constraint for the nature of the possible new physics since the conventional supersymmetric models and grand unified theory (GUT) models usually preserve the unitarity of the CKM matrix. An example of model having nonunitary CKM matrix is the model with a heavy singlet quark where the mixing between the singlet quark and ordinary quarks can violate unitarity. Another example is the model with fourth generation, where the 4×4 quark mixing matrix is unitary while 3×3 mixing matrix is not.

*Electronic address: kylee@muon.kaist.ac.kr

The nonuniversal gauge interaction is also a possible candidate leading to the nonunitarity of the CKM matrix. The quark mass matrices are diagonalized by a biunitary transformation with two independent unitary matrices V_L and V_R corresponding to the transformations of left-handed and right-handed quarks, respectively. The charged current interactions in terms of physical states are expressed by the product of the two unitary transform matrices for left-handed up-type and down-type quarks in the SM. Then the mixing matrix in the charged current interaction should be unitary since it is the product of two unitary matrices. However, if the $SU(2)$ gauge interaction is nonuniversal, the mixing matrix consists of

$$V_{\rm CKM} = V_L^{U\dagger} \begin{pmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{pmatrix} V_L^D, \tag{2}$$

which is no more unitary unless $|a|^2 = |b|^2 = |c|^2$. We consider a model which contains a nonuniversal gauge interaction to predict a nonunitary CKM matrix in this work.

We consider the model where a separate $SU(2)$ group acts only on the third generation while the first two generations couple to the usual $SU(2)$ group [4,5]. This gauge group can arise as the theory at an intermediate scale in the path of gauge symmetry breaking of noncommuting extended technicolor (ETC) models [6]. Since different $SU(2)$ gauge group is assigned on the third generation, the corresponding gauge coupling constant is different from that of the first and second generations in general, which indicates that the electroweak gauge interaction is nonuniversal. The nonuniversality of the gauge couplings leads to the violation of the unitarity in the CKM matrix, although the unitarity violating term is suppressed by the heavy scale of new physics. On the other hand, the flavour-changing neutral current (FCNC) interactions generically emerge in this model, since the neutral currents are no more simultaneously diagonalized with the quark mass matrix. Together with a new spectrum of gauge bosons, the FCNC interactions predict various new phenomena at colliders and low-energy experiments, and highly constrain the model parameters. The phenomenology of this model has been intensively studied in the literatures, using the Z-pole data [4,5,7,8] and the low-energy data [8,9].

In this work, we update the previous analysis in Ref. [7] with the recent LEP and SLC data [10]. The atomic parity violation (APV), the neutrino-nucleon scattering, $\nu e \to \nu e$ scattering, and polarized e-N scattering data are also considered to improve the constraints on the model parameters describing the new physics scale and Z-Z' mixing. We estimate the unitarity violation in the proposed model and show that the unitarity violation data presented in the Ref. [2] can be explained in this model under the constraints from $Z \to b\bar{b}$ process at the Z-pole and low-energy experiments.

The observable quantity for quark mixing is the only product of the unitary transform matrices for the left-handed up-type and down-type quarks in the SM. However in this model, the matrix elements of the individual unitary transform matrices may manifest in some processes involving the third generation and we might confront doubled parameters on the quark mixing. Thus we have to be careful in treating new parameters.

This paper is organized as follows: In section II, we briefly review the model with a separate SU(2) symmetry on the third generation and show the unitarity violation in this model. The Z-pole data in the LEP and SLC experiments and low-energy neutral current interaction experiments are analyzed to constrain the parameter space of the model in section III. The unitarity violating term is predicted under the constraints in section IV and we conclude in section V.

II. THE MODEL

We consider the model based on the extended electroweak gauge group $SU(2)_l \times SU(2)_h \times U(1)_Y$. The first and the second generations couple to $SU(2)_l$ group and the third generation couples to $SU(2)_h$ group. We assign that the left-handed quarks and leptons of the first and second generations transform as (2,1,1/3), (2,1, − 1) and those in the third generation as (1,2,1/3), (1,2, − 1) under $SU(2)_l \times SU(2)_h \times U(1)_Y$, while right-handed quarks and leptons transform as $(1,1,2Q)$ with the electric charge $Q = T_{3l} + T_{3h} + Y/2$. We write the covariant derivative as

$$D^\mu = \partial^\mu + ig_l T_l^a W_{l,a}^\mu + ig_h T_h^a W_{h,a}^\mu + ig' \frac{Y}{2} B^\mu, \quad (3)$$

where $T_{l,h}^a$ denotes the $SU(2)_{(l,h)}$ generators and Y the $U(1)$ hypercharge. Corresponding gauge bosons are $W_{l,a}^\mu$, $W_{h,a}^\mu$, and B^μ with the coupling constants g_l, g_h, and g', respectively. The gauge couplings are parametrized by

$$g_l = \frac{e}{\sin\theta\cos\phi}, \qquad g_h = \frac{e}{\sin\theta\sin\phi}, \qquad g' = \frac{e}{\cos\theta} \quad (4)$$

in terms of the electromagnetic coupling e, the weak mixing angle θ, and the new mixing angle ϕ between $SU(2)_l$ and $SU(2)_h$.

The gauge group $SU(2)_l \times SU(2)_h \times U(1)$ breaks down into the $SU(2)_{l+h} \times U(1)_Y$ by the vacuum expectation values (VEV) $\langle\Sigma\rangle = \begin{pmatrix} u & 0 \\ 0 & u \end{pmatrix}$ of the bidoublet scalar field Σ, transforming as (2,2,0) under $SU(2)_l \times SU(2)_h \times U(1)$. Sequentially the gauge group breaks down into $U(1)_{em}$ at the electroweak scale v by the VEV of the (2,1,1) scalar field Φ. We require that the first symmetry breaking scale is higher than the electroweak scale, parametrized by $v^2/u^2 \equiv \lambda \ll 1$. We demand that both $SU(2)$ interactions are perturbative so that the value of the mixing angle $\sin\phi$ is constrained $g_{(l,h)}^2/4\pi < 1$, which results in $0.03 < \sin^2\phi < 0.96$. After the symmetry breaking, the masses of the heavy gauge bosons are given by

$$m_{W'^\pm}^2 = m_{Z'}^2 = \frac{m_0^2}{\lambda\sin^2\phi\cos^2\phi}, \quad (5)$$

while the ordinary gauge boson masses given by $m_{W^\pm}^2 = m_0^2(1 - \lambda\sin^4\phi) = m_Z^2\cos^2\theta$ where $m_0 = ev/(2\sin\theta)$.

Since the couplings to the gauge bosons for the third generations are different from those of the first and second generations, we can separate the nonuniversal part from the universal part. First we consider the charged current:

$$\mathcal{L}^{CC} = \mathcal{L}_I^{CC} + \mathcal{L}_3^{CC}, \quad (6)$$

where $\mathcal{L}_I^{CC}$ denotes the universal part and $\mathcal{L}_3^{CC}$ the nonuniversal part. We consider the unitary matrices V_U and V_D diagonalizing U-type and D-type quark mass matrices, respectively. The universal part is given by

$$\mathcal{L}_I = \bar{U}_L \gamma_\mu [G_L W^\mu + G_L' W'^\mu](V_U^\dagger V_D) D_L + \text{H.c.}, \quad (7)$$

where

$$G_L = -\frac{g}{\sqrt{2}}(1 - \lambda\sin^4\phi) I G_L' \\ = \frac{g}{\sqrt{2}}(\tan\phi + \lambda\sin^3\phi\cos\phi) I \quad (8)$$

with the 3×3 identity matrix I and $U = (u, c, t)^T$ and $D = (d, s, b)^T$. We define the unitary matrix $V_{CKM}^0 \equiv V_U^\dagger V_D$ corresponding to the CKM matrix of the SM. The nonuniversal part is written in terms of mass eigenstates as:

$$\mathcal{L}_3^{CC} = (V_{31}^{U*}\bar{u}_L + V_{32}^{U*}\bar{c}_L + V_{33}^{U*}\bar{t}_L) \times \gamma^\mu (X_L W_\mu^+ \\ + X_L' W_\mu'^+)(V_{31}^D d_L + V_{32}^D s_L + V_{33}^D b_L), \quad (9)$$

where

$$X_L = -\frac{g}{\sqrt{2}}\lambda\sin^2\phi\cdot, \qquad X_L' = -\frac{g}{\sqrt{2}}\left(\frac{1}{\sin\phi\cos\phi}\right). \quad (10)$$

The quark mixing matrix is no more unitary because of the existence of $\mathcal{L}_3$. The elements of individual unitary transform matrices V_U and V_D : $\{V_{U3j}, V_{D3k}\}$ manifest in the unitarity violating term in general, which are not observable in the SM. With these elements, the number of parameters increases drastically. This property might be very interesting to predict lots of new phenomena in this model, e.g. exotic CP violation or FCNC. However, as will be shown later, the additional parameters $\{V_{U3j}, V_{D3k}\}$ do not play a role in the present analysis.

We can read out the expression for the neutral current interaction terms from the Lagrangian. The nonuniversality of the gauge couplings brings forth the FCNC interactions at tree level which does not exist in the SM. The FCNC effects on the lepton number violating processes have been discussed in Ref. [7]. For the quark sector, the FCNC interaction terms evolve with the basis used above as follows

$$\mathcal{L}_3^{\rm NC} = (V_{31}^{D*}\bar{d}_L + V_{32}^{D*}\bar{s}_L + V_{33}^{D*}\bar{b}_L), \gamma^\mu (Y_L Z_\mu^0 + Y_L' Z_\mu'^0) \times (V_{31}^D d_L + V_{32}^D s_L + V_{33}^D b_L), \tag{11}$$

where

$$Y_L = \frac{g}{2\cos\theta} \cdot \lambda \sin^2\phi, \qquad Y_L' = \frac{g}{2} \cdot \frac{1}{\sin\phi\cos\phi}, \tag{12}$$

which yield additional contributions to the $B - \bar{B}$ mixings and rare decay processes through Z and Z' exchange diagrams, and have been discussed in Ref. [9]. In this paper, we assume no lepton flavour violation to concentrate on the quark mixings.

III. EXPERIMENTAL CONSTRAINTS

A. Z-pole data

Precision measurement of the Z-pole data at LEP and SLC has provided highly accurate tests on the SM [10,11]. Still more than -2σ deviation of A_{FB}^b, the forward-backward asymmetry of $Z \to b\bar{b}$ decay from the SM prediction suggests a hint of the departure from the SM in the list of the LEP and SLC data. It is quite helpful to introduce the precision variables for the study of the new physics effects on the electroweak data because the new physics contributions to the Z-pole observables are expected to be comparable with the loop contributions of the SM. Here, we use the nonstandard electroweak precision test in terms of the ϵ variables introduced by Altarelli *et al.* [12,13] which is useful for the analysis of the Z-pole data. This ϵ analysis provides a model independent way to analyze the electroweak precision data, where the electroweak radiative corrections containing whole m_t and m_H dependencies are parametrized into the parameters ϵ's. Thus the ϵ's can be extracted from the data without specifying m_t and m_H.

The universal correction terms to the electroweak form factors $\Delta\rho$ and Δk are defined from the vector and axial-vector couplings of lepton pairs to Z boson given by [12,13]:

$$\begin{aligned} g_A &= -\frac{1}{2}\left(1 + \frac{1}{2}\Delta\rho\right), \\ x \equiv \frac{g_V}{g_A} &= 1 - \sin^2\theta_{\rm eff}^l = 1 - s_0^2(1 + \Delta k), \end{aligned} \tag{13}$$

where s_0^2 is the Weinberg angle at tree level, satisfying $s_0^2 c_0^2 = \pi\alpha(m_Z)/\sqrt{2}G_F m_Z^2$ and $\sin^2\theta_{\rm eff}^l$ is the effective Weinberg angle including the SM loop corrections to the lepton sector. Another correction term Δr_W is obtained from the mass ratio m_W/m_Z by the Eq. (1) of Ref. [13]. The parameters (ϵ_1, ϵ_2, ϵ_3) are defined by the linear combinations of correction terms as given by [12,13]

$$\begin{aligned} \epsilon_1 = \Delta\rho, \qquad \epsilon_2 &= c_0^2\Delta\rho + \frac{s_0^2 \Delta r_W}{c_0^2 - s_0^2} - 2 s_0^2 \Delta k, \\ \epsilon_3 &= c_0^2 \Delta\rho + (c_0^2 - s_0^2)\Delta k, \end{aligned} \tag{14}$$

which avoid the new physics effects being masked by the large m_t^2 corrections in ϵ_2 and ϵ_3. We note that Δr_W is irrelevant for our analysis and affects only on ϵ_2. Hence we lay aside ϵ_2 in this paper.

The parameter ϵ_b is introduced to measure the additional contribution to the $Zb\bar{b}$ vertex due to the large m_t-dependent corrections in the SM. We write the most general amplitude for $Z \to b\bar{b}$ decay as

$$\begin{aligned} M(Z \to b\bar{b}) &= \frac{g}{2\cos\theta_W} \epsilon^\mu \bar{u}\gamma_\mu (g_{bV} - g_{bA}\gamma_5) u, \\ &= \frac{g}{2\cos\theta_W} \epsilon^\mu \bar{u} (\gamma_\mu (g_{bV}^0 - g_{bA}^0 \gamma_5) \\ &\quad + \Delta_b^L \gamma_\mu P_L + \Delta_b^R \gamma_\mu P_R) u, \end{aligned} \tag{15}$$

where ϵ^μ is the polarization vector for Z boson, g_{bV} (g_{bA}) is the vector (axial-vector) coupling of b quark pair to Z boson, and g_{bV}^0 (g_{bA}^0) is the tree-level coupling in the SM. The electroweak corrections Δ_b^L and Δ_b^R depend upon m_t and m_H. The large mass of top quark gives rise to additional m_t-dependent contribution to Δ_b^L such as $\Delta_b^L(m_t^2)|_{\rm SM} \propto m_t^2/m_W^2$ through the top quark exchange diagram [14], while the contribution to Δ_b^R is suppressed by the factor of m_b^2/m_t^2. It is absent in the cases of light quarks. Since the leading contribution of the electroweak radiative correction of the SM given in Eq. (13) is left-handed in the large m_t limit, Altarelli *et al.* have defined ϵ_b through the effective couplings g_{bA} and g_{bV} in the following manner [13]:

$$\begin{aligned} g_{bA} &= -\frac{1}{2}\left(1 + \frac{1}{2}\Delta\rho\right)(1 + \epsilon_b), \\ x_b \equiv \frac{g_{bV}}{g_{bA}} &= \frac{1 - \frac{4}{3}s_0^2(1 + \Delta k) + \epsilon_b}{1 + \epsilon_b} \end{aligned} \tag{16}$$

of which asymptotic contribution is given by $\epsilon_b \approx -G_F m_t^2/4\pi^2\sqrt{2}$. The parameter ϵ_b defined in this expression is identical to $-\Delta_b^{SM}(m_t^2)$ given in Eq. (15).

Considering the new physics effect, Δ_b^R can be sizable and we have to introduce a new parameter to describe the additional right-handed current interaction effects as done in Ref. [15,16]. We define the correction terms $\Delta\rho_b$ and Δk_b in an analogous way to those of lepton sector:

$$g_{bA} = g_A(1+\Delta\rho_b) = -\frac{1}{2}\left(1+\frac{1}{2}\Delta\rho\right)(1+\Delta\rho_b),$$
$$\sin^2\theta_{\rm eff}^b = \sin^2\theta_{\rm eff}^l(1+\Delta k_b) = s_0^2(1+\Delta k)(1+\Delta k_b), \tag{17}$$

where $\Delta\rho_b$ is a deviation of g_{bA} from the axial coupling of lepton sector g_A, and Δk_b is introduced through the effective Weinberg angle for b quark sector. To avoid being masked by the electroweak radiative corrections of the SM, we define the new epsilon parameters by the relations

$$\epsilon_b \equiv \Delta\rho_b, \qquad \epsilon_b' \equiv \frac{2}{3}s_0^2(\Delta\rho_b + \Delta k_b), \tag{18}$$

with canceling $\Delta_b^{SM}(m_t^2)$ in ϵ_b'. Note that ϵ_b goes to the original definition in Eq. (16) and $\epsilon_b' = 0$ in the SM limit since $\Delta\rho_b = -\Delta k_b = -\Delta_b^{SM}(m_t^2)$. Consequently ϵ_b' purely measures the anomalous right-handed current interaction.

The parameters ϵ_1 and ϵ_3 are extracted from the inclusive partial decay width Γ_l and the forward-backward asymmetry A_{FB}^l and ϵ_b and ϵ_b' are extracted from the observables of the inclusive decay width Γ_b and forward-backward asymmetry of $b\bar{b}$ production A_{FB}^b. In consequence, the four parameters ϵ_1, ϵ_3, ϵ_b, and ϵ_b' are set to be one to one correspondent to the observables Γ_l, A_{FB}^l, Γ_b, and A_{FB}^b. The quadratic m_t dependences of the SM electroweak radiative correction appears in ϵ_1 and ϵ_b while the m_t dependence of ϵ_3 is logarithmic.

According to the definitions, we can express the observables in terms of ϵ_i's up to the linear order as given in Eq. (123) of Ref. [11], which make it easy to perform the numerical analysis. We give the modification of the linearized relations by excluding the equation of m_W^2/m_Z^2 and adding the modified equations between the observables Γ_b, A_{FB}^b, and the ϵ parameters as

$$\Gamma_b = \Gamma_b|_B(1+1.42\epsilon_1 - 0.54\epsilon_3 + 2.29\epsilon_b),$$
$$A_{FB}^b = A_{FB}^b|_B(1+17.5\epsilon_1 - 22.75\epsilon_3 + 0.157\epsilon_b) \tag{19}$$

while the relations of Γ_l and A_{FB}^l remains intact

$$\Gamma_l = \Gamma_l|_B(1+1.20\epsilon_1 - 0.26\epsilon_3),$$
$$A_{FB}^l = A_{FB}^l|_B(1+34.72\epsilon_1 - 45.15\epsilon_3),$$

with $s_0^2 = 0.2311$. The Born approximation values $\Gamma_b|_B$ and $A_{FB}^b|_B$ are defined by the tree-level results including pure QED and pure QCD corrections and consequently depend upon the values of $\alpha_s(m_Z^2)$ and $\alpha(m_Z^2)$. The QCD corrections to the forward-backward asymmetries of b quark pair in Z decays are given in the Ref. [17,18]. We obtain $\Gamma_b|_B = 379.8$ MeV, and $A_{FB}^b|_B = 0.1032$ with the values $\alpha_s(m_Z^2) = 0.119$ and $\alpha(m_Z^2) = 1/128.90$. For the experimental analysis of A_{FB}^b, a bias factor has to be introduced to scale the QCD corrections. Here we used the average value, $s_b = 0.435$, given in Ref. [17]. Through these four linear equations between (Γ_l, A_{FB}^l, Γ_b, A_{FB}^b), and (ϵ_1, ϵ_3, ϵ_b, ϵ_b'), we obtain the 4-dimensional ellipsoid in (ϵ_1, ϵ_3, ϵ_b, ϵ_b') space from the recent LEP/SLC data given in Table I, which yields

$$\epsilon_1 = (5.1 \pm 1.1)\times 10^{-3}, \qquad \epsilon_3 = (3.8\pm 1.8)\times 10^{-3},$$
$$\epsilon_b = (2.8\pm 2.9)\times 10^{-2}, \qquad \epsilon_b' = (3.5\pm 4.0)\times 10^{-2}. \tag{20}$$

We write the ϵ_i variables as $\epsilon_i = \epsilon_i^{\rm SM} + \epsilon_i^{\rm new}$. The SM prediction $\epsilon_i^{\rm SM}$ has been calculated by ZFITTER [19], given as

$$\epsilon_1^{\rm SM} = 5.8\times 10^{-3}, \qquad \epsilon_3^{\rm SM} = 5.0\times 10^{-3},$$
$$\epsilon_b^{\rm SM} = -6.6\times 10^{-2}, \qquad \epsilon_b'^{\rm SM} = 0. \tag{21}$$

The new physics contributions $\epsilon_i^{\rm new}$ are extracted from the tree-level vector and axial-vector couplings of fermions to Z boson

$$g_V = T_{3h} + T_{3l} - 2Q\sin^2\theta_W + \lambda\sin^2\phi(T_{3h}\cos^2\phi - T_{3l}\sin^2\phi),$$
$$g_A = T_{3h} + T_{3l} + \lambda\sin^2\phi(T_{3h}\cos^2\phi - T_{3l}\sin^2\phi), \tag{22}$$

where $\sin^2\theta_W$ is the shifted Weinberg angle corrected by the new contribution

$$\sin^2\theta_W = s_0^2 - \lambda\sin^4\phi\frac{c_0^2 s_0^2}{c_0^2 - s_0^2}, \tag{23}$$

up to the linear order of λ. With these shifted couplings, we obtain $\epsilon_i^{\rm new}$ in our model,

$$\epsilon_1^{\rm new} = -2\lambda\sin^4\phi, \qquad \epsilon_3^{\rm new} = -\lambda\sin^4\phi,$$
$$\epsilon_b^{\rm new} = \lambda\sin^2\phi(1+|V_{33}^D|^2), \qquad \epsilon_b'^{\rm new} = \lambda\sin^2\phi|V_{33}^D|^2, \tag{24}$$

which are expressed by three parameters, λ, $\sin\phi$, and V_{33}^D.

These parameters are constrained by the condition that the model predictions are within the experimental ellipsoid

TABLE I. Data on the precision electroweak tests, quoted in [10].

	Measurement
m_Z	91.1875 ± 0.0021 GeV
Γ_l	83.984 ± 0.086 MeV
A_{FB}^l	0.0171 ± 0.0010
R_b	0.21638 ± 0.00066
A_{FB}^b	0.0997 ± 0.0016

given in Eq. (20). We show the plots of the projected experimental ellipses for 1-σ and 2-σ confidence level and the model predictions in terms of model parameters in Fig. 1. The left upper plot is drawn on the ϵ_1-ϵ_3 plane, the right upper plot on the ϵ_1-ϵ_b plane, the left lower plot on the ϵ_1-$\epsilon_b^\prime$ plane, and the right lower plot on the ϵ_3-ϵ_b plane. The straight lines denote the model predictions and the SM predictions are expressed by the black dots. We vary the parameter $\sin^2\phi$ and fix $|V_{33}^D|$ to be 1. The parameter $|V_{33}^D|$ far from 1 is not likely since $|V_{tb}| \approx 1$.

We scan the model parameters λ, $\sin^2\phi$, and $|V_{33}^D|$ and perform the χ^2 analysis for ϵ_i variables.. The allowed parameter set $(\lambda, \sin^2\phi)$ at 90% C.L. is depicted in Fig. 2 together with the low-energy experiment bound. The best fit of χ^2/dof is 0.57 at

$$\lambda = 0.368, \qquad \sin^2\phi = 0.04, \qquad V_{33}^D = \pm 1, \qquad (25)$$

corresponding to $m_{Z'} = 676.55$ GeV. With the small value of $\sin^2\phi$, the Z' coupling is much smaller than the Z boson

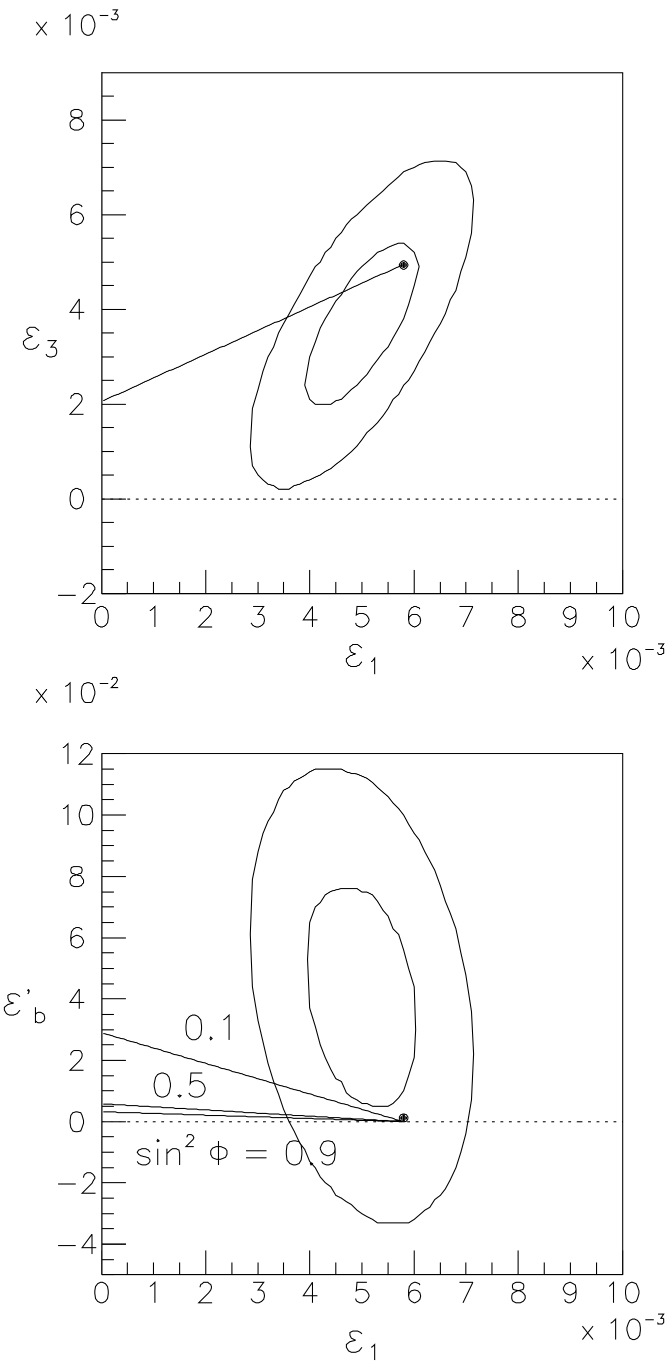

FIG. 1. Model predictions and experimental ellipses in the ϵ_i space. The dots denote the SM predictions.

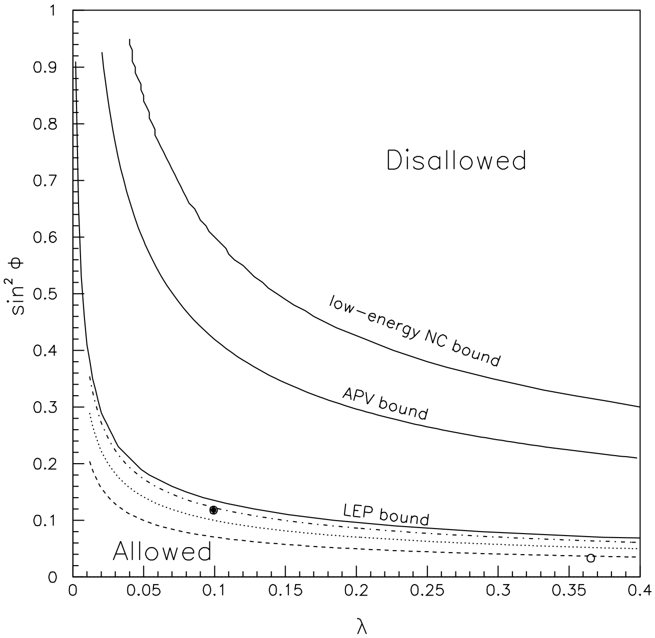

FIG. 2. Allowed model parameter set on $(\lambda, \sin^2\phi)$ plane by the low-energy neutral current data, the atomic parity violation data, and the LEP/SLC data at 90% confidence level. The white dot corresponds to the point that gives the best χ^2 fit for the LEP/SLC data and the black dot to the point for the low-energy neutral current data. The dashed line denotes $\Delta = 0.001$, the dotted line $\Delta = 0.002$, the dash-dotted line $\Delta = 0.003$.

coupling in the SM. Then the corresponding direct search bound on $m_{Z'}$ will be much lower than the present Tevatron limit for the SM type Z' boson, $m_{Z'} > 690$ GeV at 95% C.L. Thus our best fit value $m_{Z'} = 676.55$ GeV is safe compared to the $p\bar{p}$ direct limit of the Tevatron. The white dot in the Fig. 2 denotes the best fit for the LEP/SLC data. We will discuss Fig. 2 more in the later section.

B. Low-energy experiment

The low-energy neutral current interactions such as $\nu e \to \nu e$, νN scattering, and $e_{L,R}N \to e_{L,R}X$ are expressed by the effective four-fermion interactions and the coefficients of four-fermion operators have been precisely measured. The SM predictions for the coefficients are obtained including radiative corrections, while our model predictions involve both Z and Z' contributions at tree level. We perform the analysis to constrain the model parameters with the low-energy data.

For neutrino-hadron scattering, the relevant effective Hamiltonian can be written as

$$H^{\nu N} = \frac{G_F}{\sqrt{2}}\bar{\nu}\gamma^\mu(1-\gamma_5)\nu\sum_i[\epsilon_L(i)\bar{q}_i\gamma_\mu(1-\gamma_5)q_i + \epsilon_R(i)\bar{q}_i\gamma_\mu(1+\gamma_5)q_i], \tag{26}$$

where $\epsilon_{L,R}(i)$ $(i = u, d)$ are given by

$$\epsilon_{L,R}(u, d) = \epsilon^{\rm SM}_{L,R}(u, d)(1 - \lambda\sin^4\phi). \tag{27}$$

The relevant effective Hamiltonian for the scattering $\nu e \to \nu e$ at low energy can be written in the form

$$H^{\nu e} = \frac{G_F}{\sqrt{2}}\bar{\nu}\gamma^\mu(1-\gamma_5)\nu\bar{e}\gamma_\mu(g_V^{\nu e} - g_A^{\nu e}\gamma_5)e. \tag{28}$$

The coupling constants $g_V^{\nu e}$ and $g_A^{\nu e}$ are written as

$$g^{\nu e}_{V(A)} = g^{\nu e}_{V(A)}|_{\rm SM}(1 - \lambda\sin^4\phi). \tag{29}$$

For electron-hadron scattering such as $e_{L,R}N \to eX$ performed in the SLAC polarized electron experiment, the parity-violating Hamiltonian can be written as

$$H^{eN} = -\frac{G_F}{\sqrt{2}}\sum_i[C_{1i}\bar{e}\gamma^\mu\gamma_5 e\bar{q}_i\gamma_\mu q_i + C_{2i}\bar{e}\gamma^\mu e\bar{q}_i\gamma_\mu\gamma_5 q_i]. \tag{30}$$

The coefficients C_{1i} are given by

$$C_{1u,d} = C^{\rm SM}_{1u,d}(1 - \lambda\sin^4\phi), \tag{31}$$

while the coefficients C_{2i} are given by

$$\begin{aligned} C_{2u} &= C^{\rm SM}_{2u}(1 - \lambda\sin^4\phi) + 2\lambda|V^U_{31}|^2\sin^2\phi\sin^2\theta_W, \\ C_{2d} &= C^{\rm SM}_{2d}(1 - \lambda\sin^4\phi) - 2\lambda|V^D_{31}|^2\sin^2\phi\sin^2\theta_W. \end{aligned} \tag{32}$$

Note that only C_{2i} involve the additional parameters $V^{U,D}_{31}$ because of the operator structure.

The experimental values for these observables and the standard model predictions are tabulated in Table II quoted

TABLE II. Values of low-energy neutral current experimental data compared to the standard model predictions, quoted in Ref. [20].

	Experiments	SM prediction
$\epsilon_L(u)$	0.326 ± 0.012	0.3460 ± 0.0002
$\epsilon_L(d)$	-0.441 ± 0.010	-0.4292 ± 0.0001
$\epsilon_R(u)$	$-0.175^{+0.013}_{-0.004}$	-0.1551 ± 0.0001
$\epsilon_R(d)$	$-0.022^{+0.072}_{-0.047}$	0.0776
$g_V^{\nu e}$	-0.040 ± 0.015	-0.0397 ± 0.0003
$g_A^{\nu e}$	-0.507 ± 0.014	-0.5065 ± 0.0001
$C_{1u} + C_{1d}$	0.148 ± 0.004	0.1529 ± 0.0001
$C_{1u} - C_{1d}$	-0.597 ± 0.061	-0.5299 ± 0.0004
$C_{2u} + C_{2d}$	0.62 ± 0.80	-0.0095
$C_{2u} - C_{2d}$	-0.07 ± 0.12	-0.0623 ± 0.0006

from the particle data book [20]. We perform the χ^2 fit for the ten physical observables with respect to the parameters λ, $\sin\phi$, and $|V_{3j}^{U,D}|$. The allowed region of $(\lambda, \sin^2\phi)$ is shown in Fig. 2. The best fit value $\chi^2/dof = 11.33/10$ is obtained at

$$\begin{aligned} \lambda = 0.102, \qquad \sin^2\phi = 0.13, \\ |V_{31}^U| = 1, \qquad |V_{31}^D| = 0, \end{aligned} \tag{33}$$

which leads to $m_{Z'} = 749.78$ GeV. The black dot in the Fig. 2 denotes the best fit for the neutral current data.

The atomic parity violation can be described by the Hamiltonian in Eq. (30). The weak charge of an atom is defined as

$$Q_W = -2[C_{1u}(2Z + N) + C_{1d}(Z + 2N)], \tag{34}$$

where Z (N) is the number of protons (neutrons) in the atom. For ${}^{133}_{55}$Cs atom, $Z = 55$, $N = 78$, the correction to the weak charge is given by

$$\begin{aligned} \Delta Q_W &\equiv Q_W - Q_W^{SM} \\ &= -2[\Delta C_{1u}(2Z + N) + \Delta C_{1d}(Z + 2N)] \\ &= \Delta Q_W^{SM}(1 - \lambda \sin^4\phi). \end{aligned} \tag{35}$$

The recent measurements and the analyses of the weak charge for the Cs and Tl atoms give the value [21]

$$\begin{aligned} Q_W &= -72.69 \pm 0.48 \quad \text{(Cs)}, \\ Q_W &= -116.6 \pm 3.7 \quad \text{(Tl)}, \end{aligned} \tag{36}$$

while the SM predictions are $Q_W = -73.19 \pm 0.03$ (Cs), and $Q_W = -116.81 \pm 0.04$ (Tl). The APV data leads to the bound on the parameters

$$0.0003 < \lambda \sin^4\phi < 0.0134. \tag{37}$$

Figure 2 shows the bounds for the low-energy neutral current data, the APV data, and the LEP/SLC data at Z-pole at 90% confidence level in the parameter space $(\lambda, \sin^2\phi)$. Note that the constraint of the LEP/SLC data is stronger than that of the low-energy data.

IV. UNITARITY VIOLATION

We have the modified CKM matrix in the Lagrangian:

$$\begin{aligned} V_{CKM} = V_{CKM}^0 + \begin{pmatrix} V_{31}^{U*}V_{31}^D & V_{31}^{U*}V_{32}^D & V_{31}^{U*}V_{33}^D \\ V_{32}^{U*}V_{31}^D & V_{32}^{U*}V_{32}^D & V_{32}^{U*}V_{33}^D \\ V_{33}^{U*}V_{31}^D & V_{33}^{U*}V_{32}^D & V_{33}^{U*}V_{33}^D \end{pmatrix} \\ \cdot \lambda \sin^2\phi, \end{aligned} \tag{38}$$

which describes the quark mixings for the charged currents coupled to the $W^{\pm}$ bosons. The mixing matrix for $W'^{\pm}$ bosons has the same structure as above matrix except that the model parameter $\lambda \sin^2\phi$ is replaced by $1/\sin\phi\cos\phi$.

Although the quark mixing matrix is given in Eq. (37), the "observed" CKM matrix measures the effective charged current interactions through both W and W' exchange diagrams at low energies. We consider the effective Hamiltonian for extracting $|V_{uq}|$

$$\begin{aligned} \mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} \sum_{q=d,s,b} V_{uq}(\bar{u}\gamma_\mu(1 - \gamma_5)q)(\bar{\nu}\gamma^\mu(1 - \gamma_5)l) \\ + \text{H.c.}, \end{aligned} \tag{39}$$

where the measured CKM matrix element is $V_{uq} = V_{uq}^0(1 - \lambda \sin^4\phi)$ and V_{uq}^0 is the CKM matrix element in the SM. The muon decay constant G_F is identical to that in the SM if no lepton mixing is assumed [8]. The additional parameters $\{V_{U3j}, V_{D3k}\}$ are canceled in Eq. (38) when we sum the W and W' contributions in the leading order of λ. Thus the unitarity violating term defined in Eq. (1) is derived

$$\Delta = 2\lambda \sin^4\phi. \tag{40}$$

We estimate the unitarity violating term as

$$\Delta \approx 0.0012, \tag{41}$$

with the best fit value of Eq. (25) from the LEP/SLC data, and estimate as

$$\Delta \approx 0.0034, \tag{42}$$

with the best fit value of Eq. (33) from the low-energy neutral current data. Interestingly the latter is close to the reported value $\Delta = 0.0031 \pm 0.0014$ [2]. In Fig. 2, the dashed line denotes $\Delta = 0.003$, dotted line $\Delta = 0.002$, dash-dotted line $\Delta = 0.001$,

V. CONCLUDING REMARKS

We explained the recent measurement on the unitarity violation of the CKM matrix by introducing the new physics with the separate $SU(2)$ gauge symmetry on the third generation. The additional $SU(2)$ symmetry leads to the nonuniversality of the electroweak gauge coupling, which yields the unitarity violating term Δ of order $\mathcal{O}(v^2/u^2)$ in the quark mixing matrix. We performed the updated analysis on the model parameters with the recent LEP + SLC data, low-energy neutral current data, and the atomic parity violation data. We find that the Z-pole data gives much stronger bound than the low-energy data. Thus the combined analysis of the LEP + SLC data and low-energy data just provides the same result as Eq. (25) and we present them separately. Our model predictions of Δ with the best χ^2 fit of low-energy neutral current experiment agrees well with the value reported in the Ref. [1,2] and still allowed by the LEP + SLC data. It is very interesting and instructive to find new physics model to explain the violation of unitarity in the CKM matrix even though the violation is still controversial. In conclusion, we show that the nonuniversal gauge interaction model is a good candidate for the new physics with the unitarity violation of the CKM matrix. The updated measurement on the CKM matrix elements will probe the unitarity more precisely in the future.

ACKNOWLEDGMENTS

This work was supported by Korea Research Foundation Grant No. KRF-2003-050-C00003.

[1] *Workshop Proceedings of Quark-Mixing, CKM-Unitarity, Heidelberg, 2002*, edited by H. Abele and D. Mund (Mattes-Verlag, Heidelberg, 2003).
[2] H. Abele *et al.*, Eur. Phys. J. C **3**, 3 (2004).
[3] A. Czarnecki, W. J. Marciano, and A. Sirlin, Phys. Rev. D **70**, 093006 (2004) and references therein.
[4] E. Malkawi, T. Tait, and C.-P. Yuan, Phys. Lett. B **385**, 304 (1996).
[5] D. J. Muller and S. Nandi, Phys. Lett. B **383**, 345 (1996).
[6] R. S. Chivukula, E. H. Simmons, and J. Terning, Phys. Lett. B **331**, 383 (1994); Phys. Rev. D **53**, 5258 (1996).
[7] J. C. Lee, K. Y. Lee, and J. K. Kim, Phys. Lett. B **424**, 133 (1998); J. C. Lee and K. Y. Lee, in *APCTP Workshop Proceedings: Pacific Particle Physics Phenomenology, Seoul, Korea, 1997* (World Scientific, Singapore, 1998), p. 214.
[8] E. Malkawi and C.-P. Yuan, Phys. Rev. D **61**, 015007 (2000).
[9] K. Y. Lee and J. C. Lee, Phys. Rev. D **58**, 115001 (1998).
[10] LEP Electroweak Working Group, Report No. CERN-EP/2003-091.
[11] G. Altarelli, hep-ph/9811456.
[12] G. Altarelli, R. Barbieri, and S. Jadach, Nucl. Phys. B **369**, 3 (1992); G. Altarelli, R. Barbieri, and F. Caravaglios, Phys. Lett. B **349**, 145 (1995).
[13] G. Altarelli, R. Barbieri, and F. Caravaglios, Nucl. Phys. B **405**, 3 (1993).
[14] A. A. Akhundov, D. Yu Bardin, and T. Riemann, Nucl. Phys. B **276**, 1 (1986).
[15] K. Y. Lee, Phys. Lett. B **472**, 366 (2000).
[16] J. Chay, K. Y. Lee, and S.-h. Nam, Phys. Rev. D **61**, 035002 (2000).
[17] D. Abbaneo *et al.* (LEP Heavy Flavor Working Group), Eur. Phys. J. C **4**, 185 (1998).
[18] V. Ravindran and W. L. van Neerven, Phys. Lett. B **445**, 214 (1998).
[19] D. Bardin *et al.*, Report No. CERN-TH 6443/92.
[20] S. Eidelman *et al.*, Phys. Lett. B **592**, 1 (2004).
[21] C. S. Wood *et al.*, Science **275**, 1759 (1997); N. H. Edwards *et al.*, Phys. Rev. Lett. **74**, 2654 (1995); P. A. Vetter *et al.*, Phys. Rev. Lett. **74**, 2658 (1995).