

Cosmological bounds on the equation of state of dark matter

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In this exploratory study, we investigate the bounds on the equation of state of dark matter. Modeling dark matter as a fluid component, we take into account both positive and negative fixed equations of state. Using CMB, supernovae Ia and large scale structure data we find constraints on the equation of state in a modified Λ CDM cosmology. We obtain $-1.50 \times 10^{-6} < w_{\text{dm}} < 1.13 \times 10^{-6}$ if the dark matter produces no entropy and $-8.78 \times 10^{-3} < w_{\text{dm}} < 1.86 \times 10^{-3}$ if the adiabatic sound speed vanishes, both at 3σ confidence level.

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I. INTRODUCTION

In this paper, we would like to present bounds on the dark matter equation of state derived from cosmological observations. Here, we do not want to limit ourselves to a non-negative equation of state since the nature of dark matter is not yet clear. There are a number of particle dark matter candidates and numerous experiments attempting to detect dark matter particles (see [1] and references therein). However, dark matter may not be a particle at all. We will try to keep an open mind and check if a negative equation of state can be ruled out by cosmological observations. Previous studies focused on the power spectrum properties of warm dark matter [2] and mixed dark matter models [3], but we would like to emphasize that the present work does not consider the bounds on warm dark matter. We will show that the equation of state of dark matter is already strongly constrained by current observations of the CMB, supernovae Ia and large scale structure. We will not concern ourselves with bounds from other than cosmological observations.

We will work only from a fluid perspective and leave the question open how one could obtain a negative equation of state of dark matter. For simplicity, we will only allow for a constant equation of state w_{dm} .

In models where dark matter interacts with other components of the universe, such as in coupled quintessence models [4], one may obtain a negative equation of state for dark matter. It may be possible to obtain $w_{\text{dm}} < 0$ by other methods as well, but we are not aware of any such model.

In this work we will use two simple models for dark matter: one with no entropy production and one with vanishing adiabatic sound speed.

We obtain bounds for a constant equation of state of dark matter of $-1.50 \times 10^{-6} < w_{\text{dm}} < 1.13 \times 10^{-6}$ if there is no entropy production and $-8.78 \times 10^{-3} < w_{\text{dm}} < 1.86 \times 10^{-3}$ if the adiabatic sound speed vanishes, both at 3σ confidence level.

For this investigation we will assume that the universe is flat and contains a cosmological constant type dark energy component with equation of state $w_{\text{de}} = -1$, dark matter with a variable equation of state w_{dm} , baryons, photons and

massless neutrinos. We do not include the tensor part in our analysis. In the conclusions, we will address the issue what will change if we relax the flatness assumption and if we have an equation of state different from -1 .

The plan of this paper is as follows: in Sec. II, we will introduce the perturbation equations for dark matter with an arbitrary equation of state (which we will from now on call modified dark matter for brevity). In Sec. III we introduce a model with vanishing entropy production, in Sec. IV we give a different model with vanishing adiabatic sound speed. In Sec. V we present the bounds on w_{dm} obtained from a Markov Chain Monte Carlo simulation, followed by our conclusions.

II. PERTURBATION EQUATIONS

In the following, we will use the notation of [5]. The full energy momentum tensor for a fluid with equation of state w may expressed by (species indices suppressed)

$$T_0^0 = -\bar{\rho}(1 + \delta Q), \quad (1)$$

$$T_0^i = -\bar{\rho}(1 + w)vQ^i, \quad (2)$$

$$T_i^0 = \bar{\rho}(1 + w)(v - B)Q_i, \quad (3)$$

$$T_j^i = \bar{p}[(1 + \pi_L Q)\delta_j^i + \Pi Q_j^i], \quad (4)$$

where the $Q(\mathbf{k}, \mathbf{x})$ are eigenfunctions of the Laplace-Operator, $\nabla^2 Q_k(\mathbf{x}) = -k^2 Q_k(\mathbf{x})$. The (gauge dependent) variables δ, v, B, π_L and Π are defined by these equations. These quantities may then be combined to form the gauge independent quantities Δ_g , the density perturbation on hypersurfaces of constant curvature perturbation, V , the velocity and Π , the anisotropic stress component. We will construct a gauge-invariant form of the pressure perturbation π_L later.

The perturbation equations for the dark matter component expressed in gauge-invariant variables are then [5,6]¹:

¹Please note that w is the equation of state of dark matter and that the species indices are suppressed.

$$\dot{\Delta}_g + 3(c_s^2 - w)\frac{\dot{a}}{a}\Delta_g + kV(1+w) + 3\frac{\dot{a}}{a}w\Gamma = 0, \quad (5)$$

$$\begin{aligned} \dot{V} = & \frac{\dot{a}}{a}(3c_s^2 - 1)V + k(\Psi - 3c_s^2\Phi) + \frac{c_s^2 k}{1+w}\Delta_g \\ & + \frac{wk}{1+w}\left(\Gamma - \frac{2}{3}\Pi\right), \end{aligned} \quad (6)$$

$$a^2 \sum_{\alpha} (\rho_{\alpha} + p_{\alpha})V_{\alpha} = 2M_P^2 k \left(\frac{\dot{a}}{a} \Psi - \dot{\Phi} \right), \quad (7)$$

where the sum is over all present species. We will assume that the anisotropic stress vanishes for dark matter, $\Pi_{\text{dm}} = 0$. Φ and Ψ are the gravitational potentials where $\Phi = -\Psi - \sum_{\alpha} \Omega_{\alpha} \Pi_{\alpha}$. The sound speed is given by $c_s^2 = \dot{p}/\dot{\rho}$. Note that this is not the adiabatic sound speed, which is defined by $c_{\text{ad}}^2 = \delta p / \delta \rho$. Γ is the entropy production rate and is given by

$$\Gamma = \pi_L - \frac{c_s^2}{w} \delta. \quad (8)$$

This may also be expressed as the difference between “background” sound speed and adiabatic sound speed,

$$w\Gamma = (c_{\text{ad}}^2 - c_s^2)\delta. \quad (9)$$

It will be useful to formulate Γ in terms of gauge-invariant variables. From [5] it may be verified that

$$\tilde{\pi}_L := \pi_L + 3\frac{c_s^2}{w}(1+w)\frac{\dot{a}}{a}k^{-1}\sigma_g, \quad (10)$$

is the gauge-invariant pressure perturbation. Hence

$$\Gamma = \tilde{\pi}_L - \frac{c_s^2}{w}[\Delta_g - 3(1+w)\Phi]. \quad (11)$$

III. DARK MATTER WITH NO ENTROPY PRODUCTION

In this section, we will assume that $\Gamma = 0$. For a constant equation of state of dark matter, the sound speed is given by

$$c_s^2 = \dot{p}/\dot{\rho} = w. \quad (12)$$

If we allow for negative equation of state the square of the background sound speed may thus become negative. There is of course a question whether or not this assumption is reasonable, but given the unknown nature of dark matter we may consider this case and see how well constrained w is. Before we discuss the numerical solutions of the perturbation equations, it will be helpful to consider the solutions for a universe filled only with dark matter in the subhorizon limit, $k^2 \gg (\frac{\dot{a}}{a})^2$. Equations (5)–(7) may be combined to eliminate V :

$$\ddot{\Delta}_g - (3w - 1)\frac{\dot{a}}{a}\dot{\Delta}_g + wk^2\Delta_g + k^2(1+w)(\Psi - 3w\Phi) = 0, \quad (13)$$

$$\dot{\Phi}\left(\frac{\dot{a}}{a}\right) + \left(\frac{5}{2} + \frac{3}{2}w\right)\left(\frac{\dot{a}}{a}\right)^2\Phi + \frac{k^2}{3}\Phi = \frac{\Delta_g}{2}\left(\frac{\dot{a}}{a}\right)^2. \quad (14)$$

The background solution for a universe filled only with a modified dark matter component yields

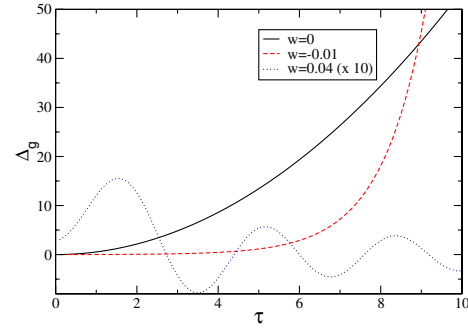


FIG. 1 (color online). Evolution of the energy density perturbation of dark matter Δ_g in the subhorizon regime for several equations of state in a universe containing only dark matter with $\Gamma = 0$. In the case of $w = 0$, the energy density perturbation grows $\propto \tau^2$ (black, straight). For $0 < w < 1/3$ (blue, dotted) one obtains decaying oscillations while for $w < 0$ the density perturbation grows rapidly (red, dashed).

$$\frac{\dot{a}}{a} = \frac{2}{\tau + 3\tau w}. \quad (15)$$

For $w = 0$ the solution of (13) and (14) in the subhorizon limit is $\Delta_g = a(\tau) = \tau^2$ and $\Phi = \text{const.}$, as is well known. For the superhorizon regime, $\Delta_g = \text{const.}$ The solutions to these equations in the subhorizon limit if $w \neq 0$ are plotted in Fig. 1. It can be seen that for $0 < w < 1/3$ we obtain decaying oscillations for Δ_g while for $w = 1/3$ one gets oscillations with constant amplitude. For $w < 0$ we find that Δ_g increases rapidly. This is obviously due to the negative sound speed, which leads to a growing gravitational potential. At superhorizon scales, $\Delta_g = \text{const.}$, regardless of the equation of state.

We have computed the resulting CMB and power spectrum in Fig. 2 with the CMBEASY software [7] (which is

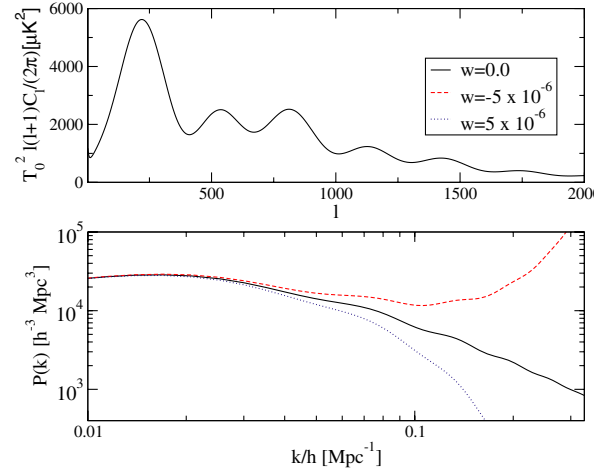


FIG. 2 (color online). CMB and matter power spectra for a Λ CDM model with modified dark matter, $\Gamma = 0$. The power on small scales is much larger than in the $w = 0$ case if the equation of state is negative (red, dashed). One can see the decaying oscillations for the case of $w > 0$ (blue, dotted). As expected, there is no difference in the power spectra at very large scales. The impact on the CMB spectrum is not noticeable at this level.

based on CMBFAST [8]), using the parameters of a best fit Λ CDM model: $\Omega_b h^2 = 0.230$, $\Omega_m h^2 = 0.144$, $n_s = 0.961$, $\tau = 0.114$, $h = 0.69$.

As one can verify the impact of varying w on the power spectrum is very marked on small scales. We therefore expect to obtain tight constraints on w .

IV. DARK MATTER WITH VANISHING ADIABATIC SOUND SPEED

Given the results of the last section and the problematic assumption of negative sound speed, we may choose a different approach. In fact, Γ measures the “difference” between adiabatic sound speed $c_{ad}^2 = \delta p / \delta \rho$ and background sound speed $c_s^2 = \dot{p} / \dot{\rho}$. In the previous example, we enforced $c_{ad}^2 = c_s^2$ by the requirement $\Gamma = 0$. A different requirement would be that the adiabatic sound speed vanishes.

The first problem that arises is that c_{ad}^2 is not gauge-invariant. We must therefore first specify what we mean by vanishing adiabatic sound speed. We may choose a hypersurface such that $c_{ad}^2 = 0$ has a definite meaning. Since $c_{ad}^2 = 0$ implies $\delta p = 0$ and $\pi_L = \delta p / p$ this leads to the requirement that $\pi_L = 0$ on a certain set of hypersurfaces. For simplicity, we choose the Newtonian slicing, giving the shear free hypersurfaces, $\sigma_g = 0$. We therefore obtain $\pi_L^{(newt)} = 0$, which with the definition Eq. (10) gives

$$\tilde{\pi}_L = 0. \quad (16)$$

This is true in any gauge because $\tilde{\pi}_L$ is gauge-invariant. Hence choosing

$$\Gamma = 3(1+w)\Phi - \Delta_g, \quad (17)$$

we enforce that the adiabatic sound speed vanishes on

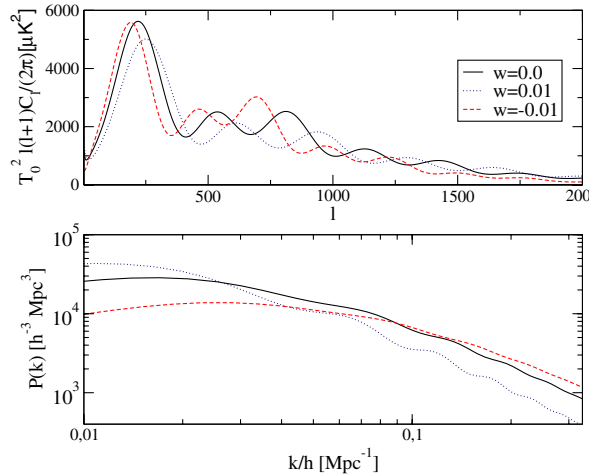


FIG. 3 (color online). CMB and power spectrum for a Λ CDM model with modified dark matter, $c_{ad}^2 = 0$. For a positive equation of state (blue, dotted) the peak positions of the CMB are shifted to smaller scales and have less power than in the $w = 0$ case (black, straight). For a negative w (red, dashed), the peak positions are shifted to smaller l . Note also the different peak ratios in these cases.

hypersurfaces of isotropic expansion rate. Of course, this choice is by no means preferred over any other choice of hypersurfaces; we have chosen this one merely for computational simplicity.

Solving the perturbation equations for a universe filled only with modified dark matter, we obtain evolution of superhorizon modes if $w \neq 0$. There is no exponential growth for subhorizon modes if $w < 0$. We may conclude that this model is well-behaved compared to the $\Gamma = 0$ case.

We have plotted the power and CMB spectra in Fig. 3 (model parameters as for the $\Gamma = 0$ case). As expected, the modification has a huge impact on the growth behavior of fluctuations.

V. BOUNDS ON THE EQUATION OF STATE OF DARK MATTER

In order to quantify the bounds on the equation of state of dark matter, we ran a Markov Chain Monte Carlo (MCMC) simulation for $\Gamma = 0$ and $c_{ad}^2 = 0$ with the ANALYZETHIS! package [9] using WMAP TT and TE spectra [10,11], VSA [12], CBI [13] and ACBAR [14] data up to $l = 2000$ as well as the SDSS power spectrum [15] (all points with $k/h < 0.15 \text{ Mpc}^{-1}$) and the SNe Ia data of Riess *et al.* [16]. Each run contained $\sim 50\,000$ points after burn-in removal. The model used was a Λ CDM cosmology the priors used were $-4 \times 10^{-6} < w_{dm} < 5 \times 10^{-6}$ for the $\Gamma = 0$ model and $-0.02 < w_{dm} < 0.02$ for the $c_{ad}^2 = 0$ model.

The resulting one-dimensional marginalized likelihoods are displayed in Fig. 4. The confidence intervals for the equation of state are displayed in Table I.

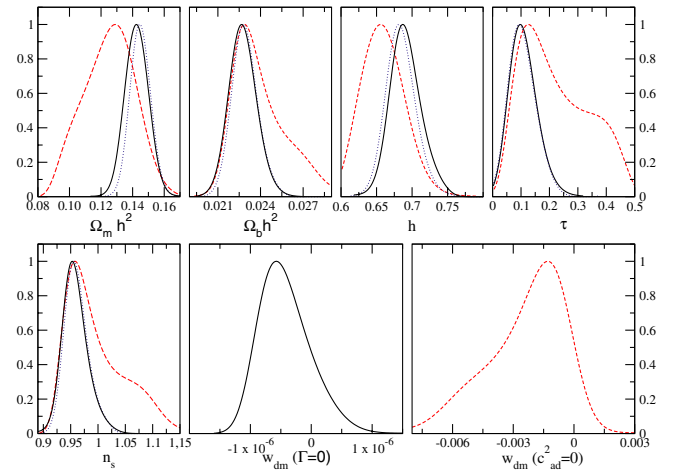


FIG. 4 (color online). One-dimensional marginalized likelihoods of the MCMC simulation for the model parameters, using CMB, SNe Ia and LSS data. The results are for the $\Gamma = 0$ (black, straight) and the $c_{ad}^2 = 0$ (red, dashed) model. We have plotted the results for a pure Λ CDM model with $w = 0$ for comparison (blue, dotted). Note the large difference in constraints on w for the two models. Also, the $\Gamma = 0$ model has nearly the same parameter distributions as a pure Λ CDM model.

TABLE I. Confidence intervals

	68.3%	95.4%	99.7%
w_{dm} for $\Gamma = 0$ ($\times 10^{-6}$)	-0.929	-1.19	-1.50
	-0.114	+0.462	+1.133
w_{dm} for $c_{ad}^2 = 0$ ($\times 10^{-3}$)	+0.098	+0.890	+1.86
	-3.59	-6.58	-8.78

The equation of state is quite strongly constrained if $\Gamma = 0$, at a level of 10^{-6} . What is somewhat surprising is that the likelihood is centered not on $w = 0$ but on a slightly negative equation of state. This may be traced to the fact that the SDSS data set we used does not encompass very small scales and therefore the observations are blind to the very strong increase in power at very small scales if $w < 0$. The constraints for the $c_{ad}^2 = 0$ case are less stringent, in line with our expectations. Here, w is only constrained at a level of 10^{-3} .

VI. CONCLUSION

Since this is an exploratory study we have not attempted to formulate a model with realistic variation in the equation of state but chose to show the main effects and bounds derived from current observations using a constant equation of state. For the $\Gamma = 0$ model, it is clear that the main constraint comes from the matter power spectrum. If we would include measurements at smaller scales, the constraints on the equation of state would be even more restrictive. We may conclude that the $\Gamma = 0$ model is unlikely to be realistic. The situation is not so clear for the $c_{ad}^2 = 0$ model. More accurate measurements of the CMB, especially in the large multipole region, should give tighter constraints. Based on the data we cannot conclude that this model is ruled out.

How strongly dependent are these results on our assumption of flatness and dark energy equation of state

$w_{de} = -1$? From Fig. 2 we can readily see that the main constraint on the $\Gamma = 0$ model comes from the large scale structure data; since the CMB spectrum does not change much in the allowed parameter range, we may conclude that relaxing the flatness assumption would not make much difference on the constraints. The same is true for an equation of state different from -1 . From previous studies it is known that for $w_{de} > -1$, structure growth is suppressed at small scales [17–19]. But for the $\Gamma = 0$ model, this can only make a small difference, since the growth suppression cannot ameliorate the strong deviation from the LSS measurements at small scales as is readily apparent in Fig. 2.

The situation for the $c_{ad}^2 = 0$ case is different. Here, relaxing the flatness assumption will lead to a weaker constraint on w_{dm} . The first peak position is sensitive to the geometry of the universe, but increasing or decreasing w_{dm} can in principle shift this peak to be in agreement with the WMAP data, as may be seen in Fig. 3. We therefore expect also that the constraint on the total energy Ω_{tot} will be weaker than in the standard Λ CDM case. If $w_{de} > -1$ the constraints on the dark energy equation of state and w_{dm} will be less stringent. As mentioned above, the main impact of $w_{de} > -1$ is through a suppression of structure growth at small scales, but this may be compensated by decreasing w_{dm} (see Fig. 3).

Relaxing the flatness and $w_{de} = -1$ will therefore have a negligible impact for the $\Gamma = 0$ model but may lead to a significant relaxation of constraints for the $c_{ad}^2 = 0$ model.

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