

Gluon propagation in space-time dependent fields

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The propagator for gluons in a space-time dependent field is derived. This is accomplished by solving the equation of motion for the gluonic Green's functions. Subsequently, a relationship between the quark and the gluon propagator is presented. With its help, characteristics of the bosonic propagator can be obtained from those of the fermionic and vice versa. Finally, this relation is discussed for the special case of ultrarelativistic collisions in the semiclassical limit.

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I. INTRODUCTION

In Refs. [1–3], the full retarded fermion propagators in the presence of arbitrarily space-time-dependent classical fields were studied. Here the generalization to the corresponding gluon propagator is presented. Correlators of this kind are, for example, needed in order to study the production of the respective particles by vacuum polarization [1–8] or calculate induced currents and condensates. Such phenomena are of importance for the physics of the early universe [9] and ultrarelativistic heavy-ion collisions together with the quark-gluon plasma (QGP) [10]. The strong classical fields are the common feature in these systems, while the detailed characteristics of the field configurations differ. After providing the general formalism, this paper focuses on particle propagation in ultrarelativistic collisions.

A lot of effort is made to study the QGP's production and equilibration [11] in nuclear collision experiments at the Relativistic Heavy-Ion Collider (RHIC) and the Large Hadron Collider (LHC), currently under construction. It is a widely used assumption that the initial state in heavy-ion collisions is dominated by gluons. Because of their large occupation number and as a first approximation, one can begin the system's description with a classical background field. Quantum fluctuations have to be investigated subsequently. Those considerations have also lead to the McLerran-Venugopalan (MV) model [12]. There, the projectiles are represented by classically interacting color-charge distributions on the two branches of the light cone. One observes that quantum corrections are enhanced by kinematical factors. They can be included into the distributions by means of the Jalilian-Marian–Iancu–McLerran–Weigert–Leonidov–Kovner (JIMWLK) equation [12].

In any case, the high occupation-number bosonic fields are that strong that multiple couplings to the classical field are not suppressed even for a small coupling constant g . In the absence of other scales, they have to be taken into account to all orders. The leading quantum processes con-

cern terms in the classical action of second order in the fields (fermions, antifermions, and bosonic quanta). From those terms, the inverse of the respective two-point Green's functions can be read off. Their solutions give the propagators of the corresponding particles to all orders in the classical field. Depending on the boundary conditions that are imposed, the correlator will serve to obtain expectation values (in-in-formalism, e.g., retarded and advanced propagator) or probabilities (in-out-formalism, e.g., Feynman propagator). In the following, the first possibility will be mostly pursued.

The paper is organized as follows: Section II contains the derivation of the equation of motion for the gluon Green's functions within the framework of the background-field method of QCD. Section III derives the retarded gluon propagator as a solution of the equation of motion in background-field Feynman gauge. Section IV gives the retarded Green's function solution for the quadratic Dirac operator. In Sec. V the interrelation between the retarded gluon propagator and the retarded quadratic Dirac propagator is worked out. Additionally, the connection to the retarded linear Dirac propagator is shown. These results are used in order to translate the known characteristics of the two-point function for the first-order Dirac equation into those of the other two correlators. Section VI applies the findings of the previous sections to ultrarelativistic heavy-ion collisions. The cases of nucleus-nucleus and nucleus-nucleon collisions are addressed as well as that of the propagation of gluons with a large momentum scale. In the last situation, all terms for the retarded propagator are spelled out and the Feynman propagator is available, too. Section VII summarizes the results.

Throughout the paper the metric tensor is given by $g^{\mu\nu} = \text{diag}(1, -1, -1, -1)$, angular momenta are measured in units of $\hbar$, and velocities in fractions of the speed of light c . From here on, the coupling constant is included in the definition of the classical field: $gA_{\text{old}}^\mu = A_{\text{new}}^\mu$. $\vec{v}$ represents the three-vector of the spatial components of any four-vector v .

II. THE EQUATION OF MOTION FOR GLUON GREEN'S FUNCTIONS

In order to derive the requested equation of motion for the gluonic Green's functions, let us start with the classical action for the bosonic sector of the system:

$$S = \frac{1}{g^2} \int d^4x \mathcal{L}_G, \quad (1)$$

with the Lagrangian density for the gauge field,

$$\mathcal{L}_G = \frac{1}{4} F^a_{\mu\nu} F^{\mu\nu a}, \quad (2)$$

where the field tensor is defined as

$$F_{\mu\nu} = i[D_\mu, D_\nu], \quad (3)$$

with the covariant derivatives

$$D_\mu = \partial_\mu - i\check{A}_\mu. \quad (4)$$

The field tensor is explicitly given by

$$F_{\mu\nu}^a[\check{A}] = \partial_\mu \check{A}_\nu^a - \partial_\nu \check{A}_\mu^a + f^{abc} \check{A}_\mu^b \check{A}_\nu^c. \quad (5)$$

The splitting of the gauge field $\check{A}_\mu$ into a classical A_μ and a quantum Q_μ field,

$$\check{A}_\mu = A_\mu + Q_\mu, \quad (6)$$

allows one to express the field tensor as a functional of two fields:

$$F_{\mu\nu}^a[A + Q] = F_{\mu\nu}^a[A] + D_\mu^{ab}[A]Q_\nu^b - D_\nu^{ab}[A]Q_\mu^b + f^{abc}Q_\mu^b Q_\nu^c. \quad (7)$$

$D_\mu^{ab}[A]$ stands for the covariant derivative as a functional of the classical field A_μ only. The Lagrangian density is composed of a sum of terms containing different powers of the quantum fields. The terms of second order determine the equation of motion for the propagator of the quantum fluctuations in the classical background.¹ It is given by

$$\mathcal{L}_G^{Q^2} = \frac{1}{2} \{ (D_\mu^{ab}[A]Q_\nu^b)(D^{ac\nu}[A]Q^{c\mu}) - (D_\mu^{ab}[A]Q_\nu^b) \times (D^{ac\mu}[A]Q^{c\nu}) - f^{abc}F_{\mu\nu}^a Q^{b\mu} Q^{c\nu} \}. \quad (8)$$

Additional terms of second order in the quantum fluctuations arise from the gauge-fixing term. The gauge-fixing term to be added to the Lagrangian density reads as follows for the background-field Feynman gauge:

$$\mathcal{L}_{GF} = \mathcal{L}_G^{Q^2} = -\frac{1}{2} (D_\mu^{ab}[A]Q^{b\mu})(D^{ac}[A]Q^{c\nu}). \quad (9)$$

As the Lagrangian density is always integrated over in order to obtain the action, the expressions for the second order terms can be transformed by virtue of partial inte-

grations. The representation as Lagrangian density is kept for the sake of simplicity:

$$\mathcal{L}_G^{Q^2} = \frac{1}{2} Q^{b\mu} \{ D_\lambda^{ba}[A] D^{ac\lambda}[A] g_{\mu\nu} - D_\mu^{ba}[A] D_\nu^{ac}[A] \} Q^{c\nu}, \quad (10)$$

$$\mathcal{L}_{GF}^{Q^2} = \frac{1}{2} Q^{b\mu} D_\mu^{ba}[A] D_\nu^{ac}[A] Q^{c\nu}. \quad (11)$$

Finally, the differential operator for the equation of motion for the gluonic Green's functions can be read off by omitting the quantum fields. Thus, the equation of motion for the Green's function $\Gamma_{\mu\nu}$ reads

$$\Gamma^{-1}(x) \Gamma_{\mu\nu}(x, y) = \delta^{(4)}(x - y) g_{\mu\nu}, \quad (12)$$

with the differential operator:²

$$\Gamma^{-1} = D[A] \cdot D[A]. \quad (13)$$

Further, the contribution to the Lagrangian density originating from the Faddeev-Popov ghosts in background-field Lorenz gauge is given by

$$\mathcal{L}_{FP} = -(D_\mu^{ab}[A] \chi^{b\dagger})(D_\mu^{ac}[A + Q] \chi^c). \quad (14)$$

Partial integration yields

$$\mathcal{L}_{FP} = \chi^{b\dagger} D_\mu^{ba}[A] D_\mu^{ac}[A + Q] \chi^c. \quad (15)$$

Therefore, the equation of motion for the ghost propagator Γ in the classical field reads

$$\Gamma^{-1}(x) \Gamma(x, y) = \delta^{(4)}(x - y). \quad (16)$$

Note the connection between the quantum gluon and the ghost propagator in this gauge:

$$\Gamma_{\mu\nu} = \Gamma g_{\mu\nu}, \quad (17)$$

whence they can be used as synonymous to each other for most practical purposes.

III. THE RETARDED GLUON PROPAGATOR

First, look at the homogeneous equation for the gluon (ghost) propagator:

$$\{D[A(x)] \cdot D[A(x)]\} \Gamma_H(x, y) = 0. \quad (18)$$

Explicitly one has

$$\{ \partial_0^2 - 2iA_0 \partial_0 - \Delta + 2i\vec{A} \cdot \vec{\partial} - i(\partial \cdot A) - A \cdot A \}(x) \Gamma_H(x, y) = 0, \quad (19)$$

where the time derivatives have been singled out, because the boundary conditions for the retarded propagator are given for equal time. The differential operator is taken at the space-time point x , which is only denoted once for the sake of brevity. Δ stands for the Laplace operator. In the

¹The terms linear in the quantum field Q_μ do not contribute to the equation of motion, because its expectation value vanishes per definition.

²The covariant derivatives as well as the operator itself are elements of $U(N)$.

term $(\partial \cdot A)$, the derivative does not act across the rounded brackets. The previous equation can be reexpressed as

$$\hat{\Gamma}^{-1}(x)\hat{\Gamma}_H(x, y) = 0, \quad (20)$$

with

$$\hat{\Gamma}^{-1} = \sigma_0 \partial_0 + \bar{\Gamma}^{-1}, \quad (21)$$

where

$$\begin{aligned} \bar{\Gamma}^{-1} = & -i\sigma_0 A_0 - \sigma_+ \\ & + \sigma_- \{-\Delta + 2i\vec{A} \cdot \vec{\partial} - i(\partial \cdot A) - A \cdot A\} + i\sigma_3 A_0, \end{aligned} \quad (22)$$

while

$$\begin{aligned} 2\hat{\Gamma}_H = & \sigma_0(\Gamma_H^{(1)} + \partial_0 \Gamma_H^{(2)}) + 2\sigma_+ \Gamma_H^{(2)} + 2\sigma_- \partial_0 \Gamma_H^{(1)} \\ & + \sigma_3(\Gamma_H^{(1)} - \partial_0 \Gamma_H^{(2)}), \end{aligned} \quad (23)$$

where $\Gamma_H^{(1)}$ and $\Gamma_H^{(2)}$ are two independent solutions of Eq. (18). σ_j with $j \in \{1, 2, 3\}$ are the Pauli matrices and σ_0 the corresponding unit matrix. $\sigma_{\pm}$ are defined as $2\sigma_{\pm} = \sigma_1 \pm i\sigma_2$. Equation (20) is solved by

$$\hat{\Gamma}_H(x, y) = \mathcal{P} \exp \left\{ \int_{y_0}^{x_0} d\xi_0 \bar{\Gamma}^{-1}(\xi_0, \vec{x}) \right\}. \quad (24)$$

Choosing a different boundary condition than $\hat{\Gamma}_H(x, y) = \sigma_0$ at $x_0 = y_0$ would only lead to overall factors to the right of the path-ordered exponential, which, for a homogeneous differential equation, does not lead to independent solutions.

The retarded gluonic propagator is to vanish for negative time differences: $\Gamma(x, y) = 0$ for $x_0 - y_0 < 0$. Further, it is to be continuous at $x_0 = y_0$ and there it is to have a discontinuous first temporal derivative: $\lim_{x_0 \rightarrow y_0 + 0} \partial_0(x) \Gamma(x, y) - \lim_{x_0 \rightarrow y_0 - 0} \partial_0(x) \Gamma(x, y) = \delta^{(3)}(\vec{x} - \vec{y})$. Hence, the retarded propagator can be expressed as

$$\Gamma_R(x, y) = \hat{\Gamma}_H^{(2)}(x, y) \theta(x_0 - y_0) \delta^{(3)}(\vec{x} - \vec{y}). \quad (25)$$

As in [1,3], this solution is fit to be analyzed in various configurations with the help of the general resummation formula (35) and the group property (36). However, here information on the characteristics of the gluonic two-point function is to be obtained by comparing it to quark correlators.

IV. THE RETARDED QUADRATIC FERMION PROPAGATOR

Starting out from the linear Dirac equation

$$\{i\gamma \cdot D[a(x)] - m\}G(x, y) = \delta^{(4)}(x - y), \quad (26)$$

in the presence of the gauge field a in fundamental representation, the quadratic differential equation is obtained through the substitution

$$G(x, y) = \{-i\gamma \cdot D[a(x)] - m\}g(x, y), \quad (27)$$

and one finds

$$\{D[a] \cdot D[a] - \sigma \cdot F/2 + m^2\}g(x, y) = \delta^{(4)}(x - y), \quad (28)$$

where use has been made of the definition of the field tensor (3) and with the commutator of the γ matrices $2\sigma^{\mu\nu} = -i[\gamma^\mu, \gamma^\nu]$.

In analogy to the gluonic case, the homogeneous part of this equation is equivalent to

$$\hat{g}^{-1}(x)\hat{g}_H(x, y) = 0, \quad (29)$$

with

$$\hat{g}^{-1} = \sigma_0 \partial_0 + \bar{g}^{-1}, \quad (30)$$

where

$$\begin{aligned} \bar{g}^{-1} = & -i\sigma_0 a_0 - \sigma_+ + \sigma_- \{-\Delta + 2i\vec{a} \cdot \vec{\partial} - i(\partial \cdot a) \\ & - a \cdot a - \sigma \cdot F[a]/2 + m^2\} + i\sigma_3 a_0, \end{aligned} \quad (31)$$

while

$$\begin{aligned} 2\hat{g}_H = & \sigma_0(g_H^{(1)} + \partial_0 g_H^{(2)}) + 2\sigma_+ g_H^{(2)} + 2\sigma_- \partial_0 g_H^{(1)} \\ & + \sigma_3(g_H^{(1)} - \partial_0 g_H^{(2)}), \end{aligned} \quad (32)$$

where $g_H^{(1)}$ and $g_H^{(2)}$ are two independent homogeneous solutions of Eq. (28). Then Eq. (29) is solved by

$$\hat{g}_H(x, y) = \mathcal{P} \exp \left\{ \int_{y_0}^{x_0} d\xi_0 \bar{g}^{-1}(\xi_0, \vec{x}) \right\}. \quad (33)$$

The boundary conditions in the fermionic case are the same as in the bosonic, i.e., $g(x, y) = 0$ for $x_0 - y_0 < 0$, $\lim_{x_0 \rightarrow y_0 + 0} g(x, y) = \lim_{x_0 \rightarrow y_0 - 0} g(x, y)$, and $\lim_{x_0 \rightarrow y_0 + 0} \partial_0(x) g(x, y) - \lim_{x_0 \rightarrow y_0 - 0} \partial_0(x) g(x, y) = \delta^{(3)}(\vec{x} - \vec{y})$, whence the retarded propagator can be expressed as

$$\hat{g}_R(x, y) = \hat{g}_H^{(2)}(x, y) \theta(x_0 - y_0) \delta^{(3)}(\vec{x} - \vec{y}). \quad (34)$$

V. INTERRELATION BETWEEN THE PROPAGATORS

With the help of the general resummation formula derived in [3],

$$\begin{aligned}
\mathcal{P} \exp \left\{ \int_{y_0}^{x_0} d\xi_0 [B(\xi_0) + C(\xi_0)] \right\} &= \mathcal{P} \exp \left\{ \int_{y_0}^{x_0} d\xi_0 B(\xi_0) \right\} \\
&\times \mathcal{P} \exp \left[\int_{y_0}^{x_0} d\xi_0 \mathcal{P} \exp \left\{ \int_{\xi_0}^{y_0} dz_0 B(z_0) \right\} C(\xi_0) \mathcal{P} \exp \left\{ \int_{y_0}^{\xi_0} dz_0 B(z_0) \right\} \right] \\
&= \mathcal{P} \exp \left[\int_{y_0}^{x_0} d\xi_0 \mathcal{P} \exp \left\{ \int_{\xi_0}^{x_0} dz_0 B(z_0) \right\} C(\xi_0) \mathcal{P} \exp \left\{ \int_{x_0}^{\xi_0} dz_0 B(z_0) \right\} \right] \\
&\times \mathcal{P} \exp \left\{ \int_{y_0}^{x_0} d\xi_0 B(\xi_0) \right\},
\end{aligned} \tag{35}$$

which is based on the group property valid for path-ordered exponentials,

$$\mathcal{P} \exp \left\{ \int_{y_0}^{x_0} d\xi_0 B(\xi_0) \right\} = \mathcal{P} \exp \left\{ \int_{z_0}^{x_0} d\xi_0 B(\xi_0) \right\} \times \mathcal{P} \exp \left\{ \int_{y_0}^{z_0} d\xi_0 B(\xi_0) \right\}, \tag{36}$$

the bosonic propagator can be identified in the fermionic. This is achieved by making the choice $C = c = \sigma_- \{-\sigma F[a]/2 + m^2\}$. The homogeneous solution of the quadratic Dirac equation can then be reexpressed as

$$\hat{g}_H(x_0, y_0; \vec{x}) = \mathcal{P} \exp \left\{ + \int_{y_0}^{x_0} d\xi_0 \hat{\Gamma}'_H(x_0, \xi_0; \vec{x}) c(\xi_0, \vec{x}) \hat{\Gamma}'_H(\xi_0, x_0; \vec{x}) \right\} \hat{\Gamma}'_H(x_0, y_0; \vec{x}). \tag{37}$$

The $\vec{y}$ in all the arguments has been left out because, due to the $\delta^{(3)}(\vec{x} - \vec{y})$ present in the propagator, it is always taken equal to $\vec{x}$ in the end. The bosonic function has to be taken in the fundamental representation. This supersymmetric replacement has been indicated by the prime.

Ultimately, the inverse relation is required, whence the task of identifying the fermionic propagator in the bosonic arises. To this end, start out with the bosonic homogeneous solution (24). First multiply with unity in the space of the γ matrices everywhere. The inverse operation is taking one-quarter of the trace over the elements of the Clifford algebra, which has not been denoted separately for the sake of brevity. Afterwards, the application of the resummation formula (35) with $C = -c$ results in

$$\hat{\Gamma}_H(x_0, y_0; \vec{x}) = \mathcal{P} \exp \left\{ - \int_{y_0}^{x_0} d\xi_0 \hat{g}'_H(x_0, \xi_0; \vec{x}) c'(\xi_0, \vec{x}) \hat{g}'_H(\xi_0, x_0; \vec{x}) \right\} \hat{g}'_H(x_0, y_0; \vec{x}). \tag{38}$$

The primes indicate that the fermionic functions are constructed in the adjoint representation. The inserted matrix c' can be split into the contributions from the mass and the field tensor, respectively. Resumming all terms involving the mass corresponds to replacing every massive quark function $\hat{g}'_H$ by a massless one $\hat{g}'_h$:

$$\hat{\Gamma}_H(x_0, y_0; \vec{x}) = \mathcal{P} \exp \left\{ \frac{1}{2} \int_{y_0}^{x_0} d\xi_0 \hat{g}'_h(x_0, \xi_0; \vec{x}) \sigma F[A(\xi_0, \vec{x})] \sigma_- \hat{g}'_h(\xi_0, x_0; \vec{x}) \right\} \hat{g}'_h(x_0, y_0; \vec{x}). \tag{39}$$

Again this expression is fit to be studied with the help of Eqs. (35) and (36) along the lines of Refs. [1,3]. Here we shall concentrate on an expansion in the insertions of the field tensor. To lowest order, this yields the following for the relevant σ_+ component:

$$\hat{\Gamma}_H^{[0](2)}(x_0, y_0; \vec{x}) = \hat{g}_h^{(2)}(x_0, y_0; \vec{x}). \tag{40}$$

The corrections arising in higher orders of the expansion include further couplings to the field tensor. The σ_+ component of the first order reads

$$\hat{\Gamma}_H^{[1](2)}(x_0, y_0; \vec{x}) = \frac{1}{2} \int_{y_0}^{x_0} d\xi_0 \hat{g}_h^{(2)}(x_0, \xi_0; \vec{x}) \sigma F[A(\xi_0, \vec{x})] \hat{g}_h^{(2)}(\xi_0, y_0; \vec{x}), \tag{41}$$

where use has been made of the relations $\sigma_\pm \sigma_\mp \sigma_\pm = \sigma_\pm$ and $(\sigma_\pm)^2 = 0$. In fact, to all orders only the σ_+ component of $\hat{g}'_h$ contributes to $\hat{\Gamma}_H^{(2)}$. Multiplying the previous two equations with $\theta(x_0 - y_0) \delta^{(3)}(\vec{x} - \vec{y})$ in order to obtain the contributions to the retarded propagator (25) leads to

$$\Gamma_R^{[0]}(x, y) = g'_r(x, y) \tag{42}$$

and

$$\Gamma_R^{[1]}(x, y) = \frac{1}{2} \int d^4\xi g'_r(x, \xi) \sigma : F[A(\xi)] g'_r(\xi, y), \quad (43)$$

where $\theta(x_0 - y_0) = \theta(x_0 - \xi_0)\theta(\xi_0 - x_0)$ for $\xi_0 \in [x_0, y_0]$ and afterwards the definition of the massless retarded quadratic fermion propagator g_r in accordance with Eq. (34) have been exploited. In vacuum configurations of the gauge field, Eq. (42) becomes exact and the two propagators are identical. Last but not least, the retarded quadratic fermion propagator g_R can be constructed from the retarded propagator G_R for the linear Dirac equation. Here the massless case is needed:

$$g_r(x, y) = - \int d^4z G_r(x, z) G_r(z, y), \quad (44)$$

which provides the link to the results presented in Refs. [1–3]. The minus sign originates from the sign of the covariant derivative in Eq. (27).

VI. SEMICLASSICAL ULTRARELATIVISTIC HEAVY-ION COLLISIONS

The semiclassical description of an ultrarelativistic heavy-ion collision is usually begun with a current of color charges moving along the light cone [13]. In order to obtain the classical gauge field, the Yang-Mills equations have to be solved in the presence of this current. In Lorenz gauge the field consists of two different contributions. One of them is the radiation field inside the forward light cone. The other is made up of the Weizsäcker-Williams (WW) sheets $A_\pm^{\text{WW}}$. These correspond to the Coulomb fields of the different color charges boosted into the closure of the Lorentz group. Therewith they become δ distributions in the light-cone coordinates. This approximation is justified for the description of particles with low longitudinal momentum (midrapidity). On the one hand, they have a low resolution in this direction. On the other, they cannot be comovers of the charges on the light cone. For comoving particles the plane-wave expansion would not be applicable [5].

In the Lorenz gauge for the classical field, the WW contribution to the gauge field takes the form

$$\begin{aligned} A_+^{\text{WW}}(x) &= -\frac{g}{2\pi} \sum_{n_L=1}^{N_L} t_a (t_a^L)_{n_L} \delta[x_- - (b_-^L)_{n_L}] \\ &\quad \times \ln \lambda |\vec{x}_T - (\vec{b}_T^L)_{n_L}|, \\ A_-^{\text{WW}}(x) &= -\frac{g}{2\pi} \sum_{n_R=1}^{N_R} t_a (t_a^R)_{n_R} \delta[x_+ - (b_+^R)_{n_R}] \\ &\quad \times \ln \lambda |\vec{x}_T - (\vec{b}_T^R)_{n_R}|, \\ \vec{A}_T^{\text{WW}}(x) &= 0. \end{aligned} \quad (45)$$

For any four-vector v , one defines the light-cone components $\sqrt{2}v_\pm = v_0 \pm v_3$. $\vec{v}_T$ stands for the two-vector of its transverse components. λ is an arbitrary constant, regularizing the logarithm, which does not appear in quantities like the field tensor. The t_a are the generators of $\text{SU}(3)_c$. The $(t_a^{L,R})_n$ represent the color of the charges.

In general, the presence of a second nucleus leads to a precession of the charges of the first nucleus and vice versa, which is a manifestation of the covariant conservation of the current. This causes further sheetlike contributions to the gauge field which can be combined with the WW fields by modifying the charges. Up to the next-to-leading order in perturbation theory, they read [13]

$$\begin{aligned} (t_a^L)_{n_L} &= (t_a^L)_{n_L}^{\text{in}} - \alpha_S \sum_{n_R=1}^{N_R} f_{abc} (t_b^L)_{n_L}^{\text{in}} (t_c^R)_{n_R}^{\text{in}} \theta[x_+ - (b_+^R)_{n_R}] \\ &\quad \times \ln \lambda |\vec{x}_T - (\vec{b}_T^R)_{n_R}| + \mathcal{O}(\alpha_S^2), \\ (t_a^R)_{n_R} &= (t_a^R)_{n_R}^{\text{in}} + \alpha_S \sum_{n_L=1}^{N_L} f_{abc} (t_b^L)_{n_L}^{\text{in}} (t_c^R)_{n_R}^{\text{in}} \theta[x_- - (b_-^L)_{n_L}] \\ &\quad \times \ln \lambda |\vec{x}_T - (\vec{b}_T^L)_{n_L}| + \mathcal{O}(\alpha_S^2), \end{aligned} \quad (46)$$

with the initial colors $(t_a^{L,R})_{n_{L,R}}^{\text{in}}$. To all orders, the modifications amount to Wilson lines over the gauge field along the branches of the light cone. Hence, the precession terms are absent in adequate, i.e., light-cone gauges. Color neutrality requires $\sum_{n_L=1}^{N_L} t_a (t_a^L)_{n_L}^{\text{in}} = 0 = \sum_{n_R=1}^{N_R} t_a (t_a^R)_{n_R}^{\text{in}}$. Higher-order perturbative calculations for the gauge field as a solution of the Yang-Mills equations are only of value in the presence of hard energy scales or short time scales. Up to now, nonperturbative solutions for this problem have been obtained only in transverse lattice calculations [14]. In any case, the general form of the field—continuous inside the forward light cone and singular but integrable on the forward light cone—allows one to express the retarded propagator by a finite number of addends.

The final result for the massless retarded fermion propagator G_r for the linear Dirac equation in the ultrarelativistic limit³ reads [1]

$$iG_r(x, y) \gamma^0 = +G_h(x, y) \delta^{(3)}(\vec{x} - \vec{y}) \theta(x_0 - y_0), \quad (47)$$

with the homogeneous solution of the Dirac equation:

³In the ultrarelativistic limit, the “longitudinal impact parameters” $(b_\pm^{R,L})_n$ are taken to be zero because of the longitudinal contraction with the Lorentz factor $\gamma^{-1} \rightarrow 0$. While the correct limit of the propagator is thus obtained, subsequent applications of the result might be lacking terms because the order of the limits cannot be exchanged (see, for example, the discussion in [1]).

$$\begin{aligned}
G_h(x, y) = & G_h^{\text{rad}}(x, y) + \int d^4\xi G_h^{\text{rad}}(x, \xi) [T^L(\xi)\delta(\xi_-) + T^R(\xi)\delta(\xi_+)] G_h^{\text{rad}}(\xi, y) \\
& + \int d^4\xi d^4\eta G_h^{\text{rad}}(x, \xi) T^L(\xi)\delta(\xi_-) G_h^{\text{rad}}(\xi, \eta) T^R(\eta)\delta(\eta_+) G_h^{\text{rad}}(\eta, y) \\
& + \int d^4\xi d^4\eta G_h^{\text{rad}}(x, \xi) T^R(\xi)\delta(\xi_+) G_h^{\text{rad}}(\xi, \eta) T^L(\eta)\delta(\eta_-) G_h^{\text{rad}}(\eta, y),
\end{aligned} \tag{48}$$

where for an even number of scattering centers in each projectile the scattering matrices are given by

$$\begin{aligned}
T^L(x) + \rho^+ &= \rho^+ \mathcal{P} \prod_{n_L=1}^{N_L} \exp[-i\alpha_{n_L}^L(\vec{x}_T)], \\
T^R(x) + \rho^- &= \rho^- \mathcal{P} \prod_{n_R=1}^{N_R} \exp[-i\alpha_{n_R}^R(\vec{x}_T)],
\end{aligned} \tag{49}$$

with

$$2\pi\alpha_{n_{L,R}}^{L,R} = -gt_a(t_a^{L,R})_{n_{L,R}} \ln\lambda|\vec{x}_T - (\vec{b}_T^{L,R})_{n_{L,R}}|, \tag{50}$$

while the homogeneous solution in the radiation field is given by

$$G_h^{\text{rad}}(x, y) = \mathcal{P} \exp \left\{ i \int_{y_0}^{x_0} d\xi_0 \gamma^0 [i\gamma^j \partial_j(x) + \gamma \cdot A^{\text{rad}}(\xi_0, \vec{x})] \right\}. \tag{51}$$

$2\rho^\pm = 1 \pm \gamma^0 \gamma^3$ are two projectors, whence $(\rho^\pm)^2 = \rho^\pm$ and $\rho^\pm \rho^\mp = 0$. Higher orders in the T cannot contribute to Eq. (48), because lines of constant x_- or x_+ can only be crossed once [1,5].

This remains also true for the convolution of two retarded propagators in Eq. (44). There are at most contributions with one coupling to each scattering matrix in $g_r(x, y)$.

As already mentioned above, in nucleus-nucleus collisions the radiation field A^{rad} is only known numerically up to now [14]. This problem can be circumvented in proton-nucleus collisions by regarding the charge density of the proton as small [15]. The Yang-Mills equations can be solved to all orders in the charge density of the nucleus

and to the first order in the charge density of the proton.⁴ The propagator in the radiation field is replaced by its perturbative expansion up to the first order in the thus obtained radiation field. The scattering matrices for said case are obtained from the above expressions by keeping the nuclear scattering matrix $T^L(x)$ as it is and by replacing the one for the proton by the lowest nontrivial order:

$$T_{[1]}^R(x) = -i\rho^- \sum_{n_R=1}^{N_R} \alpha_{n_R}^R(\vec{x}_T). \tag{52}$$

Note that the addend, where the radiation field and the protonic scattering matrix contribute simultaneously, is discarded because it is of second order in the charge density of the nucleon.

In the case where a large momentum scale is assigned to the propagated particle, be it in nucleus-nucleus or in proton-nucleus collisions, the equations simplify further [6,13]. Also the other scattering matrix in Eq. (49) is reduced to its lowest order:

$$T_{[1]}^L(x) = -i\rho^+ \sum_{n_L=1}^{N_L} \alpha_{n_L}^L(\vec{x}_T). \tag{53}$$

The Yang-Mills equations are only solved perturbatively to the first order in each of the charge densities to yield the radiation field $A_{[1]}^{\text{rad}}$. The propagator in the radiation field is expanded to the first order in the field. At the end, only couplings to the sheetlike contributions *or* to the radiation field are kept. Taking them into account simultaneously leads to higher-order terms. Thus, the retarded linear Dirac propagator to this order is given by Eq. (47) with

$$\begin{aligned}
G_h^{\text{II}}(x, y) = & \underline{G_h^0(x-y)} + \int d^4\xi G_h^0(x-\xi) [\underline{T_{[1]}^L(\xi)\delta(\xi_-)} + \underline{T_{[1]}^R(\xi)\delta(\xi_+)} - i\gamma^0 \gamma \cdot A_{[1]}^{\text{rad}}(\xi)] G_h^0(\xi-y) \\
& + \int d^4\xi d^4\eta G_h^0(x-\xi) T_{[1]}^L(\xi)\delta(\xi_-) G_h^0(\xi-\eta) T_{[1]}^R(\eta)\delta(\eta_+) G_h^0(\eta-y) \\
& + \int d^4\xi d^4\eta G_h^0(x-\xi) T_{[1]}^R(\xi)\delta(\xi_+) G_h^0(\xi-\eta) T_{[1]}^L(\eta)\delta(\eta_-) G_h^0(\eta-y).
\end{aligned} \tag{54}$$

$G_h^0(x-y)$ stands for the free homogeneous solution of the linear Dirac equation without mass. Putting this into Eq. (44) and subsequently into Eq. (42) yields

⁴In [16] the “expansion parameter” is not the proton’s charge density but the order of hatched commutators between the WW fields of the two projectiles.

$$\begin{aligned}
 g_r^{\Pi}(x, y) = & -g_r^0(x - y) - \int d^4 z G_r^0(x - z) G_r^{\Pi}(z, y) - \int d^4 z G_r^{\Pi}(x, z) G_r^0(z - y) \\
 & + \int d^4 \xi d^4 \eta G_r^0(x - \xi) T_{[1]}^L(\xi) \delta(\xi_-) g_r^0(\xi - \eta) T_{[1]}^R(\eta) \delta(\eta_+) G_r^0(\eta - y) \\
 & + \int d^4 \xi d^4 \eta G_r^0(x - \xi) T_{[1]}^R(\xi) \delta(\xi_-) g_r^0(\xi - \eta) T_{[1]}^L(\eta) \delta(\eta_+) G_r^0(\eta - y),
 \end{aligned} \quad (55)$$

with

$$g_r^0(x, y) = - \int d^4 z G_r^0(x - z) G_r^0(z - y). \quad (56)$$

For the use with Eq. (43), the addends which are not underlined will not contribute at all. In connection with the radiated field tensor merely the free propagator in Eq. (54) is needed. Hence, one finds the following, still with some higher-order terms to be removed in the first addend:

$$\begin{aligned}
 \Gamma_R^{[1]}(x, y) = & \frac{1}{2} \int d^4 \xi g_r^0(x - \xi) \sigma : F[A^{\text{WW}}(\xi)] g_r^0(\xi - y) \\
 & + \frac{1}{2} \int d^4 \xi \underline{g}_r'(x, \xi) \sigma : F[A^{\text{WW}}(\xi)] g_r^0(\xi - y) \\
 & + \frac{1}{2} \int d^4 \xi g_r^0(x - \xi) \sigma : F[A^{\text{WW}}(\xi)] \underline{g}_r'(\xi, y) \\
 & + \frac{1}{2} \int d^4 \xi g_r^0(x - \xi) \sigma : F[A_{[1]}^{\text{rad}}(\xi)] g_r^0(\xi - y),
 \end{aligned} \quad (57)$$

with

$$\begin{aligned}
 \underline{g}_r(x, y) = & + \int d^4 \xi g_r^0(x - \xi) [T_{[1]}^L(\xi) \delta(\xi_-) \\
 & + T_{[1]}^R(\xi) \delta(\xi_+)] G_r^0(\xi - y) + \int d^4 \xi G_r^0(x - \xi) \\
 & \times [T_{[1]}^L(\xi) \delta(\xi_-) + T_{[1]}^R(\xi) \delta(\xi_+)] g_r^0(\xi - y).
 \end{aligned} \quad (58)$$

To this accuracy, the last contribution comes from the second order term of Eq. (39):

$$\begin{aligned}
 \Gamma_R^{[2]}(x, y) = & \frac{1}{4} \int d^4 \xi d^4 \eta g_r^0(x - \xi) \sigma : F[A_+^{\text{WW}}(\xi)] \\
 & \times g_r^0(\xi - \eta) \sigma : F[A_-^{\text{WW}}(\eta)] g_r^0(\eta - y) \\
 & + \frac{1}{4} \int d^4 \xi d^4 \eta g_r^0(x - \xi) \sigma : F[A_-^{\text{WW}}(\xi)] \\
 & \times g_r^0(\xi - \eta) \sigma : F[A_+^{\text{WW}}(\eta)] g_r^0(\eta - y),
 \end{aligned} \quad (59)$$

where only the field tensors as functionals of the sheetlike configurations (45) [13]

$$\begin{aligned}
 F_{+T}[A_+^{\text{WW}}] = & \frac{g}{2\pi} \sum_{n_L=1}^{N_L} t_a(t_a^L)_{n_L} \delta[x_- - (b_-^L)_{n_L}] \\
 & \times \frac{\vec{x}_T - (\vec{b}_T^L)_{n_L}}{|\vec{x}_T - (\vec{b}_T^L)_{n_L}|^2}
 \end{aligned} \quad (60)$$

and

$$\begin{aligned}
 F_{-T}[A_-^{\text{WW}}] = & \frac{g}{2\pi} \sum_{n_R=1}^{N_R} t_a(t_a^R)_{n_R} \delta[x_+ - (b_+^R)_{n_R}] \\
 & \times \frac{\vec{x}_T - (\vec{b}_T^R)_{n_R}}{|\vec{x}_T - (\vec{b}_T^R)_{n_R}|^2},
 \end{aligned} \quad (61)$$

and all other components equal to zero, contribute.

Here, as at most two field insertions of any kind per addend are allowed, the corresponding Feynman propagator can be obtained by replacing the free retarded propagators by free Feynman propagators.

Results invariant with respect to gauge transformations of the classical field A_μ can be obtained from an operator in different ways, for example, by multiplying with adequate gauge links or by averaging over all these gauges [1]. The MV-like models fall back on averaging over an ensemble of charge distributions with a gauge-invariant weight [12]. This averaging can be carried out equivalently over the WW fields [17].

In the case of a gluon propagating at a large momentum scale, the radiated field is a functional linear in A_+^{WW} and A_-^{WW} . Thus, the same holds for the gluon propagator in this situation. Because of the color neutrality of each projectile, the average over the charge densities or equivalently the WW fields is to vanish $\langle A_\pm^{\text{WW}} \rangle = 0$. Therefore, the average propagator to this order coincides with the free propagator. As the free gluonic propagator is intrinsically gauge dependent, it is evident that this method does not lead to a result invariant with respect to arbitrary transformations of the total gauge field $\check{A}_\mu$. The deviations from the free case start at the next order or if the average over an operator is taken that contains higher powers of the propagator.

Along the same lines, the propagator in the proton-nucleus case is, per definition of the approximation, a functional linear in the charge density or the WW field of the proton. This does not change if terms beyond Eq. (43) are taken into account. Hence, after taking the average over the proton imposing color neutrality, only the average over the propagator in the field of the nucleus is left. As before,

deviations originating from the presence of the nucleon only start playing a role in the following orders or in averages over powers of the propagator.

Starting out from purely classical (MV) initial conditions for the description of an ultrarelativistic collision, one observes that the emission of semihard quantum gluons is enhanced by the kinematical factor $\ln(1/x)$, where x is the longitudinal momentum fraction of the emitted particle. This enhancement spoils the construction of a perturbation series in powers of the strong coupling constant α_s , because $\alpha_s \ln(1/x)$ is no longer a small parameter. The aforementioned JIMWLK equations serve to absorb these into the charge distributions leading to an equivalent classical description of that part of the system. The new perturbative series above this background has been freed of this problem [18] unless $\alpha_s^2 \ln(1/x)$ becomes of order 1.

VII. SUMMARY

Solutions for the retarded gluonic propagator in background-field Feynman gauge and the retarded propagator for the quadratic Dirac equation have been derived to all orders in the classical field. Imposing the corresponding boundary conditions has singled out the retarded correlators. Subsequently, relations expressing each of these propagators as a functional of the other have been determined. The one constructing the bosonic two-point function from the fermionic has been appended with another one, linking the quadratic fermion propagator with the linear one. With the help of said equations, the results for Green's functions of the linear Dirac operator from Refs. [1–3] can be and have been transferred to those of the quadratic and the bosonic operator.

This fact has been exploited for the case of ultrarelativistic collisions. For nucleus-nucleus collisions all pieces of information, which do not require numerical calculations for the radiated field, have been exposed. For nucleon-nucleus collisions, one can go further, because they allow for an expansion of the Yang-Mills equations in the charge density of the proton. The building blocks for the description of this system have been provided.

As a third system, the propagation of a gluon with a large momentum scale is investigated. In this case the Yang-Mills equations can be expanded in the densities of both projectiles. All possible terms up to first order in any of the charge densities have been derived. In this case, also the Feynman propagator can be obtained. At this order, averaging the propagators over the charge distributions along the line of the MV model gives the free propagator. Deviations arise when averaging over operators containing powers of the propagator at this order or over the propagator at a higher order. In the case of a proton-nucleus collision, the averaging procedure leads to the average over the propagator in the field of the nucleus.

It will be interesting to apply the presented correlators to describe phenomena like vacuum polarization and quantities like induced currents or condensates in the gluonic case. Such investigations will be presented elsewhere.

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