

Interpreting doubly special relativity as a modified theory of measurementStefano Liberati,^{1,*} Sebastiano Sonego,^{2,†} and Matt Visser^{3,‡}¹*International School for Advanced Studies, Via Beirut 2-4, 34014 Trieste, Italy and INFN, Trieste, Italy*²*Università di Udine, Via delle Scienze 208, 33100 Udine, Italy*³*School of Mathematics, Statistics, and Computer Science, Victoria University of Wellington, P.O. Box 600, Wellington, New Zealand*

(Received 26 October 2004; published 1 February 2005)

In this article we develop a physical interpretation for the deformed (doubly) special relativity theories (DSRs), based on a modification of the theory of measurement in special relativity. We suggest that it is useful to regard the DSRs as reflecting the manner in which quantum gravity effects induce Planck-suppressed distortions in the measurement of the “true” energy and momentum. This interpretation provides a framework for the DSRs that is manifestly consistent, nontrivial, and in principle falsifiable. However, it does so at the cost of demoting such theories from the level of fundamental physics to the level of phenomenological models—models that should in principle be derivable from whatever theory of quantum gravity one ultimately chooses to adopt.

DOI: 10.1103/PhysRevD.71.045001

PACS numbers: 03.30.+p, 04.60.-m, 11.30.Cp

I. INTRODUCTION

The search for observable effects of quantum gravity has been one of the driving trends in recent years. Several results in the context of string theory [1], loop quantum gravity [2] and other candidate models [3] for quantum gravity have led researchers to focus mostly on modifications of the dispersion relations for elementary particles, leading to deviations from standard Lorentz invariance. Generically these modified dispersion relations can be cast in the form¹

$$E^2 = p^2 + m^2 + f(E, p; \kappa), \quad (1.1)$$

where κ denotes the mass scale at which the quantum gravity corrections become appreciable. Normally, one assumes that κ is of order the Planck mass: $\kappa \sim M_P \approx 1.22 \times 10^{19}$ GeV. Most interestingly, it was shown that several significant constraints can be put on the intensity of the Lorentz violating term $f(E, p; \kappa)$ using current experiments and observations [4].

An open issue is the interpretation of the origin of such deformed dispersion relations. There is an extensive literature in which an explicit breakdown of Lorentz invariance has been considered (see [5] and references therein). However, some authors have tried to see if a more conservative generalization of the Lorentz transformations could be found, in order to save the equivalence of inertial frames in special relativity [6–8]. In particular the DSRs (deformed or doubly special relativity theories) attempt to “deform” special relativity in momentum space, by introducing nonstandard “Lorentz transformations” that leave the modified dispersion relations above invariant.

Unfortunately, Lorentz invariance provides an extremely strong and rigid framework for particle physics, and while it is relatively easy to “break” Lorentz invariance, it is much more difficult to deform it without “breaking” it. This has led to considerable debate concerning the physical status of DSRs, with a strong minority of authors arguing for either the triviality [9] or internal inconsistency [10,11] of such theories.

In this paper we further investigate the DSR framework and propose an alternative interpretation that we think is both logically consistent and nontrivial. After presenting, in the next section, a very concise review of the DSR proposal and its open problems we shall focus, in Sec. III, on the momentum space DSR transformations and their mathematical meaning. This will lead us to suggest, in Sec. IV, a physical interpretation of DSR as a new theory of measurement which could stem from quantum gravity effects. In Sec. V we shall discuss how this new framework could be used to solve some of the problems pointed out by previous authors concerning the DSR proposal. Finally, Sec. VI contains a summary of the main ideas presented in the paper.

II. THE DSR FRAMEWORK

In this section we briefly outline the DSR framework and its open issues. This review is in no sense complete as more in-depth discussions are now available in several published articles (see, e.g., [12] and references therein).

A. Deformed Lorentz algebra

Consider the Lorentz algebra of the generators of rotations, L_i , and boosts, B_i :

$$\begin{aligned} [L_i, L_j] &= i\epsilon_{ijk}L_k, & [L_i, B_j] &= i\epsilon_{ijk}B_k, \\ [B_i, B_j] &= -i\epsilon_{ijk}L_k \end{aligned} \quad (2.1)$$

(Latin indices $i, j, \dots$ run from 1 to 3). Supplement this

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¹We work in units with $c = 1$.

with the following commutators between the Lorentz generators and those of translations in spacetime (the momentum operators P_0 and P_i):

$$[L_i, P_0] = 0; \quad [L_i, P_j] = i\epsilon_{ijk}P_k; \quad (2.2)$$

$$[B_i, P_0] = if_1\left(\frac{P}{\kappa}\right)P_i; \quad (2.3)$$

$$[B_i, P_j] = i\left[\delta_{ij}f_2\left(\frac{P}{\kappa}\right)P_0 + f_3\left(\frac{P}{\kappa}\right)\frac{P_iP_j}{\kappa}\right]. \quad (2.4)$$

Finally, assume

$$[P_i, P_j] = 0. \quad (2.5)$$

The commutation relations (2.3) and (2.4) are given in terms of three unspecified, dimensionless structure functions f_1 , f_2 , and f_3 , and are sufficiently general to include all known DSR proposals—the DSR1 [6], DSR2 [7], and DSR3 [8]. Furthermore, in all the DSRs considered to date, the dimensionless arguments of these functions are specialized to

$$f_i\left(\frac{P}{\kappa}\right) \rightarrow f_i\left(\frac{P_0}{\kappa}, \frac{\sum_{i=1}^3 P_i^2}{\kappa^2}\right), \quad (2.6)$$

so that rotational symmetry is completely unaffected. In order that the $\kappa \rightarrow +\infty$ limit reduce to ordinary special relativity we demand that, in that limit, f_1 and f_2 tend to 1, and that f_3 tend to some finite value.

B. A note on terminology

The “internal” commutation relations (2.1) and (2.2), among the boosts and rotations are not altered in any way—so the Lorentz subgroup is not changed at all. This underlies the claim that Lorentz invariance is not “broken” in these theories. On the other hand, the DSR group acts on the momenta in a nontrivial manner—and if we choose to label “states” by the real eigenvalues² p_μ of the momentum operators we see that the DSR group acts nontrivially on states even if it possesses the same number of symmetry generators as the Lorentz group. This leads to the nomenclature of a “deformed” Lorentz invariance.

On this terminology we feel that a brief comment is in order. In fact adopting the above DSR conventions would, if carried to their logical conclusion, also force one to declare that “spontaneous symmetry breaking” never breaks any symmetry—simply on the grounds that in spontaneous symmetry breaking the symmetry group is unaffected, while it is only the states (and, in particular, the vacuum) that then transform in a nontrivial way. While DSR does not appear to be an example of spontaneous symmetry breaking (the uncertainty arising from the fact

that we do not have a precise field theoretic description of how to implement the DSR algebra), the basic logic is clear: Based on standard particle physics usage, the Lorentz symmetry in DSR theories would be classed as “broken”, and not “deformed” (although one can usefully disagree about whether the breaking is “soft”, “hard”, “spontaneous”, or “other”).

C. Nonlinear representations of the Lorentz group

In all doubly special relativity theories, there is a claim that the Lorentz group “acts nonlinearly on energy and momentum”. This amounts to the assertion that physical energy and momentum are nonlinear functions of a fictitious pseudomomentum one-form π , whose components transform linearly under the action of the Lorentz group [13]. Indeed such behavior is automatically guaranteed if the realization of the Lorentz group on the energy-momentum space is faithful, i.e., one-to-one [14]. If it were not, then either (i) the same element of the Lorentz group would act in two different ways on energy and momentum, or (ii) two different elements of the Lorentz group would act in the same way. In case (i), one would need extra parameters, in addition to those characterising boosts and rotations, in order to fully specify the transformation. (The Lorentz transformations would then be a subgroup of the full physical transformation group.) The physical meaning of these extra parameters would be, however, totally obscure. The possibility (ii) conflicts with the simple experimental fact that, at low energies, different elements of the Lorentz group are observed to act differently. Thus, if E is the “physical” energy and p_i are the components of “physical” three-momentum, we must have

$$p_\mu = \mathcal{F}_\mu(\pi_0, \pi_1, \pi_2, \pi_3; \kappa), \quad (2.7)$$

where $p_0 \equiv -E$, and the variables π_μ transform linearly under the Lorentz group. For example, in DSR2, the specific DSR model developed by Magueijo and Smolin [7]:

$$E = \frac{-\pi_0}{1 - \pi_0/\kappa}; \quad (2.8)$$

$$p_i = \frac{\pi_i}{1 - \pi_0/\kappa}. \quad (2.9)$$

It is easy to check that while π satisfies the usual dispersion relation $\pi_0^2 - \boldsymbol{\pi}^2 = \mu_0^2$ (μ_0 is the Casimir invariant), E and p_i satisfy a modified relation

$$(1 - \mu_0^2/\kappa^2)E^2 + 2\kappa^{-1}\mu_0^2E - \mathbf{p}^2 = \mu_0^2. \quad (2.10)$$

In fact, as we said, doubly special relativity has been invented precisely in order to provide a theoretical background to anomalous dispersion relations like the one above [6]. For a general theory based on Eq. (2.7), one can write

²Greek indices from the middle of the alphabet, $\mu, \nu, \dots$, run from 0 to 3.

$$\pi_\mu = \mathcal{G}_\mu(E, \mathbf{p}; \kappa), \quad (2.11)$$

with $\mathcal{G} = \mathcal{F}^{-1}$. Then, the modified dispersion relation is

$$\eta^{\mu\nu} \mathcal{G}_\mu(E, \mathbf{p}; \kappa) \mathcal{G}_\nu(E, \mathbf{p}; \kappa) = -\mu_0^2, \quad (2.12)$$

where $\eta_{\mu\nu}$ is the metric tensor of Minkowski spacetime.

D. Open issues in DSR

If DSR is formulated as above—only in momentum space—then as we shall soon see it is an incomplete theory. Moreover, since it is always possible to introduce the new variables π_μ , on which the Lorentz group acts in a linear manner, the only way that DSR can avoid triviality is if there is some physical way of distinguishing the pseudoenergy $\epsilon \equiv -\pi_0$ from the true-energy E , and the pseudomomentum $\boldsymbol{\pi}$ from the true-momentum $\mathbf{p}$ —otherwise DSR would be no more than a nonlinear choice of coordinates on momentum space.

In view of the standard relations $E \leftrightarrow i\hbar\partial_t$, $\mathbf{p} \leftrightarrow -i\hbar\nabla$ (which will presumably be modified in some way in DSR) it is already clear that in order to physically distinguish the pseudoenergy ϵ from the true-energy E , and the pseudomomentum $\boldsymbol{\pi}$ from the true-momentum $\mathbf{p}$, one will need to have some idea of how to relate momenta to position—at a minimum, one will need to develop some notion of DSR-spacetime.

In this endeavor there have been two distinct lines of approach, one presuming commutative spacetime coordinates, the other trying to relate the DSR feature in momentum space to a noncommutative position space. In both cases several authors have pointed out major problems. In the case of commutative spacetime coordinates, some analyses have led authors to question the non-triviality [9] or internal consistency [10,11] of DSR. On the other hand, noncommutative proposals [15] are not yet well understood.

For these reasons we shall first focus on those problems, or ambiguities, which are well understood using purely the momentum space structure of DSR. In particular these include (but this list is not meant to be exhaustive):

- (i) The saturation problem (also known as the “soccer ball problem”): How can macroscopic objects, which experimentally certainly can and do have trans-Planckian total energies, fit into a DSR framework that typically exhibits a maximum energy of order the Planck energy [11]?
- (ii) Definition of particle velocities: How are particle velocities to be defined in DSR? Using phase velocity, group velocity, or something else [16]?
- (iii) Multiplicity problem: Why are there so many different realizations of DSRs?

While the last two problems can be interpreted as ambiguities related to the incompleteness of the present theory, the first issue demonstrates a much more serious problem

with the multiple particle sector of the theory. In dealing with collisions or composite objects it is natural to add linearly the pseudomomenta π_μ and then transform back to the DSR momenta p_μ , so that for N particles one gets

$$p_{\text{tot}} = \mathcal{F}\left(\sum_1^N \mathcal{G}(p_\mu; \kappa); \kappa\right). \quad (2.13)$$

Reduced to the bone, the issue is here related to the fact that the nonlinear transformation $\mathcal{F}$ maps infinity to the Planck scale (energy or momentum, depending on the particular DSR proposal). So it would seem that DSR cannot describe objects with energies (momenta) larger than the Planck scale. This prediction is already very disturbing by itself, but lies also at the origin of other unpleasant consequences of DSRs. For example, the internal energy of a gas in the thermodynamical limit $N \rightarrow +\infty$ is of order κ . More generally, one cannot formulate statistical mechanics and thermodynamics, because the partition function diverges [10].

To date the various solutions proposed for the saturation problem seem as problematic as the paradox itself. For example, it has been proposed [7] to replace (2.13) with

$$p_{\text{tot}} = \mathcal{F}_N\left(\sum_1^N \mathcal{G}(p_\mu; \kappa); \kappa\right). \quad (2.14)$$

As long as we choose $\mathcal{F}_N$ in such a way that it saturates at $N\kappa$ instead of κ , then we can indeed obtain a total energy that is extensive in the number of particles (so there is at least a hope of beginning to set up thermodynamics), and a total energy that can become arbitrarily large (so that we can at least hope to accurately describe at least the kinematics of planets, stars, and galaxies). The canonical choice at this stage is to set $\mathcal{F}_N(\pi; \kappa) = \mathcal{F}(\pi; N\kappa)$. The crucial point here is that the resolution of the paradox is obtained at the very high price of replacing a single DSR algebra with different DSR algebras acting on each N -particle sector of the Fock space. Alternatively, one might claim that it is too early to address the problem because of the lack of a proper field theory (itself due to the lack of a full comprehension of DSR in coordinate space [17]), but somehow this is tantamount to “solving” a problem with another problem.

Given the above open issues of DSR, we here wish to restart by looking at the subject from scratch. In the following we develop an interpretation of the DSRs (in terms of a modification of the theory of measurement in special relativity) which is internally consistent, mathematically and physically nontrivial, and falsifiable—three key tests that any viable physical theory must pass. Thus by adopting this interpretation we can guarantee that we are

asking (and hopefully answering) physically meaningful questions.

III. THE MATHEMATICAL MEANING OF p_μ

We want to start our investigation from what we know for sure as the defining properties of all of the DSR theories so far proposed, i.e., from the relations (2.7). Since the π_μ transform linearly under the action of the Lorentz group, there is no difficulty in identifying them as the components of a one-form in Lorentzian coordinates. But what kind of mathematical objects are the p_μ ? If, by “action of the Lorentz group”, we simply mean a change of Lorentzian coordinates, then the p_μ cannot be scalars (because they are affected by the coordinate transformation), nor can they be tensor components of some kind (because they do not transform linearly). As far as we know, no geometrical object that has yet been defined in the mathematical literature can be used to describe the p_μ . Of course, all this discussion relies heavily on the use of an ordinary space-time manifold, which one might argue is not a legitimate concept in the Planckian regime. However, with no space-time manifold (and hence no notion of tangent vectors, tensors, *etcetera*), the mathematical status of such objects as p_μ becomes even more mysterious.

This point can be further clarified trying to rewrite Eqs. (2.8) and (2.9) in an explicitly covariant form. Since the denominator $1 - \pi_0/\kappa$ that appears in both equations contains the pseudoenergy $-\pi_0$, there are only two ways in which these equations can be interpreted, given that π_0 and π_i are the components of a one-form:

- (1) Suppose that Eqs. (2.8) and (2.9) are valid in every Lorentzian chart. Then we can write

$$p_\mu = \frac{\pi_\mu}{1 - \pi_\nu \delta_0^\nu / \kappa}, \quad (3.1)$$

where δ_ν^μ is the Kronecker symbol. But by doing this one introduces the chart-dependent structure δ_0^μ , which would be regarded as meaningless in ordinary differential geometry.

- (2) In contrast, suppose that Eqs. (2.8) and (2.9) are valid only in one particular class of Lorentzian coordinates. Now we can rewrite them in a covariant form as

$$p_\mu = \frac{\pi_\mu}{1 - \pi_\nu u^\nu / \kappa}, \quad (3.2)$$

where u^μ is a four-vector that, in the preferred class of coordinates, has components $u^0 = 1$ and $u^i = 0$. But while this option is perfectly sound from a mathematical point of view, the use of the preferred vector u^μ unfortunately amounts to introducing a

preferred frame and an explicit breaking of Lorentz invariance, which is in contrast with the whole spirit inspiring DSR theories.³

One way out of this dilemma (and in fact we suspect it is the only mathematically sensible way out of this dilemma) is to reinterpret Eq. (3.2) without assuming that the four-velocity u^μ is a preferred vector in spacetime. Since the motivation for the anomalous dispersion relation (2.10) is ultimately of a phenomenological character, one may interpret E and $\mathbf{p}$ as the energy and three-momentum *measured* by a specific observer with four-velocity u^μ . Then, E and the magnitude of the three-momentum are true scalar quantities, representing the outcomes of measurements performed by one particular specified observer, and so are unaffected by coordinate changes. But in which sense, then, can one say that they “transform nonlinearly under the action of the Lorentz group?”

IV. DSR AS A NEW THEORY OF MEASUREMENT

The reformulation of E and $\mathbf{p}$ in terms of an explicit observer-dependent four-velocity amounts, basically, to changing the theory of measurement in special relativity. We now outline a modified theory of measurement in which DSRs can fit, and present a few speculations about the possible physical origin of the differences with respect to the ordinary theory.

A. “Real” versus “measured” energy-momenta

Let us begin by recalling the important distinction between coordinates (with no direct physical meaning, in general) and a *reference frame*, which is a field of tetrads $\{e_\alpha^\mu | \alpha = 0, 1, 2, 3\}$ such that

$$g_{\mu\nu} e_\alpha^\mu e_\beta^\nu = \eta_{\alpha\beta}, \quad (4.1)$$

where now $g_{\mu\nu}$ is the metric tensor, $\eta_{\alpha\beta} = \eta^{\alpha\beta} = \text{diag}(-1, 1, 1, 1)$, and e_0^μ is a future-directed vector. (Warning: The indices $\mu, \nu, \dots$ are standard tensor indices, associated with a choice of coordinates, while the indices $\alpha, \beta, \dots$ only label different vectors in the tetrad, and have nothing to do with any particular chart adopted.) Two reference frames e_α^μ and $\bar{e}_\alpha^\mu$ are related by a Lorentz matrix Λ_α^β , so

$$\bar{e}_\alpha^\mu = \Lambda_\alpha^\beta e_\beta^\mu. \quad (4.2)$$

The use of a reference frame is crucial in order to extract, from the abstract tensors of any relativistic theory, scalar quantities that could be interpreted as measurement

³It is interesting to note that this case is *not equivalent* to the ansatz considered in most of the papers on quantum gravity phenomenology [4]. In these works energy and momentum are characterized by modified dispersion relations like (1.1), but they compose in the standard way (like π_μ does in our framework). In this sense they appear as hybrid models from the point of view of the situation envisaged by Eq. (3.2).

outcomes. In particular, in the usual theory of measurement [18], if a particle has four-momentum π_μ , its energy and i -th component of three-momentum, measured in the reference frame $\{e_\alpha^\mu\}$, are given by the expressions:

$$\epsilon(\pi; e) = -\pi_\mu e_0^\mu; \quad (4.3)$$

$$\pi_i(\pi; e) = \pi_\mu e_i^\mu. \quad (4.4)$$

Equations (4.3) and (4.4) can be summarized into the single relation

$$\pi_\alpha(\pi; e) = \pi_\mu e_\alpha^\mu, \quad (4.5)$$

under the identification $\pi_0 \equiv -\epsilon$.

It is important to realize that, while the π_μ are components of a one-form in some chart, the π_α are *scalars*—i.e., they are a set of four chart-independent numbers. However, they depend on the reference frame adopted. In a new frame $\bar{e}_\alpha^\mu$, from the same one-form π_μ one obtains four different scalars $\bar{\pi}_\alpha$, related to the π_α as

$$\bar{\pi}_\alpha = \pi_\mu \bar{e}_\alpha^\mu = \pi_\mu \Lambda_\alpha^\beta e_\beta^\mu = \Lambda_\alpha^\beta \pi_\beta, \quad (4.6)$$

which is a usual linear Lorentz transformation. In particular, the $\bar{\pi}_\alpha$ are linear functions of the π_α .

In a DSR context we now suggest reformulating Eq. (2.7) as

$$\begin{aligned} p_\alpha &= \mathcal{F}_\alpha(\pi_\mu e_0^\mu, \pi_\mu e_1^\mu, \pi_\mu e_2^\mu, \pi_\mu e_3^\mu; \kappa) \\ &= \mathcal{F}_\alpha(\pi_\mu e_\beta^\mu; \kappa), \end{aligned} \quad (4.7)$$

so that the “physically measured” energy and momentum, E and p_i , become nonlinear functions of both the underlying one-form π_μ and the reference frame specified by the tetrad e_α^μ . In particular, Eqs. (2.8) and (2.9) proposed by Magueijo and Smolin [7] are to be rewritten as

$$p_\alpha = \frac{\pi_\mu e_\alpha^\mu}{1 - \pi_\mu e_0^\mu / \kappa}, \quad (4.8)$$

with $E = -p_0$, as usual. If the $\mathcal{F}_\alpha$ are nonlinear, then upon performing a Lorentz transformation the $\bar{p}_\alpha$ are also nonlinear functions of the p_α . This defines a mathematically precise and physically consistent sense in which the theory is simultaneously Lorentz-invariant (and covariant), while the physical (measured) energy and momentum do not transform linearly under a change of reference frame.

In summary, our proposal is that the one-form π_μ be interpreted as the “real” energy-momentum, and the four scalars p_α as measured energy-momenta. The transformation from one to the other additionally depends on the reference frame of the detector as encoded in the tetrad e_α^μ . That is,

$$p_\alpha = \mathcal{F}_\alpha(\pi_\mu e_\beta^\mu; \kappa), \quad \pi_\mu = g_{\mu\nu} e_\alpha^\nu \eta^{\alpha\beta} \mathcal{G}_\beta(p_\gamma; \kappa), \quad (4.9)$$

where in the last equality we have used the completeness relation for the tetrad,

$$\eta^{\alpha\beta} e_\alpha^\mu e_\beta^\nu = g^{\mu\nu}. \quad (4.10)$$

We further simplify this by defining

$$\mathcal{G}_\mu(p_\alpha; e; \kappa) := g_{\mu\nu} e_\alpha^\nu \eta^{\alpha\beta} \mathcal{G}_\beta(p_\gamma; \kappa), \quad (4.11)$$

so that

$$\pi_\mu = \mathcal{G}_\mu(p_\alpha; e; \kappa). \quad (4.12)$$

B. Physical origin of the modifications

Up to this point, we have only inquired as to whether the formalism of doubly special relativity can be made logically and mathematically consistent, and we have refrained from asking physical questions. In this section we speculate about a possible physical basis for this new interpretation of the DSRs.

How should we understand the new theory of measurement expressed by Eq. (4.7)? For the sake of clarity, let us focus on a measurement of energy (similar considerations apply to measurements of momentum components). Setting the index $\alpha = 0$ in Eq. (4.7) we find that, in general, the measurement outcome for energy (i.e., $E \equiv -p_0$) differs from the one predicted by standard measurement theory (where we would obtain $\epsilon = -\pi_\mu e_0^\mu$). We want to understand the origin of this discrepancy; namely, how is it that a measurement does not reveal directly the value ϵ , given by Eq. (4.3), but the more complicated expression given by Eq. (4.7)?

First of all, let us note that, in general, we can write

$$E = \epsilon + f(\epsilon; \kappa), \quad (4.13)$$

where f is a function such that $f(\epsilon; +\infty) = 0$. The discrepancy between ϵ and E is thus due to the finiteness of the DSR scale κ , which is usually taken to be the Planck scale, since we assume the DSRs arise through quantum gravity effects. In fact, based on the presumed existence of a smooth limit as gravity is switched off, one might plausibly expect the dimensional parameter κ to lead to a relation such as

$$E = \epsilon[1 + \tilde{f}(\epsilon/\kappa)], \quad (4.14)$$

where $\tilde{f}(0) = 0$. This expression has the benefit of reproducing the general form of the phenomenological models that have been investigated in the literature.

From these remarks, it seems plausible to identify the physical origin of the discrepancy between the usual and the modified formulas for the measured energy in the quantum gravitational effects that take place whenever one performs a measurement. If such effects were not present, the measurement outcome for a particle with four-momentum π would be ϵ , as usual. However, they are universal and nonscreenable, so they always modify the measurement outcome into E : This is why the measurement theory has to be revised.

In connection to this point, it is worth mentioning that there exists a consistent theme in the literature (see [19] and references therein), in which gravitational effects add to the standard quantum uncertainty, producing modified Heisenberg relations of the type

$$\Delta x \Delta p \gtrsim \hbar(1 + \lambda \Delta p^2 / \kappa^2), \quad (4.15)$$

with λ a numerical coefficient of order one.⁴ Now, modified uncertainty relations can be traced back, formally, to modified commutators. And the DSR variables obey modified commutation relations [20]. This appears to support our claim about the phenomenological character of energy and momentum used in DSR—the variables E and $\mathbf{p}$. If one were able to remove the additional uncertainty due to gravity, one would end up with standard Heisenberg relations, and standard commutation relations, for the variables ϵ and $\boldsymbol{\pi}$. However, whether a concrete proposal along these lines is viable is of course a matter of debate, due to our present lack of knowledge about the nature and effects of gravity in the quantum regime.

Note that, within this interpretation, the “real” energy-momentum is $\boldsymbol{\pi}$, while the p_α are only measurement outcomes. However, an effective theory formulated in terms of the p_α includes, in general, a violation of Lorentz invariance. This entails measurable physical consequences. Although quantum gravitational processes do not affect ϵ , they do affect E , hence, e.g., thresholds. More dramatically, since the $\pi_\alpha = \pi_\mu e_\alpha^\mu$ transform under the ordinary linear Lorentz group, the symmetry implies that it is these variables that will obey conservation of four-momentum [13]. It then need not be the case that the measurement outcomes p_α satisfy conservation of energy and momentum, with the possibility of Planck-suppressed violations of these conservation laws now being a real concern. Indeed, in this framework it is logical to address questions about particle reactions (e.g., thresholds) by imposing energy-momentum conservation on $\boldsymbol{\pi}$ and then expressing the result in the measured variables p_α (see, e.g., Ref. [21] for a concrete example of this procedure).

V. IMPLICATIONS OF THE PROPOSAL

Regarding DSR as a new theory of measurement in the way we suggested, leads one to reevaluate its physical consequences. Here we sketch a few implications of the newly proposed framework.

A. The saturation problem

Our working hypothesis (or more precisely, speculation) does not provide an automatic resolution to the saturation problem of DSR. However it is clear from the overall idea (that differences in the measured energies and momenta are

due to the quantum gravitational interaction) that no simple prescription for the energy and momentum composition of macroscopic bodies applies. In the case of large numbers of particles, decoherence phenomena will take place⁵ and the measurement of the mass of a classical object might not be affected by graviton exchange with the balance used. In this sense, our proposal simply implies that it should be impossible to find a coherent *quantum* system whose overall mass is larger than the Planck mass.

Indeed, we note that the most extensive Bose-Einstein condensates experimentally created to date contain about 10^6 atoms [23], corresponding to a mass of about 10^8 GeV. If the DSRs in fact represent the correct way of doing quantum gravity phenomenology, and if our interpretation of the DSRs as a modified theory of measurement is the correct one, then the “saturation problem” may be viewed as predicting a maximum attainable mass for a Bose-Einstein condensate, of order one Planck mass, corresponding to about 10^{17} Rb atoms. This is a robust qualitative prediction of the DSR framework, which is in principle testable (though technically challenging). Furthermore, since in this framework the limitation alluded to above is actually a limitation on the maximum mass of a coherent quantum system we can (more boldly and more speculatively) also tie this back to Penrose’s speculations on the gravitationally-induced collapse of the wave-function [24]. While one cannot, given the current state of knowledge, guarantee that this is the way the universe actually works, the new interpretation of the DSRs provides both a consistent logical framework, and a physical reason to suspect that such effects may be possible.

A more precise way of putting this is to realize that a measurement of the energy-momentum of some composite object depends not only on the “true” energy-momentum and on the observer’s reference frame, but also on many details of the internal structure of the composite object, its interaction with the detector, and the internal construction of the latter. Let us collectively denote these extra variables as X , and so write:

$$p_\alpha = \mathcal{F}_\alpha(\pi_\mu e_\beta^\mu; \kappa; X); \quad \pi_\mu = \mathcal{G}_\mu(p_\alpha; e; \kappa; X). \quad (5.1)$$

In particular, among the additional variables X one can place:

- (i) The total number N of elementary particles in the body whose “physical” energy-momentum is to be measured. It is vitally important to note that in the current context, where the nonlinear transform $\mathcal{F}$ represents a phenomenological description of the measurement process, an N -dependence of this type is much more physically reasonable than in theories where $\mathcal{F}$ is assumed to be “fundamental” [13].

⁴Analogous results follow from specific models for quantum gravity.

⁵This could even be caused by the gravitons themselves [22].

- (ii) The “renormalization scale” $\bar{\mu}$ typically used within the particle physics community. This has the effect of giving a sensible physical meaning to the “running” of measured energy-momentum with “renormalization scale”.
- (iii) The type of DSR (DSR1, DSR2, DSR3, ...?) appropriate to the particular detector. That is, as a by-product of this new interpretation, it is now clear why the fundamental principles of DSR theories allow so many apparently quite different DSR models—at least three standard DSR implementations are widespread in the literature. The multiplicity of DSRs is simply a reflection of the fact that there are many different classes of “detectors” one could think of building, all of which would be compatible with the basic framework of the DSR interpretation we advocate in this article.

For a shorthand that retains the key aspects of the physics, we can discard other variables and simply write:

$$\begin{aligned} p_\alpha &= \mathcal{F}_\alpha(\pi_\mu e_\beta^\mu; \kappa; N, \bar{\mu}, DSR); \\ \pi_\mu &= \mathcal{G}_\mu(p_\alpha; e; \kappa; N, \bar{\mu}, DSR). \end{aligned} \quad (5.2)$$

B. Conservation laws

Recall that in terms of the “true” energy-momenta π the transformation laws are simple, while in terms of the measured energy-momenta p_α the transformation laws are complicated. This is telling us that it is the “true” energy-momenta π that are related to whatever underlying symmetries that via Noether’s theorem lead to conservation laws. This observation, in one form or another, has led to almost universal acceptance in the literature of the fact that conservation laws should be implemented in terms of the “true” energy-momenta π —the “Judes-Visser variables” of [13].

Indeed in the “modified measurement” interpretation of the DSRs advocated in this article it is clear that there is no physical need for the measured energy-momenta to satisfy conservation laws—and this natural lack of conservation laws at high energies and momenta is a quite generic feature of the DSRs. Equally well, the occurrence of non-standard dispersion relations is no longer “unusual” or “peculiar”, but must instead be seen as quite natural and in fact inevitable.

C. Einstein’s equations

While in the highly interacting quantum gravity regime there will certainly be drastic modifications to the standard Einstein equations, there is observationally a wide range of distance and time scales in the solar system and beyond over which standard general relativity (and, in particular, the standard Einstein equations) works well. In this regime we can meaningfully ask whether the gravitational field

couples to the “true” energy-momenta π or the measured energy-momenta p_α ?

Since the solar system contains many macroscopic objects of super-Planckian mass, the Einstein equations would seem to prefer to couple to energy-momentum variables that do not suffer from any saturation problem. For instance, if we try to couple the gravitational field to measured energy-momenta p_α , then the resulting metric will depend not only on the source, but on the four velocity (and in fact the entire tetrad) of the observing apparatus (plus the particle content of the source, the resolution [“renormalization scale”] of the observer, and the internal structure of the detector). Thus, in parallel with the fact that the energy-momenta satisfies the relations

$$\begin{aligned} p_\alpha &= \mathcal{F}_\alpha(\pi_\mu e_\beta^\mu; \kappa; N, \bar{\mu}, DSR), \\ \pi_\mu &= \mathcal{G}_\mu(p_\alpha; e; \kappa; N, \bar{\mu}, DSR), \end{aligned} \quad (5.3)$$

we would now be *forced* to distinguish a “true” metric $\gamma_{\mu\nu}$ from a measured metric $g_{\mu\nu}$, with relations of the form:

$$\begin{aligned} g_{\mu\nu} &= \tilde{\mathcal{F}}_{\mu\nu}(\gamma_{\mu\nu}; e; \kappa; N, \bar{\mu}, DSR); \\ \gamma_{\mu\nu} &= \tilde{\mathcal{G}}_{\mu\nu}(g_{\mu\nu}; e; \kappa; N, \bar{\mu}, DSR). \end{aligned} \quad (5.4)$$

But, in an application of *reductio ad absurdum*, this measured metric depends not on the apparatus that is measuring the metric, but on the apparatus that is measuring the composite object that is used as the source for the Einstein equations.

The only way out of this is to apply the Einstein equations directly to the “true” metric with the “true” variables as source. One could then *independently* introduce the notion of a measured metric as in Eq. (5.4), but now depending on whatever apparatus is measuring the metric, and then interpret the “measured metric” $g_{\mu\nu}$ as a “running metric” that depends on the observer’s motion and the resolution of his (metric-measuring) apparatus. Then, the measured metric need not—and in general will not—satisfy the Einstein equations. But since in the DSR framework we know that deviations from standard physics must be both Planck-suppressed and macroscopically suppressed we expect

$$g_{\mu\nu} = \gamma_{\mu\nu}[1 + \mathcal{O}(\bar{\mu}/\kappa, N)], \quad (5.5)$$

so that any deviations from the metric expected on the basis of the usual Einstein equations should also be greatly suppressed.

To (hopefully) clarify the situation a little further: Suppose someone tells you that at position x she has measured the presence of a particle with four-momentum p_α . After making enquiries regarding the structure of the particle detector, one would invert the nonlinear transform $\mathcal{F}$ to determine the “true” four-momentum π_μ . This can then be inserted into the Einstein equation to determine the “true” metric at some other point y . After making enqui-

ries regarding the structure of the metric detector placed at y , one can apply the appropriate nonlinear transform $\tilde{\mathcal{F}}$ (distinct from the previous one) to determine the measured metric at y .

In a similar vein one can now think of a parallel “ κ -deformed phenomenology” that would apply to all branches of physics—for instance there would be DSR-distorted electric and magnetic fields, *etcetera*. While calculating the specific form of these DSR distortions in any given situation would be quite horribly complicated, the present interpretation has the great virtue of being logically consistent and allowing us to ask physically meaningful questions.

VI. CONCLUSIONS

The key point to be taken from the present article is that by viewing the DSRs as a modified theory of measurement, we can provide a mathematically precise, logically coherent, and physically nontrivial interpretation for the DSRs. The previous lack (apart from the considerations of [25]) of any such coherent physical interpretation has seriously hampered developments in the field. Key features of the “measurement” interpretation of the DSRs are:

- (i) There does not seem to be any pressing need to go to noncommuting coordinates. At least for the time being, ordinary differential geometry based on Lorentzian manifolds seems quite sufficient as a framework.
- (ii) Conservation laws, and the Einstein equations, seem to preferentially couple to the “true” energy-momenta, which transform linearly under the Lorentz group.
- (iii) Measured energy and momenta do not only transform nonlinearly under the Lorentz group, but are now quite naturally seen to obey nonstandard dis-

persion relations, to not satisfy standard conservation laws, and to not directly act as sources for the Einstein equations.

- (iv) The DSRs are now to be viewed as phenomenological theories, that depend on the measurement apparatus. In a limited sense this may be viewed as a “demotion”, but in another sense this new point of view now guarantees that the existence of DSR-like effects is both natural and ubiquitous—as is quantum gravity itself.
- (v) With this new “measurement” interpretation there can no longer be any doubt about the falsifiability of DSR effects and scientific status of specific DSR theories, so there is a clear path to experimentally testing the DSRs. Without the interpretation we have argued for in this article, or something closely related thereto, the DSRs run the very real risk of amounting to physically empty mathematical manipulations akin to the coordinate transformations of general relativity.

In summary, we feel that the considerable confusion in the current literature regarding the questions of consistency, triviality, and physical acceptability of the DSRs is largely the result of misinterpreting what the DSRs are trying to say. Viewed as a modified theory of measurement, the DSRs make perfectly sensible statements about empirical reality that can (at least in principle) be tested in the usual scientific manner.

ACKNOWLEDGMENTS

We are grateful to Daniel Grumiller for comments on the manuscript. S.L. would also like to thank Aurelio Grillo for valuable discussions. The research of Matt Visser was supported by the Marsden fund administered by the Royal Society of New Zealand.

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