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Energy Dependence of the Average Charged-Particle Multiplicity in Hadronic Collisions*

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Previous results pertaining to the growth of the pion-production cross section at 90° in the center-of-mass frame are used to infer the dependence of the charged-particle multiplicity on the momentum of the projectile particle.

Recent measurements of the average charged-particle multiplicity ($\langle n_{\rm ch}\rangle$) in pp collisions at high scattering energies[1] have stimulated theoretical speculation concerning the specific dependence of $\langle n_{\rm ch}\rangle$ on the momentum of the incident projectile in the laboratory (p).[2] Suggestions, for example, have been made that, at low energies, $\langle n_{\rm ch}\rangle$ should grow as a polynomial in p, while at higher values of beam momentum the dependence of $\langle n_{\rm ch}\rangle$ on p should become logarithmic.[2] In this note we will use a recently obtained result[3] pertaining to the growth of the invariant cross section for pion production at $x = p_l^*/p_{l,\max}^* = 0$ (p_l^* and $p_{l,\max}^*$ being, respectively, the momentum of the pion and of the projectile particle in the center-of-mass system) to infer the dependence of $\langle n_{\rm ch}\rangle$ on p.

It is now firmly established that Feynman scaling at $x=0$ does not set in below energies comparable to that of the CERN Intersecting Storage Rings – the ISR (equivalent to $\lesssim 1500$ GeV/c for the system with target at rest).[4] The approach to scaling at $x=0$, in fact, appears to be consistent with an energy dependence expected on the basis of Mueller-Regge phenomenology.[3] It is also known that the single-particle distribution function in the rapidity variable

$$y \equiv \tfrac{1}{2}\ln\frac{E^* + p_l^*}{E^* - p_l^*}$$

for the reaction

$$pp \to \pi^- + \text{anything} \tag{1}$$

develops a plateau in the range of ISR energies.[4] The number of produced π^- mesons can thus be taken as proportional to the product of the height and width of the plateau:

$$n_{\pi^-} \sim \frac{d\sigma}{dy}\bigg|_{y=0} \times (\text{range of } y)\,. \tag{2}$$

Since the range of the y variable is proportional to $\ln p$, asymptotically we can rewrite

$$n_{\pi^-} \sim \sigma_T C_{\pi^-} \ln p + \alpha_0\,, \tag{3}$$

where C_{π^-} represents the height of the plateau in $d\sigma/dy$ normalized by the total pp cross section σ_T (i.e., $C_{\pi^-} = \sigma_T^{-1} d\sigma_{\pi^-}/dy|_{y=0}$), and α_0 is a constant dependent on the units of p.

If Feynman scaling held at all energies then C would be an energy-independent constant (this is essentially the definition of Feynman scaling). In a previous note[3] we determined that C_{π^-} changes with energy in a specific manner[5]:

$$C_{\pi^-}(p) = 0.75 - \frac{0.93}{p^{1/4}}\,. \tag{4}$$

Hence, if we assume that Eq. (2) is approximately valid even in the 5-GeV/c momentum range (recall that there is no clear plateau in $d\sigma/dy$ for $p \lesssim 30$ GeV/c), and that the growth of $\langle n_{\rm ch}\rangle$ is proportional to n_{π^-}, we can write

$$\langle n_{\rm ch}\rangle = \alpha + \beta\left(1 - \frac{1.24}{p^{1/4}}\right)\ln p\,, \tag{5}$$

where α and β are taken as arbitrary parameters. (We note that, naively, charge conservation requires β to be consistent with 1.5 ± 0.1, i.e., with twice the limiting value for C_{π^-}; α is related to the scale and units used for p.)

In Fig. 1 we display the results of a recent compilation[1] of the values for $\langle n_{ch}\rangle$ in pp collisions as a function of the incident laboratory momentum. (When error bars are not shown they are typically as large as the data points.) The solid curve drawn on the figure is the two-parameter expression given in Eq. (5) with $\alpha = 2.2$ and $\beta = 1.5$. The energy dependence of the data is reproduced remarkably well by our expression.[6] The dashed curve, which clearly does not give as good an account of the data, is also a two-parameter overall fit to $\langle n_{ch}\rangle$ using a simpler $\ln p$ dependence, namely,

$$\langle n_{ch}\rangle = 0.48 + 1.27 \ln p\,. \tag{6}$$

Although Eq. (5) describes the dependence of $\langle n_{ch}\rangle$ on incident momentum from, essentially, threshold to the TeV range of energies, we wish to caution that we have made certain simplifying assumptions in our discussion of the dependence of $\langle n_{ch}\rangle$ on momentum; these assumptions may have to be reexamined once better data at high energies become available. We have assumed, for example, that there exists a uniform plateau in the y variable at all energies; this clearly tends to overestimate n_{π_-} at smaller values of p. We have also ignored details of $K^\pm$ and p-$\bar{p}$ production; these processes may have a dependence which differs substantially from the one used in Eq. (5).[7]

Finally, we stress that although Eq. (5) is not consistent with the specific form for the growth of $\langle n_{ch}\rangle$ in the Mueller-Regge picture,[8] our present

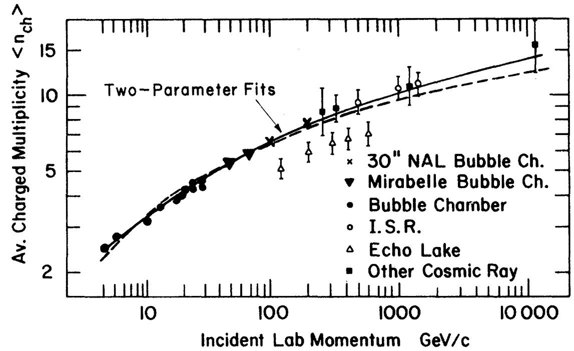

FIG. 1. Average inelastic charged-particle multiplicity in pp collisions as a function of projectile momentum in the laboratory system. For explanation of curves see text.

result does not exclude an asymptotic form which emerges more naturally using this framework, namely, $(a + b/p^{1/2}) \ln p + c$.[9] Nevertheless, it is clear that an expression such as Eq. (5), with only two free parameters, gives an excellent phenomenological description of the dependence of $\langle n_{ch}\rangle$ on incident momentum.

Note added in proof. After the completion of this report we became aware of similar work done by S. Rai Choudhury (Delhi and Michigan) and by P. K. Malhotra *et al.* (Delhi).

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[1]For a compilation of the latest results see the report by D. R. O. Morrison, in Proceedings of the Third International Colloquium on Multiparticle Reactions, Zakopane, 1972, edited by T. Hofmokl (unpublished). We will use in our Fig. 1 data presented in Fig. 7.2 of this report; in addition, we include the recent results from the 30-in. bubble chamber at NAL: at 200 GeV/c, G. Charlton *et al.*, Argonne report (unpublished); at 100 GeV/c, results from the Michigan-Rochester Collaboration (unpublished).

[2]See, for example, the discussion and references given by E. L. Berger in the Argonne Report ANL/HEP 7148 (unpublished). For a specific model see E. L. Berger and A. Krzywicki, Phys. Letters 36B, 380 (1971).

[3]T. Ferbel, Phys. Rev. Letters 29, 448 (1972).

[4]See the reports by J. C. Sens, D. R. O. Morrison, and E. Lillethun, in Proceedings of the Fourth International Conference on High Energy Collisions, Oxford, 1972 (unpublished).

[5]We thank V. Blobel for communicating to us the latest results for the reactions $pp \to \pi^- +$anything from the 12 GeV/c and 24 GeV/c Bonn-DESY (Hamburg)-Munich Collaboration.

[6]We have ignored the apparently incorrect results from the Echo Lake experiment [L. W. Jones *et al.*, Nucl. Phys. B43, 477 (1972)].

[7]See, for example, the report of T. Ferbel in Proceedings of the Colloquium on Multiparticle Reactions, Zakopane, 1972, edited by T. Hofmokl (unpublished).

[8]M. S. Chen and F. E. Paige (unpublished). I thank L.-L. Wang and A. H. Mueller for clarifying discussions.

[9]Private communication from L.-L. Wang. In this regard see H. Meyer and W. Struczinski [DESY report (unpublished)] who suggest that Feynman scaling at $x = 0$ is approached as $\approx 1/p^{1/2}$.