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Method for Calculating Electromagnetic Mass Differences of Hadrons: Application to the Pion and Kaon*

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A method is presented for the calculation of hadronic electromagnetic mass differences and is applied to the calculation of $m_{\pi^\pm}-m_{\pi^0}$ and $m_{K^\pm}-m_{K^0}$. Our results, although far from conclusive, include an explanation of the observed sign and general magnitude of $m_{K^\pm}-m_{K^0}$. In contrast with the familiar Cottingham approach, the method apparently fails for $m_{\pi^\pm}-m_{\pi^0}$.

I. INTRODUCTION

The observed values of the $\Delta I=1$ electromagnetic mass differences have not yet yielded to a convincing calculation,[1] in contrast to the known $\Delta I=2$ differences, $m(\pi^+)-m(\pi^0)$ and $m(\Sigma^+)+m(\Sigma^-)-2m(\Sigma^0)$, which appear relatively eager to reveal their origin.[2] An appealing reason for this dichotomy, as first proposed by Harari,[3] is based upon the relative enhancement of the strong-interaction dynamics in channels with $I=1$. In particular, Harari observed that the existence of the $I=1$ A_2 meson implies the necessity of subtraction terms in the Cottingham[4] formulas for the $\Delta I=1$ mass differences. Although there exist arguments[5] that these subtraction terms can be obtained from observed electroproduction data, they are based on ideas of doubtful validity, and their implementation requires information about delicate features of these cross sections[6] beyond the reach of present experiments.

In this paper we present observations and results which have emerged from an attempt to calculate the pion and kaon electromagnetic mass differences in a manner designed to avoid these difficulties. We have chosen these particular mass shifts because they are natural ones to test our method of calculation, and because we feel that an understanding of the kaon mass difference is basic to an understanding of the observed $\Delta I = 1$ electromagnetic mass differences of the baryons.

In contrast to most other efforts,[2] we have not been able to obtain reasonable results for the pion, while the results for the kaon are encouraging. A possible reason why our approach might not be expected to be successful for the $\Delta I = 2$ mass differences is discussed in Sec. IV.

Our scheme of calculation, including a statement of the assumptions and approximations employed, is presented in Sec. II. It proceeds from the observation that the mass difference between two adjacent members of an isospin multiplet is simply related to the zero-momentum-transfer (squared) matrix element of the divergence of the isospin current. In this respect our approach is similar to that of Li and Pagels,[7] who have applied this same observation primarily to the study of the breaking of SU(3)×SU(3), although they have also deduced from it relations among various electromagnetic mass differences. Also, in the sense that our approach views the mass difference between two particles (say, the K^+ and K^0) as the consequence of the dynamics in the channel of $K^+\overline{K}^0$ scattering, it overlaps with the work of Ball and Zachariasen,[8] who attempt to explain the neutron-proton mass difference from features of the $p\overline{p}$ and $n\overline{n}$ scattering amplitudes. In Sec. III the relations derived in Sec. II are applied specifically to the kaon mass-difference problem.

Our method requires the construction of a model for the process $K^+\overline{K}^0 \to \gamma\rho^+$. More specifically, we are required to supply an expression for the imaginary s-wave part of this amplitude in the region $t = (K^+ + \overline{K}^0)^2 < 4M_K^2$. Such terms arise through the crossed-channel exchange of kaons and strange mesons of higher spin. Our principal finding is that significant contributions arise only from the pseudoscalar and spin-2 [$K_N(1420)$] exchanges, and it is the quantitative dominance of the latter which leads to the correct sign and approximate magnitude of the K^+K^0 mass difference. These qualitative features of our result are independent of any assumption concerning the details of the strong interactions, although they are also less than compelling because of the approximate nature of the calculation and because the explicit form of our result involves a divergent integral. However, as discussed in Sec. IV, we feel that this divergence arises from our crude method of approximating the crossed-channel exchanges, and that in fact the formal expression [see Eq. (20)] which we obtain for the mass difference converges.

On the basis of a particular (Veneziano) model of the $I = 1$ $K\overline{K}$ elastic amplitude, we also compute rescattering corrections in Sec. IV and show how the strong interactions can provide an effective cutoff to the integral expression for the K^+K^0 mass difference. Some general features of dual-resonance-model predictions for $K^+\overline{K}^0 \to \gamma\rho$ are discussed along with difficulties encountered in attempting to apply similar methods to the analogous pion problem.

In Sec. V our results are summarized, and a variant of our calculation for $m_{K^+} - m_{K^0}$ is suggested which is more internally consistent and which is likely to lead to essentially the same numerical conclusions.

II. METHOD OF CALCULATION

In this section the method of calculation is illustrated by applying it specifically to the K^+-K^0 mass difference. We begin by observing that if V_μ^+ is the charge-raising component of isospin current whose matrix element between the $K^+\overline{K}^0$ state and the vacuum is given by

$$\langle K^+\overline{K}^0 \,|\, V_\mu^+ \,|\, 0\rangle = f(t)(K^+ - \overline{K}^0)_\mu + g(t)(K^+ + \overline{K}^0)_\mu \,, \tag{1}$$

where $t \equiv (K^+ + \overline{K}^0)^2 \equiv Q^2$, then the Ward identity, which guarantees that $f(0)$ is unity except for corrections of order α ($\approx \frac{1}{137}$), has the consequence that the matrix element of the divergence of V_μ^+,

$$\langle K^+\overline{K}^0 | -i\partial^\mu V_\mu^+ \,|\, 0\rangle \equiv F(t) \,, \tag{2}$$

leads to an equality between $F(0)$ and $m^2(K^+) - m^2(K^0)$,

$$m_{K^+}{}^2 - m_{K^0}{}^2 = F(0) \,, \tag{3}$$

which is valid to first order in the fine-structure constant α. The function $F(t)$ is analytic in t except for a right-hand cut, where the discontinuity is given by

$$\mathrm{Im}F(t) = \tfrac{1}{2}(2\pi)^4 \sum_n \delta^4(Q - P_n)\langle \pi^+ \,|\, j_0^{K*} \,|n\rangle\langle n\,| -i\partial^\mu V_\mu^+ \,|0\rangle \,. \tag{4}$$

Here j_0^K is the renormalized source current of the neutral kaon, which arises from contracting out the $\overline{K}^0$ in Eq. (2).

There are two kinds of intermediate states which contribute to the sum in Eq. (4): those which contain only hadrons and those which also contain a photon. Thus we can write

$$\mathrm{Im}F = \mathrm{Im}_h F + \mathrm{Im}_\gamma F\,, \tag{5}$$

where the two contributions are indicated by the appropriate subscripts. We now make our first approximation and neglect all the intermediate states $|n\rangle$ in $\mathrm{Im}_h F$ except the elastic rescattering states, whose contribution to the sum in Eq. (4) gives for $t > 4m_K^2$

$$\mathrm{Im}_h F(t) = \rho(t) h^*(t) F(t)\,, \tag{6}$$

where $F(t)$ is the upper boundary value on the cut of the $F(t)$ defined in Eq. (2), $\rho(t)$ is the phase-space factor

$$\rho(t) = \left(1 - \frac{4m_K^2}{t}\right)^{1/2}, \tag{7}$$

and $h(t)$ is the $I = 1, 0^+$ partial-wave $K\bar{K}$ scattering amplitude given in terms of its phase shift by

$$h(t) = \rho^{-1}(t) e^{i\delta} \sin\delta(t)\,. \tag{8}$$

By explicitly inserting a photon into the intermediate states occurring in Eq. (4), we obtain

$$\mathrm{Im}_\gamma F = \tfrac{1}{2}(2\pi)^4 \sum_n \int \frac{d^3k}{(2\pi)^3 2k_0} \sum_\lambda \delta^4(Q - P_n - k) \langle K^+ | j_0^{K*} | nk\lambda\rangle \langle nk\lambda | -i\partial^\mu V_\mu^+ | 0\rangle\,, \tag{9}$$

where in Eq. (9) the index n refers only to hadronic states. The matrix element on the right-hand side of this expression can be simplified by using

$$\partial^\mu V_\mu^+ = ieA^\mu V_\mu^+\,, \tag{10}$$

which is valid[9] in any field theory in which the photon is minimally coupled and in which the isospin current is conserved to zeroth order in α,[10] to contract out the photon and obtain

$$\langle nk\lambda | -i\partial^\mu V_\mu^+ | 0\rangle = e\epsilon_\gamma^\mu \langle n | V_\mu^+ | 0\rangle\,, \tag{11}$$

where ϵ_γ^μ is the photon polarization vector. The form of the right-hand side of Eq. (11) indicates that the states $|n\rangle$ which are important in Eq. (9) are those which are coupled strongly to the vacuum by the isovector current. The notions of vector dominance suggest our second approximation, which is to include in the sum over n in Eq. (9) only the state containing a single, positively charged ρ meson.[11,12] The expression in Eq. (9) then reduces to

$$\mathrm{Im}_\gamma F = \frac{\sqrt{2}\,\alpha m_\rho^2}{16\pi} \rho_\gamma(t) h_\gamma^{K*}(t)\,, \tag{12}$$

where the phase-space factor ρ_γ is equal to $t^{-1}(t - m_\rho^2)$, where the factor of m_ρ^2 arises from the vector-dominance relation

$$\langle q\lambda_\rho | V_\mu^+ | 0\rangle = \sqrt{2}\, g_{\rho\pi\pi}^{-1} m_\rho^2 \epsilon_\mu^\rho\,, \tag{13}$$

and where h_γ^K is a 0^+ amplitude for $K^+ + \bar{K}^0 \to \gamma + \rho$ defined by

$$h_\gamma^K(t) = \int d\Omega_\gamma \left(g^{\mu\nu} - \frac{q^\mu q^\nu}{m_\rho^2}\right) M_{\mu\nu}^K(q,k)\,, \tag{14}$$

with

$$e g_{\rho\pi\pi} M_{\mu\nu}^K \epsilon_\gamma^\mu \epsilon_\rho^\nu = \langle q\lambda_\rho k\lambda_\gamma | j_0^K | K^+\rangle\,. \tag{15}$$

The expression for $\mathrm{Im}F$ in Eq. (5), with the two parts of the right-hand side of Eq. (5) given by Eqs. (6) and (12), is essentially an Omnès equation, which can be solved by standard techniques. In particular,

$$\mathrm{Im}(DF) = \tfrac{1}{16}\alpha m_\rho^2 \rho_\gamma D h_\gamma^K\,, \tag{16}$$

where D is the D function for the $I = 1, 0^+$ $K^+\bar{K}^0$ partial-wave amplitude; i.e.,[13]

$$D(t) = \exp\left[-\frac{t - t_0}{\pi}\int_{4m_K^2}^\infty \frac{dt'\,\delta(t')}{(t' - t_0)(t' - t)}\right], \tag{17}$$

where t_0 is arbitrary. If we now assume that DF goes to zero as t approaches infinity, we can write an unsubtracted dispersion relation for DF, which from Eq. (16) implies

$$F(t) = \frac{\sqrt{2}\,\alpha m_\rho^2}{16\pi^2 D(t)} \int_{m_\rho^2}^\infty \frac{dt'}{t' - t} \rho_\gamma(t') D(t') h_\gamma^K(t')\,. \tag{18}$$

The combination Dh_γ^K occurring in the integral has no discontinuity across the right-hand unitarity cut running between $4m_K^2$ and infinity. Since we have already assumed that Dh_γ^K vanishes for large t, we can write

$$D(t') h_\gamma^K(t') = \frac{1}{\pi}\int_{\mathrm{LHC}} \frac{dt''}{t'' - t'} D(t'') \mathrm{Im} h_\gamma^K(t'')\,, \tag{19}$$

where LHC denotes the left-hand cut. If this expression is substituted inside the right-hand side of Eq. (18) and if the integral over t' is performed explicitly, we obtain upon setting $t = 0$ and using Eq. (3)

$$m_{K^+}^2 - m_{K^0}^2 = -\frac{\alpha}{32\pi^3}\frac{1}{D(0)} \times \int_{\mathrm{LHC}} dt' K(t', 0) D(t') \mathrm{Im} h_\gamma^K(t')\,, \tag{20}$$

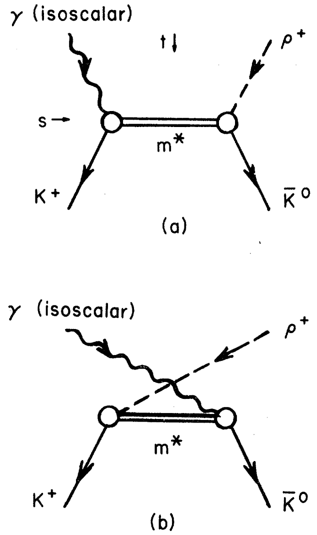

FIG. 1. Driving amplitudes for the mass differences.

where

$$K(t',t)=-\frac{2m_\rho^2}{t'-t}\left[\left(1-\frac{m_\rho^2}{t'}\right)\ln\left(1-\frac{t'}{m_\rho^2}\right)-\left(1-\frac{m_\rho^2}{t}\right)\ln\left(1-\frac{t}{m_\rho^2}\right)\right]. \tag{21}$$

It should be noted that Eq. (20) has been deduced by ignoring the $\pi^+\eta$ intermediate state which can also contribute to the $I=1$, $J^P=0^+$ channel appropriate for the K-meson mass difference. We could have included this state in the formalism of this section by introducing an $F(t)$ and an $h^\gamma(t)$ for the $\pi^+\eta$ state in analogy with Eqs. (2) and (14), and by reinterpreting Eq. (18) to be a matrix equation with the 2×2 matrix $D^{-1}(t)D(t')$ connecting the two component objects F and h^γ. To keep the calculation here as simple as possible, we have not used this two-channel formalism in our efforts in Secs. III and IV to evaluate the right-hand side of Eq. (20).

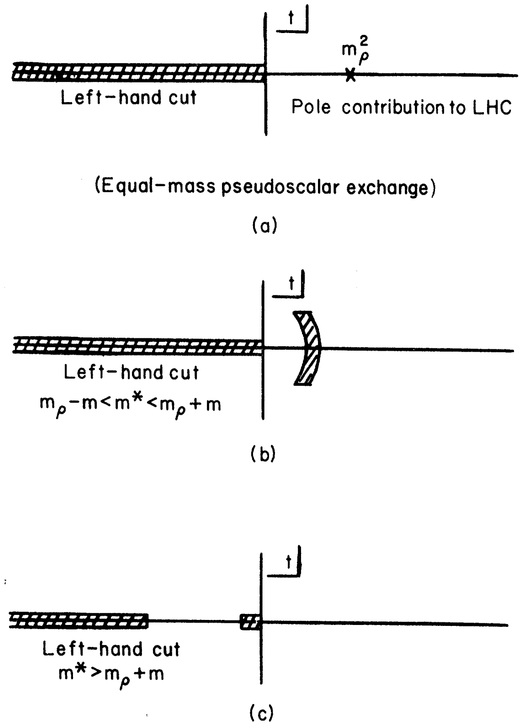

FIG. 2. Integration regions labeled LHC (left-hand cut) for various exchanges contributing to $K^+\bar{K}^0\to\gamma\rho^+$.

Weak justification for the reasonableness of this simplification could be based on the observation that since $D^{-1}(0)D(t')$ in Eq. (20) is evidently diagonal at $t'=0$, the important region of t' in the integral in Eq. (20) might not extend so far from $t'=0$ to require us to retain the off-diagonal components of this matrix product. More significantly, however, in Sec. V a modified version of the present calculation is suggested which does include the effects of the $\pi^+\eta$ channel and which, as discussed there, is likely to lead to numerical results for $m_{K^+}-m_{K^0}$ similar to those we obtain here.

III. FORMAL CONTRIBUTIONS TO THE KAON ELECTROMAGNETIC MASS DIFFERENCE

The left-hand discontinuities of h_γ^K which appear in Eq. (20) are given by the various exchanges indicated by m^* in Fig. 1. When the exchanged object of mass m^* is a K^+, the graph in Fig. 1(a) yields an expression for h_γ^K as defined in Eqs. (14) and (15) which is the sum of a part having a discontinuity along the negative[14] t axis and a part which a pole at $t=m_\rho^2$ (see Fig. 2). This pole, which would be a short cut if the photon were given a small mass, arises from the kinematic possibility that at $t=m_\rho^2$ the two incident kaons can combine directly into a ρ meson after one of the kaons emits a zero-momentum photon. Upon evaluation this pole dominates the part of the mass difference arising from K-meson exchange in Fig. 1. It gives a contribution to $\operatorname{Im}h_\gamma^K$ of the form

$$\operatorname{Im}h_\gamma^K|_{\text{pole}}=-\delta(t-m_\rho^2)\,\frac{8\pi^2}{\sqrt2}\,m_\rho^2\left(\frac{4m_K^2}{m_\rho^2}-1\right)^{1/2}\tan^{-1}\left(\frac{4m_K^2}{m_\rho^2}-1\right)^{1/2}. \tag{22}$$

By substituting this pole part of $\mathrm{Im}\,h_\gamma^K$ into Eq. (20) we can easily calculate the pole part of the kaon exchange contribution to the kaon mass difference to obtain

$$(m_{K^+} - m_{K^0})|_{\rm pole} = \frac{\alpha m_\rho^2}{4\pi m_K}\left(\frac{4m_K^2}{m_\rho^2} - 1\right)^{1/2} \tan^{-1}\left(\frac{4m_K^2}{m_\rho^2} - 1\right)^{1/2} D^{-1}(0)D(m_\rho^2)\,. \tag{23}$$

Insertion of the observed values for the various masses into the right-hand side of Eq. (23) gives

$$(m_{K^+} - m_{K^0})|_{\rm pole} = 0.40 D^{-1}(0)D(m_\rho^2) \text{ MeV}\,. \tag{24}$$

It is amusing to note that since the D-function ratio is positive [see Eq. (17)], Eq. (23) taken by itself predicts opposite signs for meson mass differences depending upon whether $4m^2 \lessgtr m_\rho^2$; i.e., Eq. (23) and its analog for $m_{\pi^+} - m_{\pi^0}$ predict precisely the wrong signs for both pion and kaon mass differences.

As we have mentioned there is also a small contribution from the left-hand continuum portion of the integral in Eq. (20) when the exchanges particle is the kaon. The algebraic form for the corresponding part of the $\mathrm{Im}\,h_\gamma^K$ is given by

$$\mathrm{Im}\,h_\gamma^K|_{K\text{ contin}} = -\frac{4\pi^2}{\sqrt{2}}\,\frac{m_\rho^2 - 4m_K^2 + \frac{1}{2}(t - m_\rho^2)}{(t - m_\rho^2)(1 - 4m_K^2/t)^{1/2}}\,. \tag{25}$$

Contributions from the vector and axial-vector exchanges are generally small. To facilitate their evaluation we assume SU(3) symmetry of the couplings, which may then be obtained from the measured partial decay widths of the vector and pseudovector mesons. For the K^* exchange, calculation of the (d-type) couplings involves three ingredients: vector dominance at the $\gamma K^* K$ vertex; obtaining the pure octet part of the coupling in terms of the $\rho\omega\pi$ coupling constant (ω-ϕ mixing[12] and the vanishing of the $\phi\rho\pi$ coupling are assumed); and, finally, calculating the $\rho\omega\pi$ coupling constant from the 3π decay of ω.[15] The result is

$$\mathrm{Im}\,h_\gamma^K|_{K^*} = \frac{4\pi^2}{3(1.06)^2\sqrt{2}}\left|\frac{g_{\rho\omega\pi}}{g_{\rho\pi\pi}}\right|^2 \frac{m_{K^*}^2(t - m_\rho^2) + \frac{1}{2}(m_{K^*}^2 - m_K^2)(m_{K^*}^2 + m_\rho^2 - m_K^2)}{(t - m_\rho^2)(1 - 4m_K^2/t)^{1/2}}\,, \tag{26}$$

with

$$\left|\frac{g_{\rho\omega\pi}}{g_{\rho\pi\pi}}\right|^2 = 9.4/\text{GeV}^2\,. \tag{27}$$

For the pseudovector exchanges

$$\mathrm{Im}\,h_\gamma^K|_{PV} = -\frac{8\pi^2}{\sqrt{2}}\left|\frac{g_{VAP}}{g_{\rho\pi\pi}}\right|^2 \frac{1 + (m_\rho^2 - m_K^2 + m_A^2)^2/8m_A^2 m_\rho^2}{(m_\rho^2 - t)(1 - 4m_K^2/t)^{1/2}}\,. \tag{28}$$

The vector-pseudovector-pseudoscalar couplings are taken to be pure s-wave. Values for the $K_A(1242)$ and Q mesons[16] are obtained in a straightforward way from the measured widths[17] (the Q region can be fitted by two Breit-Wigner shapes):

$$K_A(1242):\quad \Gamma = 0.127 \text{ GeV},\quad \left|\frac{g_{VAP}^{1242}}{g_{\rho\pi\pi}}\right|^2 = 0.76 \text{ GeV}^2 \tag{29}$$

$$Q(1250):\quad \Gamma = 0.182 \text{ GeV},\quad \left|\frac{g_{VAP}^{1250}}{g_{\rho\pi\pi}}\right|^2 = 1.08 \text{ GeV}^2 \tag{30}$$

$$Q(1400):\quad \Gamma = 0.220 \text{ GeV},\quad \left|\frac{g_{VAP}^{1400}}{g_{\rho\pi\pi}}\right|^2 = 1.12 \text{ GeV}^2\,. \tag{31}$$

Finally, the $K_N(1420)$, which is the principal contributor, gives

$$\mathrm{Im}\,h_\gamma^K| = \frac{3\pi^2}{2\sqrt{2}}\left|\frac{g_{VTP}}{g_{\rho\pi\pi}}\right|^2 m_N^2\,\frac{t^2 + Bt + C}{(m_\rho^2 - t)(1 - 4m_K^2/t)^{1/2}}\,, \tag{32}$$

where

$$\begin{aligned} B &= \frac{(m_N^2 - m_K^2)(m_N^2 - m_K^2 + m_\rho^2)}{m_N^2} - 2m_\rho^2\,,\\ C &= \frac{1}{6m_N^4}\{[(m_N^2 - m_K^2)(m_N^2 - m_K^2 + m_\rho^2) - 2m_N^2 m_\rho^2]^2 + 2m_K^2 m_N^2 m_\rho^4\}\,, \end{aligned} \tag{33}$$

and

$$\left|\frac{g_{VTP}}{g_{\rho\pi\pi}}\right|^2 = 13.3/\mathrm{GeV}^4 . \tag{34}$$

Again, the coupling constant g_{VTP} has been obtained from the measured partial decay widths for $K_N \to K^*\pi$.

The form for $\mathrm{Im}h_\gamma^K$ in Eq. (20) is obtained by summing the expressions in Eqs. (22), (25), (26), (28), and (32).

IV. EFFECTS OF THE STRONG INTERACTIONS

The elastic t-channel strong-interaction effects are formally summarized by the appearance of the D-function ratio in Eq. (20). If this ratio were taken to be unity, the contributions to $\mathrm{Im}h_\gamma^K$ arising from the pseudoscalar, vector, and spin-2 meson exchanges calculated in Sec. III would lead to divergent integrals. We feel that these divergences would not be present for the physical $\mathrm{Im}h_\gamma^K$, but that rather they are reflections of the inadequacy of single resonance exchanges to give reasonable expressions for the distant regions of the left-hand cut. In fact, it is not difficult to construct dual resonance models for $K\bar{K} \to \gamma\rho$ for which the integral in Eq. (20) converges (we will come back to this point later). In this section we argue that the D-function ratio in Eq. (20) can reasonably provide a convergence factor to the integral which effectively eliminates the dependence of the resulting mass difference upon the large-t behavior of $\mathrm{Im}h_\gamma^K$, where the implications of single resonance exchanges, as calculated in Sec. III, are inadequate.

Let us now turn to a specific (Veneziano) model for the $I=1$ $K\bar{K}$ elastic scattering amplitude which we will use as a guide for studying possible features of the strong interactions as manifested by the D-function ratio appearing in Eq. (20).

We have projected out the $J=0$ partial wave in a simple model[18] containing the A_2 and ϕ trajectories in the t and s channels, respectively, and used the result which is depicted in Fig. 3 to calculate via Eq. (17) the D-function ratio occurring in Eq. (20). This exercise leads to an approximate relation

$$D^{-1}(0)D(t) \simeq \left(1-\frac{t}{4m_K^2}\right)^{-1/4}\left(1-\frac{at}{1+am_\rho^2}\right)^{-3/4}\left[\prod_{k=2}^{n-1}\left(1-\frac{at}{k+am_\rho^2}\right)^{-1}\right]\left(1-\frac{at}{n+am_\rho^2}\right)^{-1/2} \tag{35}$$

valid for $t < m_K^2$. (a is the universal Regge trajectory slope.) The general form of this function which is illustrated in Fig. 4 suggests a damping effect of the strong interactions for distant regions of the left-hand cut, as mentioned above.

A value for the parameter n in Eq. (35) which determines the asymptotic form of $D^{-1}(0)D(t)$ is related in this model to the intercept of the s-channel ϕ trajectory in a rather complicated manner.[19] The simple model we have considered predicts a very large value of n for physically realistic values of the ϕ trajectory parameters ($a \simeq 0.89$, $b \simeq 0.08$) and therefore effectively fails to give any prediction at all insofar as our hypothesis of two-particle unitarity already assumes the dominance of near-threshold effects. A value of $n \simeq 3$ follows from the assumption that the important region in the sense of dispersion theory for the strong rescattering effect is $(2m_K)^2 \lesssim t \lesssim (4m_K)^2$. The same value of n can be obtained from the Lovelace model by assuming $b \simeq 0.2$.

Using this model for $D^{-1}(0)D(t)$ we can evaluate Eq. (20) for each of the contributions listed in Eqs. (22), (25), (26), and (28)–(32). The results are

$$m_{K^+} - m_{K^0}|_{K\ \mathrm{pole}} = +0.82\ \mathrm{MeV}, \qquad m_{K^+} - m_{K^0}|_{Q(1250)} = +0.39\ \mathrm{MeV},$$

$$m_{K^+} - m_{K^0}|_{K\ \mathrm{contin}} = +0.13\ \mathrm{MeV}, \qquad m_{K^+} - m_{K^0}|_{Q(1400)} = +0.16\ \mathrm{MeV},$$

$$m_{K^+} - m_{K^0}|_{K^*} = -0.34\ \mathrm{MeV}, \qquad m_{K^+} - m_{K^0}|_{K_N(1420)} = -4.45\ \mathrm{MeV},$$

$$m_{K^+} - m_{K^0}|_{K_A(1242)} = +0.25\ \mathrm{MeV}.$$

Total: $m_{K^+} - m_{K^0} = -3.0$ MeV.

As we emphasized previously, the quantitatively relevant damping mechanism has been supplied in our calculation by the strong interactions. We list below some results for other choices of the parameter n which controls the damping strength in our model for strong rescattering effects (specifically, the $K_N(1420)$-exchange part, which is most sensitive numerically to strong damping effects):

n	$[m_{K^+} - m_{K^0}]_{K_N}$ (MeV)
4	−1.73
3.5	−2.84
3	−4.45
2.5	−7.47
2	−16.1

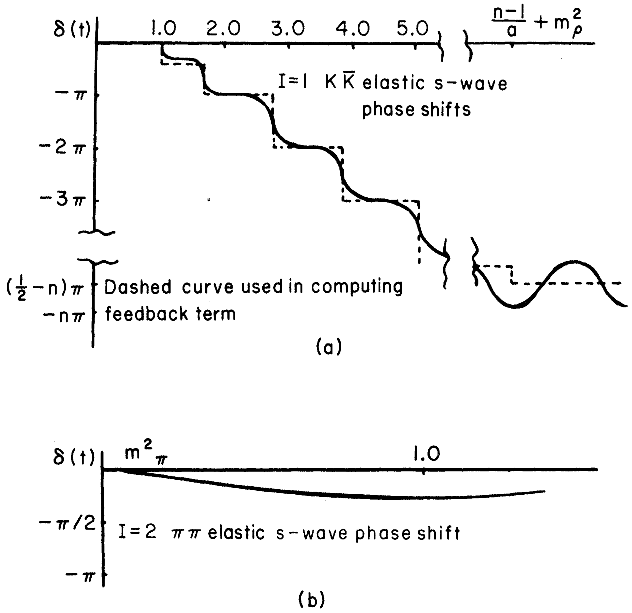

FIG. 3. Elastic scattering phase shifts determined from simple Veneziano models.

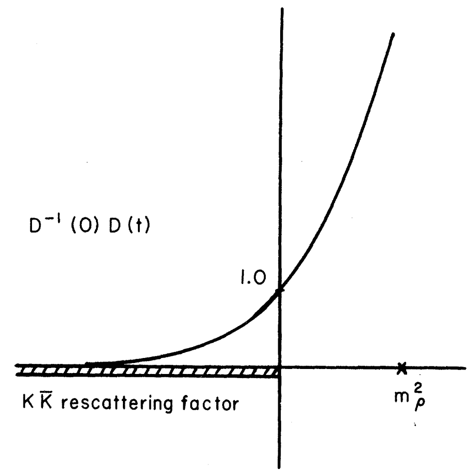

FIG. 4. Elastic rescattering factor.

We wish again to emphasize that when the damping factor n becomes small, the integral in Eq. (20) is determined increasingly by distant regions of the t axis, where we would expect higher-mass Regge recurrences to play an important and in fact dominant role. For $n=3$, about 65% of the total contribution comes from the region where we would also expect contributions from the first Regge recurrence of the $K_N(1420)$.

In our discussion of the quantitative aspects of the kaon mass-difference evaluation up to this point we have emphasized the simplifications brought about as a result of suppression of the region of large $-t$ in Eq. (20) by strong, t-channel rescattering effects. We may expect therefore that the analogous pion-mass-difference problem may prove to be less tractable insofar as the $I=2$ t-channel strong-interaction effects are small. We have in fact done a calculation of the D-function ratio for pions from a Veneziano model[18] with simultaneous s- and u-channel poles (we assume the absence of exotic t-channel resonances), and we find as expected that there is too little deviation from unity to be of any essential help in suppressing the high $-t$ region. Single-resonance exchange models for $\pi^+\pi^0\to\gamma\rho^+$ therefore lead to divergent expressions for the mass difference, and it is necessary to introduce *ad hoc* cutoffs to render the expressions finite. The individual contributions appear to be generally large (the ω and A_2 exchanges with reasonable cutoffs are both larger than the physical mass difference) and almost invariably of the wrong sign. Finally, there is no natural mechanism for suppressing higher-spin resonance exchanges, and it is entirely possible that in our approach such terms may be important or even the dominant contributions.

We have also attempted to construct dual resonance models for $\pi^+\pi^0\to\gamma\rho$, with the hope of obtaining convergent integral expressions analogous to Eq. (20). We find that models with simultaneous s- and u-channel exchanges generally lead to integrals which diverge exponentially. This is in sharp contrast with models containing simultaneous s- and t-channel or u- and t-channel poles, which may lead to convergent integral expressions, as in the case of the kaon mass-difference problem.

Thus, short of appealing to the presence of t-channel exotic mesons, we are unable to construct a dual resonance model for $\pi^+\pi^0\to\gamma\rho^+$ which would lead to a finite $\pi^+\pi^0$ mass difference. In connection with this point a number of authors have recognized that ordinary partial-wave dispersion relations with a finite number of subtractions may be violated in models with indefinitely rising Regge trajectories.[20] On the basis of these considerations we may conjecture that our dispersion relation (19) is valid only when the t channel has $I=1$, i.e., it is valid for the kaon but not for the pion problem.

V. CONCLUSIONS

We have outlined a method for calculating electromagnetic mass differences and applied it to the kaon and pion mass-difference problems. The resulting formal expression is given by Eq. (20). Although the reliability of our assumptions and approximations is far from certain, they do lead to a self-consistent explanation of the correct sign and approximate magnitude of the kaon mass difference. The major contribution comes from $K_N(1420)$ exchange; the kaon exchange term has the opposite sign and a magnitude smaller by a factor of 5; vector and axial-vector exchanges are small. Given the basic correctness of the calculation, a better number for the kaon mass difference could be obtained if reliable knowledge of the $I=1$ $K\bar{K}$ elastic s-wave phase shift was available.

Our attempts to apply the same technique to the pion mass difference have been unsuccessful. The individual exchange contributions appear to be large, cutoff-dependent, and generally of the wrong sign. Also, it is not possible to argue, as was done in Sec. IV for the kaon mass difference, that the strong rescattering corrections generate a D function which vanishes sufficiently rapidly for large $-t$ to restrict the effective integration region in Eq. (20) to the region of small $-t$, where the elementary-particle exchanges yield a good approximation for $\mathrm{Im}h^{\gamma}$.

The suggestion which emerges from these considerations is that the method of calculation pursued here may be appropriate for the octet parts of the electromagnetic mass differences, where the effects of the rescattering corrections are large – and that it is inapplicable, either in principle or in practice, for the parts which transform as the components of a 27-plet under SU(3). It is suggested, therefore, that the present scheme of calculation should be used to calculate $F_8(t)$ obtained from Eq. (2) by replacing the $K^+\bar{K}^0$ state by the octet combination of the $K^+\bar{K}^0$ and $\pi^+\eta$. To complete the determination of $m_{K^+}-m_{K^0}$ it would also be necessary to calculate $F_{27}(0)$, but this could be done with reasonable expectation of success by following the procedure of Cottingham.

We are presently attempting to recalculate $m_{K^+}-m_{K^0}$ from this alternative viewpoint. However, we do not expect to obtain numerical results which differ significantly from those which have been reported in this paper. In fact, it is fairly easy to see from the form of Eq. (18) that to the extent that both F and h^{γ} are pure octets the expression in Eq. (20) and the results of Secs. III and IV follow (except that the D function should be understood as the D function for the octet channel).

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[10]The presence of an additional term in Eq. (10) originating from possible strong isospin-violating forces which are comparable in magnitude to the electromagnetic forces does not materially affect our result. Matrix elements appearing in Eq. (9) contain photons which couple with strength e through the electromagnetic field. Terms containing the residual isospin-violating strong force will have an approximate magnitude α^2, and we may conscientiously neglect them in our calculation of order α.

[11]M. Gell-Mann and F. Zacharaisen, Phys. Rev. 124, 953 (1961).

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[13]Our D function (in contrast to the D function of Ref. 8) is defined without a possible polynomial factor. Such a factor would imply a zero of D and thus [see Eq. (18)] a pole in F which is not associated with an observed particle.

[14]The general boundaries for the integration region as a function of exchanged mass m^* are given by (see Fig. 2)

$$t_B=m_\rho{}^2-\tfrac{1}{2}(m^{*2}-m^2)\frac{m^{*2}-m^2+m_\rho{}^2}{m^{*2}}\times\left[1\mp\left(1-\frac{4m^{*2}m_\rho{}^2}{(m^{*2}-m^2+m_\rho{}^2)^2}\right)^{1/2}\right].$$

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[17]These values can be interpreted as upper limits for the g_{VAP} coupling. We are assuming that the only decay mode contributing to the pseudovector resonance width is $K_A \to K^* \pi$.

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[19]We have obtained an approximate relation between n and b based upon a geometrical, semiquantitative analysis of the expression

$$\tan\delta(t) = -\frac{f_\rho^2}{16\pi}\frac{1}{[t(t-4m_K^2)]^{1/2}}\frac{1}{\Gamma(\alpha(t))}\frac{\pi}{\sin\pi\alpha(t)} \times \int_{4m^2-t}^{0} ds\,\frac{\Gamma(1-\alpha_s(s))}{\Gamma(1-\alpha_s(s)-\alpha_t(t))},$$

where $\alpha(t)$ is the A_2 trajectory and $\alpha_\phi = as+b$ is the ϕ trajectory. The minimum value of $\delta(t)$ is $(\frac{1}{2}-n)\pi$. [See Fig. 3(a).] Our relationship is

$$x+(L+b)(2-L-b) = -\frac{b^2}{x}n^{(b-x)}\left[\frac{\Gamma(2+x)}{\Gamma(1+b)}\frac{(1+b/n)^{b+1/2}}{(1+x/n)^{x+1/2}}\right].$$

The bracketed term is nearly unity and can be neglected to first approximation.

$$L \equiv 4am_K^2 - a\,m_\rho^2$$

$$x \equiv \frac{-[2+\ln(\frac{1}{2}n)] + \{[2+\ln(\frac{1}{2}n)]^2 - 4\ln(\frac{1}{2}n)\}^{1/2}}{2\ln(\frac{1}{2}n)}.$$

Specific values of n vs b are obtained numerically from these highly nonlinear relations. Strictly speaking, these relations are valid only in the limit of large n.

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Spontaneously Broken Gauge Theories of Weak Interactions and Heavy Leptons*

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Branching ratios and production cross sections are calculated for the heavy leptons which occur in a class of spontaneously broken gauge theories of weak interactions. Several examples of such theories are constructed.

I. INTRODUCTION

The recent developments in unified gauge theories of weak and electromagnetic interactions[1-7] have already been fruitful in focusing attention on the experimental question of the existence of leptonic[8] and hadronic[9] neutral currents. Such currents arise because in some models[1,2,7] a neutral heavy boson Z^0 must exist in addition to charged intermediate bosons $W^{\pm}$. In other models,[4-6] no neutral currents are needed, but additional heavy leptons are required (along, probably, with "charmed" heavy hadrons as well). It is probable that in any renormalizable theory of weak and electromagnetic interactions either neutral Z's or heavy leptons, or both, will be required. This assertion gains credibility when one considers the process $e^+e^- \to W^+W^-$, which proceeds via the diagrams of Fig. 1.

The high-energy behavior of this amplitude in the $J=1$ partial wave violates the unitarity condition.[10] In a renormalizable theory with small coupling constants, phase shifts must not grow large, except near narrow resonances. In the present case, there appears to be no alternative to large phase shifts other than introduction of additional particle-exchange poles into the amplitude, as in Fig. 2. The s-channel poles have the quantum numbers of the Z^0, and t- or u-channel poles have the quantum numbers of neutral or doubly charged heavy leptons, probably with spin $\frac{1}{2}$ (in order to keep higher-order processes renormalizable).

Thus, most renormalizable theories will contain heavy leptons, and in any case it is of interest to understand the phenomenology of such particles. It is the purpose of this paper to outline observable consequences of the existence of such heavy leptons in the context of these renormalizable gauge theories. The particles we consider are E^+ and E^0 (M^+ and M^0), $J=\frac{1}{2}$ fermions with the same lepton number assignment as the e^- (μ^-). In Sec. II we consider the decay modes of such particles, and in Sec. III we discuss their production. We