$$\cos[\lim_{s\to+\infty}\delta(s)]=0$$

implies

$$\lim_{s\to+\infty}\delta(s)=\sigma=-\tfrac{1}{2}\pi\ .$$

Then by (A6) $0\leqslant\tau-\sigma\leqslant\pi$, which is inconsistent with (II). Therefore $\tau-\sigma=\pi$ and from (A4) we get $\alpha(s)\sim s$. This result along with the assumed linearity of the real part implies $\mathrm{Im}\alpha(s)\lesssim s$.

These conclusions justify the fact that we have written once-subtracted dispersion relations in (2) and (3) of Sec. II.

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[4]The presence of poles is obviously excluded because $\alpha(s)$ must be bounded in the finite s plane. Otherwise the t-channel amplitude $A(s,t)\simeq t^{\mathrm{Re}\alpha(s)}$ would not be polynomially bounded, which contradicts very general properties.

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[6]P. D. B. Collins and E. J. Squires, in *Regge Poles in Particle Physics*, edited by G. Höhler (Springer, Berlin, 1968), p. 73.

[7]It is well known from the theory of integral equations that if the square integrability of the solution is not required (i.e., if we allow $\epsilon=0$ exactly) then the solution of the integral equation (12) is not unique. Here we determine the unique L^2 solution which will be seen to be physically acceptable because it gives rise to reasonable widths.

[8]B. R. Desai, P. Kaus, R. T. Park, and F. Zachariasen, Phys. Rev. Letters 25, 1389 (1970); 25, 1686(E) (1970).

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Pion-Deuteron Scattering at High Energies*

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Glauber theory, including the complications of spin and isospin, is applied to high-energy πd scattering. We compare theoretical predictions with recent experimental results on total and differential elastic cross sections. Particular attention is given to the suggestion of new experiments which bear on possible modifications of the Glauber picture.

I. INTRODUCTION

The classic picture of high-energy hadron-nucleus collisions developed by Glauber[1] has by now reached a high degree of sophistication, and is well known to account for the basic features observed experimentally.[2] At the same time, Glauber theory has yet to be given a justification in terms of quantum field theory in the cloak of Feynman diagrams,[3] and periodic suggestions are made for its improvement. The history of the subject discourages the hope that a swift theoretical conclusion might be forthcoming, so we have undertaken a benchmark calculation of total and elastic differential cross sections for $\pi^\pm d$ scattering in terms of the "best available" pion-nucleon scattering amplitudes and deuteron wave functions. In so doing we have been able to make quantitative comparisons with recently published data and to recognize the need for additional experimental studies.

Our calculations of elastic differential cross sections represent in part merely an updating of earlier work by Michael and Wilkin[4] and by Alberi and Bertocchi.[5] In addition we treat briefly some modifications to Glauber theory proposed lately by Cheng and Wu.[6] A comparison of our calculated total cross sections with data obtained at Serpukhov allows us to draw some tentative conclusions about the inelastic screening corrections suggested by Gribov.[7] By means of a simple example we assess the plausible magnitude of such contributions and refute some misconceptions promulgated by Gurvits and Marinov.[8]

In Sec. II we present the formalism used in the computations. Thereafter we treat in turn total cross sections (Sec. III) and elastic differential cross sections (Sec. IV). The work is summarized in Sec. V.

II. FORMALISM FOR πd SCATTERING

In general we have followed the computational procedure outlined by Michael and Wilkin,[4] but in order to correct some minor errors and misprints in their work, we shall give a full description here.

A. Model of the Deuteron

The deuteron wave function of spin projection M is

$$\Phi_M(r)=\frac{u(r)}{r}Y_{00}(\hat r)\langle \tfrac{1}{2}\tfrac{1}{2}m_1 m_2|1M\rangle\chi^p_{m_1}\chi^n_{m_2}$$
$$+\frac{w(r)}{r}Y_{2,M-m_1-m_2}(\hat r)\langle 2\,1\,M-m_1-m_2\;m_1+m_2|1M\rangle\langle\tfrac{1}{2}\tfrac{1}{2}m_1m_2|1\,m_1+m_2\rangle\chi^p_{m_1}\chi^n_{m_2}, \qquad (1)$$

where $\chi^p_{m_1}$ and $\chi^n_{m_2}$ are proton and neutron Pauli spinors of projection m_1 and m_2, and a summation over m_1 and m_2 is implicit. The S-wave and D-wave radial wave functions are chosen real and normalized by

$$\int_0^\infty dr\,(u^2+w^2)=1\,. \qquad (2)$$

All realistic models of the deuteron contain 6–7% D-wave probability to reproduce the finite quadrupole moment of the deuteron. We have investigated the following three models:

(1) Humberston's model.[9] In this refinement of the Hamada-Johnston potential[10] the radial wave functions are parametrized as

TABLE I. Parameters for Humberston's wave functions.

$\mu=0.7082$ F^{-1} $\alpha=0.2317$ F^{-1} $r_c=0.343/\mu$	
$\delta=5.4$ F^{-1}	$\rho=3.6$ F^{-1}
$c_1=0.88515$ F$^{-1/2}$	$d_1=0.02339$ F$^{-1/2}$
$c_2=-0.07080$ F$^{-1/2}$	$d_2-0.05448$ F$^{-1/2}$
$c_3=-0.86471$ F$^{-1/2}$	$d_3=0.01663$ F$^{-1/2}$
$c_4=0.05132$ F$^{-1/2}$	$d_4=0.03774$ F$^{-1/2}$
$c_5=-0.65931$ F$^{-1/2}$	$d_5=-0.01139$ F$^{-1/2}$
$c_6=-10.462$ F$^{-1/2}$	$d_6=-0.01580$ F$^{-1/2}$
$c_7=26.243$ F$^{-1/2}$	
$c_8=-16.124$ F$^{-1/2}$	

$$u(r)=e^{-\alpha r}(1-e^{-\delta(r-r_c)})\sum_{l=1}^{8}c_l e^{-(l-1)\mu r},$$
$$w(r)=e^{-\alpha r}\left(1+\frac{3}{\alpha r}+\frac{3}{\alpha^2 r^2}\right)(1-e^{-\rho(r-r_c)})\times\sum_{l=1}^{6}d_l e^{-(l-1)\mu r}. \qquad (3)$$

Both functions vanish for $r<r_c$. The parameters have the values given in Table I. These wave functions yield a quadrupole moment $Q=0.2844$ F^2, and D-wave probability $P_D=6.953\%$.

(2) Reid's models.[11] In these models, the deuteron potential is assumed to vanish beyond $\mu r=x_p$, where $\mu=0.7$ F^{-1}. The wave functions for a hard-core (HC) and a soft-core (SC) model are tabulated in Ref. 11. In Table II we list the cutoff and observables for each model.

Results obtained from our computations differed only slightly from one of these wave functions to the next. We shall therefore report detailed results only for Reid's hard-core model, which is, according to the experts,[12] the most realistic.

TABLE II. Parameters of Reid's models.

	HC	SC
x_p	10.38383	10.01
Q (F^2)	0.27700	0.27964
P_D (%)	6.4970	6.4696

B. Single-Scattering Amplitudes

It is convenient to work in the laboratory frame, in which the pion-nucleon amplitudes take the form

$$F_{\pi N}(\vec{q}) = a(\vec{q}) + \vec{\sigma}\cdot\vec{q}\times\hat{k}b(\vec{q}), \tag{4}$$

where the incident pion momentum $\vec{k}$ defines the y direction[13] and $\vec{q}$ is the momentum transfer. The impulse approximation to πd scattering is given by

$$T^S_{M'M}(\vec{q}) = \int d^3r\, \Phi^*_{M'}(r)[F_{\pi p}(\vec{q}) + F_{\pi n}(\vec{q})]\, e^{i\vec{q}\cdot\vec{r}/2}\Phi_M(r). \tag{5}$$

If we let $\vec{q}$ lie along the z axis, then

$$e^{i\vec{q}\cdot\vec{r}/2} = \sum (2l+1) i^l j_l(\tfrac{1}{2}qr) P_l(\cos\theta). \tag{6}$$

Inserting (6) and (1) into (5), we find

$$\begin{aligned}
T^S_{11} &= (a_{\pi p} + a_{\pi n})[\phi_a(\tfrac{1}{2}q) + \phi_b(\tfrac{1}{2}q) - \sqrt{2}\,\phi_c(\tfrac{1}{2}q) + \tfrac{1}{2}\phi_d(\tfrac{1}{2}q)],\\
T^S_{00} &= (a_{\pi p} + a_{\pi n})[\phi_a(\tfrac{1}{2}q) + \phi_b(\tfrac{1}{2}q) + 2\sqrt{2}\,\phi_c(\tfrac{1}{2}q) - \phi_d(\tfrac{1}{2}q)],\\
T^S_{10} &= [q(b_{\pi p} + b_{\pi n})/\sqrt{2}\,][-\phi_a(\tfrac{1}{2}q) + \tfrac{1}{2}\phi_b(\tfrac{1}{2}q)\\
&\qquad - 2\sqrt{2}\,\phi_c(\tfrac{1}{2}q) - \tfrac{1}{2}\phi_d(\tfrac{1}{2}q)],\\
T^S_{1,-1} &= 0,
\end{aligned} \tag{7}$$

where we have defined the form factors

$$\begin{aligned}
\phi_a(\tfrac{1}{2}q) &= \int_0^\infty dr\, j_0(\tfrac{1}{2}qr)|u(r)|^2,\\
\phi_b(\tfrac{1}{2}q) &= \int_0^\infty dr\, j_0(\tfrac{1}{2}qr)|w(r)|^2,\\
\phi_c(\tfrac{1}{2}q) &= \int_0^\infty dr\, j_2(\tfrac{1}{2}qr)u(r)w(r),\\
\phi_d(\tfrac{1}{2}q) &= \int_0^\infty dr\, j_2(\tfrac{1}{2}qr)|w(r)|^2,\\
\phi_e(\tfrac{1}{2}q) &= \int_0^\infty dr\, j_4(\tfrac{1}{2}qr)|w(r)|^2.
\end{aligned} \tag{8}$$

The remaining πd scattering amplitudes may be expressed in terms of those in (7) by means of the relations

$$T_{M'M} = T_{MM'} = T_{-M',-M}, \tag{9}$$

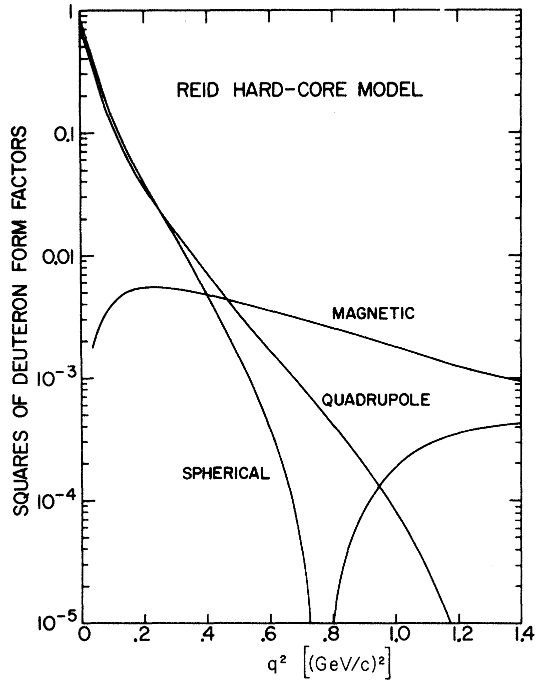

FIG. 1. Squares of deuteron form factors given by Reid's hard-core model (Ref. 11), which was used in the detailed computations reported.

which follow from parity conservation and time-reversal invariance.

Finally, it is enlightening to define spherical, quadrupole, and magnetic form factors as

$$\begin{aligned}
\phi_S &= \phi_a + \phi_b,\\
\phi_Q &= 2\phi_c - \tfrac{1}{2}\sqrt{2}\,\phi_d,\\
\phi_M &= \phi_a - \tfrac{1}{2}\phi_b + \tfrac{1}{2}\sqrt{2}\,\phi_c + \tfrac{1}{2}\phi_d,
\end{aligned} \tag{10}$$

in terms of which we may write the unpolarized differential cross section (in impulse approximation) as

$$\begin{aligned}
\frac{d\sigma}{d\Omega_{\text{lab}}} &= \tfrac{1}{3}\sum_{M,M'} |T_{M'M}|^2\\
&= |a_{\pi p} + a_{\pi n}|^2[\phi_S{}^2(\tfrac{1}{2}q) + \phi_Q{}^2(\tfrac{1}{2}q)]\\
&\quad + \tfrac{2}{3}q^2|b_{\pi p} + b_{\pi n}|^2\phi_M{}^2(\tfrac{1}{2}q).
\end{aligned} \tag{11}$$

The squares of the form factors (10) are plotted in Fig. 1.

C. Double-Scattering Amplitudes

The contribution of double scattering to the elastic scattering amplitude is given by

$$T^D_{M'M} = \frac{i}{4\pi k}\int d^2q'\, \mathcal{T}_{M'M}(\vec{q}, \vec{q}'), \tag{12}$$

where

$$\mathcal{T}_{M'M}(\vec{q},\vec{q}\,')=\int d^3r\,\Phi^*_{M'}(r)e^{i\vec{q}\,'\cdot\vec{r}}[F_{\pi n}(\tfrac{1}{2}\vec{q}-\vec{q}\,')F_{\pi p}(\tfrac{1}{2}\vec{q}+\vec{q}\,')+F_{\pi p}(\tfrac{1}{2}\vec{q}-\vec{q}\,')F_{\pi n}(\tfrac{1}{2}\vec{q}+\vec{q}\,')]\,\Phi_M(r)\,. \tag{13}$$

The loop momentum $\vec{q}\,'$ is confined to the transverse (x-z) plane, and may be parametrized as

$$\vec{q}\,'=\begin{cases} q'(\sin\alpha, 0, \cos\alpha), & q'_x>0 \\ q'(-\sin\alpha, 0, \cos\alpha), & q'_x<0 \end{cases} \tag{14}$$

where the polar angle α runs between 0 and π. [In Ref. 4, α was apparently integrated from 0 to 2π. Only minor changes result when the proper expression is evaluated.] After dropping terms which vanish in the integration over α, we have the following explicit forms:

$$\begin{aligned}\mathcal{T}_{11}=&\ \phi_a(q')(A-q'^2B\sin^2\alpha)+\phi_b(q')[A+B(\tfrac{3}{40}q^2-\tfrac{3}{10}q'^2-\tfrac{1}{10}q'^2\sin^2\alpha)]\\ &-\phi_c(q')\sqrt{2}\,[AP_2(\cos\alpha)+B(\tfrac{3}{8}q^2+\tfrac{1}{2}q'^2)\sin^2\alpha]\\ &+\phi_d(q')[\tfrac{1}{2}AP_2(\cos\alpha)+(\tfrac{3}{112}q^2B)(7\sin^2\alpha-2)+(\tfrac{1}{28}q'^2B)(6+7\sin^2\alpha)]\\ &+\phi_e(q')15B\sqrt{4\pi}\,\{(\tfrac{1}{4}q^2-q'^2\cos^2\alpha)[2Y_{4,2}(\alpha,0)/35\sqrt{10}-\tfrac{2}{175}Y_{4,0}(\alpha,0)]-\tfrac{4}{175}q'^2\sin^2\alpha\ Y_{4,0}(\alpha,0)\\ &\qquad+2q'^2\sin2\alpha\ Y_{4,-1}(\alpha,0)/35\sqrt{5}\,\},\end{aligned} \tag{15}$$

$$\begin{aligned}\mathcal{T}_{00}=&\ \phi_a(q')\{A+B[\tfrac{1}{4}q^2+q'^2(1-2\cos^2\alpha)]\}\\ &+\phi_b(q')\{A+B[\tfrac{1}{10}q^2-\tfrac{1}{5}q'^2(1+\cos^2\alpha)]\}\\ &+\phi_c(q')\sqrt{2}\,\{2AP_2(\cos\alpha)+B[\tfrac{1}{2}q^2P_2(\cos\alpha)-q'^2(1+\cos^2\alpha)]\}\\ &+\phi_d(q')\{-AP_2(\cos\alpha)+B[q^2(21\sin^2\alpha-8)/58+\tfrac{1}{14}q'^2(1+7\cos^2\alpha)]\}\\ &+\phi_e(q')15B\sqrt{4\pi}\,\{(q^2/4-q'^2\cos^2\alpha)[-2\sqrt{2}\,Y_{4,2}(\alpha,0)/35\sqrt{5}+\tfrac{4}{175}Y_{4,0}(\alpha,0)]\\ &\qquad-\tfrac{8}{175}q'^2\sin^2\alpha\ Y_{4,0}(\alpha,0)-4q'^2\sin2\alpha\ Y_{4,-1}(\alpha,0)/35\sqrt{5}\,\},\end{aligned} \tag{16}$$

$$\begin{aligned}\mathcal{T}_{10}=&\ \phi_a(q')[-qG+2q'\cos\alpha\,H]+\phi_b(q')[\tfrac{1}{2}qG-q'\cos\alpha\,H]\\ &+\phi_c(q')\sqrt{2}\,[-\tfrac{1}{2}q(1-3\sin^2\alpha)G+q'\cos\alpha H]+\phi_d(q')[-\tfrac{1}{2}q(1-3\sin^2\alpha)G+q'\cos\alpha\,H],\end{aligned} \tag{17}$$

$$\begin{aligned}\mathcal{T}_{1,-1}=&\ \phi_a(q')[B(\tfrac{1}{4}q^2-q'^2\cos^2\alpha)]+\phi_b(q')[B(\tfrac{1}{4}q^2-\tfrac{1}{10}q'^2\cos\alpha)]\\ &+[\phi_c(q')/\sqrt{2}\,]\{-3A\sin^2\alpha+B[-\tfrac{1}{2}q^2P_2(\cos\alpha)+q'^2(3-\cos^2\alpha)]\}\\ &+\phi_d(q')\{\tfrac{3}{4}A\sin^2\alpha+\tfrac{1}{28}B[\tfrac{1}{4}q^2(15\sin^2\alpha-4)+q'^2(7\cos^2\alpha-3)]\}\\ &+\phi_e(q')15B\sqrt{4\pi}\,\{(\tfrac{1}{4}q^2-q'^2\cos^2\alpha)[\tfrac{1}{175}Y_{4,0}(\alpha,0)-\sqrt{2}\,Y_{4,2}(\alpha,0)/35\sqrt{5}+\sqrt{2}\,Y_{44}(\alpha,0)/5\sqrt{35}]\\ &\qquad-4q'^2\sin^2\alpha\ Y_{4,2}(\alpha,0)/35\sqrt{10}-q'^2\sin2\alpha\,[Y_{4,-1}(\alpha,0)/35\sqrt{5}-Y_{4,-3}(\alpha,0)/5\sqrt{35}\,]\}.\end{aligned} \tag{18}$$

In Eqs. (15)–(18) we used the symbols

$$A=2[a_{\pi p}(\tfrac{1}{2}q+q')a_{\pi n}(\tfrac{1}{2}q-q')+(p\leftrightarrow n)], \tag{19}$$

$$B=2[b_{\pi p}(\tfrac{1}{2}q+q')b_{\pi n}(\tfrac{1}{2}q-q')+(p\leftrightarrow n)], \tag{20}$$

$$G=[a_{\pi p}(\tfrac{1}{2}q+q')b_{\pi n}(\tfrac{1}{2}q-q')+b_{\pi p}(\tfrac{1}{2}q+q')a_{\pi n}(\tfrac{1}{2}q-q')+(p\leftrightarrow n)]/\sqrt{2}, \tag{21}$$

$$H=[a_{\pi p}(\tfrac{1}{2}q+q')b_{\pi n}(\tfrac{1}{2}q-q')-b_{\pi p}(\tfrac{1}{2}q+q')a_{\pi n}(\tfrac{1}{2}q-q')+(p\leftrightarrow n)]/\sqrt{2}. \tag{22}$$

D. Amplitudes for Pion-Nucleon Scattering

In our calculations we have used the parametrization of the invariant amplitudes $A'^{(\pm)}$ and $B^{(\pm)}$ obtained by Barger and Phillips[14] in their five-pole fit to the pion-nucleon scattering data. This Regge-pole model, determined by simultaneous analysis of high-energy data and continuous moment sum rules, provides an excellent description of the sub-30 GeV/c cross section and polarization data. Moreover, the amplitude structure in the intermediate energy range has been confirmed by the recent amplitude analyses performed at 6 GeV/c.[15] At energies above 30 GeV, the Barger-Phillips amplitudes seriously underestimate the total cross sections measured at Serpukhov,[16,17] as we indicate in Fig. 2. This flaw will obviously damage our ability to make

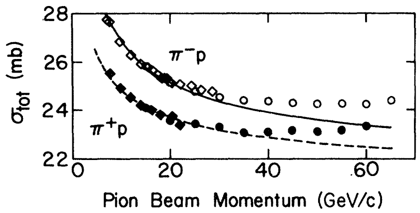

FIG. 2. Comparison of the Barger-Phillips five-pole parametrization (Ref. 14) with high-energy data on pion-nucleon total cross sections. The π^-p points are taken from Ref. 18 (◇) and Ref. 17 (○); the π^+p points from Ref. 18 (◆) and Ref. 16 (●).

predictions for πd scattering in the Serpukhov energy regime.

The invariant amplitudes are related to the laboratory amplitudes a and b by

$$a=(1-t/4M^2)^{1/2}A'/4\pi J^{1/2}, \tag{23}$$

$$b=(1-t/4M^2)^{-1/2}ikB/8\pi MJ^{1/2}, \tag{24}$$

where M is the nucleon mass, k is the laboratory momentum of the incident pion, and

$$J\equiv\frac{k^2}{\pi}\left|\frac{\partial\Omega_{\rm lab}}{\partial t}\right|=\frac{k^2}{p^{*2}}\left|\frac{\partial\cos\theta_{\rm lab}}{\partial\cos\theta^*}\right|. \tag{25}$$

In Eq. (25), p^* is the c.m. momentum of the incident pion.

III. TOTAL CROSS SECTIONS

The $\pi^\pm d$ system is particularly well suited to a critical experimental study of the Glauber theory because (under the assumption of charge symmetry) the component πp and πn processes may be studied individually in experiments using hydrogen targets. However, the fact that $\pi^\pm n$ cross sections may be measured cleanly in $\pi^\mp p$ interactions has apparently encouraged neglect of the πd system at energies below 30 GeV where the $\pi^\pm p$ total cross sections are precisely known. The results presented in this section indicate that high-precision measurements of πd total cross sections in the 5 to 30 GeV energy range are prerequisite to refinement or confirmation of the Glauber picture.

A. "Conventional" Glauber Theory

The total cross section for πd scattering is given by

$$\sigma_t(\pi d)=\frac{4\pi}{3k}\mathrm{Im}\left(\sum_M T_{MM}(t=0)\right), \tag{26}$$

which may be recast as

$$\sigma_t(\pi d)=\sigma_t(\pi p)+\sigma_t(\pi n)-\delta\sigma, \tag{27}$$

with

$$\delta\sigma=-\frac{4\pi}{3k}\mathrm{Im}\left(\sum_M T^D_{MM}(t=0)\right). \tag{28}$$

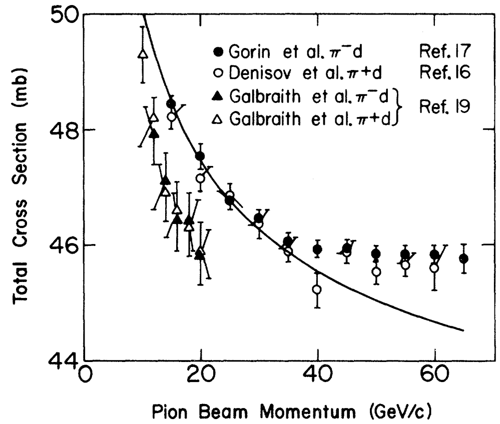

FIG. 3. Prediction for $\pi^\pm d$ total cross sections is compared with the high-energy data. Notice that whereas the $\pi^\pm p$ total cross sections shown in Fig. 2 are essentially constant between 30 and 60 GeV/c, the $\pi^\pm d$ cross sections continue to decrease.

Our calculation, based on the pion-nucleon scattering amplitudes of Barger and Phillips,[14] is compared with the existing high-energy data in Fig. 3. For energies below about 30 GeV, where the πN amplitudes are known to be reliable, the theoretical curve is in excellent agreement with the recent measurements carried out at Serpukhov.[16,17] The earlier measurements by Galbraith *et al.*[19] lie systematically lower than the calculated values. This is consistent with the observation that the $\pi^\pm p$ total cross sections reported in Ref. 19 lie below those quoted in Ref. 18, to which the Barger-Phillips amplitudes are closely tied.

Above 30 GeV, the situation is less satisfactory. As we should expect from Fig. 2, the theoretical curve falls below the experimental points. However, unlike the $\pi^\pm p$ cross sections, which remain roughly constant above 30 GeV, the πd cross sections continue to fall. To the extent that the πp cross sections are constant, the decrease of the πd cross sections must be laid to increasing screening corrections. Gorin *et al.*[17] determined the amount of screening directly from the high-energy data as

$$\delta\sigma=\sigma_t(\pi^+p)+\sigma_t(\pi^-p)-\tfrac{1}{2}[\sigma_t(\pi^+d)+\sigma_t(\pi^-d)]. \tag{29}$$

Their results are shown in Fig. 4 together with the corresponding data from Ref. 19. The (solid) theoretical curve in Fig. 4 reflects the decreasing $\pi^\pm p$ cross sections given by the Barger-Phillips parametrization. The measured screening corrections are clearly increasing with energy; the

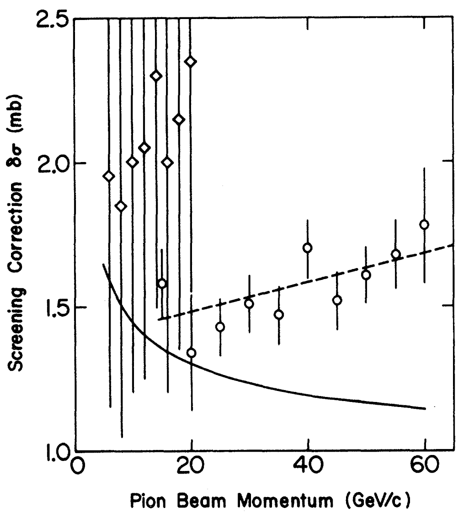

FIG. 4. Experimental results for the screening correction $\delta\sigma$ are shown together with our calculation (solid curve). Data are from Ref. 19 (◇) and Ref. 17 (○). The dashed line is a best fit of the form $A + Bp^C$ to the Serpukhov data.

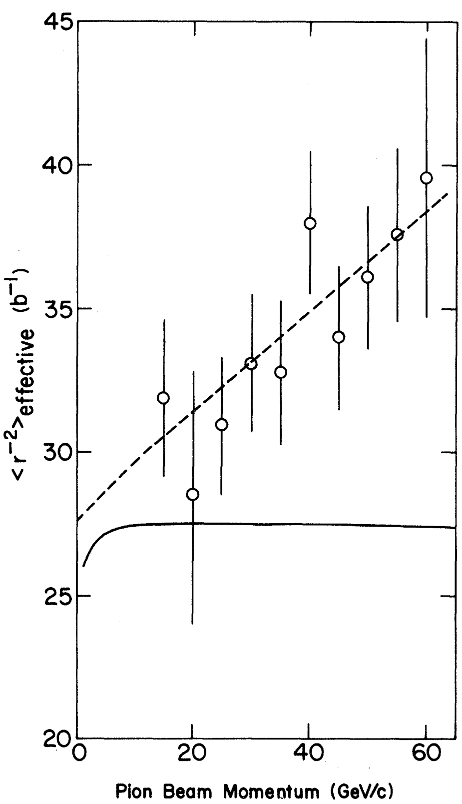

FIG. 5. Experimental results for the effective deuteron radius are compared with the results of our calculation (solid curve). Data are from Ref. 17. The dashed line is a best fit of the form $A + Bp^C$ to the data.

Serpukhov points are well fitted by the form

$$\delta\sigma = \{1.39 + 0.004[p_{\text{lab}}/(1\ \text{GeV}/c)]^{1.05}\}\ \text{mb}$$

(the dashed line in Fig. 4). Even if we were to hold the theoretical prediction constant above 30 GeV, the gap between theory and experiment would continue to grow.

This may be seen in another way in Fig. 5. The magnitude of screening corrections can be characterized by an effective deuteron radius, defined through[20]

$$\langle r^{-2}\rangle_{\text{effective}} = \frac{4\pi\delta\sigma}{\sigma_t(\pi^+p)\sigma_t(\pi^-p)}. \tag{30}$$

In the high-energy regime, conventional Glauber theory yields a roughly constant value for $\langle r^{-2}\rangle_{\text{effective}}$ (solid curve in Fig. 5), whereas the Serpukhov data imply that $\langle r^{-2}\rangle_{\text{effective}}$ is an increasing function of momentum (dotted curve) which is well fitted by the form

$$\langle r^{-2}\rangle_{\text{effective}} = \{27.6 + 0.225[p_{\text{lab}}/(1\ \text{GeV}/c)]^{0.95}\}\ \text{b}^{-1}.$$

Taken literally, the Serpukhov results seem to indicate the presence of additional screening corrections, above and beyond those predicted by Glauber theory.

B. Inelastic Screening Contributions

Gribov[7] has argued that for incident momenta $\gtrsim 10$ GeV/c the screening effect should undergo a marked change as inelastic rescattering becomes competitive with the elastic rescattering responsible for the Glauber screening correction. Similar suggestions have been made by Pumplin and Ross,[21] by Alberi and Bertocchi,[22] and by Harrington.[23] Estimates of the effect of inelastic rescattering on the screening correction have been made by Gurvits and Marinov[8] who based their work on experimental diffraction dissociation data and by Kancheli and Matinyan[24] who used the triple-Regge techniques introduced by Kancheli.[25] In the case of inelastic quasi-two-body reactions, detailed calculations of the absorptive corrections due to inelastic intermediate states have been undertaken by the Michigan group[26] and Ter-Martirosyan and collaborators.[27] The merit of strong-absorption models remains a topic of contention.

Encouraged by the new data,[17] we have made an estimate of the inelastic screening correction to the πd total cross section more quantitative than those published previously. The rescattering corrections we consider are indicated in Fig. 6: We include as intermediate states all those multipion states which may be reached from the incident pion by diffractive excitation. Neglecting spin and as-

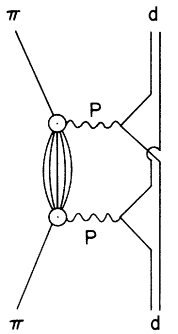

FIG. 6. Diagram responsible for inelastic rescattering corrections in a diffractive excitation model.

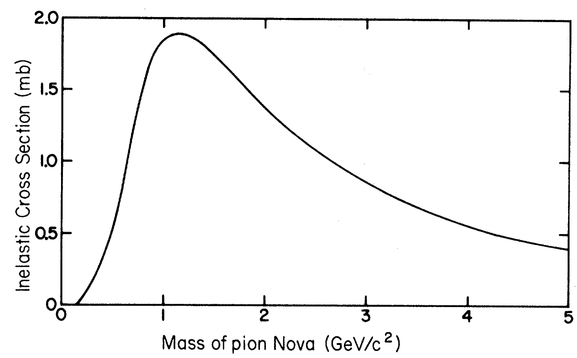

FIG. 7. Inelastic cross section as a function of the mass of the pion nova.

suming all diffractive amplitudes to be purely positive imaginary, we may write the inelastic screening correction as[7]

$$\delta\sigma_{\text{inelastic}} \approx 2\sum_l \int dt\,\rho(4t)\,d\sigma_l/dt\,, \qquad (31)$$

where $\rho(t)$ is the deuteron form factor and $d\sigma_l/dt$ is the differential cross section for production of the lth intermediate state. For simplicity we parametrize the differential cross sections as exponentials:

$$d\sigma_l/dt = A_l\sigma_l \exp\{A_l[t - t_l(p_{\text{lab}})]\}\,, \qquad (32)$$

where t_l is the minimum squared momentum transfer required to produce the state l from an incident beam of momentum p_{lab}, and A_l is the slope of the differential cross section. Approximating the deuteron form factor by an exponential as well, $\rho(t) = e^{at/4}$, we may simplify (31) to

$$\delta\sigma_{\text{inelastic}} \approx 2\sum_l A_l\sigma_l(a + A_l)^{-1}\exp[at_l(p_{\text{lab}})]\,, \qquad (33)$$

which is a form suitable for computation.

We take as intermediate states all channels containing an odd number of pions (>1) and assign them the cross sections suggested by the "nova model" for inclusive distributions.[28] Whether or not it resembles an ultimate theory for inclusive reactions, the nova model does provide a successful parametrization for single-particle spectra. Labeling the inelastic channels by the number of pions produced in addition to the beam pion (thus a positive even integer), we may write[28]

$$\sigma_l \propto e^{-4.2/l}/l^2\,. \qquad (34)$$

The mass of the $(l+1)$-pion state is taken as

$$M(l) = (l/2.1)\ \text{GeV}/c^2 + m_\pi\,. \qquad (35)$$

Finally, Jacob and Slansky[28] have deduced that the inelastic cross section contributed by pion novas, $\sum_l \sigma_l$, should be approximately 7 mb. The nova spectrum thus normalized is shown in Fig. 7. We characterize the deuteron form factor by the slope $a = 43\ (\text{GeV}/c)^{-2}$; results are insensitive to the precise value.

The choice of a slope A_l to characterize the differential cross section for nova production is more problematical, for it is not a parameter known (or for that matter, directly measurable) experimentally. To simplify our model still further, we chose $A_l = A$, independent of l. On the basis of the trends present in existing data on diffractive production,[29] we have taken the value $A = 2.5\ (\text{GeV}/c)^{-2}$ as representative of the entire mass spectrum. Unfortunately $\delta\sigma_{\text{inelastic}}$ is roughly proportional to A, so the magnitude of the inelastic screening corrections is sensitive to the value chosen. We would, however, find it difficult to entertain more than a factor of 2 uncertainty from this source.

The result of our model calculation is shown in Fig. 8. The energy dependence is in agreement

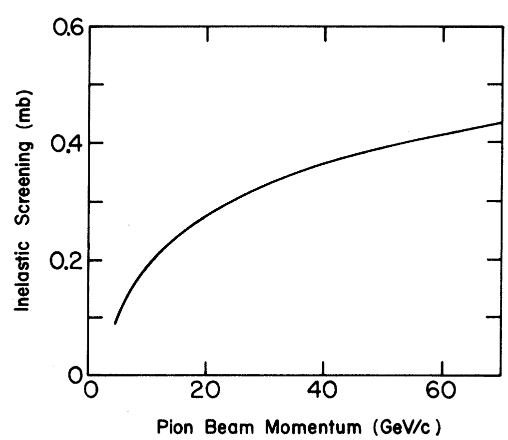

FIG. 8. The additional screening corrections arising from inelastic rescattering for the model discussed in the text. Compare the experimental results displayed in Fig. 4.

with the qualitative discussion given by Kancheli and Matinyan,[24] and is similar to the energy dependence of what we tentatively identified as an inelastic screening in the data of Fig. 4. The energy dependence strongly disagrees with the expectations of Gurvits and Marinov,[8] who predicted that inelastic rescattering effects should diminish above 20 GeV/c. Their conclusion was based on the identification of a decreasing cross section as "diffractive," and hence with a purely imaginary amplitude, in conflict with the Phragmén-Lindelöf theorem, and need not be taken seriously. As we have already observed, the magnitude of the inelastic screening we find should be regarded only as illustrative. It is, however, in rough agreement with the implications of Fig. 4.

Our crude estimate indicates that the idea of inelastic rescattering contributions should indeed be taken seriously. Precise measurements of $\delta\sigma$ in the 5 to 30 GeV/c momentum range are urgently needed to refine the conclusions tentatively drawn from the Serpukhov data.

IV. ELASTIC SCATTERING

Our calculations of the differential cross sections for elastic πd scattering are confronted with the data of the CERN-Trieste Group[30] at 9.0, 13.0, and 15.2 GeV/c in Figs. 9, 10, and 11. In the single-scattering regime $[-t < 0.3\ (\mathrm{GeV}/c)^2]$ there is excellent quantitative agreement between theory and experiment. Also, in the region of the break $[0.3 < -t < 0.5\ (\mathrm{GeV}/c)^2]$, where the cross section is determined by the contribution of the quadrupole form factor,[4,5,31,32] the theory provides a good description of the data. However, at larger angles, in the double-scattering regime, there appears a systematic tendency for the calculated curve to lie above the data by an amount appreciably more than the ±20% uncertainty in experimental normalization. The discrepancy seems most pronounced at the lowest energy. Our results and conclusions do not differ essentially from those reported earlier, in Refs. 4, 5, and 30, which were based on other

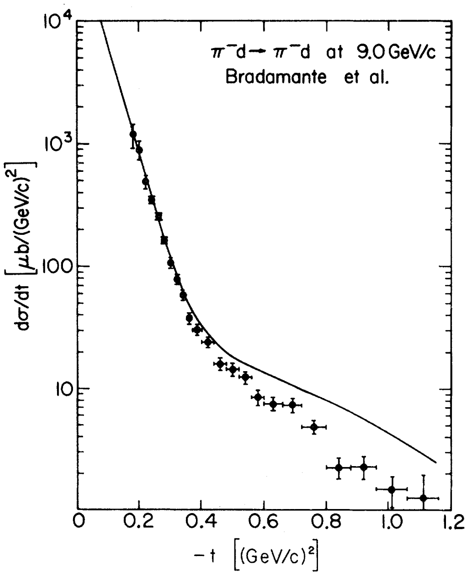

FIG. 9. Calculated differential cross section for $\pi^- d$ elastic scattering at 9 GeV/c is compared with the data of the CERN-Trieste Group (Ref. 30). In addition to the statistical errors shown, the data carry an absolute normalization error of ±20%.

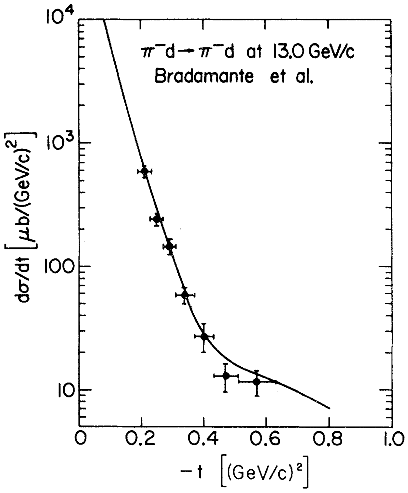

FIG. 10. Same as Fig. 9, at 13.0 GeV/c.

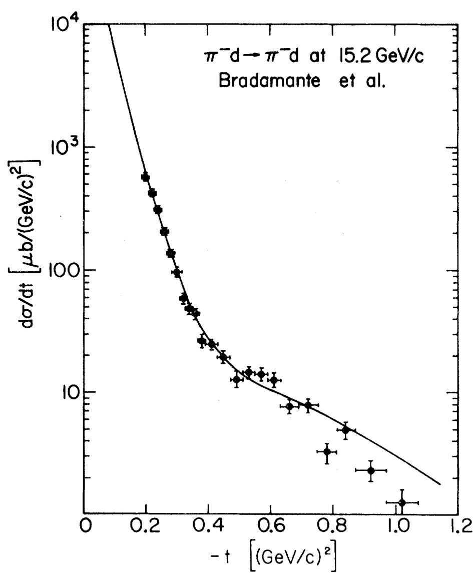

FIG. 11. Same as Fig. 9, at 15.2 GeV/c.

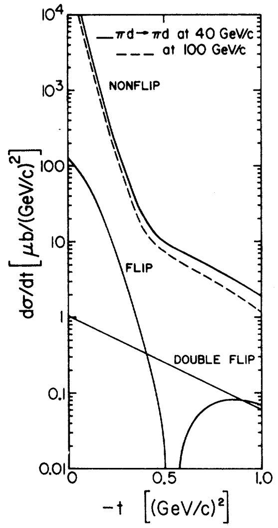

FIG. 12. Extrapolations of the present calculations to 40 and 100 GeV/c. At 40 GeV/c the contributions of the over-all nonflip, single-flip, and double-flip amplitudes are shown separately. The full cross section differs insignificantly from the nonflip contribution. At 100 GeV/c only the full cross section is shown.

πN scattering amplitudes, and, as we have remarked in Sec. II, do not depend strongly on our choice of deuteron wave function. These computations support the inference of the CERN-Trieste Group[33] that the disagreement in the double-scattering regime (which is also observed in pd scattering) cannot be ascribed to the uncertainty in our present knowledge of the hadron-nucleon scattering amplitudes.

Although we have omitted the contributions of πN charge exchange amplitudes from the cross sections presented here, we verified that over the range of energy and angle considered the additional diagrams would alter the results by less than two percent. In Fig. 12 we show extrapolations of our conventional Glauber-theory calculations to 40 and 100 GeV/c. At 40 GeV/c we indicate the contributions of the various amplitudes: nonflip (T_{11} and T_{00}), flip (T_{10}), and double flip ($T_{1,-1}$).

How would inelastic rescattering affect differential cross sections? If the diffractive production amplitudes are dominantly positive imaginary, with increasing energy we should expect to find

(i) a decrease in the forward differential cross section,

(ii) movement of the break in $d\sigma/dt$ to smaller values of t,

(iii) an increase in the larger-angle cross section, compared with what is given by elastic rescattering alone.

In our formulation of the theory we have regarded the deuteron as an object at rest both before and after the collision. A number of authors [e.g., Fäldt[34]] have emphasized that for "large" momentum transfers this is certainly not the case, and that recoil effects must be considered in computing the overlap integral between initial and final deuteron wave functions. Recently Cheng and Wu[6] have suggested a fully relativistic scheme for incorporating deuteron motion. The general conclusion of their work is that the cross section in the double-scattering regime will be reduced by approximately 20%, in agreement with Fäldt's estimates. To combine these refinements with the detailed calculations we have presented would require the numerical evaluation of fivefold integrals, which would exact a price in computer (not to mention human) time that seems to us presently unjustified.

V. SUMMARY

We have presented a systematic comparison of Glauber theory with high-energy pion-deuteron scattering data. To the surprise of no one, the theory successfully accounts for the basic features of πd total cross sections and of differential cross sections for elastic scattering. However, it appears possible that further refinements may be required to conform with experimental details.

The different behavior of πN and πd total cross sections above 30 GeV/c suggests the presence of inelastic rescattering corrections in addition to conventional Glauber screening. A rudimentary estimate indicates that corrections of the kind envisaged by Gribov could account for the experimental results. There is a persistent hint from our work and from earlier publications that Glauber theory overestimates the elastic scattering cross section in the large-angle regime. We have not pursued in detail suggestions that proper account of deuteron recoil will result in better agreement of theory with experiment.

A number of new experiments are called for. Precise measurements of the screening correction in the 5 to 30 GeV/c momentum range will be useful in pursuing the idea of inelastic rescattering. In addition, measurements of the differential cross sections for πN and πd elastic scattering at Serpukhov and NAL energies may be expected to shed light on the magnitude of double-scattering amplitudes as well as on the influence of inelastic intermediate states. Extension of the CERN-IHEP Boson Spectrometer measurements[35] of the reaction $\pi^- p \to p +$anything to additional values of squared momentum transfer would make possible much more reliable theoretical estimates of inelastic screening.[36]

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†The authors hold guest appointments at Brookhaven National Laboratory.

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