Study of Low-Energy $\Sigma^{\pm}n$ Interaction in K^-d Capture at Rest*

Tai Ho Tan

Department of Physics and Astrophysics, University of Colorado, Boulder, Colorado 80302

(Received 19 August 1971)

The investigation is carried out via a final-state interaction process from the reaction $K^-d \to \pi^{\mp}\Sigma^{\pm}n$. In the zero-range approximation, we have found two acceptable sets of complex scattering lengths for the $I=\frac{1}{2}$ $^3S_1$ state to be $-(7.5\pm 1.0)+i(3.0\pm 1.0)$ F and $+(2.8\pm 1.0)+i(3.0\pm 1.0)$ F. The presence of a Σ^+n bound state remains a distinct possibility. The interaction in the $I=\frac{3}{2}$ $^3S_1$ state is found to be much weaker, with the integrated cross section less than 8% that of the $I=\frac{1}{2}$ state.

I. INTRODUCTION

The study of the low-energy ΣN interaction is very interesting and will reveal information on nuclear forces involving strangeness-nonzero baryons. Because of short hyperon lifetimes very few data have been collected on $\Sigma^{\pm}p$ scattering and are hitherto restricted only to the momentum region above[1] 100 MeV/c. In this paper, we present an investigation which is based on a different approach. We have studied $\Sigma^{\pm}n$ scatterings in a pure $^3S_1$ state via a two-step final-state interaction process from the reaction $K^-d \to \pi^{\pm}\Sigma^{\mp}n$ at rest, as represented by Fig. 1(b). The dominance of the final-state ΣN interaction in the studied reactions has been demonstrated experimentally by the results of our previous investigation.[2] There, the presence of a striking enhancement in the Λp mass spectrum at 2128.7 ± 0.2 MeV from the reaction $K^-d \to \pi^-\Lambda p$ can best be interpreted as due to a two-step process: $K^-p(n) \to \pi^-\Sigma^+(n)$ followed by $\Sigma^+(n) \to \Lambda p$. In addition, the Σ^+n collision is found to occur primarily in the $^3S_1$ state. Such overwhelming presence of the absorptive process automatically implies the presence of the Σ^+n elastic channel. The type of interaction considered here is also without the complication of Coulomb interference.

Our analysis to extract the Σ^+n scattering parameters is based on the same formalism[3,4] employed in our previous work to understand the inelastic data. There, the deduction of the parameters was inconclusive. The complex scattering lengths $a+ib$ were found to be $a=0.8\pm 0.2$ F and $b=1.8\pm 0.4$ F, quite independent of the normalization value used in the calculation of the reaction rate. The small but positive real part would imply a bound state with a very large binding energy (~59 MeV). This is clearly in contradiction to the apparently observed value of 0.2 ± 0.2 MeV obtained by fitting the data to a Breit-Wigner function. Obviously, a simple way out of this dilemma is to ascribe the difficulty to the inadequacy of the theoretical formalism employed in dealing with the absorption below the physical Σ^+n threshold mass. This is very probable because of the large $\Sigma\Lambda$ mass difference (~74 MeV) and in view of the fact that a substantial amount of the absorptive events do occur below the threshold. However, based on the success of the investigation of the nucleon-nucleon scattering in the complex nuclei, the formalism can be expected to work for ΣN elastic scattering. Hence, the analysis should yield reliable scattering parameters.

II. EXPERIMENTAL PROCEDURE

Using the same exposure as in the previous investigation, more than six thousand candidates were measured and reconstructed to study the following reactions:

$$K^-d \to \pi^+\Sigma^-n, \tag{1}$$

$$K^-d \to \pi^-\Sigma^+n. \tag{2}$$

To reduce the measurement errors these events are further selected by a series of restrictions. A fiducial volume is imposed so that there is at least 10 cm of measured path length for the incident K^- beam track before interaction and for all the other nonstopping and nondecaying secondary tracks. The dipping angle of each measured track must not exceed 30°. The extrapolated momentum of the K^- to the vertex of interaction by measurement of curvature must be within one standard deviation from zero value to ensure that the event is a K^- capture at rest. A sample of 738 events from type (1) and 833 events from type (2) survive the test. Approximately similar numbers of $\pi^-p\Sigma^0$ events have also been reconstructed. But, unfortunately, because of the bias due to the missing of very short recoil protons, they cannot be used for this study.

Since the neutrons are not measured in the reconstruction of reactions (1) and (2), we do not expect any biases in the data arising from the low-momentum spectator nucleons. On the other hand,

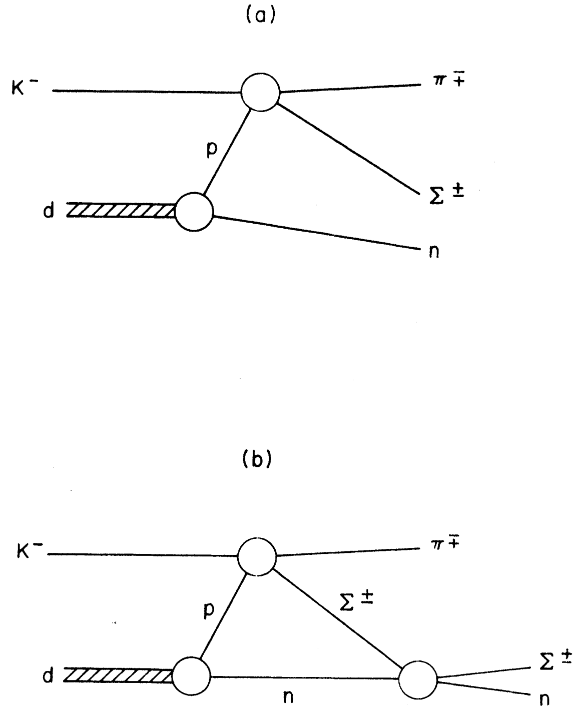

FIG. 1. Schematic diagrams showing (a) impulse interaction and (b) final-state $\Sigma^{\pm}n$ interaction.

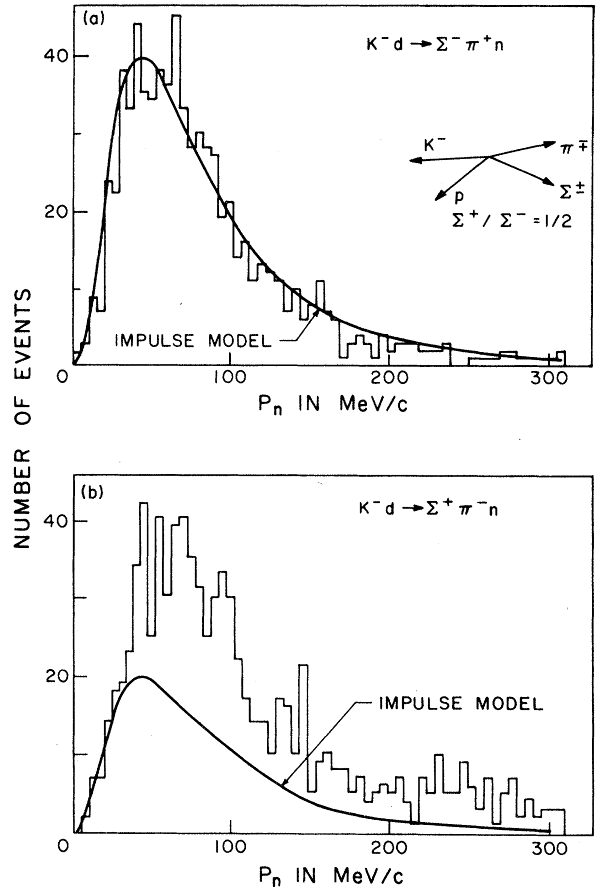

FIG. 2. Momentum distribution of the neutrons in the laboratory system from reactions (a) $K^-d \rightarrow \pi^+\Sigma^-n$ and (b) $K^-d \rightarrow \pi^-\Sigma^+n$.

each Σ is measured and many are reconstructed with very short path lengths. It is therefore necessary to determine if any biases due to kinematic reconstruction are present in our sample of data that might subsequently affect our conclusion.

The decay time of each Σ is calculated by using the measured path length and fitted momentum. The lifetimes for both Σ^+ and Σ^- obtained from the distribution agree very well with the known values. These distributions plotted on the semilogarithmic papers are fairly linear. Only events with very short decay times are missing. They account for five percent more events in reaction (1) and ten percent more events in reaction (2). If there are no significant biases in reconstructing the short Σ's, these missing events should be due entirely to those cases in which the Σ's decay before traveling an observable distance (<1 mm). Otherwise, the extrapolated number from the time distribution will be greater than the actual number of the "zero-length" events.

Almost all of the Σ^-'s decay into π^-n. About half of the Σ^+'s decay into π^+n, while the other half decay into π^0p. Therefore, for those events with the Σ's decaying inside the "zero-length" interval, half of these events in reaction (2) cannot be distinguished from those in reaction (1). They will both be seen in two-prong topology with visibly identifiable outgoing π^+ and π^- tracks. Inferred from the decay-time distribution, each reaction contributes about equally to these two-prong ($\pi^+\pi^-$) events. No other source of contribution appears possible. From a scanning sample containing about 1050 events of type (2) and 960 events of type (1) with visible Σ's, we observe about 110 two-prong ($\pi^+\pi^-$) events. The ratio of the number of "zero-length" Σ's to the number of Σ's with visible track length is approximately 0.05 for reaction (1) and 0.1 for reaction (2). These agree remarkably well with the values estimated from the decay-time distribution. The consistency in these ratios clearly implies that our data are not noticeably biased in the reconstruction.

We have also looked into the possible differences between the momentum distribution from these "zero-length" Σ's and from those visible Σ's. An investigation of decay-length distribution as a function of Σ's momentum indicates that there may be a slightly higher concentration of lower-momentum Σ's in the "zero-length" events. However, the deviation of the momentum distribution due to the "zero-length" Σ's represents only a very small fraction of the entire sample. Its effect on the subsequent study of the ΣN systems is not expected to be greater than that already accounted for by the statistical errors.

III. RESULTS

Figure 2(a) exhibits the momentum distribution of the neutrons from reaction (1). It agrees remarkably well with the prediction of the impulse model [see Fig. 1(a)], as is indicated by the solid curve. Similar distribution from reaction (2) as shown in Fig. 2(b) deviates markedly from the prediction of the model. This is undoubtedly due to the expected presence of the $\Sigma^+n \to \Sigma^+n$ final-state interaction. To understand the significance of these distributions we must first investigate the possible contribution from the pion-nucleon system. Since the pion-nucleon scattering in the low-energy region is known to be dominated by the P_{33} partial wave, any significant amount of final-state interaction would have been manifested by a more prominent presence of the π^+n state over the π^-n state. The π^+n and π^-n mass spectra from the two reactions are illustrated, respectively, in Figs. 3(a) and 3(b). Clearly, there are no differences in their shapes and sizes except for a slight mass shift due to the mass difference of Σ^+ and Σ^-. Hence, the contribution from $\pi^{\pm}n$ final-state interaction is expected to be insignificant.[5]

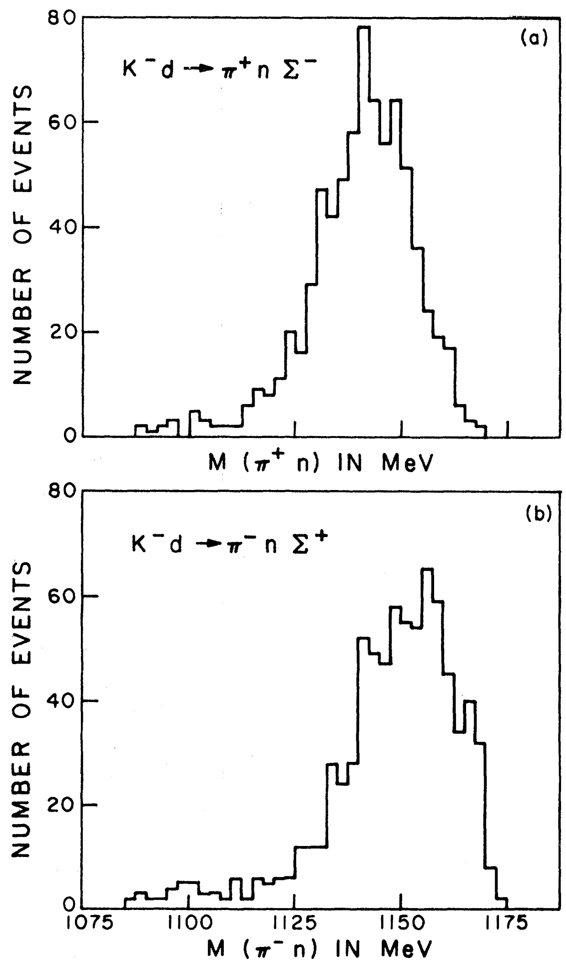

FIG. 3. Combined mass distribution for (a) π^+n system and (b) π^-n system.

The agreement between the data and the impulse model in Fig. 2(a) can therefore be construed as due to the absence of any final-state interaction in reaction (1). This allows us to conclude that the ΣN interaction cross section in the $I=\frac{3}{2}$ $^3S_1$ state is very small and is consistent with being zero in comparison with the $I=\frac{1}{2}$ state. Such a result was suggested previously from the low-energy Σ^+p scattering data on the basis of the assumption of SU_3 invariance combined with pp scattering data.[6,7] More recently Downs and Iddings, considering the hyperon-nucleon potential as arising from boson exchange, deduce the cross section of the $I=\frac{3}{2}$ state to be less than 1/1800 times that of the $I=\frac{1}{2}$ state for ΣN interaction in the $^3S_1$ system.[8] If we take the number of the excess events in each bin and square it, sum up all the bins, and take the square root, we find the deviation from the impulse model to be less than 2 percent. Following the procedure that will be delineated in the analysis of the $I=\frac{1}{2}$ state, we estimate the amount of $I=\frac{3}{2}$ $^3S_1$ cross section to be less than 8 percent that of the $I=\frac{1}{2}$ state.

The above findings reduce the production mechanisms involved in reaction (2) to two types. The first type is the direct production as represented by Fig. 1(a) and the second type is due to the ΣN final-state interaction. Contributions from the final-state interaction can be attributed to the Σ^+n elastic scattering [Fig. 1(b)] and to the charge-exchange process [$K^-n(p) \to \pi^-\Sigma^0(p)$ followed by $\Sigma^0(p) \to \Sigma^+n$]. Investigations of the Λp interaction cross section in the region of Σ^+n threshold precludes any significant contribution from $K^-n(p) \to \pi^-\Lambda(p) \to \pi^-\Sigma^+n$ process. The production rate can then be expressed in terms of the energy M and the scattering angle $\cos\theta$ of the Σ^+n system in its center-of-mass frame as

$$\frac{d^2N}{dM\, d\cos\theta} = |\alpha + \beta f_{\Sigma\Sigma} + \gamma f_{\Sigma'\Sigma}|^2 \rho\,, \qquad (3)$$

where α, $\beta f_{\Sigma\Sigma}$, and $\gamma f_{\Sigma'\Sigma}$ represent the matrix elements of the above three processes, respectively.

$f_{\Sigma\Sigma}$ and $f_{\Sigma'\Sigma}$ equal, respectively, $\frac{2}{3}$ and $\sqrt{2}/3$ times $I=\frac{1}{2}$ state ΣN elastic-scattering amplitude, which is then expressed in terms of the complex scattering lengths $a+ib$ in the standard manner. α, β, and γ differ only slightly by a kinematic factor, and all contain the deuteron wave function as well as the coupling information in the K^-N vertex. Here, the information on K^-N vertex is incorporated by taking Kim's solutions[9] for the low-energy K^-N interaction using the charge-independence formulation. The effects of the Fermi motion are automatically included in the formulation. ρ denotes the phase space. Since the ratio of Σ^+ to Σ^- production from K^-p capture averages to be about $\frac{1}{2}$ in the presently considered low-energy region, integration over $|\alpha|^2$ should yield about half the known rate of reaction (1). This provides the exact normalization factor for Eq. (3) to convert the calculated rate into the actual number of events observed. Accurate knowledge of this factor is crucial in the selection of the proper sets of the scattering parameters.

Figure 4(a) shows the angular distribution of Σ^+ scattering in the Σ^+n center-of-mass frame as illustrated. The dashed curve is the corresponding Σ^- angular distribution from reaction (1) with the height of each bin scaled down to half. This represents the portion due to direct Σ^+ production and can be fitted to the square of a polynomial function $f(\cos\theta)$. After integrating over the energy M, Eq. (3) can be simplified to the form

$$\frac{dN}{d\cos\theta}=|f(\cos\theta)+c|^2. \qquad (4)$$

An excellent fit to the data is obtained when c equals 2.5 ± 0.3, and is indicated by the solid curve. The fact that c is a constant implies S-wave Σ^+n scattering. Such agreement with the expectation provides the necessary consistency required in our procedure before we can justifiably proceed to extract the details of the ΣN scattering information.

Figure 4(b) exhibits the mass distribution for the Σ^+n combined system. Equation (3) is integrated over the angle and fitted by a least-squares method to the data. The fit to the elastic data is very sensitively dependent on the real part a and depends only weakly on the imaginary part b, while the fit to the absorptive channel is very sensitive to the value of b. Two best solutions, each having a χ^2 value of about 1.0, are obtained:

$$a=-7.5\pm1.0\text{ F},\quad b=3.0\pm1.0\text{ F} \qquad \text{(Solution I)}$$

and

$$a=+2.8\pm1.0\text{ F},\quad b=3.0\pm1.0\text{ F}. \qquad \text{(Solution II)}$$

The contribution derived from the interference between the different diagrams eliminates the usual

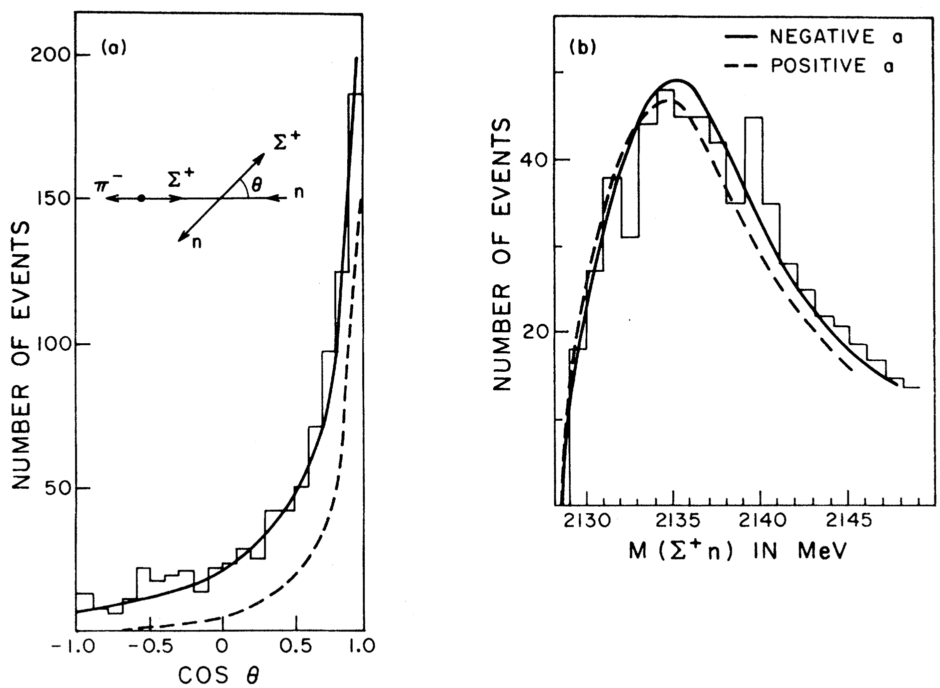

FIG. 4. (a) Angular distribution of Σ^+ production in the Σ^+n rest frame; (b) Σ^+n combined mass distribution from reaction $K^-d\to\pi^-\Sigma^+n$.

ambiguity in the sign of a. The error in a comes mainly from two sources: (i) large statistical and possibly systematic errors near the threshold region; and (ii) a 10% uncertainty in the estimate of the impulse production rate.

IV. DISCUSSION

The positive value of a in solution II implies a bound Σ^+n state with a binding energy of $5.1^{+6.7}_{-2.5}$ MeV. This is much closer to the apparent value of 0.2 ± 0.2 MeV observed and certainly appears to be more reasonable than the solution obtained by fitting the absorption data alone.[10] As mentioned earlier, the apparent mass and width of the enhancement may deviate considerably from their true values depending on the dynamics of the interaction process. The direction of the mass shift cannot be simply predicted because of the uncertainty associated with the virtual intermediate state. Hence, based on binding-energy data alone, it is not possible to choose one set of the solution over the other. In fact it is entirely possible that the observed enhancement results from the combination of a lower-mass bound state plus the threshold cusp effect.

Some cross sections for Σ^-p interactions in the momentum region 100–200 MeV/c have been measured and compiled in Ref. 1. If the singlet state can be neglected, the triplet-state cross sections, which are then equal to $\frac{4}{3}$ the measured values, can be compared with those deduced from the above solutions.[11] Restricting ourselves first to elastic scattering ($\Sigma^-p\to\Sigma^-p$), we find that solution I is one standard deviation higher while solution II is about one standard deviation lower than the measured values. It must be noted that there are only three measured points available and each is attached with a large error bar. There are more measured points available for the inelastic channel ($\Sigma^-p\to\Lambda n$), and they all have substantially smaller error bars. Here, comparison indicates that solution II seems to coincide with the measured values, while solution I is about half a standard deviation lower.

Our results can be compared with the fits obtained by Gell *et al.*[12] It appears that the selection of the proper set of the scattering lengths can be achieved only either by employing a more suitable theoretical formalism that will provide more consistent interpretation to the Σ^+n final-state interaction near and below the threshold-energy region, or by performing in the future experiments that are capable of measuring Σ^-p cross sections more precisely and closer to the threshold-energy region.

V. CONCLUSION

By studying ΣN interaction via the reaction $K^-d\to\pi^\mp\Sigma^\pm n$, the interaction in the $I=\frac{3}{2}$ $^3S_1$ state is found to be much weaker, with the integrated cross section equalling less than 8 percent that of the $I=\frac{1}{2}$ state. Two acceptable sets of complex zero-range scattering lengths for $I=\frac{1}{2}$ $^3S_1$ state are found to be: $-(7.5\pm1.0)+i(3.0\pm1.0)$ F and $+(2.8\pm1.0)+i(3.0\pm1.0)$ F.

ACKNOWLEDGMENTS

The author wishes to express appreciation to Professor B. Downs and Professor C. Iddings for illuminating discussions.

---

*Work supported by the U. S. Atomic Energy Commission.

[1]Particle Data Group, LBL Report No. UCRL-20000 YN, 1970 (unpublished). See F. R. Stannard, Phys. Rev. 121, 1513 (1961); H. G. Dosch *et al.*, Phys. Letters 21, 236 (1966); H. A. Rubin *et al.*, Phys. Rev. 159, 1149 (1967); R. Engelmann *et al.*, Phys. Letters 21, 587 (1966); S. S. Yamamoto *et al.*, AEC Report No. NY03651-11, 1969 (unpublished).

[2]T. H. Tan, Phys. Rev. Letters 23, 395 (1969).

[3]R. Karplus and L. Rothberg, Phys. Rev. 115, 1058 (1959).

[4]T. Kotani and M. Ross, Nuovo Cimento 14, 1282 (1959).

[5]Also, just as the interaction in the YN system, the final $\pi^\pm n$ system is expected to be forced into the S state which of course has a very small interaction cross section.

[6]P. D. DeSouza, G. A. Snow, and S. Meshkov, Phys. Rev. 135, B565 (1964).

[7]H. A. Rubin and R. A. Burnstein, Phys. Rev. 159, 1149 (1967).

[8]B. Downs and C. Iddings (private communication).

[9]J. K. Kim, Phys. Rev. Letters 14, 29 (1965).

[10]This value is derived from an experiment with an energy resolution of 1.0 MeV (Ref. 2). A 3-MeV binding energy has been reported by D. Cline, R. Laumann, and J. Mapp [Phys. Rev. Letters 20, 1452 (1968)] from a different experiment. However, their energy resolution is very much poorer.

[11]The measured Σ^-p cross section contains contributions from both the singlet and the triplet states, i.e., $\sigma_{\text{total}}=\frac{1}{4}\sigma_s+\frac{3}{4}\sigma_t$.

[12]Y. Gell, G. Alexander, and I. Stumer, Nucl. Phys. B22, 583 (1970).