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Restrictions on Nonlocal Hidden-Variable Theory*

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In a recent experiment, Freedman and Clauser have found that Bell's inequality for local hidden-variable theories is violated by over six standard deviations. We derive an inequality similar to Bell's for nonlocal hidden-variable theories in which the hidden-variable density distribution of the particles measured is nonlocal, but where the detectors are contained within some finite volume. The measurement of Freedman and Clauser is used to put a lower limit on the nonlocality of the hidden-variable density distributions of the photons detected in their experiment.

While there has been little dispute over the usefulness of quantum theory, there has been considerable disagreement over interpretation. One of the most interesting alternatives to the widespread Copenhagen school of thought is hidden-variable theory, where the statistical results of quantum theory are generated by averaging over a set of experimentally undetermined (i.e., hidden) variables. In 1965, Bell[1] demonstrated that any local hidden-variable theory leads to predictions which differ with the predictions of quantum mechanics. It was not until 1969 that Clauser, Horne, Shimony, and Holt[2] proposed an experiment, based on Bell's inequality, which would differentiate between a causal description of nature, i.e., strictly local hidden-variable theory, and the statistical description of quantum mechanics. Various groups[3,4] are now completing the proposed experiment, including Freedman and Clauser,[3] who have very recently reported experimental results consistent with quantum predictions, but over six experimental standard deviations from the closest predictions allowed by local hidden-variable theory.

In an attempt to retain the general notion of hidden variables, we shall introduce nonlocal hidden variables and examine the constraints imposed by experiment. As we shall see, these constraints provide strong argument against such nonlocal hidden-variable theory.

In the experiments referred to above, polarization coincidence ratios are measured for the two photons emitted from a simple cascade transition in an excited atom. In the local hidden-variable theories which these measurements test, it is assumed that the photons propagate as localized particles, and that the influence of the polarizers and detectors may be confined to a finite region of space. However, in some cases, e.g., interference phenomena, photons are usually (and most easily) considered in terms of wave properties which extend over macroscopic distances. In this paper we therefore abandon the assumption that the photon is localized by considering a nonlocal density distribution of hidden variables. However, we retain the idea that each detector-polarizer is confined within some macroscopic volume exclusive of the other detector-polarizer. This represents the simplest nonlocal generalization of local hidden-variable theory.

In order to determine restrictions on nonlocality, we derive an inequality similar to Bell's[1] inequality. Let us consider an ensemble of correlated pairs of photons, such that one from each pair enters apparatus I_a and the other II_b, where a and b are adjustable parameters such as the setting of the axis of a polarizer. In each apparatus the particle must make a binary decision, e.g., the photon either does or does not pass through the polarizer. We represent the result by $A(a)$ and $B(b)$, where $A(a)$ [or $B(b)$] is either +1 or −1. According to hidden-variable theory we assume that A and B depend on the set of hidden variables, λ, which describe the system. It is conventional in local theories and convenient in the present work to assume that $A(a, \lambda)$ is independent of the setting b, and likewise $B(b, \lambda)$ is independent of a. In a strictly local hidden-variable theory, the photon is precisely characterized by the hidden variables, λ, and a nonnegative but otherwise arbitrary probability distribution $\rho(\lambda)$, which due to the locality of the photon is independent of a and b. We introduce nonlocality by allowing $\rho(\lambda)$ to depend on a and b, i.e., we consider $\rho(\lambda; a, b)$. There are, of course, other nonlocal generalizations where A and B become nonlocal. We have chosen this particular generalization because it is the simplest mathematically and conceptually, and because we may use the experiment of Freedman and Clauser to place a lower bound on the nonlocality of ρ. It might be pointed out that in order to avoid transmitting information at speeds greater than the speed of light, one must think of $\rho(a, b; \lambda)$ as containing retardation, i.e., when a polarizer setting is changed at A, its effect at B may not be instantaneous.

We now define a correlation function given by

$$P(a,b)=\int_\Gamma A(a,\lambda)\,B(b,\lambda)\,\rho(\lambda;a,b)d\lambda$$

where

$$\int_\Gamma \rho(\lambda;a,b)d\lambda=1\,.$$

We may then write

$$\begin{aligned}|P(a,b)-P(a,d)|&=\left|\int A(a,\lambda)B(b,\lambda)\rho(\lambda;a,b)d\lambda-\int A(a,\lambda)B(d,\lambda)\rho(\lambda;a,d)d\lambda\right|\\&=\left|\int A(a,\lambda)B(b,\lambda)[1\pm A(c,\lambda)B(d,\lambda)]\rho(\lambda;a,b)d\lambda\right.\\&\quad-\int A(a,\lambda)B(d,\lambda)[1\pm A(c,\lambda)B(b,\lambda)]\rho(\lambda;a,d)d\lambda\\&\quad\left.\mp\int A(a,\lambda)A(c,\lambda)B(b,\lambda)B(d,\lambda)[\rho(\lambda;a,b)-\rho(\lambda;a,d)]d\lambda\right|\\&\leqslant 2\pm[P(c,d)+P(c,b)]+\Delta\end{aligned}$$

or

$$-2-\Delta\leqslant P(a,b)-P(a,d)+P(c,d)+P(c,b)\leqslant 2+\Delta\,,$$

where

$$\Delta=\int|\rho(\lambda;a,b)-\rho(\lambda;a,d)|\,d\lambda+\int|\rho(\lambda;a,b)-\rho(\lambda;c,d)|\,d\lambda+\int|\rho(\lambda;a,d)-\rho(\lambda;c,b)|\,d\lambda\,. \tag{1}$$

The term Δ is a nonlocal term which goes to zero in a local hidden-variable theory.

The correlation functions, $P(x,y)$, are related[5] to coincidence counting rates by

$$P(x,y)=\frac{4R(x,y)}{R_0}-\frac{2R_1(x)}{R_0}-\frac{2R_2(y)}{R_0}+1\,,$$

where $R(x,y)$ is the coincidence counting rate with both polarizers in, R_1 (and R_2) correspond to one in and one out (and vice versa), and R_0 is the rate with both polarizers out. Using this relation, inequality (1) may now be expressed[6] as

$$-1-\tfrac{1}{4}\Delta\leqslant\frac{R(a,b)}{R_0}-\frac{R(a,d)}{R_0}+\frac{R(c,d)}{R_0}+\frac{R(c,b)}{R_0}-\frac{R_1(c)}{R_0}-\frac{R_2(b)}{R_0}\leqslant\tfrac{1}{4}\Delta\,. \tag{2}$$

The rates R, R_1, R_2, and R_0 may be computed[5] quantum mechanically. In the case of local hidden-variable theory ($\Delta=0$), the quantum result violates inequality (2) if the half angle of the detector is sufficiently large and the polarizers sufficiently efficient. In the experiment of Freedman and Clauser, the maximum violation occurs[7] for

$$\frac{3R(22.5°)}{R_0}-\frac{R(67.5°)}{R_0}-\frac{R_1}{R_0}-\frac{R_2}{R_0}\leqslant\tfrac{1}{4}\Delta$$

$$\frac{3R(67.5°)}{R_0}-\frac{R(22.5°)}{R_0}-\frac{R_1}{R_0}-\frac{R_2}{R_0}\geqslant-1-\tfrac{1}{4}\Delta\,,$$

where Δ is zero for the test of local hidden-variable theory. It is assumed that $R_1(x)$ and $R_2(x)$ are independent of x; it is also assumed that $R(x,y)=R(|x-y|)$, i.e., the rates depend on the angle between polarization axes, and $\rho(\lambda;x,y)=\rho(\lambda;|x-y|)$.

From these two inequalities it may be quickly found that

$$\delta=\frac{R(22.5°)}{R_0}-\frac{R(67.5°)}{R_0}-\frac{1}{4}\leqslant\tfrac{1}{8}\Delta\,. \tag{3}$$

Freedman and Clauser experimentally measured $\delta=+0.050\pm0.008$. In the case of our nonlocal theory ($\Delta\neq0$), the experimental result gives a measure of the nonlocality, namely,

$$\begin{aligned}2.0\geqslant\tfrac{1}{2}\Delta&=\int|\rho(\lambda;22.5°)-\rho(\lambda;67.5°)|d\lambda\\&\geqslant0.2\int\rho(\lambda;|a-b|)d\lambda=0.2\,.\end{aligned} \tag{4}$$

This is a highly nonlocal density distribution.

It is also of interest to note that there are several levels of locality. First, there is strict locality, where the quantities are confined to an infinitesimal volume of space. Conserved quantities, for example, must be conserved in the strictly local sense in order to have exact conservation in all inertial frames. Local theories are also causal. Second, there is semilocality where the quantities are completely confined within some finite volume of space. One conventionally considers detectors as semilocal. Third, there is decaying nonlocality corresponding to a quantity which is dense in some region of space and whose density decreases with the distance from the region, but is not necessarily zero beyond any finite volume. Examples include the usual probability distributions for photons and electrons as well as gravitational fields. Fourth, there is a nondecaying nonlocality where the density of the quantity does not decrease. Such nondecaying quantities are usually considered nonphysical.

For geometries where the polarizers are well separated from each other and from the cascading atoms, the experiment of Freedman and Clauser can be considered a test of both strictly local and semilocal theories. The strictly local theory, of course, is the only theory which is truly causal. Furthermore, there is nothing in the quantum predictions which depends on the distance to the detectors. Hence, if quantum predictions are found to be valid for this type of experiment for detectors placed very far from the cascading atoms, then the nonlocal hidden-variable theory used to describe these results must be of the nondecaying sort. Indeed, it is difficult to think of any nonlocal theory which avoids this problem.

If one computes the length of the photon wave packet in the experiment of Freedman and Clauser, one finds that this length is much smaller than the distance from the source and the detector. Hence the wave packets do not overlap both the source and the polarizer-detector system simultaneously. In our nonlocal model, we see from inequality (4) that the nonlocality of the photon must be much greater than the size of its wave packet. The size and degree of nonlocality in the nonlocal hidden-variable theories which we have considered provide strong argument against using such nonlocal hidden-variable theories as alternatives to a quantum-mechanical description of nature.

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