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PHYSICAL REVIEW D VOLUME 7, NUMBER 2 15 JANUARY 1973

Convergence of the Sum Rules for Cluster Functions in a ϕ^3 Ladder Model*

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(Received 28 August 1972)

The convergence of the series in the sum rules of Campbell and Chang for cluster functions is investigated numerically for the ϕ^3 ladder model. It is found that in this model these sum rules diverge for a sufficiently large value of the coupling constant that the Pomeranchukon-Regge trajectory goes through $\alpha(0)=1$.

I. INTRODUCTION

The dominant part of the new high-energy data deals with highly inelastic reactions, and the complexity of such processes requires new methods of analysis, such as the use of inclusive spectra and correlations. However, to obtain a better understanding of the underlying mechanism it may be useful to turn back to reactions which are still amenable to exclusive analysis and to investigate more carefully special quantities relevant for inclusive data. For this purpose cluster-decomposition methods[1-3] might provide a useful description of exclusive data. A major problem with its application seems to arise from the unknown convergence properties of the infinite series in the sum rules. It is the purpose of this paper to investigate the convergence numerically in the ϕ^3 ladder model. The paper will have two parts: First we will introduce the notation and the general concepts, and second we will present the results of the investigation.

II. NOTATION AND GENERAL CONCEPT OF CLUSTER DECOMPOSITION

Let us first specify the general situation considered here. We concentrate on inelastic processes involving only one type of particle. Following Campbell and Chang[2] we will study only the longitudinal momenta of produced secondaries (Fig. 1) and integrate over all other variables, including the momenta of the leading particles. The distinction between produced and leading particles will be possible due to an assumed kinematic separation. The longitudinal momenta will be specified

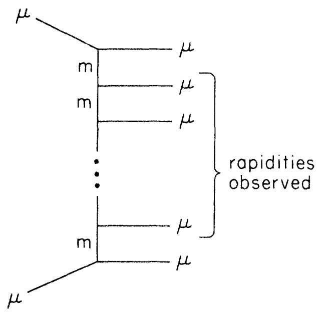


FIG. 1. The graph used in the calculation.

by rapidities which are defined by

$$z_i = \tfrac{1}{2}\ln[(q_0^i + q_\parallel^i)/(q_0^i - q_\parallel^i)]\,, \tag{1}$$

where q^i denotes the 4-momentum of the ith secondary. In terms of rapidities we can define the exclusive n-particle spectrum:

$$d\sigma_{n+2}^{\text{exclusive}}/\sigma_2 = f_n(z_1\cdots z_n)dz_1\cdots dz_n\,. \tag{2}$$

For the questions of convergence we consider it adequate to deal only with integrated quantities, and the integrated exclusive n-particle spectrum is defined by

$$\begin{aligned} F_n(s) &= \int dz_1\cdots dz_n f_n(z_1\cdots z_n) \\ &= n!\sigma_{n+2}/\sigma_2\,. \end{aligned} \tag{3}$$

Following Campbell and Chang[2] we decompose the exclusive spectra into cluster functions $\{g_n(z_1\cdots z_n)\}$:

$$\begin{aligned} &g_1(z_1) = f_1(z_1)\,, \\ &g_2(z_1,z_2) = f_2(z_1,z_2) - g_1(z_1)g_1(z_2)\,, \\ &\text{etc.} \end{aligned} \tag{4}$$

Counting the possible distributions of secondaries in clusters, we obtain the following inductive definition of integrated cluster functions $\{G_n(s)\}$:

$$\begin{aligned} &G_1(s) = F_1(s)\,, \\ &G_n(s) = F_n(s) - \sum_{\substack{k=2\cdots n \\ \{\nu_1\cdots\nu_k\} \\ \Sigma\nu_i = n}} \frac{n!}{\nu_1!\cdots\nu_n!}\,\frac{1}{D}\,G_{\nu_1}\cdots G_{\nu_n}\,, \end{aligned} \tag{5}$$

where

$$D = \prod_{i=1}^{n}(\text{number of times } \nu_i = i)!\,.$$

Campbell and Chang derive sum rules connecting cluster functions and inclusive data. The integrated version of these sum rules is given in Eqs. (6):

$$\frac{\sigma_t}{\sigma_2} = \exp\left(\sum_n \frac{1}{n!}\,G_n\right)\,, \tag{6a}$$

$$\int \rho(z)dz = \sum_n n\frac{1}{n!}\,G_n\,, \tag{6b}$$

$$\iint g(z,z')dz\,dz' = \sum_n n(n-1)\frac{1}{n!}\,G_n\,, \tag{6c}$$

where $\rho(z)$ and $g(z,z')$ are the inclusive spectrum and the correlation function. These equations have for the moment only formal meaning; the equations are valid to each order in the number of secondaries (equivalent to order in the coupling constant in a ladder model); nothing is said, however, about the convergence of the sums.

The convergence of these series is related to factorization, which is defined by

$$f_n(z_1\cdots z_n) = f_r(z_1\cdots z_r)f_{n-r}(z_{r+1}\cdots z_n) \tag{7}$$

for

$$z_1,\ldots,z_r \gg z_{r+1},\ldots,z_n\,.$$

This has a simple consequence for exclusive correlations:

$$\begin{aligned} &g_n(z_1\cdots z_n) \to 0 \\ &\qquad \text{if } z_1,\ldots,z_r \gg z_{r+1},\ldots,z_n \text{ for any } r\,, \end{aligned} \tag{8}$$

which means that cluster functions can only have nonzero values if all the rapidities lie within a certain interaction region. The asymptotic s dependence of the integrated cluster functions has to come from the phase-space integration. According to factorization (8) this integration picks up the total extent of the phase space only once, and asymptotically we therefore have the relation

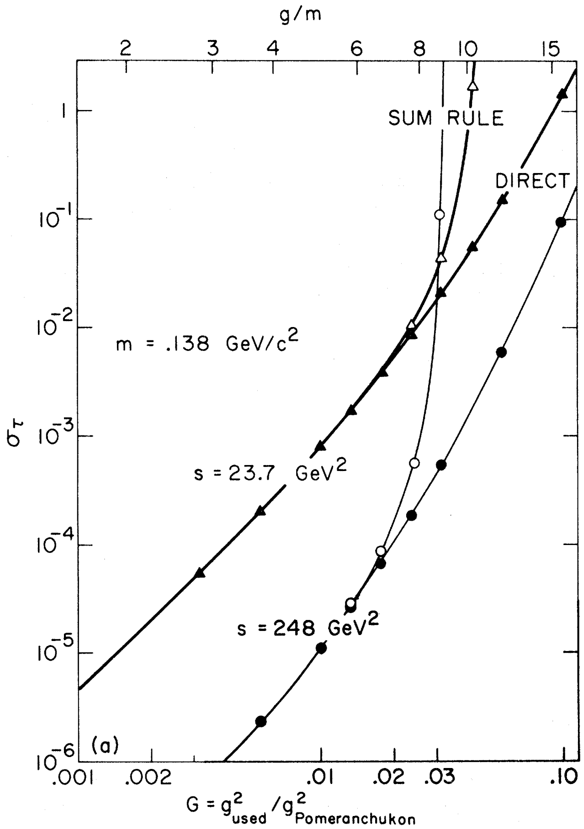
$$G_n(s) \sim \ln(s)\,. \tag{9}$$

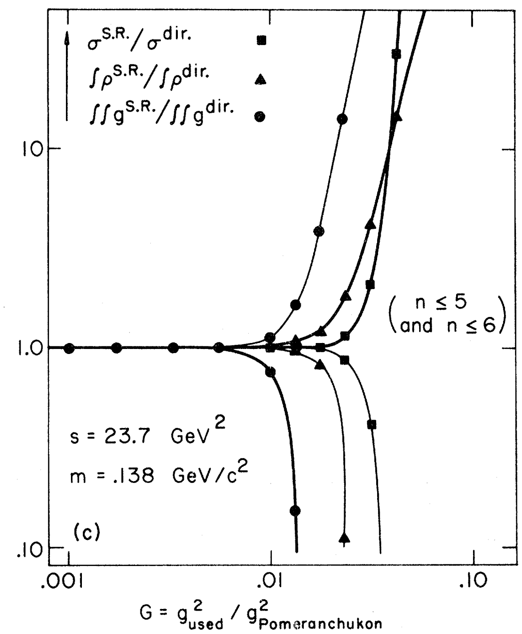
On the other hand, without factorization the power in $\ln(s)$ would have to increase with multiplicity, and the convergence of the series would get worse as s increases.

III. CLUSTER DECOMPOSITION AND ϕ^3 LADDER MODEL

To test the cluster decomposition numerically we turn to a simple ϕ^3 model, as indicated in Fig. 1. All masses of incoming and outgoing particles are identical ($\mu = 0.138$ GeV/c^2), and we will start out by also setting the exchanged mass equal to this value ($m = \mu$). The ϕ^3 multiperipheral model seems a useful example since factorization can be verified for it[2] and the question of convergence is not trivially influenced by the choice of free parameters.

For the calculation of the exclusive cross sections we used a numerical program which was written by Wyld[4] and Miller and which solves the ABFST (Amati, Bertocchi, Fubini, Stanghellini, Tonin)[5] equation by numerical iteration. Inherent





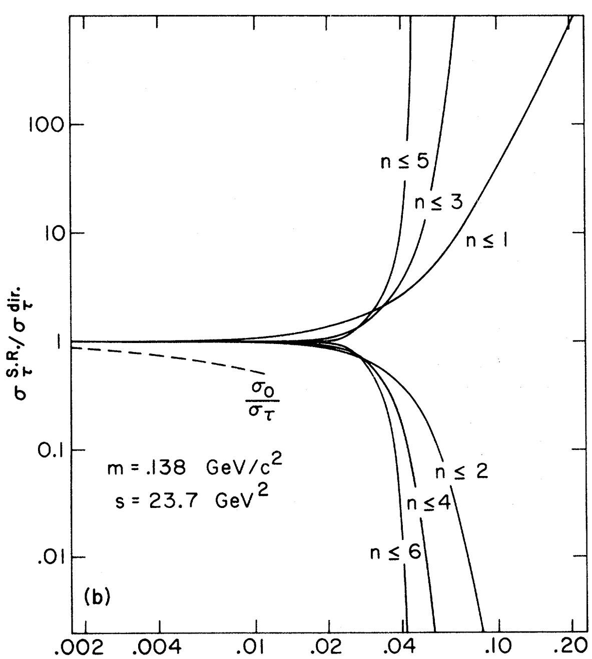
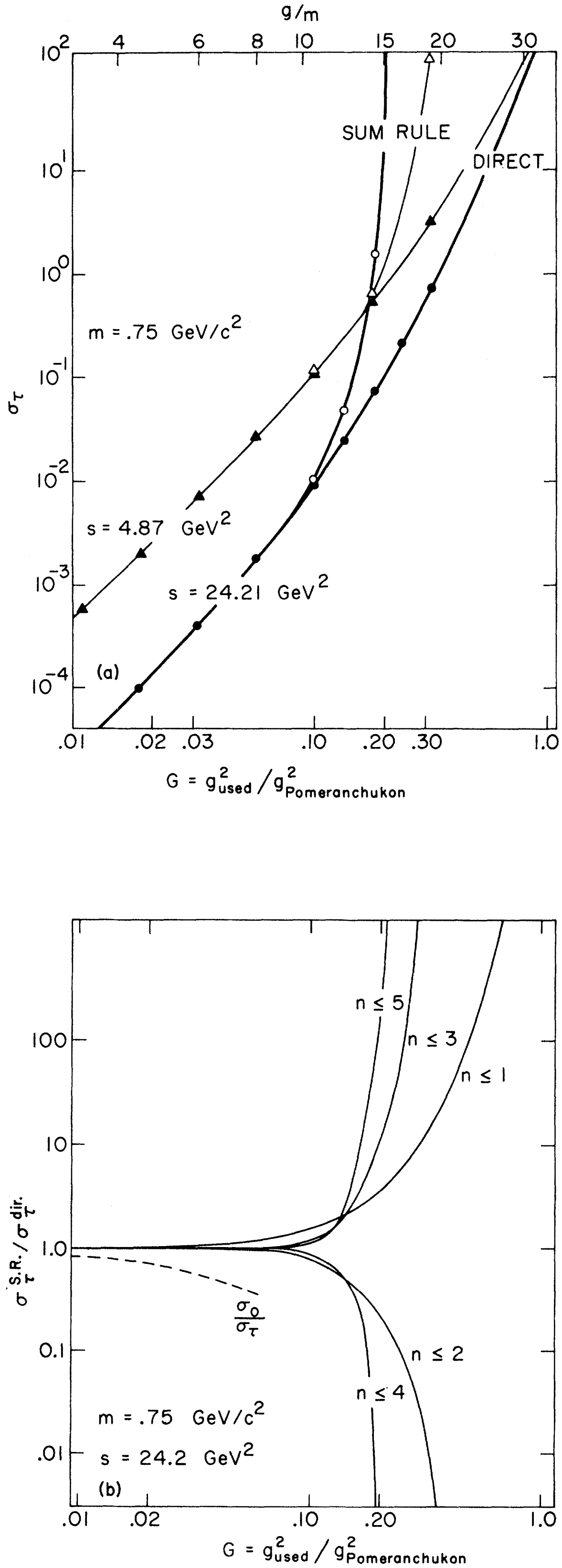


FIG. 2. The convergence of the series in the sum rules for a model with all masses equal ($\mu = m = 0.138$ GeV/c^2) as a function of the coupling ($G = g_{\text{used}}^2/g_{\text{Pomeranchukon}}^2$). (a) Both sides of the first sum rule [Eq. 6(a)] and their energy dependence. Five terms are kept in the series on the right-hand side of the sum rule. (b) The convergence of the first sum rule as it depends on the number of terms kept in the series. Shown is σ_T calculated by sum rule/σ_T calculated directly for the indicated multiplicities in the series in the sum rule. The dashed line gives σ_0/σ_T, to indicate the trivial elastic domain. (c) The convergence of the other sum rules [Eqs. 6(b) and 6(c)] for 5 and 6 terms kept in the series.

in this method is the approximation that interference terms are not considered. Numerical accuracy is important for the questions considered here, but numerical errors seem sufficiently small.

The coupling constant plays a critical role for the question of convergence. As the exclusive cluster functions have the simple g dependence of the exclusive spectra (i.e., $G_n \sim g^{2n}$), we can treat the sums as power series and check if the desired Pomeranchukon coupling constant [such that $\alpha(g^2) = 1$ at $t = 0$] lies within the radius of convergence. But the issue is not the purely mathematical problem, but the numerical question whether one can get reasonable approximations with a chosen number of terms; the good or possibly catastrophic behavior of very high terms in the sums seems of no practical interest.

To test the convergence we calculated both sides of the first sum rule, Eq. (6a), keeping five terms



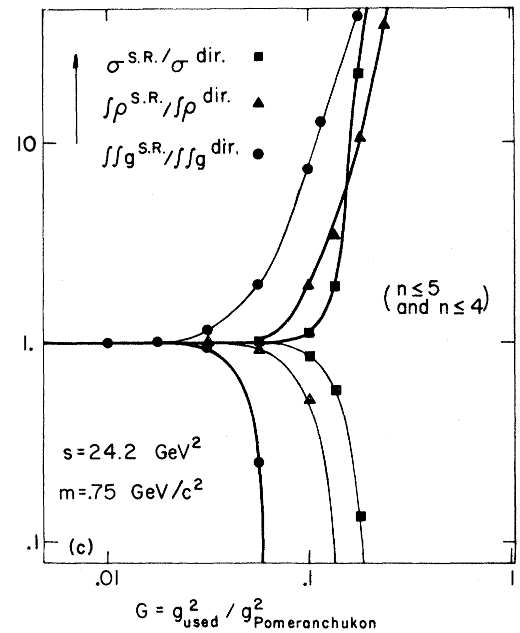


FIG. 3. The convergence of the series in the sum rules for a model with all external masses equal to the π-meson mass ($\mu = 0.138$ GeV/c^2) and all exchanged masses equal to the ρ-meson mass ($m = 0.75$ GeV/c^2). Otherwise the graphs are identical to Fig. 2.

in the series on the right-hand side of the equation, for incoming laboratory momenta of 900 and 8.6 GeV/c as functions of the coupling constant. We present the result in Fig. 2(a). We find convergence up to a large coupling strength g/m, approximately equal to 8; but the desired Pomeranchukon coupling ($g/m = 50$) lies far outside of the radius of convergence. This feature is definitely beyond the influence of possible minor errors in the determination of the Pomeranchukon coupling constant. The value of the exact coupling constant is obtained with a program written by Wyld[4] which determines the eigenvalue of the homogeneous part of the Bethe-Salpeter equation. Figure 2(b) investigates the convergence at 8.6 GeV/c in more detail. It shows the ratio of the left- and right-hand sides of the sum rule for σ_T, keeping various numbers of terms (up to $n = 6$) in the series on the right. We find oscillations as a function of n for large coupling. Successive terms in the series alternate in sign. For approximately $G \geq 0.06$ the oscillations increase with increasing n and the series converges. The convergence becomes of course trivial for a low coupling strength ($g/m \ll 5$ or $G \ll 0.01$); for orientation in this regard we indicate the contribution of elastic scattering by a dashed line giving σ_0/σ_T. Figure 2(c) shows the convergence of the other sum rules. For this purpose we plotted the ratios of the left- and the right-hand sides of Eqs. (6). The divergence starts somewhat earlier for the one-particle spectrum and considerably earlier for the two-particle correlation, which seems to be in accord with the expected effect from the factors n and $n(n-1)$.

The size of the interaction region depends on the mass of the exchanged particle. To test this influence we increased the mass m to approximately the ρ-meson mass ($m = 0.75$ GeV/c^2). Although the situation improves somewhat (approximately by a factor of 4), as is shown in Figure 3(a), the Pomeranchukon coupling stays far outside of the convergent region. Figure 3(b) investigates the cluster sums for different numbers of multiplicities considered and indicates again the contribution of the elastic cross section. If we neglect the somewhat different absolute size of the coupling, we find a detailed similarity to Fig. 2(b). Figure 3(c) tests the other sum rules; the divergence of these sum rules again starts somewhat earlier.

IV. CONCLUSION

The calculation illustrates the problem of convergence for the cluster expansion and the sum rules. For various choices of parameter values in the ϕ^3 ladder model we find that the convergence fails for large values of the coupling strength, and that the necessary value to produce a Pomeranchukon with $\alpha(0) = 1$ lies definitely in the divergent region. The existence of a nontrivial convergent region might indicate that convergence is not impossible for other models, which might require a weaker coupling strength to obtain a Pomeranchukon. This hope seems especially attractive, as the Pomeranchukon coupling needed in the pure ladder model is probably somewhat too strong. Definite conclusions as to the applicability of the cluster expansion therefore do not seem possible; the answer to this question has to come from the behavior of the experimental data.

ACKNOWLEDGMENTS

The author wishes to thank Professor H. W. Wyld, Professor S.-J. Chang, and Dr. D. K. Campbell for many helpful discussions.

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*Work supported in part by the National Science Foundation under Grant No. NSF GP 25303.

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