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Pion Mechanism for the Process $\gamma + \gamma \rightarrow \nu + \bar{\nu}$

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A mechanism involving pions in the intermediate state is considered for the process $\gamma + \gamma \rightarrow \nu + \bar{\nu}$. Fixed-momentum dispersion relation methods are employed to evaluate the cross sections and luminosities for both electron-type and muon-type neutrinos. Comparison is made between this mechanism and previous results involving intermediate bosons.

I. GENERAL DISCUSSION

The process $\gamma + \gamma \rightarrow \nu + \bar{\nu}$ vanishes to lowest order in the Fermi theory of weak interactions.¹ It has been analyzed, assuming an intermediate-boson theory of weak interactions.² In this paper, we consider a mechanism for this process in the Fermi theory. The mechanism involves charged pions in the intermediate state, specifically, direct coupling of a pion to a charged lepton and a neutrino. The coupling constant G' is known phenomenologically from the decay of the pion³:

$$G' = fG, \quad (1.1)$$

$$f = 0.95m_\pi, \quad (1.2)$$

$$G = 10^{-5}/M_p^2. \quad (1.3)$$

We employ a fixed-momentum dispersion relation approach in the manner of Ref. 2. The lowest-order contribution to the matrix element is of order ω^3 , where ω is the center-of-mass energy of each photon. For both the production of electron-type and muon-type neutrinos, the crossed-channel contributions are found to be very small com-

pared to the direct-channel contributions.

The interaction Lagrangian is

$$L_{\text{int}} = L_{e\gamma} + L_{\pi\gamma} + L_{\pi l \nu \gamma}. \quad (1.4)$$

The expression for $L_{\pi l \nu \gamma}$ is

$$L_{\pi l \nu \gamma} = \frac{G}{\sqrt{2}} (J_\alpha l_\alpha + J_\alpha^\dagger l_\alpha^\dagger), \quad (1.5)$$

where

$$J_\alpha = f \left(\frac{\partial}{\partial x_\alpha} - ieA_\alpha \right) \phi_\pi, \quad (1.6)$$

$$l_\alpha = \bar{e}\gamma_\alpha(1 + \gamma_5)\nu_e + \bar{\mu}\gamma_\alpha(1 + \gamma_5)\nu_\mu. \quad (1.7)$$

II. ABSORPTIVE PARTS

The absorptive part $A(T)$ is obtained from the S -matrix unitarity formula

$$\begin{aligned} A(T) &= -\frac{1}{2}i \langle f | T - T^\dagger | i \rangle \\ &= -\frac{1}{2}(2\pi)^4 \sum_n \langle f | T | n \rangle \langle n | T^\dagger | i \rangle \delta(P_n - P_i), \end{aligned} \quad (2.1)$$

where

$$S_{fi} = \delta_{fi} + R_{fi}, \quad (2.2)$$

$$R_{fi} = -(2\pi)^4 i \delta(P_f - P_i) T_{fi}. \quad (2.3)$$

We approximate Eq. (2.1) by including only two-particle intermediate states. Specifically, we include $|e^+(\mu^+), e^-(\mu^-)\rangle$, $|\pi^+, \pi^-\rangle$ states in the direct

channel and $|\pi, e(\mu)\rangle$ states in the crossed channel. The electrons and muons in the intermediate states are for $\langle \nu_e, \bar{\nu}_e |$ and $\langle \nu_\mu, \bar{\nu}_\mu |$ final states, respectively.

Hence, for the direct channel,

$$\begin{aligned} A(T) \cong & -\frac{1}{2}(2\pi)^4 [\sum \langle \nu, \bar{\nu} | T | l^+, l^- \rangle \langle l^+, l^- | T^\dagger | \gamma_1, \gamma_2 \rangle \delta(p_{l^+} + p_{l^-} - k_1 - k_2) \\ & + \sum \langle \nu, \bar{\nu} | T | \pi^+, \pi^- \rangle \langle \pi^+, \pi^- | T^\dagger | \gamma_1, \gamma_2 \rangle \delta(p_{\pi^+} + p_{\pi^-} - k_1 - k_2)] \\ = & -\frac{1}{2}(2\pi)^4 [\sum M_{2a} M_{1a}^\dagger \delta(p_{l^+} + p_{l^-} - k_1 - k_2) + \sum M_{2b} M_{1b}^\dagger \delta(p_{\pi^+} + p_{\pi^-} - k_1 - k_2)], \end{aligned} \quad (2.4)$$

where

$$M_{1a} = \langle \gamma_1, \gamma_2 | -iT | e^+(\mu^+), e^-(\mu^-) \rangle, \quad (2.5)$$

$$M_{1b} = \langle \gamma_1, \gamma_2 | -iT | \pi^+, \pi^- \rangle, \quad (2.6)$$

$$M_{2a} = \langle \nu, \bar{\nu} | -iT | e^+(\mu^+), e^-(\mu^-) \rangle, \quad (2.7)$$

$$M_{2b} = \langle \nu, \bar{\nu} | -iT | \pi^+, \pi^- \rangle. \quad (2.8)$$

The respective graphs are shown in Figs. 1-4.

$$M_{1a} = \frac{ie^2}{(2\omega_1)^{1/2}(2\omega_2)^{1/2}} \bar{v}(p - k_1 - k_2) \left(\not{\epsilon}_2 \frac{i(\not{p} - \not{k}_1) - m}{(p - k_1)^2 + m^2} \not{\epsilon}_1 + \not{\epsilon}_1 \frac{i(\not{p} - \not{k}_2) - m}{(p - k_2)^2 + m^2} \not{\epsilon}_2 \right) u(p), \quad (2.9)$$

$$\begin{aligned} M_{1b} = & \frac{ie^2}{(2\omega_{\pi^+})^{1/2}(2\omega_{\pi^-})^{1/2}(2\omega_1)^{1/2}(2\omega_2)^{1/2}} \left(\frac{(2p_\pi - k_1) \cdot \epsilon_1 (2p_\pi - 2k_1 - k_2) \cdot \epsilon_2}{(p_\pi - k_1)^2 + m_\pi^2} \right. \\ & \left. + \frac{(2p_\pi - k_2) \cdot \epsilon_2 (2p_\pi - 2k_2 - k_1) \cdot \epsilon_1}{(p_\pi - k_2)^2 + m_\pi^2} - 2\epsilon_1 \cdot \epsilon_2 \right), \end{aligned} \quad (2.10)$$

$$M_{2a} = \frac{iG'^2}{2} \bar{v}(p - p_\nu - p_{\bar{\nu}})(\not{p}_\nu - \not{p})(1 + \gamma_5)v(-p_{\bar{\nu}}) \frac{1}{(p_\nu - p)^2 + m_\pi^2} \bar{u}(p_\nu)(1 - \gamma_5)(\not{p}_\nu - \not{p})u(p), \quad (2.11)$$

$$M_{2b} = \frac{iG'^2}{2(2\omega_{\pi^+})^{1/2}(2\omega_{\pi^-})^{1/2}} \bar{u}(p_\nu)(1 - \gamma_5)\not{p}_\pi \frac{i(\not{p}_\nu - \not{p}_\pi) - m}{(p_\nu - p_\pi)^2 + m^2} (\not{p}_\nu + \not{p}_{\bar{\nu}} - \not{p}_\pi)(1 + \gamma_5)v(-p_{\bar{\nu}}). \quad (2.12)$$

The lepton mass m in the above equations is the electron mass for $\langle \nu_e, \bar{\nu}_e |$ final states and the muon mass for $\langle \nu_\mu, \bar{\nu}_\mu |$ final states.

For the crossed channel,

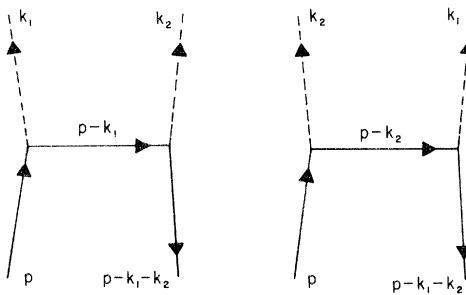


FIG. 1. Graphs for M_{1a} .

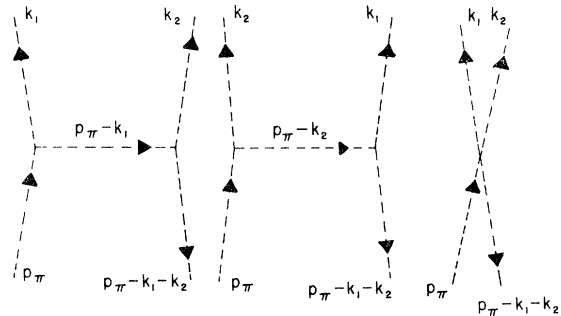
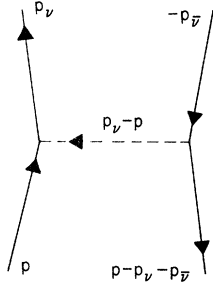
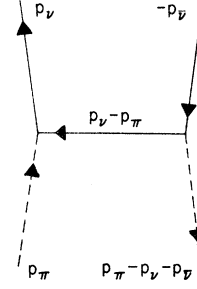


FIG. 2. Graphs for M_{1b} .

FIG. 3. Graph for M_{2a} .FIG. 4. Graph for M_{2b} .

$$A^c(T) \cong -\frac{1}{2}(2\pi)^4 \sum \langle \gamma_2, \nu | T | l, \pi \rangle \langle l, \pi | T^\dagger | \gamma_1, \nu' \rangle \delta(p_l + p_\pi - k_1 - p_{\nu'}) = -\frac{1}{2}(2\pi)^4 \sum M_2^c M_1^{c\dagger} \delta(p_l + p_\pi - k_1 - p_{\nu'}), \quad (2.13)$$

where

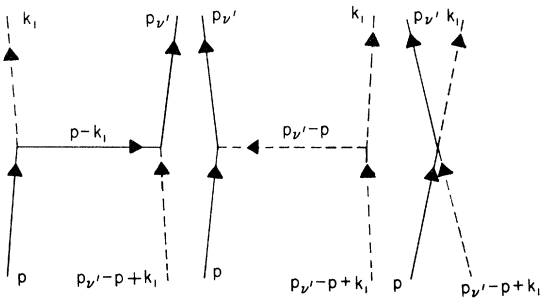
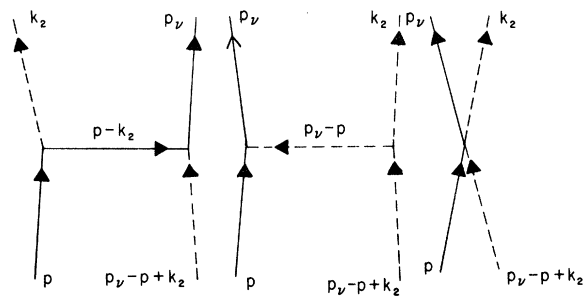
$$M_1^c = \langle \gamma_1, \nu' | -iT | e(\mu), \pi \rangle, \quad (2.14)$$

$$M_2^c = \langle \gamma_2, \nu | -iT | e(\mu), \pi \rangle. \quad (2.15)$$

The graphs for M_1^c and M_2^c are shown in Figs. 5 and 6, respectively:

$$\begin{aligned} M_1^c = & \frac{ieG'}{\sqrt{2}(2\omega_\pi)^{1/2}(2\omega_1)^{1/2}} \bar{u}(p_{\nu'}) (1 - \gamma_5) (\not{p}_{\nu'} - \not{p} + \not{k}_1) \frac{i(\not{p} - \not{k}_1) - m}{(p - k_1)^2 + m^2} \not{\epsilon}_1 u(p) \\ & + \frac{eG'}{\sqrt{2}(2\omega_\pi)^{1/2}(2\omega_1)^{1/2}} \frac{(k_1 + 2p_{\nu'} - 2p) \cdot \epsilon_1}{(p_{\nu'} - p)^2 + m_\pi^2} \bar{u}(p_{\nu'}) (1 - \gamma_5) (\not{p}_{\nu'} - \not{p}) u(p) \\ & - \frac{eG'}{\sqrt{2}(2\omega_\pi)^{1/2}(2\omega_1)^{1/2}} \bar{u}(p_{\nu'}) (1 - \gamma_5) \not{\epsilon}_1 u(p), \end{aligned} \quad (2.16)$$

$$\begin{aligned} M_2^c = & \frac{ieG'}{\sqrt{2}(2\omega_\pi)^{1/2}(2\omega_2)^{1/2}} \bar{u}(p_\nu) (1 - \gamma_5) (\not{p}_\nu - \not{p} + \not{k}_2) \frac{i(\not{p} - \not{k}_2) - m}{(p - k_2)^2 + m^2} \not{\epsilon}_2 u(p) \\ & + \frac{eG'}{\sqrt{2}(2\omega_\pi)^{1/2}(2\omega_2)^{1/2}} \frac{(k_2 + 2p_\nu - 2p) \cdot \epsilon_2}{(p_\nu - p)^2 + m_\pi^2} \bar{u}(p_\nu) (1 - \gamma_5) (\not{p}_\nu - \not{p}) u(p) \\ & - \frac{eG'}{\sqrt{2}(2\omega_\pi)^{1/2}(2\omega_2)^{1/2}} \bar{u}(p_\nu) (1 - \gamma_5) \not{\epsilon}_2 u(p). \end{aligned} \quad (2.17)$$

FIG. 5. Graphs for M_1^c .FIG. 6. Graphs for M_2^c .

Let the plane of scattering be the x - z plane, and we choose the center-of-mass frame. Then, in the direct channel, we have

$$\begin{aligned} k_1 &= (0, 0, k, i\omega), \\ k_2 &= (0, 0, -k, i\omega), \\ p_\nu &= ((1-z^2)^{1/2}k, 0, zk, i\omega), \\ p_{\bar{\nu}} &= (-(1-z^2)^{1/2}k, 0, -zk, i\omega). \end{aligned} \quad (2.18)$$

In the crossed channel,

$$\begin{aligned} k_1 &= (0, 0, k, i\omega), \\ p_\nu &= (0, 0, -k, i\omega), \\ p_\nu &= ((1-z^2)^{1/2}k, 0, zk, i\omega), \\ k_2 &= (-(1-z^2)^{1/2}k, 0, -zk, i\omega). \end{aligned} \quad (2.19)$$

In the above equations,

$$z = \cos\theta, \quad (2.20)$$

where θ is the center-of-mass scattering angle.

We also specify our calculation to be in the transverse gauge. Hence, in the direct channel one has the following four linear polarization cases.

$$\begin{aligned} \text{Case I: } \epsilon_1 &= (0, 1, 0, 0), \quad \epsilon_2 = (0, 1, 0, 0). \\ \text{Case II: } \epsilon_1 &= (1, 0, 0, 0), \quad \epsilon_2 = (-1, 0, 0, 0). \\ \text{Case III: } \epsilon_1 &= (0, 1, 0, 0), \quad \epsilon_2 = (-1, 0, 0, 0). \\ \text{Case IV: } \epsilon_1 &= (1, 0, 0, 0), \quad \epsilon_2 = (0, 1, 0, 0). \end{aligned} \quad (2.21)$$

In the crossed channel, one has the following four linear polarization cases:

$$\begin{aligned} \text{Case I: } \epsilon_1 &= (0, 1, 0, 0), \quad \epsilon_2 = (0, 1, 0, 0). \\ \text{Case II: } \epsilon_1 &= (1, 0, 0, 0), \quad \epsilon_2 = (-z, 0, (1-z^2)^{1/2}, 0). \\ \text{Case III: } \epsilon_1 &= (0, 1, 0, 0), \quad \epsilon_2 = (-z, 0, (1-z^2)^{1/2}, 0). \\ \text{Case IV: } \epsilon_1 &= (1, 0, 0, 0), \quad \epsilon_2 = (0, 1, 0, 0). \end{aligned} \quad (2.22)$$

III. EVALUATION

Expressing the matrix element as

$$M = \sum_i \alpha_i(s, t, u) \beta_i, \quad (3.1)$$

where $\alpha_i(s, t, u)$ is the i th amplitude and β_i is the corresponding i th basis, the process $\gamma + \gamma \rightarrow \nu + \bar{\nu}$ can be represented by²

$$M = A(s, t, u) \beta_A + B(s, t, u) \beta_B + C(s, t, u) \beta_C. \quad (3.2)$$

The variables s , t , and u are

$$\begin{aligned} s &= -(k_1 + k_2)^2, \\ t &= -(k_1 - k_3)^2, \\ u &= -(k_1 - k_4)^2. \end{aligned} \quad (3.3)$$

The respective bases are

$$\begin{aligned} \beta_A &= \bar{u}(p_\nu) \frac{1-\gamma_5}{2} [K^2(\epsilon_1 P \cdot \epsilon_2 + \epsilon_2 P \cdot \epsilon_1) - (P \cdot K)(\epsilon_1 K \cdot \epsilon_2 + \epsilon_2 K \cdot \epsilon_1) \\ &\quad - K(K \cdot \epsilon_1 P \cdot \epsilon_2 + P \cdot \epsilon_1 K \cdot \epsilon_2) + P \cdot K \epsilon_1 \cdot \epsilon_2 K] \frac{1+\gamma_5}{2} v(-p_{\bar{\nu}}), \end{aligned} \quad (3.4)$$

$$\beta_B = 4(F_{\mu\nu}^1 F_{\alpha\mu}^2 P_\alpha k_\rho^2 k_\nu^2 + F_{\mu\nu}^1 F_{\nu\beta}^2 P_\mu k_\rho^1 k_\beta^1) \bar{u}(p_\nu) \frac{1-\gamma_5}{2} \gamma_\rho \frac{1+\gamma_5}{2} v(-p_{\bar{\nu}}), \quad (3.5)$$

$$\beta_C = 2F_{\mu\nu}^1 F_{\alpha\beta}^2 (P_\mu k_\rho^1 \epsilon_{\alpha\nu\beta\sigma} P_\sigma + P_\alpha k_\rho^2 \epsilon_{\mu\beta\nu\sigma} P_\sigma) \bar{u}(p_\nu) \frac{1-\gamma_5}{2} \gamma_\rho \frac{1+\gamma_5}{2} v(-p_{\bar{\nu}}), \quad (3.6)$$

where

$$K = k_1 - k_2, \quad P = p_\nu - p_{\bar{\nu}}. \quad (3.7)$$

Only the basis β_A is of order ω^3 , the others being of order ω^5 . Hence, to lowest order, we have

$$M \cong A(s, t, u) \beta_A. \quad (3.8)$$

The respective direct- and crossed-channel contributions to $A(s, t, u)$ are²

$$A(s, t) = \frac{M_I + M_{II}}{2\beta_A}, \quad (3.9)$$

$$A(u, t) = \frac{M_I^c + M_{II}^c}{2\beta_A^c}, \quad (3.10)$$

where the indices I and II signify the polarization cases in the respective channels. In Eq. (3.10), we have

$$\beta'_A = \bar{u}(p_\nu) \frac{1-\gamma_5}{2} [K'^2(\not{\epsilon}_1 P' \cdot \epsilon_2 + \not{\epsilon}_2 P' \cdot \epsilon_1) - (P' \cdot K')(\not{\epsilon}_1 K' \cdot \epsilon_2 + \not{\epsilon}_2 K' \cdot \epsilon_1) - \not{K}'(K' \cdot \epsilon_1 P' \cdot \epsilon_2 + P' \cdot \epsilon_1 K' \cdot \epsilon_2) + P' \cdot K' \epsilon_1 \cdot \epsilon_2 \not{K}'] \frac{1+\gamma_5}{2} u(p_{\nu'}), \quad (3.11)$$

where

$$K' = k_1 + k_2, \quad P' = p_\nu + p_{\nu'}. \quad (3.12)$$

The absorptive parts $A'(s, t)$ and $A'(u, t)$ are obtained by substituting the respective channel absorptive parts of M , as given by expressions (2.4) and (2.13).

$$A'(s, t) = \frac{A(T)_I + A(T)_{II}}{2\beta_A}, \quad (3.13)$$

$$A'(u, t) = \frac{A^c(T)_I + A^c(T)_{II}}{2\beta'_A}. \quad (3.14)$$

The respective dispersive parts of the amplitudes are obtained from the absorptive expressions by integration:

$$A(s, t) = \frac{1}{\pi} \int_{4m_\pi^2}^{\infty} \frac{A'(s', t)}{s' - s} ds', \quad (3.15)$$

$$A(u, t) = \frac{1}{\pi} \int_{(m_\pi + m_l)^2}^{\infty} \frac{A'(u', t)}{u' - u} du'. \quad (3.16)$$

The resulting expression for $A(s, t, u)$ is⁴

$$A(s, t, u) = (4.42) \frac{ie^2 G'^2}{32\pi^2} \frac{m_e^2}{m_\pi^4} \text{ electron-type neutrinos}, \quad (3.17)$$

$$A(s, t, u) = (-14.57) \frac{ie^2 G'^2}{32\pi^2} \frac{m_\mu^2}{m_\pi^4} \text{ muon-type neutrinos}. \quad (3.18)$$

In the factor (4.42) appearing in Eq. (3.17), the contribution from the crossed channel is only (0.08). In the factor (-14.57) of Eq. (3.18), the crossed-channel contribution is only (0.15). Thus, for both types of neutrino emission the crossed-channel contributions are very small. The corresponding expression for R is

$$R = \frac{\pi^2 e^2 G'^2 \rho}{2(2\omega_1)^{1/2} (2\omega_2)^{1/2} m_\pi^4} \delta(p_\nu + p_{\bar{\nu}} - k_1 - k_2) \bar{u}(p_\nu) \frac{1-\gamma_5}{2} \times [K^2(\not{\epsilon}_1 P \cdot \epsilon_2 + \not{\epsilon}_2 P \cdot \epsilon_1) - (P \cdot K)(\not{\epsilon}_1 K \cdot \epsilon_2 + \not{\epsilon}_2 K \cdot \epsilon_1) - \not{K}(K \cdot \epsilon_1 P \cdot \epsilon_2 + P \cdot \epsilon_1 K \cdot \epsilon_2) + P \cdot K \epsilon_1 \cdot \epsilon_2 \not{K}] \times \frac{1+\gamma_5}{2} v(-p_{\bar{\nu}}), \quad (3.19)$$

where

$$\rho = 4.42 m_e^2 \text{ electron-type neutrinos}, \quad (3.20)$$

$$\rho = -14.57 m_\mu^2 \text{ muon-type neutrinos}. \quad (3.21)$$

IV. CROSS SECTION AND LUMINOSITY

The unpolarized differential cross section obtained from Eq. (3.19) is

$$\frac{d\sigma}{d\Omega} = \frac{e^4 G'^4 \rho^2}{2048\pi^6 m_\pi^8} \omega^6 (1 - \cos^4 \theta). \quad (4.1)$$

The total unpolarized cross section is

$$\sigma = \frac{e^4 G'^4 \rho^2}{640\pi^5 m_\pi^8} \omega^6. \quad (4.2)$$

Evaluating Eq. (4.2), one obtains

$$\sigma = 5.4 \times 10^{-64} \left(\frac{\omega}{M_p} \right)^6 \text{ cm}^2 \text{ electron-type neutrinos}, \quad (4.3)$$

$$\sigma = 1.1 \times 10^{-53} \left(\frac{\omega}{M_p} \right)^6 \text{ cm}^2 \text{ muon-type neutrinos}. \quad (4.4)$$

TABLE I. Neutrino cross sections and luminosities for various intermediate-boson masses. The units of σ and L are cm^2 and $\text{erg}/\text{cm}^3 \text{sec}$, respectively.

	$\sigma(\nu_e, \bar{\nu}_e)$	$\sigma(\nu_\mu, \bar{\nu}_\mu)$	$L(\nu_e, \bar{\nu}_e)$	$L(\nu_\mu, \bar{\nu}_\mu)$
$M_W = 10 \text{ GeV}$	$1.9 \times 10^{-46} (\omega/M_p)^6$	$3.5 \times 10^{-47} (\omega/M_p)^6$	$8.1 \times 10^{-8} (T/10^9)^{13}$	$1.5 \times 10^{-8} (T/10^9)^{13}$
$M_W = 50 \text{ GeV}$	$4.1 \times 10^{-49} (\omega/M_p)^6$	$1.1 \times 10^{-49} (\omega/M_p)^6$	$1.8 \times 10^{-10} (T/10^9)^{13}$	$4.7 \times 10^{-11} (T/10^9)^{13}$
$M_W = 100 \text{ GeV}$	$2.9 \times 10^{-50} (\omega/M_p)^6$	$8.5 \times 10^{-51} (\omega/M_p)^6$	$1.2 \times 10^{-11} (T/10^9)^{13}$	$3.7 \times 10^{-12} (T/10^9)^{13}$
$M_W = 500 \text{ GeV}$	$5.9 \times 10^{-53} (\omega/M_p)^6$	$2.1 \times 10^{-53} (\omega/M_p)^6$	$2.6 \times 10^{-1} (T/10^{10})^{13}$	$9.3 \times 10^{-2} (T/10^{10})^{13}$
$M_W = 1000 \text{ GeV}$	$4.1 \times 10^{-54} (\omega/M_p)^6$	$1.6 \times 10^{-54} (\omega/M_p)^6$	$1.8 \times 10^{-2} (T/10^{10})^{13}$	$6.9 \times 10^{-3} (T/10^{10})^{13}$

The corresponding blackbody neutrino luminosities are

$$L = 2.4 \times 10^{-12} \left(\frac{T}{10^{10}} \right)^{13} \text{ erg/cm}^3 \text{ sec} \quad (4.5)$$

electron-type neutrinos,

$$L = 4.7 \times 10^{-2} \left(\frac{T}{10^{10}} \right)^{13} \text{ erg/cm}^3 \text{ sec} \quad (4.6)$$

muon-type neutrinos.

In Ref. 2, the expression obtained for σ is

$$\sigma = \frac{e^4 g^4}{160\pi^5 M_W^8} \left(\frac{1}{3} \ln \frac{M_W^2}{m^2} - \frac{23}{72} \right)^2 \omega^6. \quad (4.7)$$

Table I gives the corresponding values of the cross sections and luminosities for various intermediate-boson masses M_W .⁵

Upon comparison, one finds that the expressions for $\sigma(\nu_\mu, \bar{\nu}_\mu)$ and $L(\nu_\mu, \bar{\nu}_\mu)$ of Eqs. (4.4) and (4.6) are of the order of magnitude of $\sigma(\nu_e, \bar{\nu}_e)$ and $L(\nu_e, \bar{\nu}_e)$ of Ref. 2 for $M_W = 500 \text{ GeV}$. Of particular interest is to note that in Ref. 2 the respective expressions for σ and L are always greater for electron-type neutrino emission, whereas those of Eqs. (4.3)–(4.6) are greater for muon-type emission.

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S. Deser (Gordon and Breach, New York, 1966).

⁴See the Appendix of Ref. 2 for the methods of approximation employed.

⁵A factor of $(4\pi)^2$ was omitted from the respective expressions for $M_W = 2 \text{ GeV}$ in Ref. 2.