Radiative corrections to neutrino mixing and *CP* violation in the minimal seesaw model with leptogenesis

Jian-wei Mei* and Zhi-zhong Xing†
CCAST (World Laboratory), P. O. Box 8730, Beijing 100080, China
and Institute of High Energy Physics, Chinese Academy of Sciences, P. O. Box 918 (4), Beijing 100039, China‡
(Received 12 December 2003; published 9 April 2004)

Radiative corrections to neutrino mixing and *CP* violation are analyzed in the minimal seesaw model with two heavy right-handed neutrinos. We find that the textures of the effective Majorana neutrino mass matrix are essentially stable against renormalization effects. Taking into account the Frampton-Glashow-Yanagida *Ansatz* for the Dirac neutrino Yukawa coupling matrix, we calculate the running effects of light neutrino masses, lepton flavor mixing angles, and *CP*-violating phases for both the $m_1=0$ (normal mass hierarchy) and $m_3=0$ (inverted mass hierarchy) cases in the standard model and in its minimal supersymmetric extension. Very instructive predictions for the cosmological baryon number asymmetry via thermal leptogenesis are also given with the help of low-energy neutrino mixing quantities.

DOI: 10.1103/PhysRevD.69.073003 PACS number(s): 14.60.Pq, 11.30.Er, 25.30.Pt

I. INTRODUCTION

In the standard model (SM), lepton number is conserved and neutrinos are massless Weyl particles. However, the recent Super-Kamiokande [1], SNO [2], KamLAND [3], and K2K [4] experiments have provided us with very compelling evidence that neutrinos are actually massive and lepton flavors are mixed. The most economical modification of the SM, which can both accommodate neutrino masses and allow lepton number violation to explain the cosmological baryon asymmetry via leptogenesis [5], is to introduce two heavy right-handed neutrinos $N_{1,2}$ and keep the Lagrangian of electroweak interactions invariant under $\mathrm{SU}(2)_{\mathrm{L}}\times \mathrm{U}(1)_{\mathrm{Y}}$ gauge transformation [6–9]. In this case, the Lagrangian relevant for lepton masses is described by

$$-\mathcal{L}_{\mathrm{Y(SM)}}=\overline{l_{\mathrm{L}}}Y_l e_{\mathrm{R}}H+\overline{l_{\mathrm{L}}}Y_\nu \nu_{\mathrm{R}}H^{\mathrm{c}}+\frac{1}{2}\overline{\nu_{\mathrm{R}}^{\mathrm{c}}}M_{\mathrm{R}}\nu_{\mathrm{R}}+\mathrm{H.c.}, \quad (1)$$

where l_{L} denotes the left-handed lepton doublet; e_{R} and ν_{R} stand, respectively, for the right-handed charged lepton and Majorana neutrino singlets; and H is the Higgs-boson weak isodoublet (with $H^{\mathrm{c}}\equiv i\sigma_2 H^*$). If the minimal supersymmetric standard model (MSSM) is taken into account, one may similarly write out the Lagrangian relevant for lepton masses:

$$-\mathcal{L}_{\mathrm{Y(MSSM)}}=\overline{l_{\mathrm{L}}}Y_l e_{\mathrm{R}}H_1+\overline{l_{\mathrm{L}}}Y_\nu \nu_{\mathrm{R}}H_2+\frac{1}{2}\overline{\nu_{\mathrm{R}}^{\mathrm{c}}}M_{\mathrm{R}}\nu_{\mathrm{R}}+\mathrm{H.c.}, \quad (2)$$

where $H_{1,2}$ (with hypercharges $\pm 1/2$) are the MSSM Higgs doublets.

Without loss of generality, both the heavy Majorana neutrino mass matrix M_{R} and the charged-lepton Yukawa coupling matrix Y_l can be taken to be diagonal, real, and positive. In this specific flavor basis, the Dirac neutrino Yukawa coupling matrix Y_ν is a complex 3×2 rectangular matrix:

$$Y_\nu=\begin{pmatrix} a_1 & a_2 \\ b_1 & b_2 \\ c_1 & c_2 \end{pmatrix}. \quad (3)$$

Below the mass scale of the lightest right-handed neutrino N_1 (denoted as M_1), M_{R} can be integrated out of the theory. Such a treatment corresponds to a replacement of the last two terms in $\mathcal{L}_{\mathrm{Y}}$ by a dimension-5 operator, whose coupling matrix takes the seesaw [10] form

$$\kappa\,(M_1)=Y_\nu M_{\mathrm{R}}^{-1}Y_\nu^T=\begin{pmatrix} \frac{a_1^2}{M_1} & \frac{a_1b_1}{M_1} & \frac{a_1c_1}{M_1} \\ \frac{a_1b_1}{M_1} & \frac{b_1^2}{M_1} & \frac{b_1c_1}{M_1} \\ \frac{a_1c_1}{M_1} & \frac{b_1c_1}{M_1} & \frac{c_1^2}{M_1} \end{pmatrix} + \begin{pmatrix} \frac{a_2^2}{M_2} & \frac{a_2b_2}{M_2} & \frac{a_2c_2}{M_2} \\ \frac{a_2b_2}{M_2} & \frac{b_2^2}{M_2} & \frac{b_2c_2}{M_2} \\ \frac{a_2c_2}{M_2} & \frac{b_2c_2}{M_2} & \frac{c_2^2}{M_2} \end{pmatrix}. \quad (4)$$

A very special feature of the minimal seesaw model is that $\mathrm{Det}[\kappa(M_1)]=0$ holds for arbitrary a_i, b_i, and c_i [11]. The reason for this is rather simple: the 2×2 mass matrix M_{R} is at most of rank 2, hence $\kappa\,(M_1)=Y_\nu M_{\mathrm{R}}^{-1}Y_\nu^T$ can at most have rank 2, and $\mathrm{Det}[\kappa\,(M_1)]=0$ must hold.

After spontaneous gauge symmetry breaking, the neutral component of H acquires the vacuum expectation value $v\approx 174$ GeV. Then one obtains the charged-lepton mass ma-

*Electronic address: jwmei@mail.ihep.ac.cn
†Electronic address: xingzz@mail.ihep.ac.cn
‡Mailing address.

trix $M_l = vY_l(M_Z)$ and the light (left-handed) Majorana neutrino mass matrix $M_\nu = v^2\kappa(M_Z)$ at the electroweak scale $\mu = M_Z$ in the SM. The neutral components of H_1 and H_2 may similarly acquire the vacuum expectation values $v\cos\beta$ and $v\sin\beta$ at the electroweak symmetry breaking scale. It turns out that $M_l = v\cos\beta Y_l(M_Z)$ and $M_\nu = v^2\sin^2\beta\kappa(M_Z)$ in the MSSM. Note that $\kappa(M_Z)$ and $\kappa(M_1)$ can be related to each other via the following one-loop renormalization-group equation (RGE) [12]:

$$16\pi^2\frac{\mathrm{d}\kappa}{\mathrm{d}t} = \alpha\kappa + C[(Y_lY_l^\dagger)\kappa + \kappa(Y_lY_l^\dagger)^T], \qquad (5)$$

where $t = \ln(\mu/M_1)$ with μ being the renormalization scale. In the SM [13] or in its minimal supersymmetric extension [14],

$$C_{\rm SM} = -\frac{3}{2},$$

$$\alpha_{\rm SM} = -3g_2^2 + 6f_t^2 + \lambda \qquad (6)$$

or

$$C_{\rm MSSM} = 1,$$

$$\alpha_{\rm MSSM} = -\frac{6}{5}g_1^2 - 6g_2^2 + 6f_t^2, \qquad (7)$$

where $g_{1,2}$ denote the gauge couplings, f_t denotes the top-quark Yukawa coupling, and λ denotes the Higgs-boson self-coupling in the SM.[1] If $M_1 \gg M_Z$ holds, some deviation of $\kappa(M_Z)$ from $\kappa(M_1)$ must take place.

Apparently, it is $\kappa(M_Z)$ or M_ν that governs the low-energy phenomenology of neutrino masses and lepton flavor mixing, because Y_l stays diagonal in the RGE running from M_1 to M_Z. In all recent analysis of the minimal seesaw model [6–9], however, the quantum corrections to κ at the electroweak scale have been neglected for the sake of simplicity and illustration. The importance of RGE effects for the evaluation of baryogenesis via leptogenesis in the *bottom-up* approach (i.e., from low energies to the mass scale of heavy right-handed neutrinos) has been pointed out by some authors [15], but a careful analysis of such effects in the minimal seesaw model has not been done.

The purpose of this paper is to examine the stability of κ against radiative corrections in the minimal seesaw model. We find that the texture of κ is essentially stable in the RGE evolution from M_1 to M_Z. To be specific, we calculate the running effects of neutrino masses, lepton flavor mixing angles, and *CP*-violating phases by taking into account the Frampton-Glashow-Yanagida (FGY) *Ansatz* for Y_ν [6]. The cosmological baryon number asymmetry via leptogenesis is also calculated at the scale $\mu = M_1$ with the help of low-energy neutrino mixing quantities.

II. RGE RUNNING EFFECTS FROM M_1 TO M_Z

In the flavor basis chosen above, one may simplify the RGE in Eq. (5) and get the radiative corrections to κ at M_Z. Let us define the evolution functions

$$I_\alpha = \exp\left[-\frac{1}{16\pi^2}\int_0^{\ln(M_1/M_Z)}\alpha(t)dt\right],$$

$$I_l = \exp\left[-\frac{C}{16\pi^2}\int_0^{\ln(M_1/M_Z)}f_l^2(t)dt\right], \qquad (8)$$

where f_l (for $l = e, \mu, \tau$) denote the Yukawa coupling eigenvalues of charged leptons. Then we solve Eq. (5) and arrive at

$$\kappa(M_Z) = I_\alpha\begin{pmatrix} I_e & 0 & 0 \\ 0 & I_\mu & 0 \\ 0 & 0 & I_\tau \end{pmatrix}\kappa(M_1)\begin{pmatrix} I_e & 0 & 0 \\ 0 & I_\mu & 0 \\ 0 & 0 & I_\tau \end{pmatrix}. \qquad (9)$$

The overall factor I_α affects only the magnitudes of light neutrino masses, while I_l (for $l = e, \mu, \tau$) can modify both neutrino masses and lepton flavor mixing parameters [16]. The strong mass hierarchy of three charged leptons (i.e., $f_e < f_\mu < f_\tau$) implies that $I_e < I_\mu < I_\tau$ (SM) or $I_e > I_\mu > I_\tau$ (MSSM) holds below the scale M_1.

Two comments on the consequences of Eq. (9) are in order.

(1) The determinant of κ, which vanishes at the scale M_1, keeps vanishing at the scale M_Z. This point can clearly be seen from the relation

$$\mathrm{Det}[\kappa(M_Z)] = I_\alpha^3 I_e^2 I_\mu^2 I_\tau^2\,\mathrm{Det}[\kappa(M_1)]. \qquad (10)$$

Because of $|\mathrm{Det}[\kappa(M_Z)]| = m_1m_2m_3$, where m_i (for $i = 1,2,3$) denote the masses of three light neutrinos, one may conclude that one of the three neutrino masses must vanish. Considering that $m_2 > m_1$ is required by current solar neutrino oscillation data [2], we are left with either $m_1 = 0$ (normal mass hierarchy) or $m_3 = 0$ (inverted mass hierarchy).

(2) Comparing between Eqs. (4) and (9), we find that the radiative correction to κ can effectively be expressed as the RGE running effects in a_i, b_i, and c_i of Y_ν (in the assumption that M_1 stays unchanged):

$$a_i(M_Z) = I_e\sqrt{I_\alpha}a_i(M_1),$$

$$b_i(M_Z) = I_\mu\sqrt{I_\alpha}b_i(M_1),$$

$$c_i(M_Z) = I_\tau\sqrt{I_\alpha}c_i(M_1), \qquad (11)$$

where $i = 1$ or 2. These simple relations imply that possible texture zeros of κ at M_1 remain the same at M_Z, at least at

[1] In the expression for $\alpha_{\rm SM}$, we have neglected very small contributions from the lighter quarks and charged leptons. Similarly, the up- and charm-quark Yukawa couplings have been neglected in the expression for $\alpha_{\rm MSSM}$.

the one-loop level of RGE evolution. Hence the texture of κ is essentially stable against quantum corrections from M_1 to M_Z.

For illustration, we typically take $m_t(M_Z)\approx 181$ GeV [17] to calculate the evolution functions I_α and I_l (for $l = e,\mu,\tau$). It is found that $I_e \approx I_\mu \approx 1$ is an excellent approximation both in the SM and in the MSSM. Thus the RGE running of κ is mainly governed by I_α and I_τ. The behaviors of I_α and I_τ changing with M_1 are shown in Fig. 1. One can see that $I_\tau \approx 1$ is also a good approximation, in particular in the SM. Hence the evolution of light neutrino masses must be dominated by I_α, which may significantly deviate from unity in the SM with reasonable values of the Higgs-boson mass m_H and in the MSSM with large values of $\tan\beta$.

The effect of I_τ in the running of κ will certainly lead to the evolution of the lepton flavor mixing matrix V. At the electroweak scale, V is defined to diagonalize the neutrino mass matrix M_ν in the chosen flavor basis; i.e., $V^\dagger M_\nu V^* = \mathrm{Diag}\{m_1,m_2,m_3\}$. A convenient parametrization of V is [18]

$$V=\begin{pmatrix} c_x c_z & s_x c_z & s_z \\ -c_x s_y s_z - s_x c_y e^{-i\delta} & -s_x s_y s_z + c_x c_y e^{-i\delta} & s_y c_z \\ -c_x c_y s_z + s_x s_y e^{-i\delta} & -s_x c_y s_z - c_x s_y e^{-i\delta} & c_y c_z \end{pmatrix} \times \begin{pmatrix} e^{i\rho} & 0 & 0 \\ 0 & e^{i\sigma} & 0 \\ 0 & 0 & 1 \end{pmatrix}, \tag{12}$$

where $s_x \equiv \sin\theta_x$, $c_x \equiv \cos\theta_x$, and so on. Note that the Dirac phase δ measures the strength of *CP* violation in normal neutrino oscillations, and its location in V is different from that advocated by the Particle Data Group [19]. The advantage of our present phase assignment is that δ itself does not appear in the effective Majorana mass term of the neutrinoless double beta decay, whose magnitude depends crucially on the Majorana phases ρ and σ of V [20]. Current data on solar, atmospheric, and reactor neutrino oscillations yield $\theta_x \approx 32°$ and $\theta_y \approx 45°$ (best-fit values [21]) as well as $\theta_z < 12°$ [22]. The typical mass-squared differences of solar and atmospheric neutrino oscillations read as $\Delta m^2_{\rm sun} \equiv m_2^2 - m_1^2 \approx 7.13\times 10^{-5}$ eV^2 and $\Delta m^2_{\rm atm} \equiv |m_3^2 - m_2^2| \approx 2.6\times 10^{-3}$ eV^2 (best-fit values [21]), respectively. Following Ref. [23] and Ref. [24], one may derive the RGEs of (m_1,m_2,m_3), $(\theta_x,\theta_y,\theta_z)$, and (δ,ρ,σ) with the help of Eq. (5). The relevant analytical results can be considerably simplified if the smallness of s_z and $\Delta m^2_{\rm sun}/\Delta m^2_{\rm atm}$ is taken into account. It should be noted that the parametrization of V used here is somewhat different from that taken in Ref. [24]. To be complete and explicit, we present our approximate analytical expressions for $(\dot m_1,\dot m_2,\dot m_3)$, $(\dot\theta_x,\dot\theta_y,\dot\theta_z)$, and $(\dot\delta,\dot\rho,\dot\sigma)$ in the Appendix. Because $m_1=0$ or $m_3=0$ must hold in the minimal seesaw model under consideration, it is actually possible to obtain much simpler results from Eqs. (A1)–(A4).

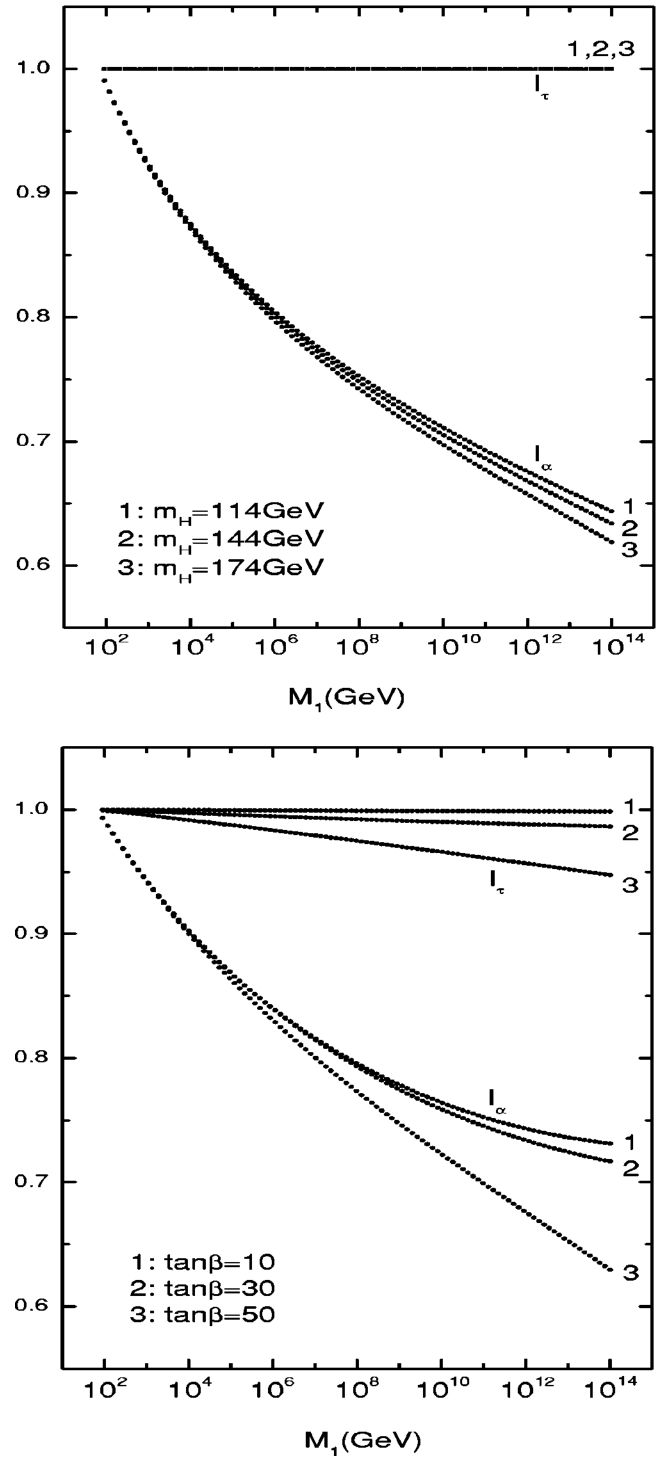

FIG. 1. Numerical illustration of the evolution functions I_α and I_τ changing with M_1 and m_H in the SM (top) or with M_1 and $\tan\beta$ in the MSSM (bottom).

III. FGY ANSATZ AND RADIATIVE CORRECTIONS

The minimal seesaw model itself has no restriction on the structure of Y_ν. In Ref. [6], Frampton, Glashow, and

Yanagida conjectured that Y_ν may have two texture zeros, $a_2=c_1=0$ or $a_2=b_1=0$, which could stem from an underlying horizontal flavor symmetry. The texture of $\kappa(M_1)$ in Eq. (4) can then be simplified:

$$\kappa(M_1)=\begin{pmatrix} \frac{a_1^2}{M_1} & \frac{a_1b_1}{M_1} & \mathbf{0} \\ \frac{a_1b_1}{M_1} & \frac{b_1^2}{M_1}+\frac{b_2^2}{M_2} & \frac{b_2c_2}{M_2} \\ \mathbf{0} & \frac{b_2c_2}{M_2} & \frac{c_2^2}{M_2} \end{pmatrix} \tag{13}$$

in the $a_2=c_1=0$ case; or

$$\kappa(M_1)=\begin{pmatrix} \frac{a_1^2}{M_1} & \mathbf{0} & \frac{a_1c_1}{M_1} \\ \mathbf{0} & \frac{b_2^2}{M_2} & \frac{b_2c_2}{M_2} \\ \frac{a_1c_1}{M_1} & \frac{b_2c_2}{M_2} & \frac{c_1^2}{M_1}+\frac{c_2^2}{M_2} \end{pmatrix} \tag{14}$$

in the $a_2=b_1=0$ case. Note that the $a_1=c_2=0$ and $a_1=b_2=0$ cases are respectively equivalent to the $a_2=c_1=0$ and $a_2=b_1=0$ cases in phenomenology. Note also that the $b_2=c_1=0$ (or $b_1=c_2=0$) case, in which the (2,3) and (3,2) matrix elements of $\kappa(M_1)$ vanish, is not favored by current neutrino oscillation data [11] and will not be taken into account in the subsequent discussions. In addition, the two instructive patterns of $\kappa(M_1)$ in Eqs. (13) and (14) are expected to have very similar consequences on neutrino masses, lepton flavor mixing angles, and *CP*-violating phases at low-energy scales (see Ref. [8] for some detailed discussions). We shall therefore concentrate later on only on the FGY *Ansatz* given in Eq. (13).

It is worth remarking that a_1, $b_{1,2}$, and c_2 in Eq. (13) are all complex parameters. Given the specific parametrization of V in Eq. (12), it is straightforward to obtain

$$\kappa(M_Z)=\frac{M_\nu}{\Omega}=V\begin{pmatrix} \frac{m_1}{\Omega} & 0 & 0 \\ 0 & \frac{m_2}{\Omega} & 0 \\ 0 & 0 & \frac{m_3}{\Omega} \end{pmatrix}V^T, \tag{15}$$

where $\Omega\equiv v^2$ in the SM and $\Omega\equiv v^2\sin^2\beta$ in the MSSM. Then the phases of a_1, $b_{1,2}$, and c_2 can be determined in terms of δ, ρ, and σ. In Refs. [6,8], the phase convention $\arg(a_1)=\arg(b_2)=\arg(c_2)=0$ has been taken. We do not adopt this phase convention in the present work. The physical results predicted by the FGY ansatz are certainly independent of any specific phase convention.

Note that $\kappa(M_Z)$ takes the same texture as $\kappa(M_1)$, and their corresponding matrix elements are related to each other via Eq. (11). This observation implies that the *bottom-up* approach should be more convenient for the numerical evaluation of radiative corrections—namely, we determine the parameters of the FGY *Ansatz* at low energies by using current neutrino oscillation data, and then run them to the mass scale M_1 to examine the size of the renormalization effects.

A. Normal neutrino mass hierarchy ($m_1=0$)

It is rather obvious that $m_1=0$ leads to $m_2=\sqrt{\Delta m^2_{\rm sun}}\approx 8.4\times 10^{-3}$ eV and $m_3=\sqrt{\Delta m^2_{\rm sun}+\Delta m^2_{\rm atm}}\approx 5.2\times 10^{-2}$ eV. Furthermore, $m_1=0$ implies that only the Majorana phase σ is physically meaningful. Taking account of the texture zero $(M_\nu)_{13}=0$, one can determine both δ and σ in terms of the flavor mixing angles $(\theta_x,\theta_y,\theta_z)$ and the mass ratio $\xi\equiv m_2/m_3\approx 0.16$ [8]:

$$\delta=\arccos\left[\frac{c_y^2s_z^2-\xi^2s_x^2(c_x^2s_y^2+s_x^2c_y^2s_z^2)}{2\xi^2s_x^3c_xs_yc_ys_z}\right],$$

$$\sigma=\frac{1}{2}\arctan\left[\frac{c_xs_y\sin\delta}{s_xc_ys_z+c_xs_y\cos\delta}\right]. \tag{16}$$

Because $|\cos\delta|\leqslant 1$ must hold, we find that θ_z is restricted to a very narrow range: $4.0°\lesssim\theta_z\lesssim 4.4°$ (i.e., $0.070\lesssim s_z\lesssim 0.077$). The implication of this result is that the FGY *Ansatz* with $m_1=0$ will simply be excluded, if the experimental value of θ_z does not really lie in the predicted region.

The running of m_i (i.e., $\dot{m}_i$) is proportional to m_i itself (for $i=1,2,3$) at the one-loop level [12–14]. As a direct consequence of $m_1=0$, the RGEs of m_1, m_2, and m_3 in Eq. (A1) may be simplified to

$$\dot{m}_1=0,$$

$$\dot{m}_2\approx\frac{1}{16\pi^2}(\alpha+2Cf_\tau^2c_x^2s_y^2)m_2,$$

$$\dot{m}_3\approx\frac{1}{16\pi^2}(\alpha+2Cf_\tau^2c_y^2)m_3. \tag{17}$$

One can see that $m_1=0$ holds at any energy scale between M_Z and M_1, and the running behaviors of m_2 and m_3 are essentially identical (dominated by the term proportional to α). To illustrate, we show the ratio $R\equiv m_2(M_Z)/m_2(M_1)$ changing with m_H in the SM or with $\tan\beta$ in the MSSM in

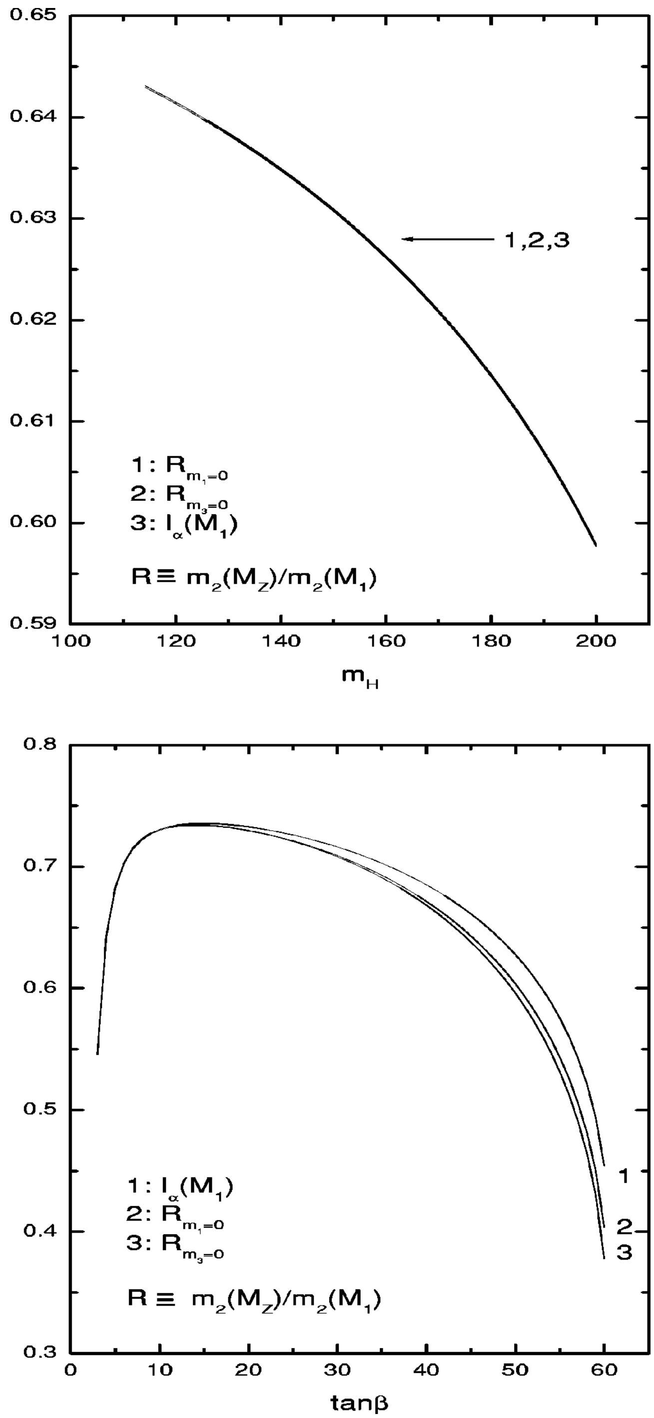

FIG. 2. The ratio of m_2 at M_Z to its value at $M_1 = 10^{14}$ GeV as a function of m_H in the SM (top) or of $\tan\beta$ in the MSSM (bottom) for the FGY *Ansatz* with $m_1 = 0$ or $m_3 = 0$.

Fig. 2, where $M_1 = 10^{14}$ GeV is typically taken. It becomes clear that $R_{m_1=0} \approx I_\alpha$ is an excellent approximation in the SM, and it is also a good approximation in the MSSM.

We remark that m_2/m_3 is approximately unchanged in the RGE evolution from M_Z to M_1 [12–14]—in other words, $\xi \approx 0.16$ is nearly a constant. It is then possible to simplify the RGEs of $(\theta_x, \theta_y, \theta_z)$ and (δ, σ) in Eqs. (A2) and (A3) up to $\mathcal{O}(\xi)$ or $\mathcal{O}(s_z)$:

$$\dot{\theta}_x \approx -\frac{Cf_\tau^2}{16\pi^2} s_x c_x s_y^2,$$

$$\dot{\theta}_y \approx -\frac{Cf_\tau^2}{16\pi^2} s_y c_y (1 + 2\xi c_x^2 \cos\delta),$$

$$\dot{\theta}_z \approx -\frac{Cf_\tau^2}{8\pi^2} \xi s_x c_x s_y c_y \tag{18}$$

and

$$\dot{\sigma} \approx \frac{Cf_\tau^2}{8\pi^2} \left(s_x c_x s_y c_y \frac{\xi}{s_z} \right) \xi \sin\delta,$$

$$\dot{\delta} \approx \frac{Cf_\tau^2}{8\pi^2} \left[s_x c_x s_y c_y \frac{\xi}{s_z} + c_x^2 (c_y^2 - s_y^2) \right] \xi \sin\delta. \tag{19}$$

In obtaining Eqs. (18) and (19), we have considered the fact that $\xi \sim 2 s_z$ holds at M_Z. As s_z has been constrained to the range $0.070 \lesssim s_z \lesssim 0.077$, the ξ/δ term in Eq. (19) is unable to drastically enhance the running of δ and σ. One can see that the running effects of three mixing angles and two *CP*-violating phases are all governed by f_τ^2. Because $f_\tau^2 \approx 10^{-4}$ in the SM, the evolution of $(\theta_x, \theta_y, \theta_z)$ and (σ, δ) is negligibly small. When $\tan\beta$ is sufficiently large (e.g., $\tan\beta \sim 50$) in the MSSM, however, $f_\tau^2 \approx 10^{-4}/\cos^2\beta$ can be of $\mathcal{O}(0.1)$ and even close to unity—in this case, some small variation of $(\theta_x, \theta_y, \theta_z)$ and (σ, δ) due to the RGE running from M_Z to M_1 will appear. Let us define $\Delta\theta_i \equiv \theta_i(M_1) - \theta_i(M_Z)$ (for $i = x, y, z$), $\Delta\delta \equiv \delta(M_1) - \delta(M_Z)$, and $\Delta\sigma \equiv \sigma(M_1) - \sigma(M_Z)$. The numerical results for $\Delta\theta_i/\theta_i(M_Z)$, $\Delta\delta/\delta(M_Z)$, and $\Delta\sigma/\sigma(M_Z)$ are shown in Fig. 3 for the MSSM with different values of M_1 and $\tan\beta$. We see that the ratio $\Delta\theta_z/\theta_z(M_Z)$ is most sensitive to the RGE running, but its magnitude is less than 10% even if $M_1 = 10^{14}$ GeV and $\tan\beta = 50$ are taken. Thus we conclude that the RGE effects on three flavor mixing angles and two *CP*-violating phases are practically negligible in the FGY *Ansatz* with $m_1 = 0$.

B. Inverted neutrino mass hierarchy ($m_3 = 0$)

If $m_3 = 0$ holds, we will arrive at $m_1 = \sqrt{\Delta m_{\rm atm}^2 - \Delta m_{\rm sun}^2} \approx 5.0 \times 10^{-2}$ eV and $m_2 = \sqrt{\Delta m_{\rm atm}^2} \approx 5.1 \times 10^{-2}$ eV. In this case, only the difference of two Majorana *CP*-violating phases $\sigma - \rho \equiv \sigma'$ is physically meaningful. Again, the texture zero $(M_\nu)_{13} = 0$ allows us to determine δ and σ' in terms of the flavor mixing angles $(\theta_x, \theta_y, \theta_z)$ and the mass ratio $\zeta \equiv m_1/m_2 \approx 0.98$ [8]:

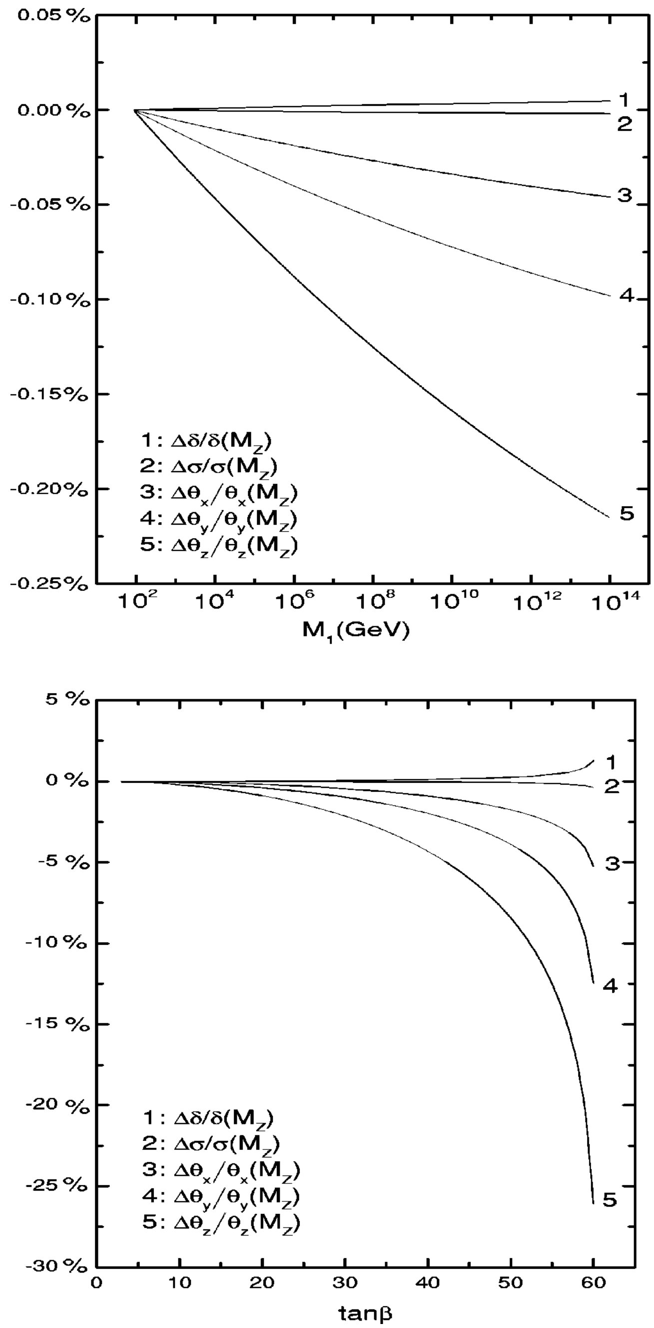

FIG. 3. The RGE evolution of lepton flavor mixing angles and *CP*-violating phases for the FGY *Ansatz* with $m_1=0$ in the MSSM. We take $\tan\beta=10$ to illustrate the running effects changing with M_1 (top), and take $M_1=10^{14}$ GeV to illustrate the running effects changing with $\tan\beta$ (bottom). Note that $\theta_z(M_Z)\approx 4.3°$ has typically been input.

$$\delta=\arccos\left[\frac{(\zeta^2c_x^4-s_x^4)c_y^2s_z^2+(\zeta^2-1)s_x^2c_x^2s_y^2}{2s_xc_x(s_x^2+\zeta^2c_x^2)s_yc_ys_z}\right],$$

$$\sigma'=-\frac{1}{2}\arctan\left[\frac{s_yc_ys_z\sin\delta}{s_xc_x(s_y^2-c_y^2s_z^2)+(s_x^2-c_x^2)s_yc_ys_z\cos\delta}\right]. \tag{20}$$

Differently from the $m_1=0$ case, $|\cos\delta|\leq 1$ does not impose significant constraints on the magnitude of θ_z via Eq. (20), except that it requires $\theta_z>0.36°$ [8]. Hence the FGY *Ansatz* with $m_3=0$ is not very sensitive to the measurement of θ_z.

As a straightforward consequence of $m_3=0$, the RGEs of m_1, m_2, and m_3 in Eq. (A1) can be simplified to

$$\dot{m}_1\approx\frac{1}{16\pi^2}(\alpha+2Cf_\tau^2s_x^2s_y^2)m_1,$$

$$\dot{m}_2\approx\frac{1}{16\pi^2}(\alpha+2Cf_\tau^2c_x^2s_y^2)m_2,$$

$$\dot{m}_3=0. \tag{21}$$

Again, $m_3=0$ holds at any energy scale between M_Z and M_1; and the running behaviors of m_1 and m_2 are essentially the same. Figure 2 illustrates the ratio $R\equiv m_2(M_Z)/m_2(M_1)$ as a function of m_H in the SM or of $\tan\beta$ in the MSSM. We see that $R_{m_3=0}\approx R_{m_1=0}\approx I_\alpha$ holds to an excellent degree of accuracy in the SM and to a good degree of accuracy in the MSSM. These numerical results confirm that the evolution of three neutrino masses is dominated by I_α, as observed in Sec. II.

While ζ is approximately a constant in the RGE running from M_Z to M_1, it is not small. In this case, we simplify Eqs. (A2) and (A3) up to $\mathcal{O}(s_z)$ so as to get the leading-order RGEs of $(\theta_x,\theta_y,\theta_z)$ and (δ,σ') as follows:

$$\dot{\theta}_x\approx-\frac{Cf_\tau^2}{16\pi^2}\left(\frac{1+\zeta^2+2\zeta\cos 2\sigma'}{1-\zeta^2}\right)s_xc_xs_y^2,$$

$$\dot{\theta}_y\approx\frac{Cf_\tau^2}{16\pi^2}s_yc_y,$$

$$\dot{\theta}_z\approx\frac{Cf_\tau^2}{16\pi^2}c_y^2s_z \tag{22}$$

and

$$\dot{\sigma}'\approx\frac{Cf_\tau^2}{8\pi^2}\left(\frac{c_x^2-s_x^2}{1-\zeta^2}\right)\zeta s_y^2\sin 2\sigma',$$

$$\dot{\delta}\approx\frac{Cf_\tau^2}{8\pi^2}\left(\frac{1}{1-\zeta^2}\right)\zeta s_y^2\sin 2\sigma'. \tag{23}$$

Unlike in the $m_1=0$ case, the RGE running effects of θ_x, δ, and σ' are enhanced by a factor of $1/(1-\zeta^2)\approx 25$ in the $m_3=0$ case. One might naively expect that the magnitudes of $\Delta\theta_x\equiv\theta_x(M_1)-\theta_x(M_Z)$, $\Delta\delta\equiv\delta(M_1)-\delta(M_Z)$, and $\Delta\sigma'\equiv\sigma'(M_1)-\sigma'(M_Z)$ are appreciable. Because of $\sin\sigma'\sim\mathcal{O}(s_z)$, however, $\dot{\delta}$ and $\dot{\sigma}'$ in Eq. (23) are actually suppressed. Therefore, only θ_x is likely to be sensitive to the RGE evolution from M_Z to M_1. We plot the numerical results of $\Delta\theta_i/\theta_i(M_Z)$ (for $i=x,y,z$), $\Delta\delta/\delta(M_Z)$, and

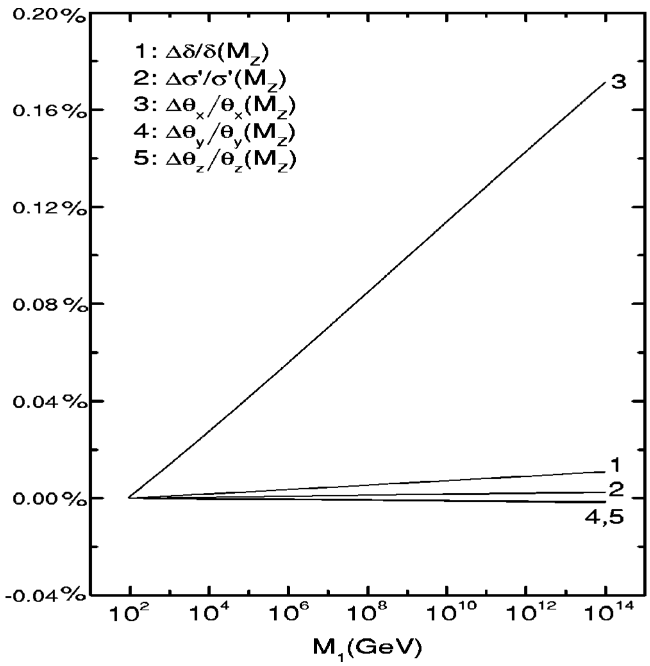

FIG. 4. The RGE evolution of lepton flavor mixing angles and *CP*-violating phases from M_Z to M_1 for the FGY *Ansatz* with $m_3=0$ in the SM. We have typically input $\theta_z(M_Z)\approx 4.3°$, and found that the influence of m_H is negligible.

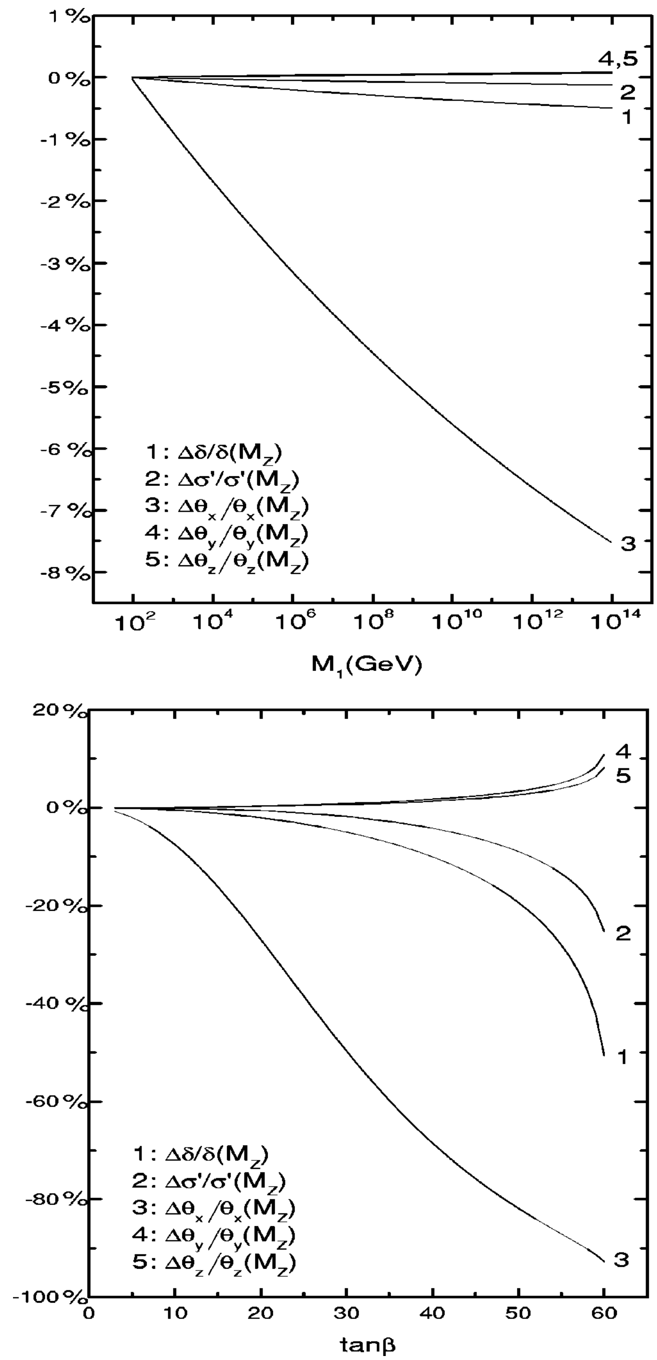

FIG. 5. The RGE evolution of lepton flavor mixing angles and *CP*-violating phases for the FGY *Ansatz* with $m_3=0$ in the MSSM. We take $\tan\beta=10$ to illustrate the running effects changing with M_1 (top), and take $M_1=10^{14}$ GeV to illustrate the running effects changing with $\tan\beta$ (bottom). Note that $\theta_z(M_Z)\approx 4.3°$ has typically been input.

$\Delta\sigma'/\sigma'(M_Z)$ in Fig. 4 for the SM and in Fig. 5 for the MSSM. One can see that the RGE effects on three flavor mixing angles and two *CP*-violating phases are negligibly small in the SM, but they may become significant in the MSSM if both M_1 and $\tan\beta$ are sufficiently large. In the latter case, $\theta_x(M_1)$ can even approach zero—this point has indeed been observed by some authors beyond the minimal seesaw model [25]. We conclude that the near degeneracy between m_1 and m_2 in the $m_3=0$ case may give rise to significant RGE running effects on the mixing angle θ_x in the MSSM, and the evolution of *CP*-violating phases δ and σ' can also be appreciable if both M_1 and $\tan\beta$ take properly large values.

IV. COSMOLOGICAL BARYON NUMBER ASYMMETRY

Lepton number violation induced by the third term of $\mathcal{L}_{\rm Y}$ in Eq. (1) or Eq. (2) allows decays of the heavy Majorana neutrinos N_i (for $i=1$ and 2) to happen: $N_i\to l+h$ and $N_i\to \overline{l}+h^{\rm c}$, where $h=H$ in the SM or $h=H_2^{\rm c}$ in the MSSM. Because each decay mode occurs at both the tree and one-loop levels (via the self-energy and vertex corrections), the interference between these two decay amplitudes may result in a *CP*-violating asymmetry ε_i between $N_i\to l+h$ and its *CP*-conjugated process [5]. For the sake of simplicity, we consider only the case in which the masses of N_1 and N_2 are hierarchical (i.e., $M_1\ll M_2$) [6]. If the interactions of N_1 are in thermal equilibrium when N_2 decays, the asymmetry ε_2 can be erased before N_1 decays. Then only the asymmetry ε_1 produced by the out-of-equilibrium decay of N_1 survives. In the flavor basis chosen above, we have

$$\varepsilon_1\equiv\frac{\Gamma(N_1\to l+h)-\Gamma(N_1\to\overline{l}+h^{\rm c})}{\Gamma(N_1\to l+h)+\Gamma(N_1\to\overline{l}+h^{\rm c})}\approx\frac{C'}{8\pi}\cdot\frac{M_1}{M_2}\cdot\frac{{\rm Im}[(Y_\nu^\dagger Y_\nu)_{12}]^2}{(Y_\nu^\dagger Y_\nu)_{11}}, \quad (24)$$

where $C'_{\rm SM}=-3/2$ and $C'_{\rm MSSM}=-3$ [26]. Taking account of Eq. (3) with $a_2=c_1=0$, we immediately arrive at

$$(Y_\nu^\dagger Y_\nu)_{11}=|a_1(M_1)|^2+|b_1(M_1)|^2 \approx \frac{1}{I_\alpha}[|a_1(M_Z)|^2+|b_1(M_Z)|^2],$$

$$(Y_\nu^\dagger Y_\nu)_{12}=b_1^*(M_1)\cdot b_2(M_1) \approx \frac{1}{I_\alpha}[b_1^*(M_Z)\cdot b_2(M_Z)], \tag{25}$$

where $I_e \approx I_\mu \approx 1$ has been used as an excellent approximation.

In the literature, ε_1 was calculated by neglecting the RGE running effects of neutrino masses, lepton flavor mixing angles, and *CP*-violating phases from M_Z to M_1 (i.e., $I_\alpha \approx 1$ was naively taken). Such an oversimplification leads to the *CP*-violating asymmetry

$$\hat{\varepsilon}_1 \approx \frac{C'}{8\pi\Omega}\cdot\frac{M_1|(M_\nu)_{12}|^2|(M_\nu)_{23}|^2}{[|(M_\nu)_{11}|^2+|(M_\nu)_{12}|^2]|(M_\nu)_{33}|}\sin\Phi, \tag{26}$$

where $\Omega=v^2$ (SM) or $\Omega=v^2\sin^2\beta$ (MSSM), and [8]

$$\Phi \approx \begin{cases} \arctan\left(\dfrac{\xi s_x^2 c_z^2 \sin 2\sigma}{s_z^2+\xi s_x^2 c_z^2\cos 2\sigma}\right) & (m_1=0), \\ -2\delta & (m_3=0). \end{cases} \tag{27}$$

One can see that $\hat{\varepsilon}_1$ is actually independent of M_2, as long as $M_2 \gg M_1$ is satisfied. Then it is straightforward to obtain $\varepsilon_1 \approx \hat{\varepsilon}_1/I_\alpha$ at the mass scale M_1. Although I_α is always smaller than unity for $M_1>M_Z$ (as already shown in Fig. 1), it remains of $\mathcal{O}(1)$ only if reasonable values of M_1 and m_H (SM) or $\tan\beta$ (MSSM) are taken. Hence the previously oversimplified approximation $\varepsilon_1 \approx \hat{\varepsilon}_1$ is unable to cause any quantitative disaster.

A nonvanishing *CP*-violating asymmetry ε_1 may result in a net lepton number asymmetry $Y_{\rm L}\equiv n_{\rm L}/\mathbf{s}=\varepsilon_1 d/g_*$, where $g_*=106.75$ (SM) or 228.75 (MSSM) is an effective number characterizing the relativistic degrees of freedom which contribute to the entropy $\mathbf{s}$ of the early universe, and d accounts for the dilution effects induced by the lepton-number-violating wash-out processes [26]. If the effective neutrino mass parameter $\tilde{m}_1 \equiv (Y_\nu^\dagger Y_\nu)_{11}\Omega/M_1$ [27] lies in the range 10^{-2} eV$\leq \tilde{m}_1 \leq 10^3$ eV, one may estimate the value of d by using the following approximate formula [28]:[2]

$$d \approx 0.3\left(\frac{10^{-3}\ \text{eV}}{\tilde{m}_1}\right)\left[\ln\left(\frac{\tilde{m}_1}{10^{-3}\ \text{eV}}\right)\right]^{-0.6}. \tag{28}$$

The lepton number asymmetry $Y_{\rm L}$ is eventually converted into a net baryon number asymmetry $Y_{\rm B}$ via the nonperturbative sphaleron processes [29]: $Y_{\rm B}\equiv n_{\rm B}/\mathbf{s}\approx -0.55Y_{\rm L}$ in the SM or $Y_{\rm B}\equiv n_{\rm B}/\mathbf{s}\approx -0.53Y_{\rm L}$ in the MSSM. A generous range $0.7\times 10^{-10} \lesssim Y_{\rm B} \lesssim 1.0\times 10^{-10}$ has been drawn from the recent Wilkinson Microwave Anisotropy Probe (WMAP) observational data [30].

Clearly, ε_1 and $Y_{\rm B}$ involve three free parameters in the SM (M_1, θ_z, and m_H) and three free parameters in the MSSM (M_1, θ_z, and $\tan\beta$). Note, however, that I_α is not very sensitive to m_H in the SM,[3] and its sensitivity to $\tan\beta$ is mild in the MSSM (see Fig. 1 for illustration). Therefore the size of $Y_{\rm B}$ calculated at M_1 is mainly dependent on how big M_1 and θ_z are. The numerical dependence of $Y_{\rm B}$ on M_1 and θ_z is illustrated in Fig. 6, where $m_H=120$ GeV (SM) and $\tan\beta=50$ (MSSM) have typically been taken. Some comments are in order.

(1) In the $m_1=0$ case, current observational data of $Y_{\rm B}$ require $M_1>3.1\times 10^{10}$ GeV (SM) or $M_1>3.4\times 10^{10}$ GeV (MSSM) for the allowed range of $\theta_z(M_Z)$. Once this mixing angle is precisely measured at low energies, it is possible to fix the ballpark magnitude of M_1 in most cases. However, to distinguish between the minimal seesaw SM and its supersymmetric version needs other experimental information (e.g., the MSSM-motivated lepton-flavor-violating processes $\mu\to e\gamma$, $\tau\to\mu\gamma$, and so on [31]).

(2) In the $m_3=0$ case, $M_1>2.25\times 10^{13}$ GeV (SM) or $M_1>2.5\times 10^{13}$ GeV (MSSM) is required by current observational data of $Y_{\rm B}$. The magnitude of $Y_{\rm B}$ increases monotonically with $\theta_z(M_Z)$ for any given value of M_1. Once θ_z is measured at low energies, it is possible to determine the rough value of M_1. Again, other experimental information is needed in order to distinguish between the minimal seesaw SM and its supersymmetric version.

(3) We remark that there is a potential conflict between achieving successful thermal leptogenesis and avoiding overproduction of gravitinos in the minimal seesaw model with supersymmetry [31]. If the mass scale of gravitinos is of $\mathcal{O}(1)$ TeV, one must have $M_1 \lesssim 10^8$ GeV. This limit is completely disfavored in the FGY *Ansatz* with $M_1 \ll M_2$, because it would lead to $Y_{\rm B} \ll 10^{-10}$. If M_1 and M_2 were almost degenerate, a special case which has been discussed in Ref. [26], it would be possible to simultaneously accommodate

[2]In view of Eq. (25), we can easily obtain $\tilde{m}_1(M_1) \approx \tilde{m}_1(M_Z)/I_\alpha$. It is proper to use $\tilde{m}_1(M_1)$ instead of $\tilde{m}_1(M_Z)$ to evaluate the dilution factor d via Eq. (28), but the numerical discrepancy between $\tilde{m}_1(M_1)$ and $\tilde{m}_1(M_Z)$ is actually insignificant.

[3]We have noticed that the numerical results for the dependence of I_α on m_H are not fully consistent with one another in the literature [15], and our result is not completely compatible with them either. It may deserve some effort to clarify the existing discrepancies.

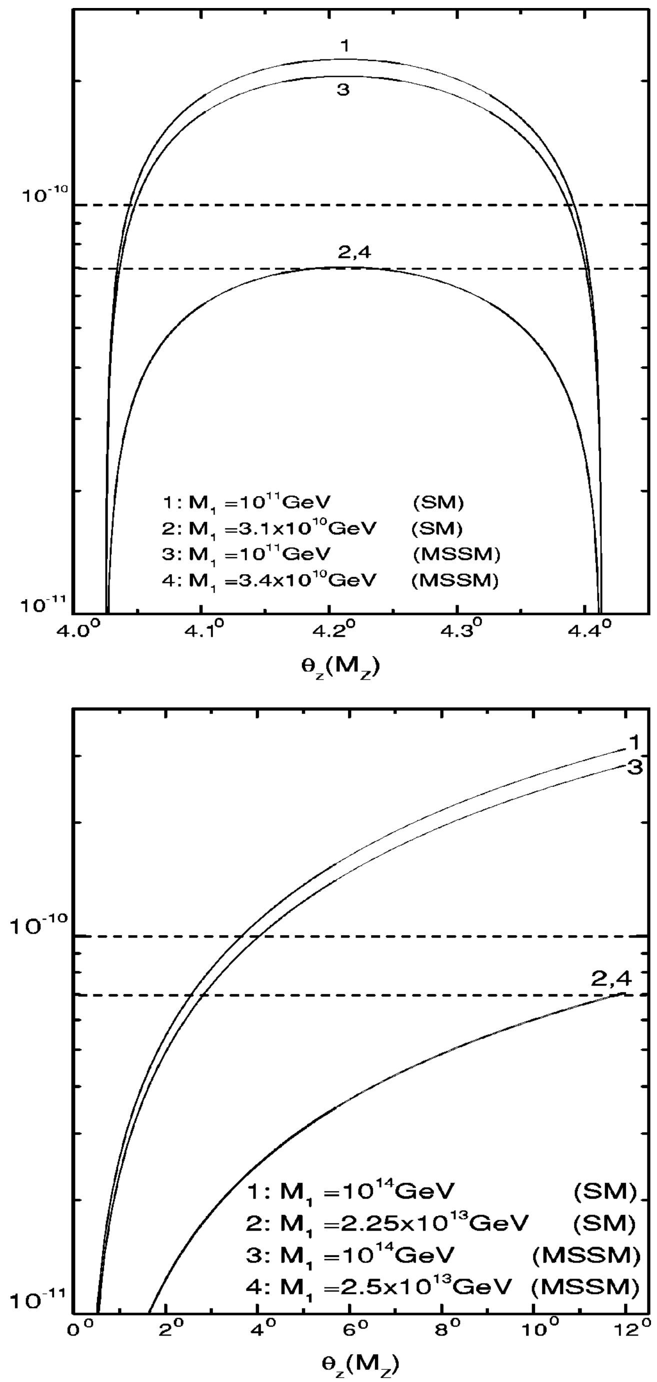

FIG. 6. Numerical illustration of Y_B changing with $\theta_z(M_Z)$ for the FGY *Ansatz* with $m_1=0$ (top) or $m_3=0$ (bottom) in the SM ($m_H=120$ GeV) and its minimal supersymmetric extension ($\tan\beta=50$). The region between the two dashed lines in each graph corresponds to the range of Y_B allowed by current observational data.

$m_{\tilde{G}}\sim\mathcal{O}(1)$ TeV and $M_1\lesssim 10^8$ GeV in the generic supergravity models with minimal seesaw and thermal leptogenesis.

V. SUMMARY

We have analyzed the radiative corrections to neutrino mixing and *CP* violation in the minimal seesaw model with two heavy right-handed neutrinos. It is shown that textures and vanishing eigenvalues of the effective Majorana neutrino mass matrix are essentially stable against renormalization effects. Taking account of the Frampton-Glashow-Yanagida *Ansatz* for the Dirac neutrino Yukawa coupling matrix, we have calculated the RGE running effects of light neutrino masses, lepton flavor mixing angles, and *CP*-violating phases from M_Z to M_1 for both $m_1=0$ and $m_3=0$ cases in the SM and its minimal supersymmetric extension. We find that such quantum corrections are not always negligible, and they should be taken into consideration in order to quantitatively test the FGY *Ansatz*. We have also discussed thermal leptogenesis in the minimal seesaw model with $M_1 \ll M_2$. Very instructive predictions for the cosmological baryon number asymmetry are obtained with the help of low-energy neutrino mixing quantities. We conclude that a precise measurement of the mixing angle θ_z in reactor- and accelerator-based neutrino oscillation experiments will be extremely helpful to examine the FGY scenario and other presently viable *Ansätze* of lepton mass matrices.

ACKNOWLEDGMENT

We are grateful to W. L. Guo for very useful discussions and help. This work was supported in part by the National Nature Science Foundation of China.

APPENDIX: ANALYTICAL APPROXIMATIONS OF THE RGES FOR NEUTRINO MASSES AND LEPTON FLAVOR MIXING PARAMETERS

Following Ref. [23] and taking into account $M_\nu = V\mathrm{Diag}\{m_1,m_2,m_3\}V^T$ with the parametrization of V given in Eq. (12), we have derived the one-loop RGEs of (m_1,m_2,m_3), $(\theta_x,\theta_y,\theta_z)$, and (δ,ρ,σ) with the help of Eq. (5). Our analytical results are consistent with those obtained in Ref. [24], where a somehow different parametrization of V was used. For simplicity, only the approximate expressions of $(\dot{m}_1,\dot{m}_2,\dot{m}_3)$, $(\dot{\theta}_x,\dot{\theta}_y,\dot{\theta}_z)$, and $(\dot{\delta},\dot{\rho},\dot{\sigma})$ up to $\mathcal{O}(s_z)$ are presented below.

(1) The running of three neutrino masses:

$$\dot{m}_1=\frac{1}{16\pi^2}[\alpha+2Cf_\tau^2(s_x^2 s_y^2+O(s_z))]m_1,$$

$$\dot{m}_2=\frac{1}{16\pi^2}[\alpha+2Cf_\tau^2(c_x^2 s_y^2+\mathcal{O}(s_z))]m_2,$$

$$\dot{m}_3=\frac{1}{16\pi^2}[\alpha+2Cf_\tau^2 c_y^2 c_z^2]m_3. \tag{A1}$$

(2) The running of three lepton flavor mixing angles:

$$\dot{\theta}_x = -\frac{Cf_\tau^2}{16\pi^2}\left[\frac{m_1^2+m_2^2+2m_1m_2\cos 2(\rho-\sigma)}{m_2^2-m_1^2}s_xc_xs_y^2 + \mathcal{O}(s_z)\right],$$

$$\dot{\theta}_y = -\frac{Cf_\tau^2}{16\pi^2}\left[\frac{m_2^2+m_3^2+2m_2m_3\cos 2(\delta-\sigma)}{m_3^2-m_2^2}c_x^2s_yc_y + \frac{m_1^2+m_3^2+2m_1m_3\cos 2(\delta-\rho)}{m_3^2-m_1^2}s_x^2s_yc_y + \mathcal{O}(s_z)\right],$$

$$\dot{\theta}_z = -\frac{Cf_\tau^2}{16\pi^2}\left[\left(\frac{2m_2m_3\cos(\delta-2\sigma)}{m_3^2-m_2^2} - \frac{2m_1m_3\cos(\delta-2\rho)}{m_3^2-m_1^2} + \frac{m_2^2-m_1^2}{m_3^2-m_2^2}\cdot\frac{2m_3^2\cos\delta}{m_3^2-m_1^2}\right)s_xc_xs_yc_y + \mathcal{O}(s_z)\right]. \quad \text{(A2)}$$

(3) The running of three *CP*-violating phases:

$$\dot{\rho} = -\frac{Cf_\tau^2}{16\pi^2}\left[\frac{A}{s_z} - \frac{2m_2m_3\sin 2\sigma}{m_3^2-m_2^2}s_x^2c_y^2 - \frac{2m_1m_3\sin 2\rho}{m_3^2-m_1^2}c_x^2c_y^2 + \frac{2m_1m_2\sin 2(\rho-\sigma)}{m_2^2-m_1^2}s_x^2s_y^2 + \mathcal{O}(s_z)\right],$$

$$\dot{\sigma} = -\frac{Cf_\tau^2}{16\pi^2}\left[\frac{A}{s_z} - \frac{2m_2m_3\sin 2\sigma}{m_3^2-m_2^2}s_x^2c_y^2 - \frac{2m_1m_3\sin 2\rho}{m_3^2-m_1^2}c_x^2c_y^2 + \frac{2m_1m_2\sin 2(\rho-\sigma)}{m_2^2-m_1^2}c_x^2s_y^2 + \mathcal{O}(s_z)\right],$$

$$\dot{\delta} = -\frac{Cf_\tau^2}{16\pi^2}\left[\frac{A}{s_z} + B + \mathcal{O}(s_z)\right], \quad \text{(A3)}$$

where

$$A = s_xc_xs_yc_y\left[\frac{2m_2m_3\sin(\delta-2\sigma)}{m_3^2-m_2^2} - \frac{2m_1m_3\sin(\delta-2\rho)}{m_3^2-m_1^2} - \frac{m_2^2-m_1^2}{m_3^2-m_2^2}\cdot\frac{2m_3^2\sin\delta}{m_3^2-m_1^2}\right],$$

$$B = \frac{2m_1m_2s_y^2\sin 2(\rho-\sigma)}{m_2^2-m_1^2} - \frac{2m_2m_3}{m_3^2-m_2^2}[s_x^2c_y^2\sin 2\sigma + c_x^2(c_y^2-s_y^2)\sin 2(\delta-\sigma)] - \frac{2m_1m_3}{m_3^2-m_1^2}[c_x^2c_y^2\sin 2\rho + s_x^2(c_y^2-s_y^2)\sin 2(\delta-\rho)]. \quad \text{(A4)}$$

[1] For a review, see C.K. Jung, C. McGrew, T. Kajita, and T. Mann, Annu. Rev. Nucl. Part. Sci. **51**, 451 (2001).

[2] SNO Collaboration, Q.R. Ahmad *et al.*, Phys. Rev. Lett. **89**, 011301 (2002); **89**, 011302 (2002).

[3] KamLAND Collaboration, K. Eguchi *et al.*, Phys. Rev. Lett. **90**, 021802 (2003).

[4] K2K Collaboration, M.H. Ahn *et al.*, Phys. Rev. Lett. **90**, 041801 (2003).

[5] M. Fukugita and T. Yanagida, Phys. Lett. B **174**, 45 (1986).

[6] P.H. Frampton, S.L. Glashow, and T. Yanagida, Phys. Lett. B **548**, 119 (2002).

[7] T. Endoh, S. Kaneko, S.K. Kang, T. Morozumi, and M. Tanimoto, Phys. Rev. Lett. **89**, 231601 (2002); R. Kuchimanchi and R.N. Mohapatra, Phys. Rev. D **66**, 051301 (2002); B. Datta and R.N. Mohapatra, *ibid.* **68**, 056006 (2003); P. Chankowski and K. Turzynski, Phys. Lett. B **570**, 198 (2003); H.K. Dreiner, H. Murayama, and M. Thormeier, hep-ph/0312012.

[8] W.L. Guo and Z.Z. Xing, hep-ph/0310326.

[9] M. Raidal and A. Strumia, Phys. Lett. B **553**, 72 (2003); V. Barger, D.A. Dicus, H.J. He, and T. Li, hep-ph/0310278; R.G. Felipe, F.R. Joaquim, and B.M. Nobre, hep-ph/0311029; A. Ibarra and G.G. Ross, hep-ph/0312138.

[10] T. Yanagida, in *Proceedings of the Workshop on Unified Theory and the Baryon Number of the Universe*, edited by O. Sawada and A. Sugamoto (KEK, Tsukuba, 1979); M. Gell-Mann, P. Ramond, and R. Slansky, in *Supergravity*, edited by F. van Nieuwenhuizen and D. Freedman (North-Holland, Amsterdam, 1979); R.N. Mohapatra and G. Senjanovic, Phys. Rev. Lett. **44**, 912 (1980).

[11] Z.Z. Xing, Phys. Rev. D **69**, 013006 (2004).

[12] For a review with extensive references, see P. Chankowski and S. Pokorski, Int. J. Mod. Phys. A **17**, 575 (2002).

[13] S. Antusch, M. Drees, J. Kersten, M. Lindner, and M. Ratz, Phys. Lett. B **519**, 238 (2001).

[14] P. Chankowski and Z. Pluciennik, Phys. Lett. B **316**, 312 (1993); K.S. Babu, C.N. Leung, and J. Pantaleone, *ibid.* **319**, 191 (1993); S. Antusch, M. Drees, J. Kersten, M. Lindner, and M. Ratz, *ibid.* **525**, 130 (2002).

[15] R. Barbieri, P. Creminelli, A. Strumia, and N. Tetradis, Nucl. Phys. **B575**, 61 (2000); S. Antusch, J. Kersten, M. Lindner, and M. Ratz, *ibid.* **B674**, 401 (2003); G.F. Giudice, A. Notari, M. Raidal, A. Riotto, and A. Strumia, hep-ph/0310123.

[16] J.R. Ellis and S. Lola, Phys. Lett. B **458**, 310 (1999); Z.Z. Xing, Phys. Rev. D **63**, 057301 (2001).

[17] H. Fritzsch and Z.Z. Xing, Prog. Part. Nucl. Phys. **45**, 1 (2000); H. Fusaoka and Y. Koide, Phys. Rev. D **57**, 3986 (1998).

[18] H. Fritzsch and Z.Z. Xing, Phys. Lett. B **517**, 363 (2001); Z.Z. Xing, *ibid.* **530**, 159 (2002).
[19] Particle Data Group, K. Hagiwara *et al.*, Phys. Rev. D **66**, 010001 (2002).
[20] Z.Z. Xing, Int. J. Mod. Phys. A **19**, 1 (2004).
[21] See, e.g., J.N. Bahcall and C. Peña-Garay, J. High Energy Phys. **11**, 004 (2003); M. Maltoni *et al.*, Phys. Rev. D **68**, 113010 (2003); P.V. de Holanda and A.Yu. Smirnov, hep-ph/0309299; and references therein.
[22] CHOOZ Collaboration, M. Apollonio *et al.*, Phys. Lett. B **420**, 397 (1998); Palo Verde Collaboration, F. Boehm *et al.*, Phys. Rev. Lett. **84**, 3764 (2000).
[23] J.A. Casas, J.R. Espinosa, A. Ibarra, and I. Navarro, Nucl. Phys. **B573**, 652 (2000); and references therein.
[24] Antusch *et al.* [15].
[25] See, e.g., S. Antusch, hep-ph/0208179.
[26] For recent reviews of leptogenesis with extensive references, see W. Buchmüller and M. Plümacher, Int. J. Mod. Phys. A **15**, 5047 (2000); A. Pilaftsis and T.E.J. Underwood, hep-ph/0309342; Giudice *et al.* [15].
[27] W. Buchmüller and M. Plümacher, Phys. Lett. B **431**, 431 (1998); W. Buchmüller, P. Di Bari, and M. Plümacher, Nucl. Phys. **B643**, 367 (2002).
[28] E.W. Kolb and M.S. Turner, *The Early Universe* (Addison-Wesley, Reading, MA, 1990); H.B. Nielsen and Y. Takanishi, Phys. Lett. B **507**, 241 (2001); E.Kh. Akhmedov, M. Frigerio, and A.Yu. Smirnov, J. High Energy Phys. **09**, 021 (2003). A different analytical approximation for the dilution factor has been presented by Giudice *et al.* [26], but it has no significant numerical inconsistency with Eq. (17).
[29] V.A. Kuzmin, V.A. Rubakov, and M.E. Shaposhnikov, Phys. Lett. **155B**, 36 (1985).
[30] D.N. Spergel *et al.*, Astrophys. J., Suppl. Ser. **148**, 175 (2003).
[31] See, e.g., J. Ellis and M. Raidal, Nucl. Phys. **B643**, 229 (2002); Raidal and Strumia [9], and references therein.