

Neutrino opacity in magnetized hot and dense nuclear matter

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We study the neutrino interaction rates in hot matter at high densities in the presence of a uniform magnetic field. The neutrino cross sections involving both the charged current absorption and neutral current scattering reactions on baryons and leptons have been considered. We have in particular considered the interesting case when the magnetic field is strong enough to completely polarize the protons and electrons in supernovae and neutron stars. The opacity in such a situation is considerably modified and the cross section develops anisotropy. This has implications for phenomena invoked in the literature to explain the observed pulsar kicks.

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I. INTRODUCTION

Knowledge of neutrino transport in supernovae cores, in neutron stars, and in collapsing stars is an essential prerequisite for an understanding of a host of interesting phenomena such as the mechanism of supernovae explosions, structure of protoneutron stars, observed pulsar kicks, etc. The important theoretical input required for the study of neutrino transport is neutrino opacity calculations in dense, hot matter and the equation of state (EOS). There has been a lot of work on neutrino interactions in matter at high densities [1] involving both the charge current absorption and neutral current scattering reactions on baryons and leptons. The dominant processes for energy and lepton number transport are the neutrino-nucleon scattering and the neutrino absorption. It has been shown that neutral current processes on charged leptons, though subdominant with regard to total opacity are important for the equilibrium of the neutrino number density.

Recently there has been a lot of interest in the study of neutrino transport at high densities in the presence of strong magnetic fields reported to be present in young pulsars. The presence of strong magnetic fields of strengths $\sim 10^{14}$ G have been suggested in neutron stars and recent observations of soft gamma ray repeaters and spinning x-ray pulsars even suggest the existence of magnetic fields greater than 10^{14} G in supernovae remnants [2]. A study of neutrino opacities in magnetized high density, hot matter is required to investigate the asymmetric neutrino emission from protoneutron stars as a possible explanation of observed pulsar kicks [3]. Among

various mechanisms suggested for pulsar kicks, the notable ones relevant to the present study are the models in which the magnetic field induces an asymmetry directly in the neutrino emission [4] giving rise to the observed kicks and those which rely on resonant Mikheyev-Smirnov-Wolfenstein (MSW) flavor transformation on account of changed resonance condition in the magnetic field [5] or on neutrino magnetic moment [6]: This requires a careful, systematic study of neutrino interactions in hot, dense, magnetized matter in various stages of degeneracy. For example, electron-neutrino absorption and scattering cross sections on nucleons and leptons would not only shift the location of the ν_e sphere but will also distort it due to asymmetry in the cross section. This effect on the dominant reaction $\nu_e + n \rightarrow p + e$ at densities near the electron-neutrino sphere was calculated by multiplying the phase space distribution of the final state electrons only and leaving the matrix elements unchanged [7]. However, it was noted [8] that in the presence of large magnetic field $\sim 10^{16}$ G even the motion of degenerate electrons, at densities likely to be present near the neutrino sphere, is quantized and on account of charge neutrality, viz. $\bar{n}_e = n_p$, the protons too are forced to occupy the lowest Landau level. In this situation, the matrix elements for absorption and scattering get modified and have to be calculated by using exact wave functions for electrons and protons by solving the Landau level $\nu=0$ [8]. Further, in order to make numerical estimates of neutrino mean free paths in magnetized dense and hot nuclear matter in the density range $\sim 10^{10} - 10^{15}$ g/cc and temperatures up to 60 MeV, we require the composition of nuclear matter. The effect of magnetic field on the composition of nuclear matter at low densities has been extensively studied in the literature [9]. In recent years the effect of strong magnetic field on cold, charge neutral, superdense in-

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teracting nuclear matter in β equilibrium has been studied in a relativistic mean field theoretical frame work [10–12]. In some of these studies [11,12] not only the effect of Landau quantization but also the contribution of anomalous magnetic moments of nucleons was incorporated in a relativistic description and it was found that this effect cannot be ignored at low densities in the presence of superstrong magnetic field $\geq 10^{18}$ G. Here following Ref. [11] we consider electrically neutral nuclear matter composed of nucleons, electrons and trapped neutrinos in β equilibrium. Hadronic interactions are incorporated through $(\rho - \omega - \sigma)$ meson exchange in the framework of relativistic nuclear mean field theory in the presence of magnetic field. When considering very intense magnetic fields we should be careful about the effect of magnetic field on strong interactions because in this situation, the nucleons and mesons interact both with themselves, and with the magnetic field through their charges and magnetic moments. However, for fields not greater than 10^{18} G we do not have to worry about this problem.

In this paper, we calculate neutrino opacity for magnetized, interacting dense nuclear matter for the following limiting cases: (a) nucleons and electrons, highly degenerate with or without trapped neutrinos; (b) nondegenerate nucleons, degenerate electrons and no trapped neutrinos; and finally (c) when all particles are nondegenerate. The important neutrino interaction processes which contribute to opacity are the neutrino absorption process

$$\nu_e + n \rightarrow p + e \quad (1)$$

and the scattering processes

$$\nu_e + N \rightarrow \nu_e + N, \quad (2)$$

$$\nu_e + e \rightarrow \nu_e + e. \quad (3)$$

In Sec. II we calculate the cross sections for these processes in the presence of magnetic field. In Sec. III the cross sections are calculated in the polarized medium when the magnetic field is very strong and density is such that the electrons and protons are confined to the lowest Landau level. For the purpose of calculating reaction rates we treat the nucleons nonrelativistically and the leptons in extreme relativistic limit. In Sec. IV we discuss the results.

II. NEUTRINO CROSS SECTION

The neutrino processes (1)–(3) get contributions from charged as well as neutral current weak interactions in the standard model. For the general process

$$\nu(p_1) + A(p_2) \rightarrow B(p_3) + l(p_4). \quad (4)$$

The cross section per unit volume of matter or the inverse mean free path is given by

$$\begin{aligned} \frac{\sigma(E_1)}{V} &= \lambda^{-1}(E_1) \\ &= \frac{1}{2E_1} \prod_{i=2,3,4} d\rho_i W_{fi} f_2(E_2) [1 - f_3(E_3)] \\ &\quad \times [1 - f_4(E_4)], \end{aligned} \quad (5)$$

where $d\rho_i = d^3p_i / (2\pi)^3 2E_i$ is the density of states of particles with four momenta $p_i = (E_i, \vec{p}_i)$, $f_i(E_i)$ are the particle distribution functions which in thermal equilibrium are given by the usual Fermi-Dirac distributions $f_i(E_i) = [1 + e^{\beta(E_i - \mu_i)}]^{-1}$ and $1 - f_i(E_i)$ accounts for the Pauli-Blocking factor for the final state particles. μ_i 's are the chemical potentials and $\beta = 1/(KT)$. The transition rate W_{fi} is

$$W_{fi} = (2\pi)^4 \delta^4(P_f - P_i) |M|^2, \quad (6)$$

where $|M|^2$ is the squared matrix element summed over the initial and final spins.

In the presence of magnetic field, the density of states of charged particles is modified and is given by

$$d\rho_i = \sum_{\nu} (2 - \delta_{\nu,0}) \int_{-\infty}^{\infty} dp_{iz} \int_{-eBL_x/2}^{eBL_x/2} \frac{dp_{iy}}{(2\pi)^2 2E_i}, \quad (7)$$

where the sum over all occupied Landau levels is to be performed. For weak magnetic fields, several Landau levels are populated and the matrix element remain essentially unchanged [14] and one needs to only account for the correct phase space factor. In the presence of strong magnetic field, the electrons occupy the lowest Landau state and the charge neutrality requirement then forces the protons into the ground state. This statement holds for cold degenerate matter. For hot matter with temperatures ≥ 10 MeV and density $10^{11} - 10^{12}$ g/cc, the protons are not so severely affected by the magnetic field. This happens because of the presence of a sizable number of positrons at these densities and temperatures, thereby modifying the charge neutrality condition. In the presence of such intense magnetic field, the matrix elements have to be calculated by using the exact wave functions of relativistic electrons and protons obtained by solving the Dirac equation.

In a gauge in which the vector potential is $\vec{A} = (0, xB, 0)$, corresponding to a constant, uniform magnetic field in the z direction, the quantum states are specified by the quantum numbers p_y , p_z , ν and S and the energy eigenvalues are given by [11,12]

$$\mathcal{E}_{\nu,s}^e = [m^2 + p_z^2 + eB(2\nu + 1 + s)]^{1/2}, \quad (8)$$

$$\mathcal{E}_{\nu,s}^p = \sqrt{m_p^{*2} + p_z^2 + eB(2\nu + 1 + s) + \kappa_p^2 B^2 + 2\kappa_p B s} \sqrt{m_p^{*2} + eB(2\nu + 1 + s)} = E_{\nu,s}^p - U_0^p, \quad (9)$$

and

$$\mathcal{E}_s^n = \sqrt{m_n^{*2} + \vec{p}^2 + \kappa_n^2 B^2 + 2\kappa_n B s} \sqrt{p_x^2 + p_y^2 + m_n^{*2}} = E_s^n - U_0^n, \quad (10)$$

where κ_p and κ_n are the anomalous magnetic moments of protons and neutrons given by

$$\kappa_p = \frac{e}{2m_p} \left[\frac{g_p}{2} - 1 \right], \quad (11)$$

$$\kappa_n = \frac{e}{2m_n} \frac{g_n}{2}, \quad (12)$$

where $g_p = 5.58$ and $g_n = -3.82$ are the Landé g factor for protons and neutrons, respectively. We have here neglected the contribution of the anomalous magnetic moment of electrons. U_0^p , U_0^n , m_N^* are given in Appendix A.

$\nu = n + 1/2 \pm s/2$ characterizes the Landau level. The electron wave function in the lowest Landau state $\nu = 0$ has the energy $E_{0,-1}^e = (m_e^2 + p_z^2)^{1/2}$ with the wave function

$$\Psi(r) = \left(\frac{eB}{\pi} \right)^{1/4} \frac{1}{\sqrt{L_y L_z}} e^{-i(E_e t - p_y y - p_z z)} e^{\xi^2/2} U_{e,1}(E_e) \quad (13)$$

and the positive energy spinors in state $\nu = 0$ are given by

$$U_{e,-1} = \frac{1}{\sqrt{E_e + m_e}} \begin{pmatrix} 0 \\ E_e + m_e \\ 0 \\ -p_z \end{pmatrix}. \quad (14)$$

Nucleons are treated nonrelativistically. In these limits the proton energy in the lowest state is given by

$$E_{0,1}^p = \frac{m_p^* - \kappa_p B}{m_p} + \frac{p_z^2}{2(m_p^* - \kappa_p B)} + U_0^p, \quad (15)$$

where

$$\bar{m}_p = m_p^* - \kappa_p B \quad (16)$$

and the wave function is

$$\Psi_p(r) = \left(\frac{eB}{\pi} \right)^{1/4} \frac{1}{\sqrt{L_y L_z}} e^{-i(E_{0,1}^p t - p_y y - p_z z)} e^{\xi^2/2} U_{p,1}(E_{0,1}^p), \quad (17)$$

where $U_{p,1}(E_{0,1}^p)$ is the nonrelativistic spin up spinor

$$U_{p,1}(E_{0,1}^p) = \frac{1}{\sqrt{2m_p}} \begin{pmatrix} \chi_+ \\ 0 \end{pmatrix} \quad (18)$$

and

$$\xi = \sqrt{eB} \left(x - \frac{p_y}{eB} \right). \quad (19)$$

For neutrons we have

$$\Psi_{n,s}(r) = \frac{1}{\sqrt{L_x L_y L_z}} e^{-ip_{n,s}(r)} U_{n,s}(E_s^n), \quad (20)$$

$$U_{n,s} = \frac{1}{\sqrt{2m_n}} \begin{pmatrix} \chi_s \\ 0 \end{pmatrix}, \quad (21)$$

and

$$E_{n,s} \simeq \frac{m_n^* - \kappa_n B s}{\bar{m}_n} + \vec{p}^2 + U_0^n, \quad (22)$$

with

$$\bar{m}_n = m_n^* - \kappa_n B s. \quad (23)$$

Neutrino wave function is the usual plane wave

$$\Psi_\nu(r) = \frac{1}{\sqrt{L_x L_y L_z}} e^{-ip_\nu(r)} U_{\nu,s}(E_\nu). \quad (24)$$

Here $U_{\nu,s}$ is the usual free particle spinor. χ_s is the spin spinor and the wave functions have been normalized in a volume $V = L_x L_y L_z$ and we have the normalization

$$\sum_{\text{Spin}} \bar{u}_\alpha u_\beta = 2m \delta_{\alpha\beta}. \quad (25)$$

We first consider the neutrino-nucleon processes (1), (2). In the presence of weak magnetic fields, the matrix element squared summed over initial and final spins in the approximation of treating nucleons nonrelativistically and leptons relativistically is given by

$$\begin{aligned} \sum |M|^2 = & 32G_F^2 \cos^2 \theta_c m_p^* m_n^* [(C_V^2 + 3C_A^2) \\ & + (C_V^2 - C_A^2) \cos \theta_{e,\nu}] E_e E_\nu, \end{aligned} \quad (26)$$

where $C_V = g_V = 1$, $C_A = g_A = 1.23$ for the absorption process; $C_V = -1$, $C_A = -1.23$ for neutrino scattering on neutrons; and $C_V = -1 + 4 \sin^2 \theta_w = 0.08$, $C_A = 1.23$ for neutrino proton scattering.

The effect of strong interactions on charged and neutral current neutrino interactions is incorporated in the framework of relativistic mean field theory. The nuclear matter consisting of neutrons, protons, electrons with or without neutrinos is considered in β equilibrium in the presence of magnetic field. Hadronic interactions are taken into account by considering the nucleons to interact by exchanging scalar (σ) and vector (ω , ρ) mesons [11]. The composition of matter is calculated for arbitrary magnetic fields and temperatures without making any approximation regarding relativity and degree of degeneracy of various particles. For the effect on weak rates, we however treat the nucleons nonrelativistically and obtain neutrino cross sections in the limits of extreme degeneracy or for nondegenerate matter. This simplifies the calculations considerably without compromising the results

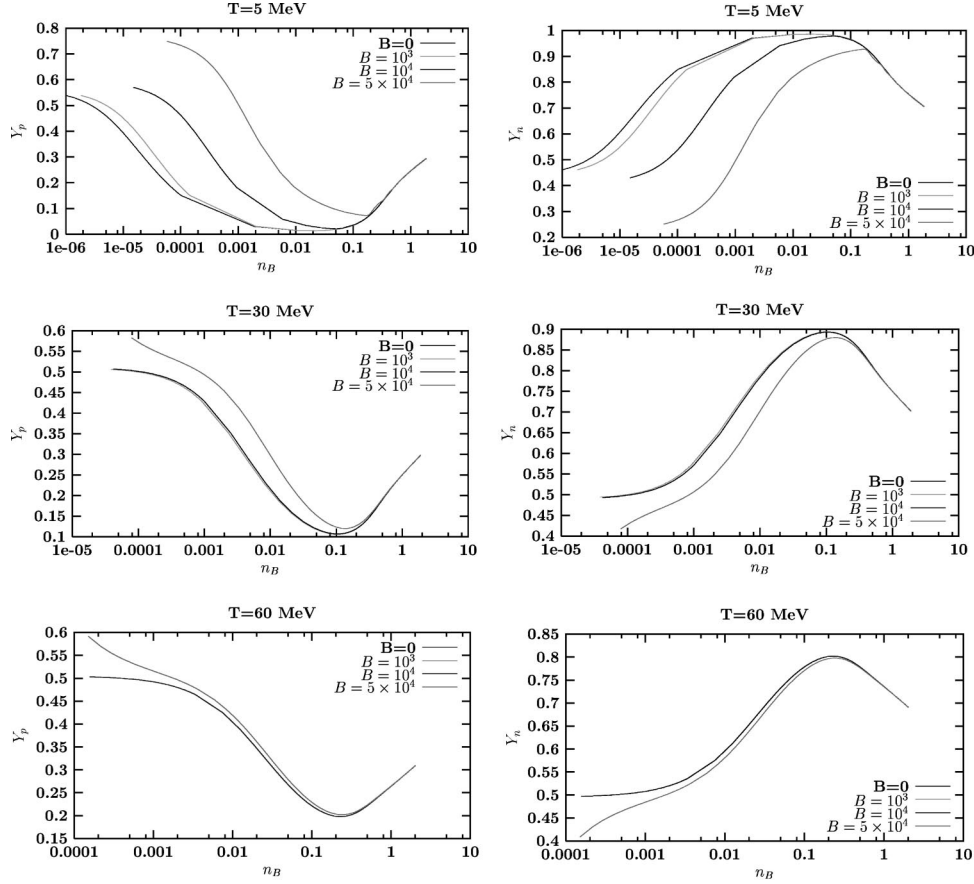


FIG. 1. Variation of Y_p and Y_n with n_B for $B=0, 10^3, 10^4$ and 5×10^4 for $T=5, 30$, and 60 MeV ($Y_{\nu_e}=0$).

in any significant way. This results in replacing the nucleon mass m by its density dependent mass m^* and the chemical potential by the appropriate effective chemical potential (see Appendix A).

We present here the results for interacting nuclear matter consisting of neutrons, protons, electrons and neutrinos (for the trapped case where lepton fraction $Y_L = Y_e + Y_{\nu_e}$ is held fixed at 0.4) for nuclear density n_B varying over a wide range ($10^{-5} - 10$) n_0 and at temperatures $T=5, 10$, and 30 MeV. In Figs. 1 and 2 we show the effect of magnetic field (measured in the units of B_e^c , viz., $B^* = B/B_e^c$ and the critical magnetic field for electrons is $B_e^c = 4.413 \times 10^{13}$ G) on the composition of nuclear matter, viz. Y_p , Y_n and Y_{ν_e} , as a function of nuclear density for $T=5, 30$, and 60 MeV for the neutrino trapped and untrapped cases, respectively. The effect of magnetic field is to raise the proton fraction and is very pronounced at low densities. At densities relevant to the core of neutron stars, one requires very high magnetic fields to change the composition. The effect of including anomalous magnetic moment also becomes significant at field strengths typically $\geq 10^5$ which can be seen in Fig. 3, where we have plotted the proton and neutron fractions Y_p and Y_n and the effective nucleon mass as a function of density for large magnetic field 5×10^5 with and without including the effect of anomalous magnetic moment of nucleons.

A. Highly degenerate matter

The absorption cross section (1) for this case can be calculated by using Eq. (14) in Eq. (5) by the usual techniques and we get, for small B ,

$$\begin{aligned} \frac{\sigma_A(E_\nu, B)}{V} &= \frac{G_F^2 \cos^2 \theta_c}{8\pi^3} (g_V^2 + 3g_A^2) m_p^* m_n^* T^2 \\ &\times \left(\pi^2 + \frac{(E_\nu - \mu_\nu)^2}{T^2} \right) \\ &\times \frac{eB}{(1 + e^{(\mu_\nu - E_\nu)/T})} \left[\theta(p_F(p) + p_F(e) \right. \\ &\quad \left. - p_F(n) - p_F(\nu)) \right. \\ &\quad \left. + \frac{p_F(p) + p_F(e) - p_F(n) + p_F(\nu)}{2E_\nu} \theta(p_F(\nu)) \right. \\ &\quad \left. - |p_F(p) + p_F(e) - p_F(n)| \right] \\ &\times \sum_{\nu=0}^{\nu_{\max}} (2 - \delta_{\nu,0}) \frac{1}{\sqrt{\mu_e^2 - m_e^2 - 2\nu eB}}, \quad (27) \end{aligned}$$

which reduces to the usual result [13] in the limit $B \rightarrow 0$. The

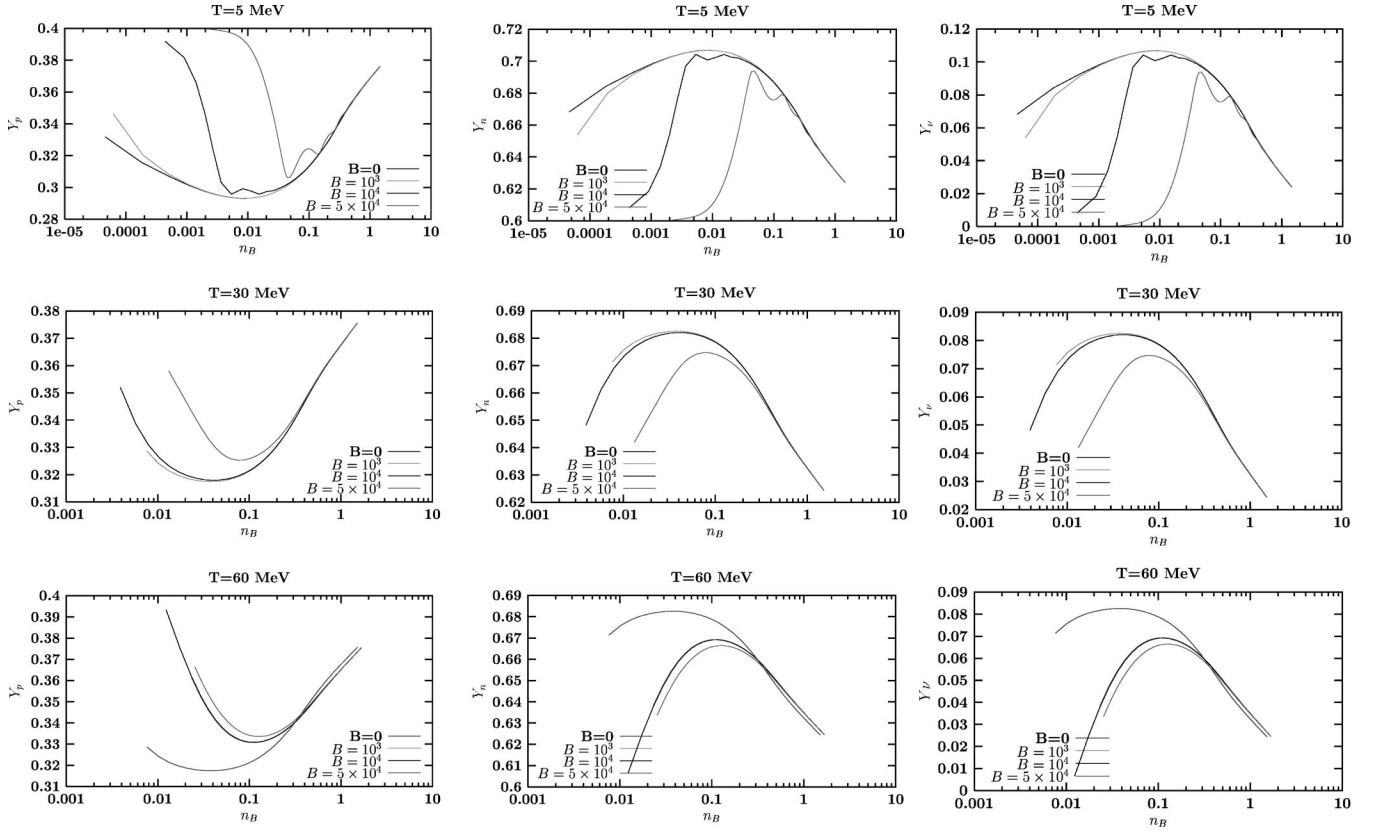


FIG. 2. Variation of Y_p , Y_n , and Y_v with n_B for $B=0, 10^3, 10^4$ and 5×10^4 for $T=5, 30$, and 60 MeV ($Y_L=0.4$).

case of freely streaming, untrapped neutrinos is obtained from Eq. (27) by putting $\mu_\nu=0$ and replacing μ_e by $(\mu_e + E_\nu)$.

When the magnetic field is much weaker than the critical field for protons, only electrons are affected and the neutrino-nucleon scattering cross-section expression remains unchanged by the magnetic field. The numerical values, however, are modified due to changed chemical composition. The cross sections are given by

$$\frac{\sigma_{\nu N}(E_\nu)}{V} = \frac{G_F^2 \cos^2 \theta_c}{16\pi^3} (C_V^2 + 3C_A^2) m_N^{*2} T^2 \mu_e \times \left(\frac{\pi^2 + \frac{(E_\nu - \mu_\nu)^2}{T^2}}{1 + \exp \frac{(E_\nu - \mu_\nu)}{T}} \right). \quad (28)$$

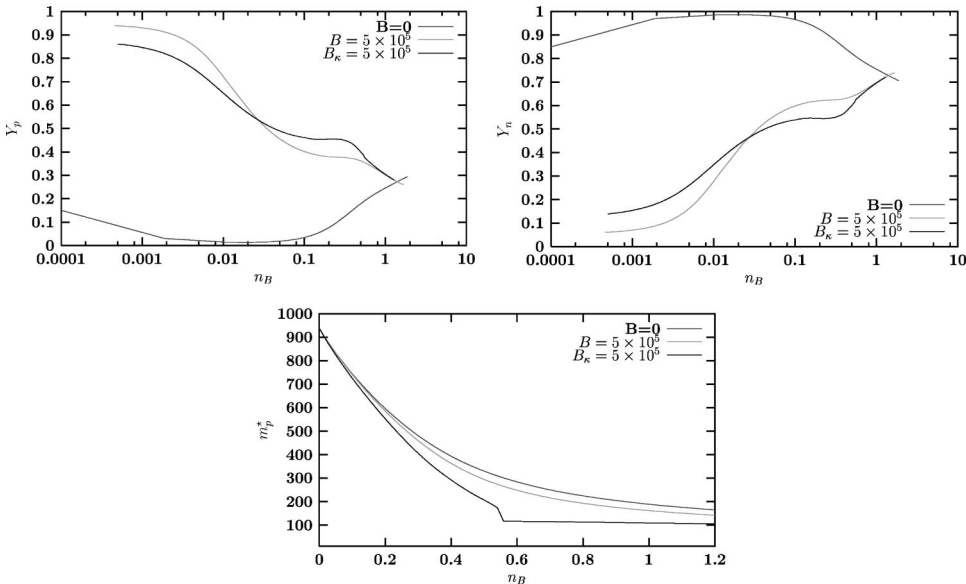


FIG. 3. Variation of Y_p , Y_n , and m_p^* with n_B for $B=0, 5 \times 10^5$ and $B_k=5 \times 10^5$ where in B_k the effect of anomalous magnetic has been included.

If neutrinos are not trapped, we get in the elastic limit

$$\frac{\sigma_{\nu N}(E_\nu)}{V} = \frac{G_F^2 \cos^2 \theta_c}{16\pi^3} (C_V^2 + 3C_A^2) m_N^{*2} T^2 E_\nu. \quad (29)$$

The neutrino electron scattering, though important for energy momentum transfer, is subdominant for neutrino opacity calculations which get major contributions from the absorption reaction in the neutrino trapped regime and by absorption and scattering processes on nucleons in the other regime. The electron-neutrino scattering in the presence of arbitrary magnetic field has been evaluated in the literature by [14]. Here we will consider the interesting case when the electrons get totally polarized by the magnetic field. For completeness, we give below the neutrino-electron scattering cross section in the absence of the field for relativistic degenerate electrons and neutrinos in the elastic approximation

$$\frac{\sigma_{\nu e}}{V} \simeq \frac{2G_F^2 \cos^2 \theta_c}{3\pi^3} (C_V^2 + C_A^2) \frac{\mu_e^2 T E_\nu^2}{1 + e^{-\beta(E_\nu - \mu_\nu)}}, \quad (30)$$

which in the untrapped regime goes over to

$$\frac{\sigma_{\nu e}}{V} \simeq \frac{2G_F^2 \cos^2 \theta_c}{3\pi^3} (C_V^2 + C_A^2) \mu_e^2 T E_\nu^2. \quad (31)$$

B. Nondegenerate matter

We now treat the nucleons as nonrelativistic nondegenerate such that $\mu_i/T \ll -1$ and thus the Pauli-blocking factor $1 - f_N(E_i)$ can be replaced by 1; the electrons are still considered degenerate and relativistic. In the approximation which is valid near the neutrino spheres for the protoneutron star in the Helmholtz-Kelvin cooling phase, the cross section

$$\begin{aligned} \frac{\sigma_A(E_\nu, B)}{V} &\simeq \frac{G_F^2 \cos^2 \theta_c}{2\pi} (C_V^2 + 3C_A^2) n_n(E_\nu + Q) \\ &\times \frac{1}{1 + e^{-\beta(E_\nu + Q - \mu_e)}} eB \sum_{\nu=0}^{\nu_{\max}} (2 - \delta_{\nu,0}) \\ &\times \frac{1}{\sqrt{(E_\nu + Q)^2 - m_e^2 - 2\nu eB}}, \end{aligned} \quad (32)$$

where n_N is the nuclear density and $Q = m_n - m_p$

$$\frac{\sigma_{\nu N}(E_\nu, B)}{V} \simeq \frac{G_F^2 \cos^2 \theta_c}{4\pi} (C_V^2 + 3C_A^2) n_N E_\nu^2. \quad (33)$$

If the electrons too are considered nondegenerate we get

$$\frac{\sigma_A}{V} \simeq \frac{G_F^2 \cos^2 \theta_c}{\pi} (g_V^2 + 3g_A^2) n_N E_\nu^2, \quad (34)$$

$$\frac{\sigma_{\nu N}}{V} \simeq \frac{G_F^2 \cos^2 \theta_c}{4\pi} (C_V^2 + 3C_A^2) n_N E_\nu^2, \quad (35)$$

$$\frac{\sigma_{\nu e}}{N} \simeq \frac{4G_F^2 \cos^2 \theta_c}{\pi^3} T^4 E_\nu (C_V + C_A)^2. \quad (36)$$

III. NEUTRINO CROSS SECTIONS IN POLARIZED MEDIUM

When the magnetic field exceeds $p_F^2(e)/2$, all the electrons in matter occupy the lowest Landau state $\nu=0$ with their spins pointing in the direction opposite to the magnetic field. In this situation charge neutrality for degenerate matter forces the nonrelativistic protons to be also in the Landau ground state but with their spins aligned along the magnetic field. In this situation matrix elements cannot be considered unchanged and should be evaluated using the exact solutions of the Dirac equation for charged particles in magnetic field. Using the wave functions given in Sec. II, the square of the matrix elements for weak processes can be evaluated in a straightforward way and we get

$$|M|_A^2 = 8G_F^2 \cos^2 \theta_c m_2^* m_3^* (E_4 + p_{4z}) [(g_V + g_A)^2 (E_1 + p_{1z}) + 4g_A^2 (E_1 - p_{1z})] \exp \left[-\frac{1}{2eB} [(p_{1x} + p_{2x})^2 + (p_{3x} + p_{4x})^2] \right], \quad (37)$$

$$|M|_{\nu p}^2 = 16G_F^2 \cos^2 \theta_c C_A^2 m_2^* (p_1 \cdot p_4 + 2p_{1z} p_{4z}) \exp \left[-\frac{1}{2eB} [(p_{4x} + p_{1x})^2 + (p_{4y} - p_{1y})^2] \right], \quad (38)$$

$$\begin{aligned}
|M|_{\nu e}^2 = & 16G_F^2 \cos^2 \theta_c [(C_V^2 + C_A^2)((E_1 E_4 + p_{1z} p_{4z})(E_2 E_3 + p_{2z} p_{3z}) - (E_1 p_{4z} + E_4 p_{1z})(E_2 p_{3z} + E_3 p_{2z})) \\
& + 2C_V C_A((E_1 E_4 + p_{1z} p_{4z})(E_2 p_{3z} + E_3 p_{2z}) - E_1 p_{4z} + E_4 p_{1z})(E_2 E_3 + p_{2z} p_{3z})] \\
& \times \exp\left(-\frac{1}{2eB}[(p_{4x} - p_{1x})^2 - (p_{4y} - p_{1y})^2]\right).
\end{aligned} \tag{39}$$

The absorption cross section is now given by

$$\begin{aligned}
\frac{\sigma_A(E_1, B)}{V} = & \frac{1}{2E_1 L_x} \int \frac{d^3 p_z}{(2\pi)^3 2E_2} \int_{-eBL_x/2}^{eBL_x/2} \int_{-\infty}^{\infty} \frac{dp_{3y} dp_{3z}}{(2\pi)^2 2E_3} \int_{-eBL_x/2}^{eBL_x/2} \int_{-\infty}^{\infty} \frac{dp_{4y} dp_{4z}}{(2\pi)^2 2E_4} (2\pi)^3 \delta(P_y) \delta(P_z) \delta(E) |M|^2 f_2(E_2) \\
& \times [1 - f_3(E_3)][1 - f_4(E_4)].
\end{aligned} \tag{40}$$

The corresponding scattering cross sections are obtained by interchanging the particle 2 with 4. Integrals over $dp_{3y} dp_{4y}$ can be performed by using the y -component momentum conserving $\delta(P_y)$ function and give eBL_x . $d\Omega_2$ integral can be performed by using $\delta(P_z) = (1/p_2) \delta(\cos \theta_2 - (p_{3z} + p_{4z} - p_{1z})/p_2)$ and gives 2π . Here for simplification we have replaced the angles appearing in the exponential factor by their average values. It is not a bad approximation since the exponential factor is indeed of order one in the high field limit. We thus obtain

$$\begin{aligned}
\frac{\sigma_A(E_1, B)}{V} \simeq & \frac{eB}{2E_1} \frac{1}{(2\pi)^3} \frac{1}{8} \int dE_2 \frac{dp_{3z}}{E_3} \frac{dp_{4z}}{E_4} \delta(E) |M|^2 \\
& f_2(E_2)[1 - f_3(E_3)][1 - f_4(E_4)].
\end{aligned} \tag{41}$$

To make further progress, we consider the cases of extreme degeneracy and nondegeneracy separately.

A. Highly degenerate matter

We first convert the integrals over z component of the electron and proton momentum into integrals over the electron and proton energy respectively by using the energy expression (8) for $\nu=0$. Further, since for strongly degenerate matter, particles at the top of their respective Fermi seas alone contribute, we replace the momenta by their Fermi momenta. In this approximation we get

$$\begin{aligned}
\frac{\sigma_A(E_1, B)}{V} \simeq & \frac{G_F^2 \cos^2 \theta_c}{(2\pi)^3} eB \frac{m_3^* m_4^*}{p_F(3) E_1} \frac{T^2}{2} \frac{\left(\pi^2 + \left(\frac{E_1}{T}\right)^2\right)}{(1 + e^{-E_\nu/T})} \int dE_2 dE_3 dE_4 \delta(E_1 + E_2 - E_3 - E_4) f_2(1 - f_3)(1 - f_4) [(g_V + g_A)^2 \\
& \times (E_1 + p_{1z}) + 4g_A^2(E_1 - p_{1z})] \left[\exp\left(-\frac{[p_F(1) + p_F(2)]^2 - [p_F(3) - p_F(4)]^2}{2eB}\right) \theta([p_F(1) + p_F(2)]^2 \right. \\
& \left. - [p_F(3) - p_F(4)]^2) + \exp\left(-\frac{[p_F(1) + p_F(2)]^2 - [p_F(3) + p_F(4)]^2}{2eB}\right) \theta([p_F(1) + p_F(2)]^2 - [p_F(3) + p_F(4)]^2) \right].
\end{aligned} \tag{42}$$

Carrying out the energy integrations

$$\begin{aligned}
\frac{\sigma_A(E_\nu, B)}{V} \simeq & \frac{G_F^2 \cos^2 \theta_c}{(2\pi)^3} eB \frac{m_n^* m_p^*}{p_F(\nu) E_\nu} \frac{T^2}{2} \frac{\left(\pi^2 + \left(\frac{E_\nu}{T}\right)^2\right)}{(1 + e^{-E_\nu/T})} [(g_V + g_A)^2(E_\nu + p_{\nu z}) + 4g_A^2(E_\nu - p_{\nu z})] \\
& \times \left[\exp\left(-\frac{[p_F(n) + p_F(\nu)]^2}{2eB}\right) \theta(p_F(n) + p_F(\nu)) \right. \\
& \left. + \exp\left(-\frac{[p_F(n) + p_F(\nu)]^2 - 4p_F^2(e)}{2eB}\right) \theta([p_F(n) + p_F(\nu)]^2 - 4p_F^2(e)) \right],
\end{aligned} \tag{43}$$

where we have made use of the fact that for electrically neutral matter $p_F(e) = p_F(p)$ and that two θ functions correspond to the fact that z-component momentum conservation $p_{nz} + p_{vz} = p_{ez} + p_{pz} = p_F(e) \pm p_F(p)$ depending on whether both electrons and protons in their Landau ground state are in the same direction or are opposite to each other. The case of freely streaming neutrons is obtained from the above by putting $p_F(\nu) = 0$ everywhere.

For the case of neutrino-proton scattering, we can perform the integrals in the degenerate limit by using $\int dp_{2z}$ as $\int dp_{2z} d\delta (p_{2z}^2/2m_2^* - p_f^2(2)/2m_2^*) d\epsilon_2$ and likewise for $\int dp_{2z}$, where $\epsilon_i = p_{iz}^2/2m_i^*$. d^3p_4 integral can be performed by choosing $\vec{p}_1 = E_1(0, \sin \theta_1, \cos \theta_1)$ and $\vec{p}_4 = E_4(\sin \theta_4 \cos \phi_4, \sin \theta_4 \sin \phi_4, \cos \theta_4)$. $d(\cos \theta_4)$ integral is performed as before by using $\delta(P_z)$ and noticing that $\cos \theta_4 = p_{1z}/p_4$ alone is allowed when initial and final protons move along the same direction, in the opposite case $\cos \theta_4 = [2p_F(p) + p_{1z}]/p_{4z} > 1$ and thus is not allowed. Approximating the exponential factor by 1 we get

$$\begin{aligned} \frac{\sigma_{vp}(E_1, B)}{V} &\simeq \frac{G_F^2 \cos^2 \theta_c}{(2\pi)^3} \frac{m_p^{*2}}{p_F^2(2)} \frac{eB}{2} \\ &\times \int d\epsilon_2 d\epsilon_3 dE_4 \left(E_4 + \frac{p_{1z}^2}{E_1} \right), \\ &\times f_2(1-f_3)(1-f_4) \delta(E_1 + \epsilon_2 - \epsilon_3 - E_4) \end{aligned} \quad (44)$$

$$\begin{aligned} &\simeq \frac{G_F^2 \cos^2 \theta_c}{(2\pi)^3} \frac{m_p^{*2}}{p_F^2(p)} g_A^2 \frac{eB}{2} \frac{T^2}{2} \\ &\times \left(\pi^2 + \left(\frac{E_\nu - \mu_\nu}{T} \right)^2 \right) \left(\mu_\nu + \frac{p_{vz}^2}{E_\nu} \right). \end{aligned} \quad (45)$$

$$\simeq \frac{G_F^2 \cos^2 \theta_c}{64\pi} \frac{m_p^{*2}}{p_F^2(p)} g_A^2 eB T^2 \mu_\nu (1 + \cos^2 \theta) \quad (46)$$

because $E_\nu \approx \mu_\nu$ and θ is the angle which the neutrino makes with the magnetic field. For the case when neutrinos are not trapped we get

$$\begin{aligned} \frac{\sigma_{vp}(E_\nu, B)}{V} &\simeq \frac{G_F^2 \cos^2 \theta_c}{(2\pi)^3} \frac{m_p^{*2}}{p_F^2(p)} g_A^2 \frac{eB}{2} \\ &\times \int \left(E_4 + \frac{p_{1z}^2}{E_1} \right) (E_1 - E_4) \frac{dE_4}{1 - e^{(E_1 - E_4)/T}} \end{aligned} \quad (47)$$

$$\simeq \frac{G_F^2 \cos^2 \theta_c}{(2\pi)^3} \frac{m_p^{*2}}{p_F^2(p)} g_A^2 T \frac{E_\nu^2}{2} \frac{eB}{2} (1 + 2 \cos^2 \theta). \quad (48)$$

Likewise for the neutrino-electron scattering we get

$$\begin{aligned} \frac{\sigma_{ve}(E_\nu, B)}{V} &\simeq \frac{G_F^2 \cos^2 \theta_c}{8\pi} T^2 eB \mu_\nu [(C_V^2 + C_A^2)(1 + \cos^2 \theta) \\ &- 4C_V C_A \cos \theta] \end{aligned} \quad (49)$$

for the trapped neutrino case and

$$\begin{aligned} \frac{\sigma_{ve}(E_\nu, B)}{V} &\simeq \frac{2G_F^2 \cos^2 \theta_c}{(2\pi)^3} T eB E_\nu^2 [(C_V^2 + C_A^2)(1 + 2 \cos^2 \theta) \\ &- 6C_V C_A \cos \theta] \end{aligned} \quad (50)$$

for the untrapped case.

B. Nondegenerate matter

Integrals can again be performed in the nondegenerate limit and we get

$$\begin{aligned} \frac{\sigma_A(E_\nu, B)}{V} &\simeq \frac{G_F^2 \cos^2 \theta_c}{4\pi} eB \cos \theta n_N \frac{1}{e^{-(E_\nu + Q - mu_e)\beta} + 1} \\ &\times [((g_V + g_A)^2 + 4g_A^2) + ((g_V + g_A)^2 - 4g_A^2)], \end{aligned} \quad (51)$$

$$\begin{aligned} \frac{\sigma_{vp}(E_\nu, B)}{V} &\simeq \frac{2G_F^2 \cos^2 \theta_c C_A^2}{(2\pi)^3} eB \int_0^\infty dp_{2z} f_2(E_2) \left(E_\nu + \frac{p_{2z}^2}{2m_p} \right) \\ &\times \left[\left(E_\nu + \frac{p_{2z}^2}{2m_p} \right) \left(2 - \frac{1}{3m_p} \left(E_\nu + \frac{p_{2z}^2}{2m_p} \right) \right) \right. \\ &\quad \left. - \frac{E_\nu^2 \cos^2 \theta + p_{2z}^2}{m_p} \right] \\ &\simeq \frac{2G_F^2 \cos^2 \theta_c C_A^2}{2\pi} E_\nu^2 n_p, \end{aligned} \quad (52)$$

where

$$n_p = \frac{eB}{2\pi^2} \int f_2(E_2) dp_{2z} \quad (53)$$

and we have ignored terms of $O(p^2/2m_p)$.

For completeness, we give below the neutrino-electron cross section for the case of nondegenerate electrons too, though here we do not consider the regime of nondegenerate electrons

$$\begin{aligned} \frac{\sigma_{ve}(E_\nu, B)}{V} &\simeq \frac{4G_F^2 \cos^2 \theta_c}{(2\pi)^3} eB [(C_V^2 + C_A^2)(4 + \cos^2 \theta) \\ &- 4C_V C_A \cos \theta] E_\nu \int_0^\infty \frac{dE_e E_e}{1 + e^{\beta(E_e - \mu_e)}} \\ &\simeq \frac{G_F^2}{\pi} n_e T E_\nu [(C_V^2 + C_A^2)(1 + \cos^2 \theta) \\ &- 4C_V C_A \cos \theta], \end{aligned} \quad (55)$$

where

$$n_e = \frac{2eB}{(2\pi)^2} \int_0^\infty e^{-\beta p_{ez}} dp_{ez} = \frac{2eBT}{(2\pi)^2} \quad (56)$$

for nondegenerate electrons.

IV. RESULTS AND DISCUSSIONS

We now present neutrino opacity results for interacting magnetized neutron star matter consisting of nucleons, electrons and trapped as well as freely streaming neutrinos for densities ranging over a wide range $(10^{-5} - 10)n_0$, temperature (5–60) MeV and magnetic field varying from $[(0-5) \times 10^5]B_e^c$. The composition of matter at these conditions has already been shown in Figs. 1–3 in Sec. II and the effect of magnetic field including anomalous magnetic moment of nucleons discussed. We first consider matter in the core of neutron stars where matter is degenerate. At these densities relativistic effects can be important and should not be neglected, however, for simplicity we have evaluated the matrix elements in the limit of nonrelativistic nucleons. Relativistic kinematic effects are expected to be typically of the order p_F/M^* in the degenerate limit and one can modify the limiting expressions (27)–(29), (43), (46) for absorption and scattering cross sections by replacing the effective mass of the nucleons (M_n^*, M_p^*) by the corresponding effective chemical potentials (μ_n^*, μ_p^*), respectively (see Ref. [1] for a discussion). In our numerical calculation of the neutrino mean free path we have made this change. In the neutrino free case ($Y_{\nu_e} = 0$), the neutrino absorption cross section due to reaction (1), the direct URCA process, is highly suppressed due to simultaneous nonconservation of energy and momentum for degenerate matter. This happens because for degenerate nuclear matter, the direct URCA process can take place only near the Fermi energies of participating particles and simultaneous energy momentum conservation sets the constraint $p_F(e) + p_F(p) \geq p_F(n)$, the proton fraction required for this constraint to be satisfied has to be greater than $\sim 11\%$. The proton fraction calculation in nuclear matter is however very sensitive to the assumption that goes into the microscopic theory of nuclear interactions which are not well known [15]. In the present work, this constraint is satisfied at moderately high densities, namely $n_B > 1.5n_0$ (see also [11,16]), whereas in some recent potential models with three-body interactions and boost corrections, this occurs at densities in excess of $\sim 4.5n_0$ [17]. Since the effect of magnetic field is to increase the proton fraction, at sufficiently high magnetic fields depending on the density, the constraint is satisfied and direct URCA process proceeds. In the case of extremely high magnetic fields capable of confining electrons and protons to the lowest Landau level, this constraint is not even required and the absorption process proceeds as can be seen from Eq. (42). Furthermore, in this situation the cross section develops anisotropy. Similarly at high temperatures (see Figs. 1 and 2), the proton fraction rises and the contribution of the direct process exceeds the neutrino-nucleon scattering process.

In the neutrino trapped regime, all the particles are degen-

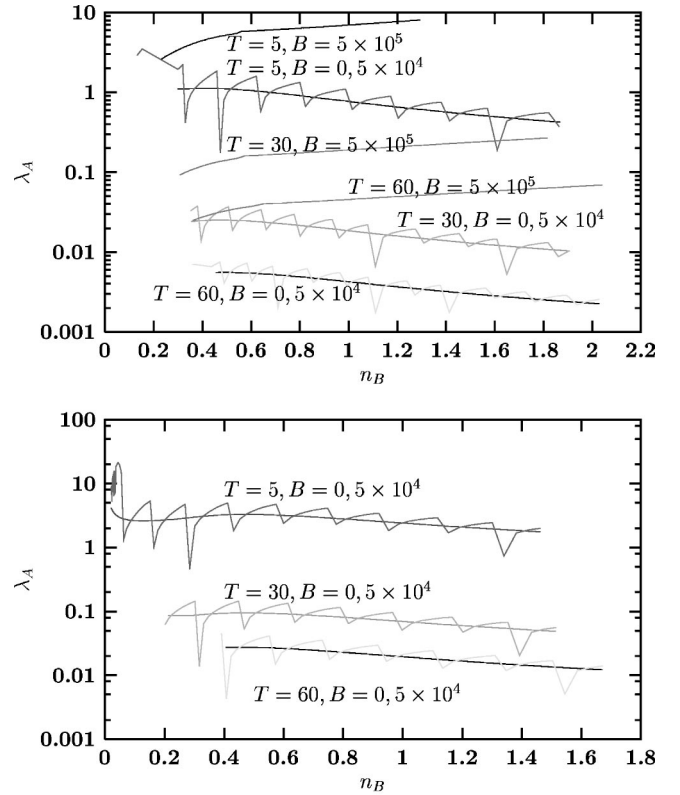


FIG. 4. The first figure shows the variation of λ_A with n_B for untrapped degenerate matter for $B=0, 5 \times 10^4$, and 5×10^5 at $T = 5, 30$, and 60 MeV. The second figure shows the variation of λ_A with n_B for trapped degenerate matter for $B=0$ and 5×10^4 at $T = 5, 30$, and 60 MeV. The effect of anomalous magnetic moment has been included for all fields.

erate and the charged current absorption process dominates over the scattering processes, the neutrino-nucleon scattering being the most important of the scattering processes. In Fig. 4 we show the neutrino absorption mean free path as a function of density for different temperatures and magnetic fields for degenerate matter. The upper and lower figures correspond to neutrino free ($Y_{\nu_e} = 0$) and neutrino trapped ($Y_L = 0.4$) matter. In the neutrino free regime, the neutrinos are thermal and we take the neutrino energy to be equal to $3T$ for calculating the mean free paths, whereas in the trapped regime, the neutrinos being degenerate, $E_\nu = \mu_\nu$ is the appropriate neutrino energy. In this high density regime, the effect of magnetic field on absorption mean free path is not significant unless one goes to extremely high magnetic fields where the matter becomes completely polarized and the lowest Landau energy level is occupied. This happens for $B \geq 5 \times 10^5 B_e^c$ in this density regime. The rapid fluctuations in absorption mean free path around the zero field value reflects the appearance of a new Landau level contributing to the reaction rate. For the case of polarizing magnetic field, we have evaluated the absorption mean free path for neutrinos propagating along the magnetic field. The asymmetry in the absorption cross section for neutrinos propagating along and opposite to the direction of magnetic field is roughly of order of 10%. In Fig. 5 we exhibit the neutrino scattering mean

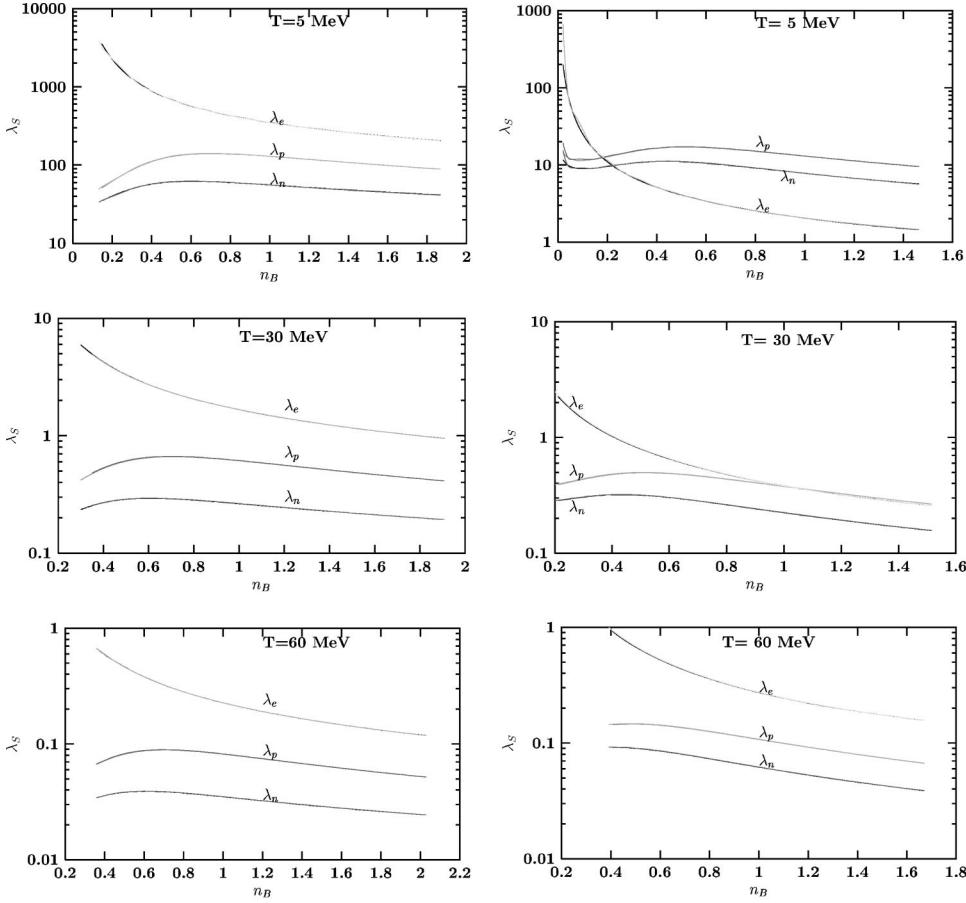


FIG. 5. Variation of neutrino mean free path λ_s ($s=n,p,e$) with n_B for degenerate matter for $T=5, 30$, and 60 MeV and for $B=0$ and 5×10^4 . The left panel is for untrapped matter ($Y_{\nu_e}=0$) and the right panel is for trapped matter ($Y_L=0.4$).

free paths λ_n , λ_p , λ_e on neutrons, protons, and electrons, respectively, for $B=0$ and 5×10^4 and at $T=5, 30$, and 60 MeV. The left panel correspond to the neutrino free and the right panel to the neutrino trapped degenerate matter. The effect of magnetic field on these scattering mean free paths is not significant at these densities. At low densities $n_B = (10^{-6} - 10^{-2})n_0$, the nucleons are nondegenerate, the electrons continue to remain degenerate except at high temperatures and the matter becomes neutrino free. Magnetic field in this regime has important effect. In Fig. 6 we have plotted the absorption mean free path as a function of density

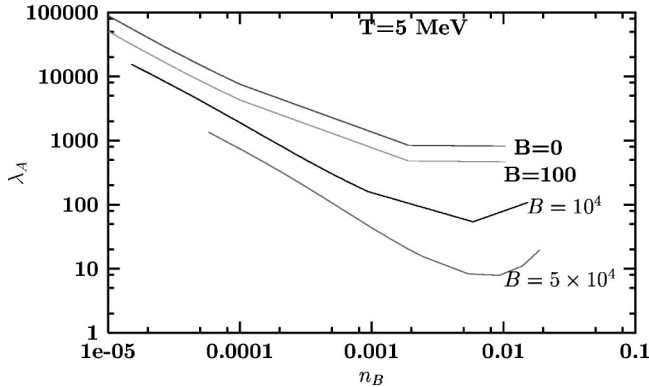


FIG. 6. Variation of λ_A with n_B for untrapped nondegenerate matter for $T=5$ MeV and $B=0, 100, 10^4$, and 5×10^4 .

at $T=5$ MeV for magnetic field varying between 0 to 5×10^4 . We find that the absorption cross section increases, i.e., mean free path decrease with the increase in magnetic field. In Fig. 7 we have plotted the neutrino scattering mean free paths λ_n , λ_p , λ_e as a function of density at $T=5, 30$, and 60 MeV for $B=0$ and $5 \times 10^4 B_c^c$. We find that whereas λ_n and λ_e increase with the magnetic field, λ_p decreases.

In summary, we have studied the neutrino interactions rates through charged as well as neutral current weak interactions in hot, dense magnetized, interacting nuclear matter over a wide range of densities and magnetic fields. The effect of including anomalous magnetic moment of nucleons on the structure of nuclear matter shows up only when the magnetic field exceeds $\sim 10^5 B_c^c$ and results in further enhancement of proton fraction at low densities and lowering of effective nucleon mass at high densities. The effect of magnetic field on neutrino mean free paths is most pronounced at low densities and results in a substantial decrease in the neutrino absorption mean free path in addition to developing anisotropy.

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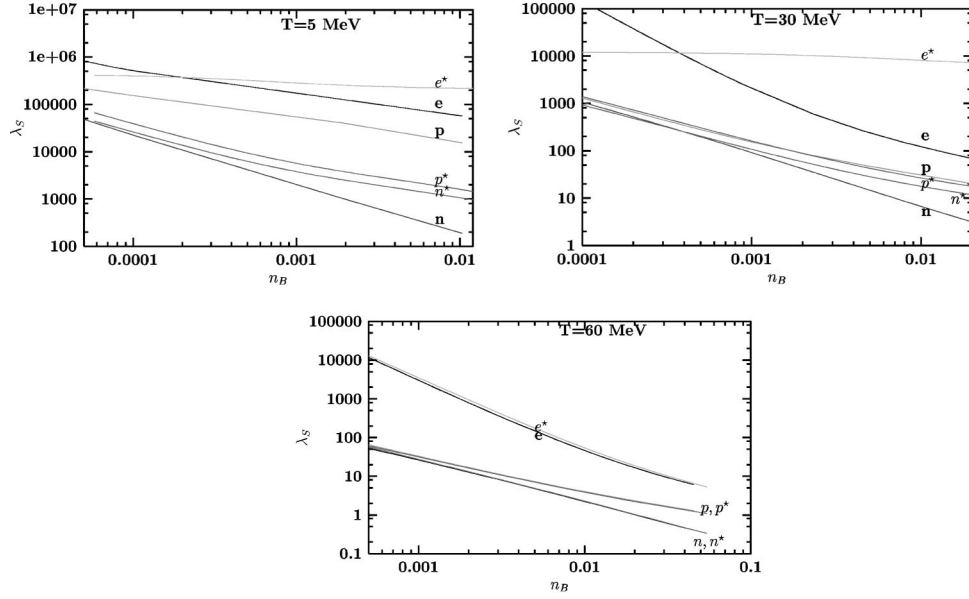


FIG. 7. Variation of neutrino scattering mean free path λ_S with n_B for untrapped nondegenerate matter for $T=5, 30$, and 60 MeV. The curves labeled n, p, e , and n^*, p^*, e^* correspond to $\lambda_n, \lambda_p, \lambda_e$ for $B=0$ and 5×10^4 , respectively.

APPENDIX

For determining the composition of dense, hot, magnetized matter, we employ a relativistic mean field theoretical approach in which the baryons (protons and neutrons) interact via the exchange of σ - ω - ρ mesons in a constant uniform magnetic field. Following Ref. [11] the thermodynamic potential of the system can be written as

$$\Omega = -\frac{1}{2} m_\omega^2 \omega_0^2 - \frac{1}{2} m_\rho^2 \rho_0^2 + \frac{1}{2} m_\sigma^2 \sigma^2 + U(\sigma) + \frac{B^2}{8\pi} + \sum_i \Omega_i, \quad (\text{A1})$$

where $i = n, p, e, \nu$

$$U(\sigma) = \frac{1}{3} b m_N (g_{\sigma N} \sigma)^3 + \frac{1}{4} c (g_{\sigma N} \sigma)^4, \quad (\text{A2})$$

and

$$\Omega_e = -\frac{1}{\beta(2\pi)^2} \sum_s \sum_\nu \int dp_z \ln[1 + e^{-\beta(\mathcal{E}_{\nu,s}^e - \mu_e^*)}], \quad (\text{A3})$$

$$\Omega_p = -\frac{1}{\beta(2\pi)^2} \sum_s \sum_\nu \int dp_z \ln[1 + e^{-\beta(\mathcal{E}_{\nu,s}^p - \mu_p^*)}], \quad (\text{A4})$$

$$\Omega_n = -\frac{1}{\beta(2\pi)^2} \sum_s \int d^3p \ln[1 + e^{-\beta(\mathcal{E}_s^n - \mu_n^*)}], \quad (\text{A5})$$

$$\Omega_\nu = -\frac{1}{\beta(2\pi)^2} \int d^3p \ln[1 + e^{-\beta(\mathcal{E}_\nu - \mu_\nu)}]. \quad (\text{A6})$$

The energy eigenvalues are given in Eqs. (8)–(10). The chemical potentials μ_N^* and the effective masses m_n^* of nucleons are given by

$$\mu_N^* = \mu_N - U_0^N, \quad (\text{A7})$$

and

$$m_N^* = m_N - g_{\sigma N} \sigma_0, \quad (\text{A8})$$

$$U_0^N = g_{\omega N} \omega_0 + g_{\rho N} \tau_{3N} \rho_0. \quad (\text{A9})$$

In the mean field approximation, the thermodynamic quantities are expressed in terms of thermodynamic averages of meson fields which are assumed to be constant and are related to the baryonic and scalar number densities through the fixed equations, viz.

$$m_\sigma^2 \langle \sigma \rangle + \frac{\partial U(\sigma)}{\partial \sigma} = g_{\sigma N} (n_n^s + n_p^s), \quad (\text{A10})$$

$$m_\omega^2 \langle \omega_0 \rangle = g_{\omega N} (n_n + n_p), \quad (\text{A11})$$

$$m_\rho^2 \langle \rho_0 \rangle = g_{\rho N} (n_p - n_n). \quad (\text{A12})$$

The number densities are given by

$$n_p = \frac{eB}{2\pi^2} \sum_s \sum_\nu \int_0^\infty \frac{dp_z}{1 + e^{\beta(\mathcal{E}_{\nu,s}^p - \mu_p^*)}}, \quad (\text{A13})$$

$$n_n = \frac{1}{(2\pi)^3} \sum_s \int \frac{d^3p}{1 + e^{\beta(\mathcal{E}_s^n - g m_n^*)}}, \quad (\text{A14})$$

$$n_p^s = \frac{eB}{2\pi^2} m_p^* \sum_s \sum_\nu \int \frac{dp_z}{\mathcal{E}_{\nu,s}^p (1 + e^{\beta(\mathcal{E}_{\nu,s}^p - \mu_p^*)})}, \quad (\text{A15})$$

$$n_n^s = \frac{m_n^*}{(2\pi)^3} \sum_\nu \int \frac{d^3p}{\mathcal{E}_s^\nu (1 + e^{\beta(\mathcal{E}_s^\nu - \mu_n^*)})}. \quad (\text{A16})$$

The net electron and neutrino number densities are given by

$$\begin{aligned} \bar{n}_e &= \frac{eB}{2\pi^2} \sum (2 - \delta_{\nu,0}) \int dp_z \\ &\times \left[\frac{1}{(1 + e^{\beta(\mathcal{E}_\nu^e - \mu_e)})} - \mu_{e \leftrightarrow}(-\mu_e) \right] \end{aligned} \quad (\text{A17})$$

and

$$\bar{n}_{\nu_e} = \frac{1}{(2\pi)^3} \int d^3p \left[\frac{1}{(1 + e^{\beta(\mathcal{E}_\nu^e - \mu_\nu)})} - \mu_{\nu \leftrightarrow}(-\mu_\nu) \right]. \quad (\text{A18})$$

All the thermodynamic quantities are now obtained by solving the field equations [(A1)–(A18)] along with the condition of charge neutrality

$$n_p = \bar{n}_e \quad (\text{A19})$$

and trapped lepton fraction

$$Y_L = Y_e + Y_{\nu_e}, \quad (\text{A20})$$

where $Y_e = \bar{n}_e/n_B$, $Y_{\nu_e} = \bar{n}_{\nu_e}/n_B$ and the condition of β equilibrium

$$\mu_n = \mu_p + \mu_e - \mu_\nu \quad (\text{A21})$$

self-consistently for a given baryon density

$$n_B = n_p + n_n. \quad (\text{A22})$$

The coupling constants $g_{\sigma N}$, $g_{\omega N}$, $g_{\rho N}$, b and c are taken from Glendenning and Moszkowski [18] by reproducing the nuclear matter properties and the numerical values are given by

$$\frac{g_{\sigma N}}{m_\sigma} = 3.434 \text{ fm}^{-1},$$

$$\frac{g_{\omega N}}{m_\omega} = 2.694 \text{ fm}^{-1},$$

$$\frac{g_{\rho N}}{m_\rho} = 2.1 \text{ fm}^{-1},$$

$$b = 0.00295,$$

$$c = -0.00107. \quad (\text{A23})$$

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